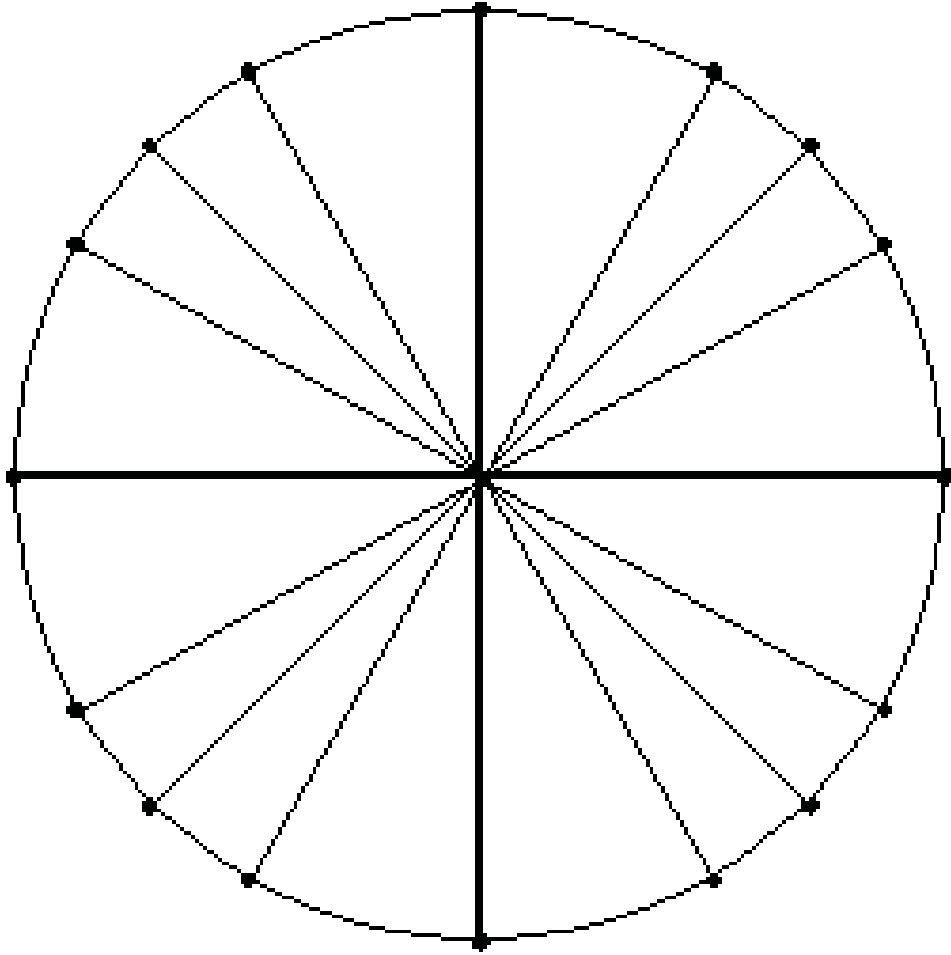
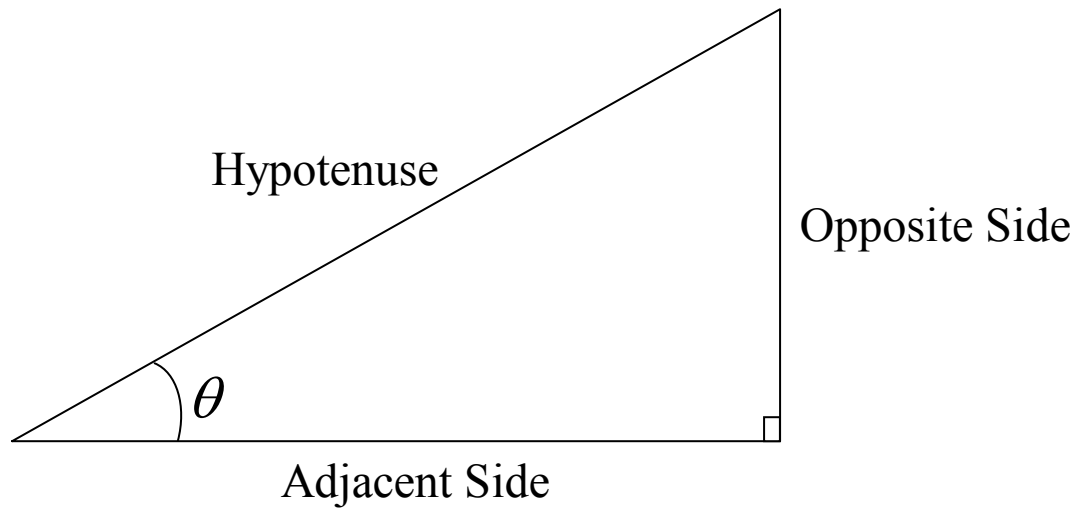


The Unit Circle



Right Triangle Trigonometry



S	O	H	C	A	H	T	O	A
i	p	y	o	d	y	a	p	d
n	p	p	s	j	p	n	p	j

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite side}}$$

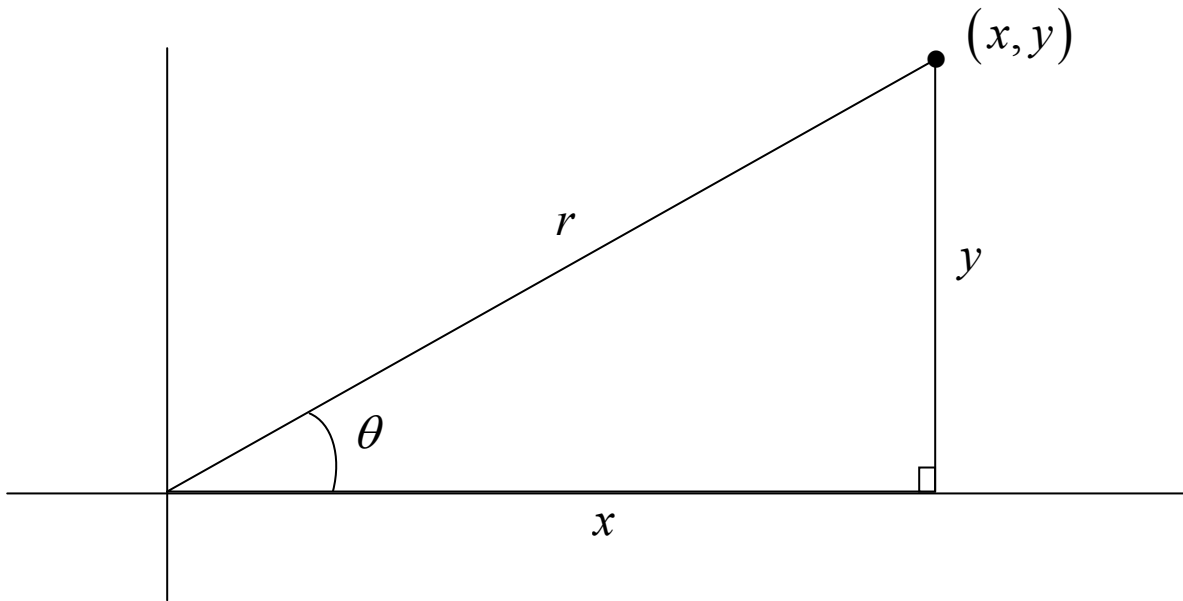
$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$$

Right Triangle Trigonometry



$$x^2 + y^2 = r^2 \Rightarrow r = \sqrt{x^2 + y^2}$$

Reciprocal
Relations

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{r}{x} = \frac{1}{\cos \theta}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y} = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y} = \frac{1}{\tan \theta}$$

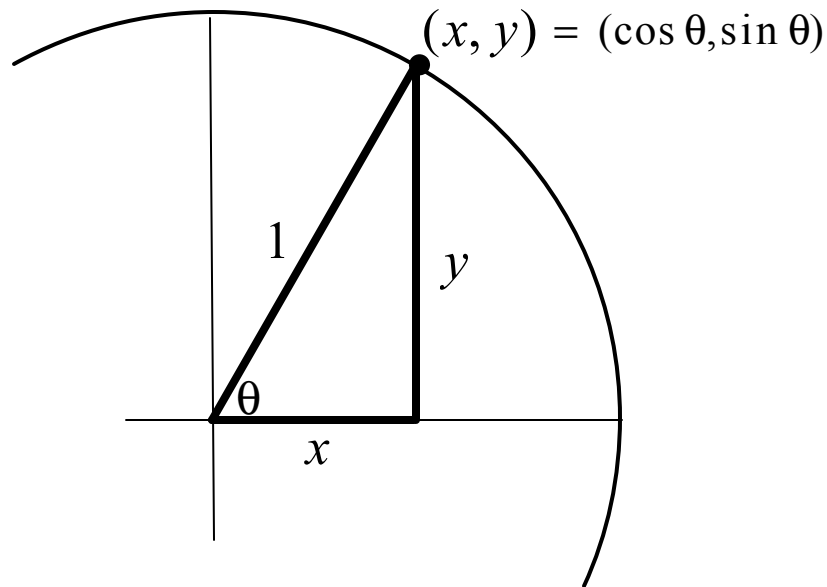
Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Unit Circle / Triangle Trigonometry



$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{1} = x$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{1}{x}$$

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y}{1} = y$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{1}{y}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{y}{x}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{x}{y}$$

By the Pythagorean Theorem:

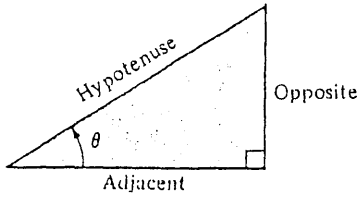
$$x^2 + y^2 = 1$$

$$(\cos \theta)^2 + (\sin \theta)^2 = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

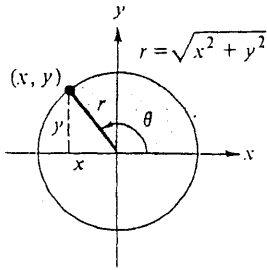
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.



$$\begin{aligned} \sin \theta &= \frac{\text{opp.}}{\text{hyp.}} & \csc \theta &= \frac{\text{hyp.}}{\text{opp.}} \\ \cos \theta &= \frac{\text{adj.}}{\text{hyp.}} & \sec \theta &= \frac{\text{hyp.}}{\text{adj.}} \\ \tan \theta &= \frac{\text{opp.}}{\text{adj.}} & \cot \theta &= \frac{\text{adj.}}{\text{opp.}} \end{aligned}$$

Circular function definitions, where θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Reciprocal Identities

$$\begin{aligned} \sin u &= \frac{1}{\csc u} & \sec u &= \frac{1}{\cos u} & \tan u &= \frac{1}{\cot u} \\ \csc u &= \frac{1}{\sin u} & \cos u &= \frac{1}{\sec u} & \cot u &= \frac{1}{\tan u} \end{aligned}$$

Tangent and Cotangent Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\begin{aligned} \sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u & 1 + \cot^2 u &= \csc^2 u \end{aligned}$$

Cofunction Identities

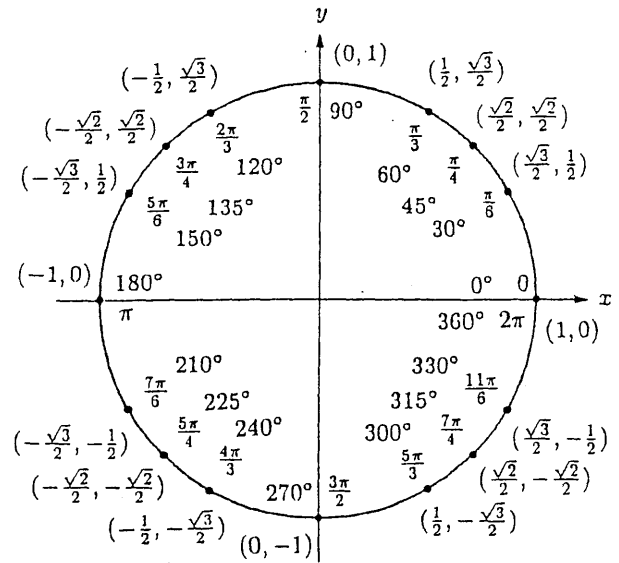
$$\begin{aligned} \sin\left(\frac{\pi}{2} - u\right) &= \cos u & \cos\left(\frac{\pi}{2} - u\right) &= \sin u \\ \csc\left(\frac{\pi}{2} - u\right) &= \sec u & \tan\left(\frac{\pi}{2} - u\right) &= \cot u \\ \sec\left(\frac{\pi}{2} - u\right) &= \csc u & \cot\left(\frac{\pi}{2} - u\right) &= \tan u \end{aligned}$$

Negative Angle Identities

$$\begin{aligned} \sin(-u) &= -\sin u & \cos(-u) &= \cos u \\ \csc(-u) &= -\csc u & \tan(-u) &= -\tan u \\ \sec(-u) &= \sec u & \cot(-u) &= -\cot u \end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$



Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$

Trigonometric Identities

Circle: $x^2 + y^2 = r^2$

$$\sin \theta = \frac{y}{r} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{x}{r} = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{y}{x} = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$$

Reciprocal Relations

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\csc^{-1} x = \sin^{-1} \left(\frac{1}{x} \right)$$

$$\sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right)$$

$$\cot^{-1} x = \tan^{-1} \left(\frac{1}{x} \right)$$

Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Opposite Angle Relations

Even Relations $\cos(-\theta) = \cos \theta$

$$\sec(-\theta) = \sec \theta$$

Odd Relations $\sin(-\theta) = -\sin \theta$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

$$\csc(-\theta) = -\csc \theta$$

Double Angle Relations

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Sum Relations

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Difference Relations

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Reducing Powers

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Half Angle Relations

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

Product-to-Sum Relations

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

Sum-to-Product Relations

$$\sin A + \sin B = 2 \sin \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right)$$

$$\sin A - \sin B = 2 \cos \left(\frac{A + B}{2} \right) \sin \left(\frac{A - B}{2} \right)$$

$$\cos A + \cos B = 2 \cos \left(\frac{A + B}{2} \right) \cos \left(\frac{A - B}{2} \right)$$

$$\cos A - \cos B = -2 \sin \left(\frac{A + B}{2} \right) \sin \left(\frac{A - B}{2} \right)$$

Table of Values for $\cos\theta$, $\sin\theta$ and $\tan\theta$

Degree	Radian			
θ	θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
0°	0	1	0	0
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	0	1	$\frac{1}{0}$ is undefined

REFERENCE ANGLES

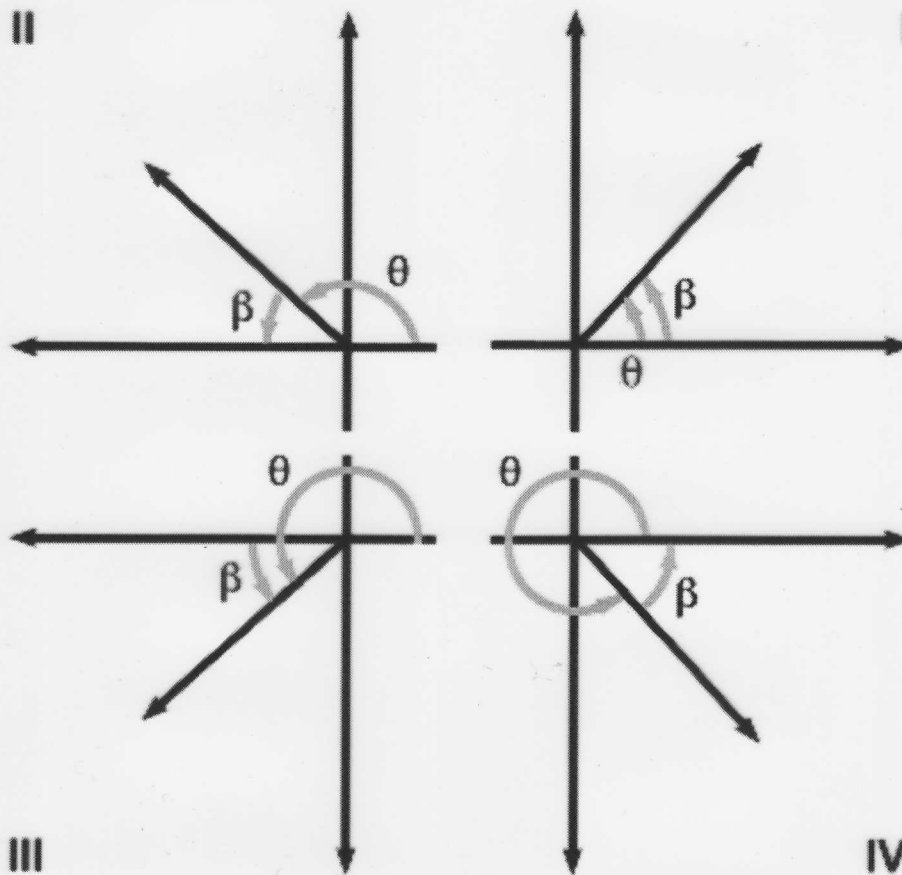


Figure 4.1: In each drawing, β is the reference angle for θ .

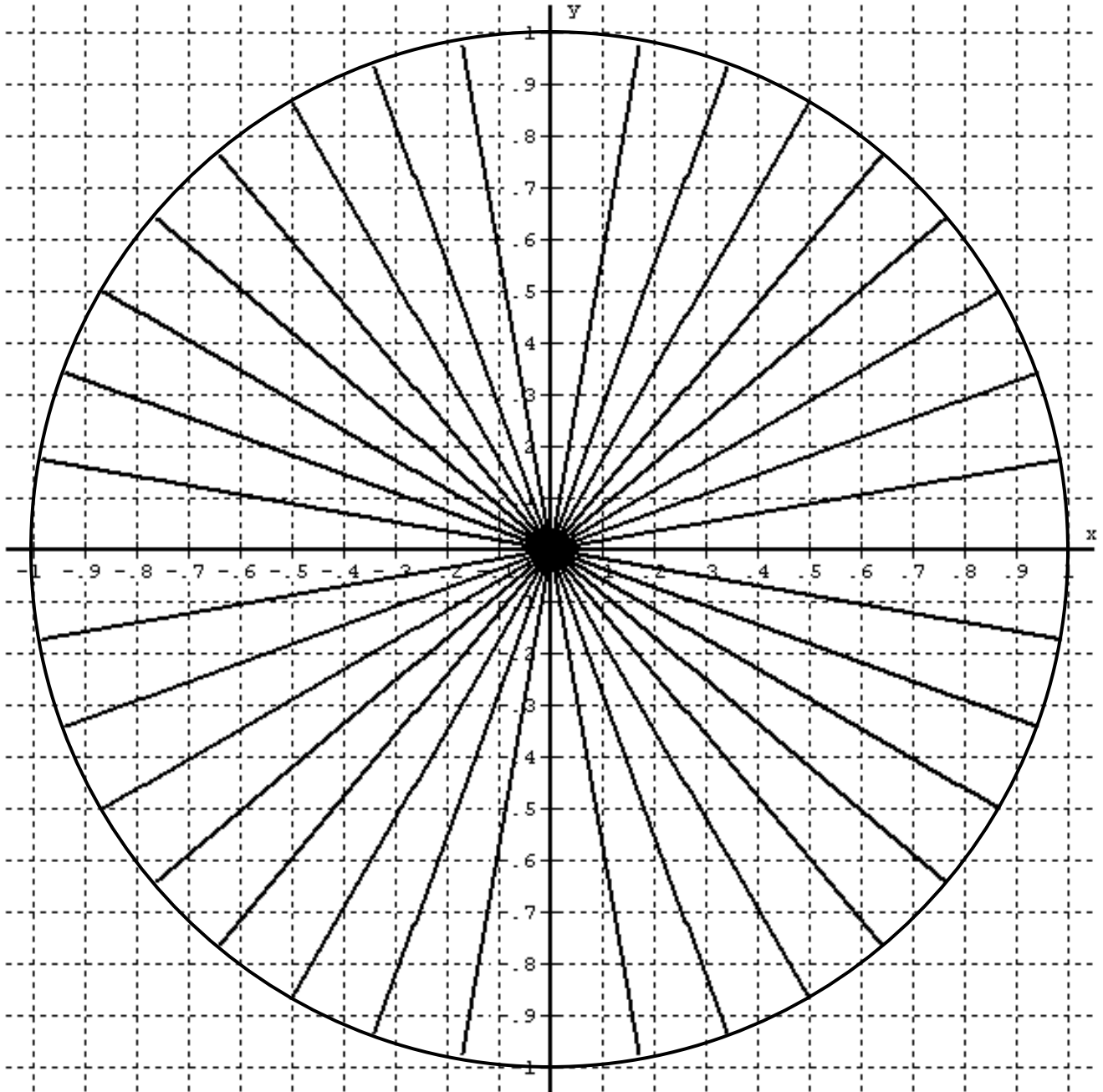
Below is a chart that will help in the easy calculation of reference angles. For angles in the first quadrant, the reference angle β is equal to the given angle θ . For angles in other quadrants, reference angles are calculated this way:

quadrant	β (reference angle)
I	$\beta = \theta$
II	$\beta = 180 - \theta$
III	$\beta = \theta - 180$
IV	$\beta = 360 - \theta$

Figure 4.2: How to calculate the reference angle β for any angle θ between 0 and 2π radians.

The Unit Circle

$\cos(\theta)$ and $\sin(\theta)$ Calculator



Each Angle Increases by 10°

MAC1114 Sample Test 1 Part 1

Name _____

Fill in the following table with the appropriate values on a Unit Circle. **No Calculators Allowed.**

	θ in degrees	θ in radians	Reference Angle (in degrees)	Coordinates (x, y)	$\sin \theta$	$\cos \theta$	$\tan \theta$
1)	60°				XXX	XXX	XXX
2)		$\frac{7\pi}{6}$			XXX	XXX	XXX
3)				$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$	XXX	XXX	XXX
4)	120°						
5)		$\frac{\pi}{4}$					
6)			XXX	(0,1)			
7)					$\sin \theta = \frac{1}{2}$ θ in QII		
8)						$\cos \theta = \frac{\sqrt{3}}{2}$ θ in QIV	
9)							$\tan \theta = -1$ θ in QII
10)	240°						
11)		$\frac{5\pi}{3}$					
12)					$\sin \theta = \frac{-\sqrt{2}}{2}$ θ in QIV		

MAC1114 Sample Test 1 Part 2

Name _____

- 13) A) Convert 200° to radian measure.
- B) Convert $\frac{\pi}{10}$ to degree measure.
- 14) Convert $57^\circ 42' 15''$ to decimal degrees. Round to 3 decimal places.
- 15) Convert 132.74° to Degrees/Minutes/Seconds
- 16) The terminal side of θ in standard position passes through the point $(12, 5)$.
- A) Find the values of the **six** trigonometric functions of θ .
- B) Find the value of θ rounded to 2 decimal places.
- 17) Use your calculator to find the values of the following. Round answers to 2 decimal places.
- A) $\sin(36.4^\circ)$
- B) $\cos(25^\circ 32')$
- C) $\sec(168^\circ)$
- D) $\cot(280^\circ)$

MAC1114 Sample Test 1 Part 2

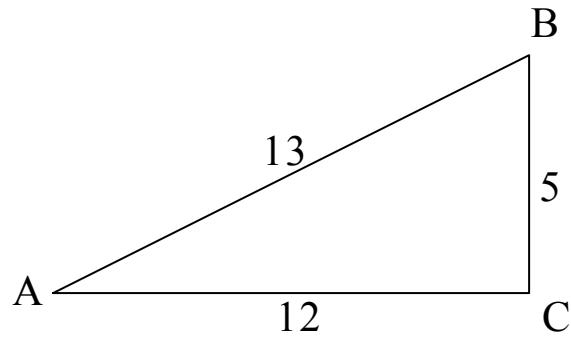
18) Find the following trig ratios, based on the triangle to the right.

A) $\cos A =$

B) $\tan B =$

C) $\sin A =$

D) $\sec B =$

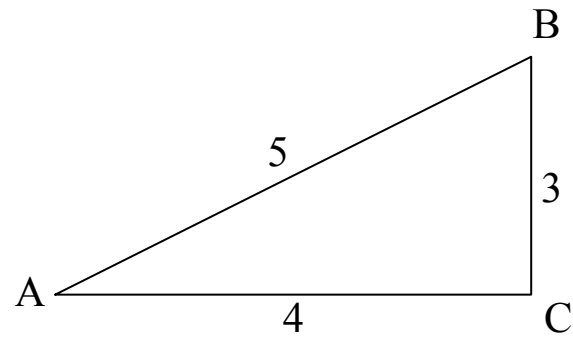


Solve the following right triangles

$A =$ $a =$

19) $B =$ $b =$

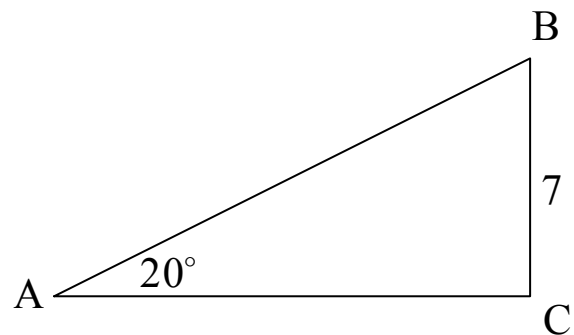
$C =$ $c =$



$A =$ $a =$

20) $B =$ $b =$

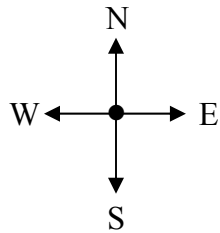
$C =$ $c =$



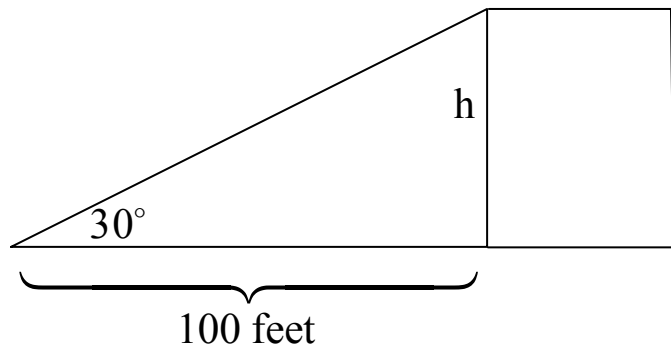
MAC1114 Sample Test 1 Part 2

- 21) A six foot tall man casts a 15 foot long shadow. A tree casts a 100 foot long shadow. How tall is the tree?

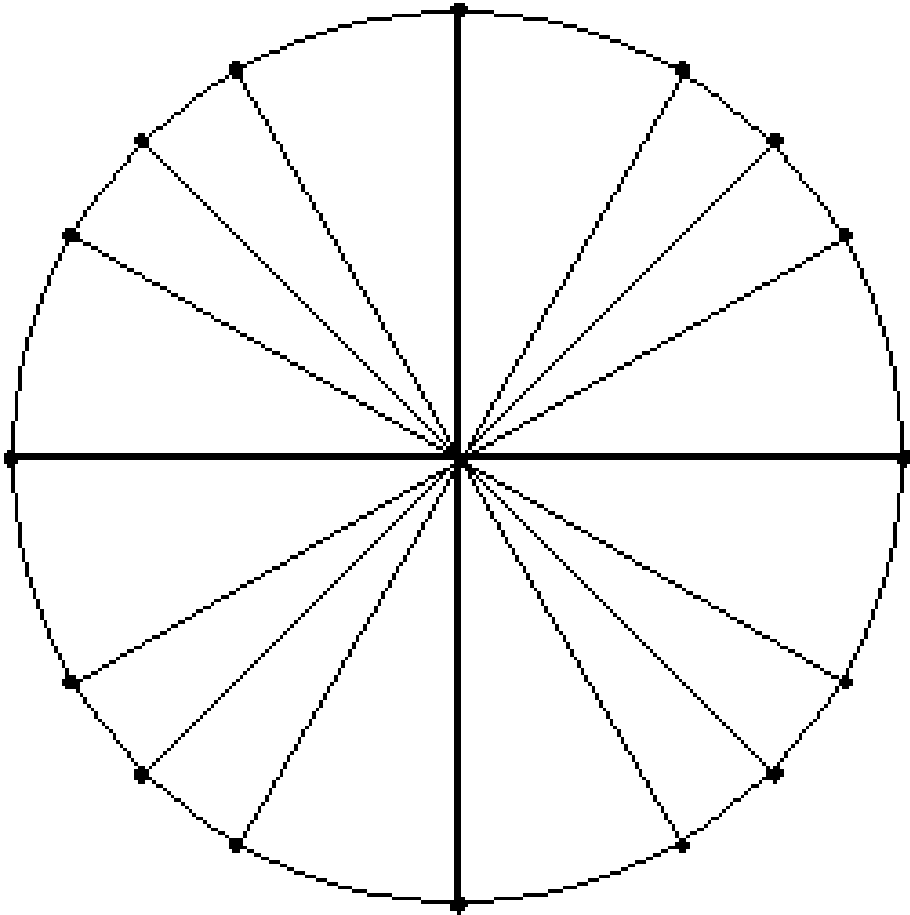
- 22) A hiker leaves camp, looks on his compass, and walks on a bearing of 300° . If the dot \bullet represents camp, draw a ray showing the direction the hiker starts walking.



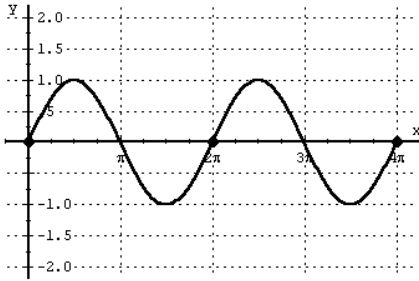
- 23) Find the height of the building



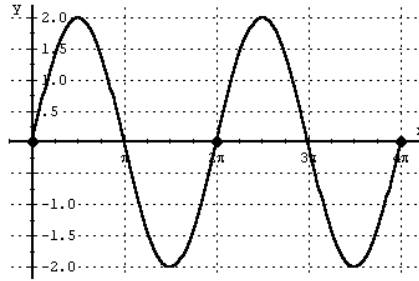
**The following Unit Circle is given for your use and reference only.
No points are given for values filled out on the circle.**



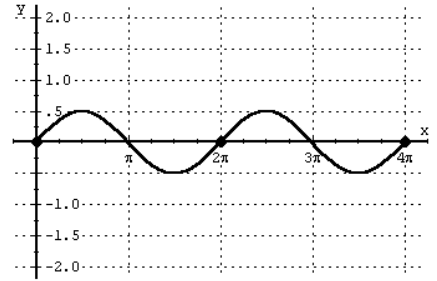
Transformations of $y = f(x) = \sin x$



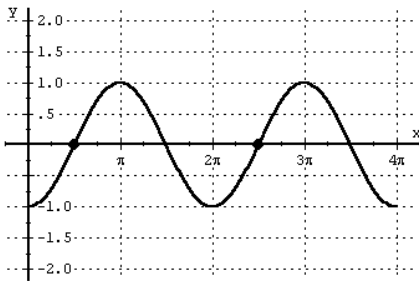
Parent Function
 $y = f(x) = \sin x$



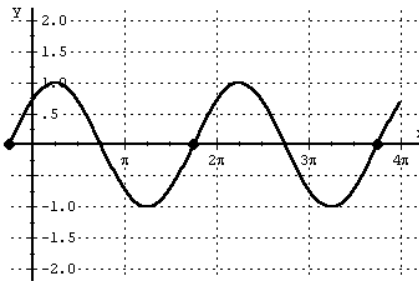
Vertical Stretch
 $y = 2 f(x) = 2 \sin x$



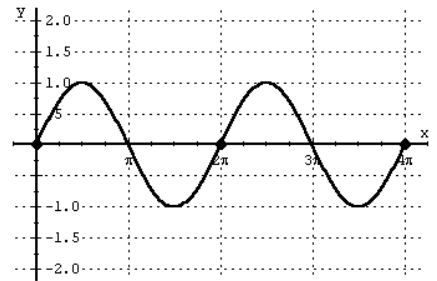
Vertical Compression
 $y = \frac{1}{2} f(x) = \frac{1}{2} \sin x$



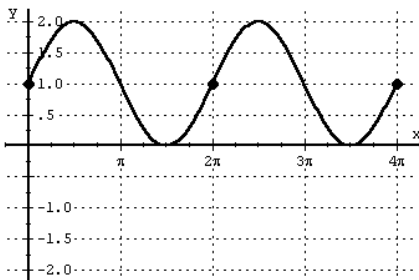
Horizontal Shift – Right $\frac{\pi}{2}$
 $y = f(x - \frac{\pi}{2}) = \sin(x - \frac{\pi}{2})$



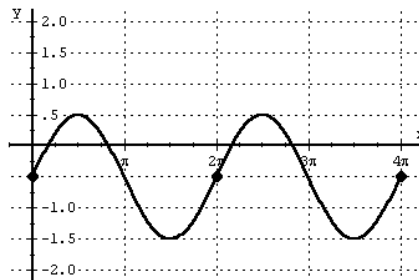
Horizontal Shift – Left $\frac{\pi}{4}$
 $y = f(x + \frac{\pi}{4}) = \sin(x + \frac{\pi}{4})$



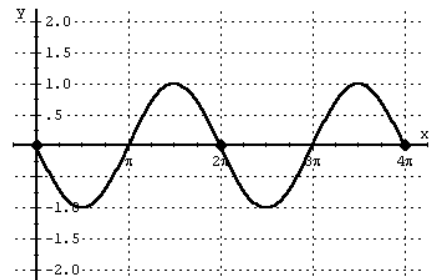
Horizontal Shift – Right 2π
 $y = f(x - 2\pi) = \sin(x - 2\pi)$



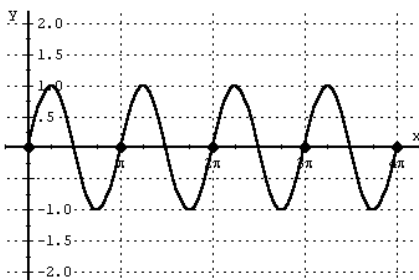
Vertical Shift – Up 1
 $y = f(x) + 1 = 1 + \sin x$



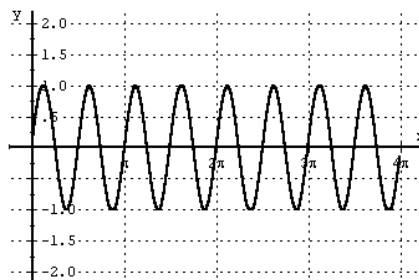
Vertical Shift – Down $\frac{1}{2}$
 $y = f(x) - \frac{1}{2} = -\frac{1}{2} + \sin x$



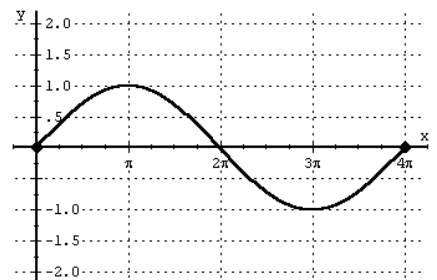
Reflection about the x-axis
 $y = -f(x) = -\sin x$



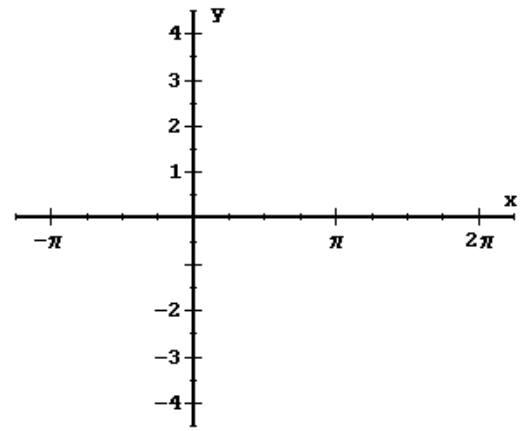
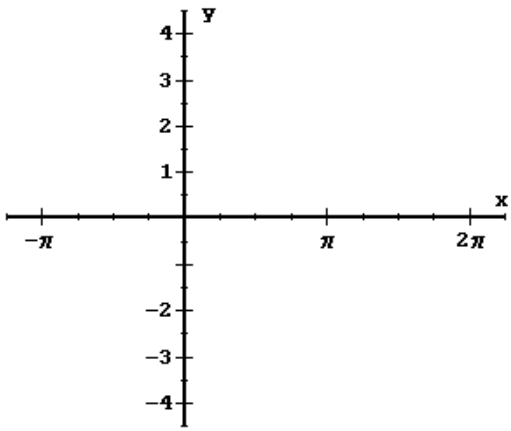
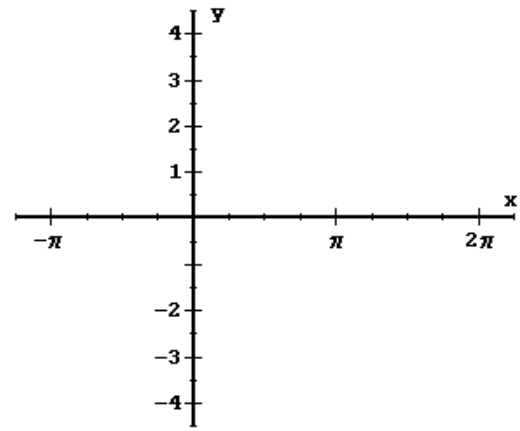
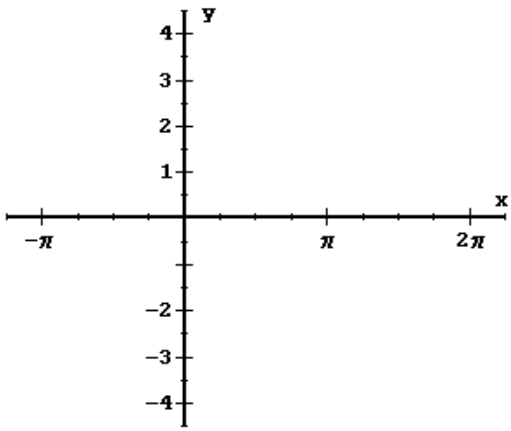
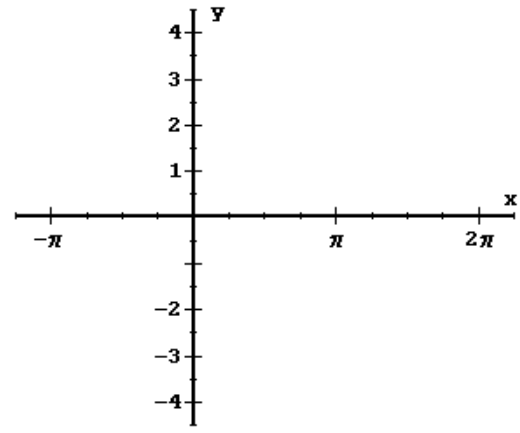
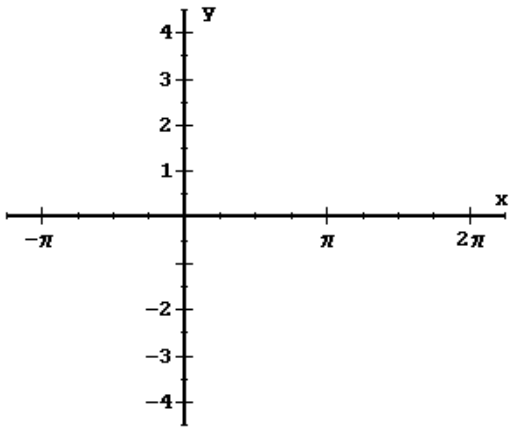
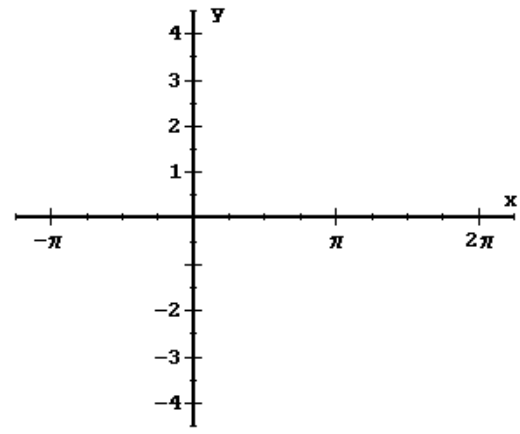
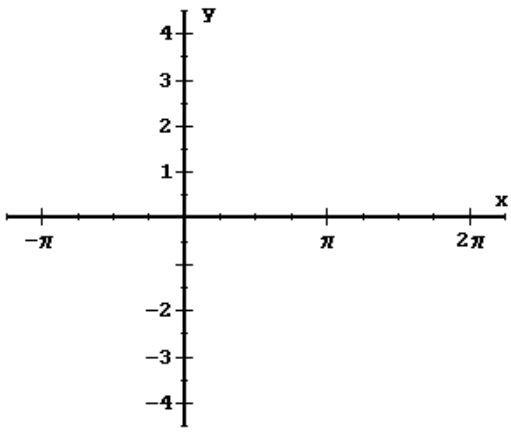
Horizontal Compression
 $y = f(2x) = \sin 2x$

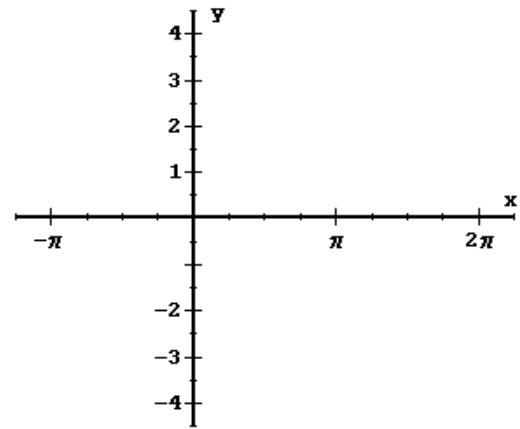
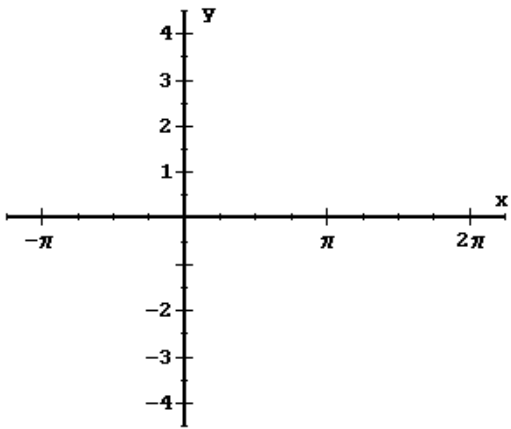
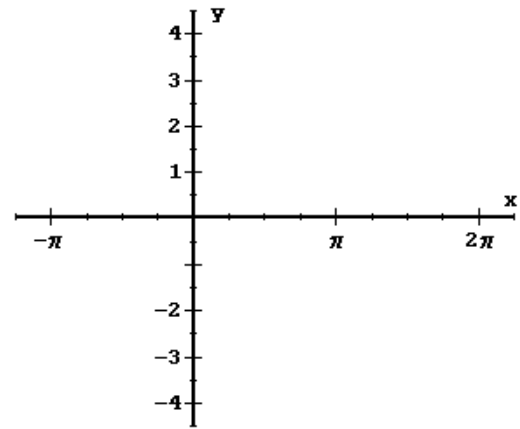
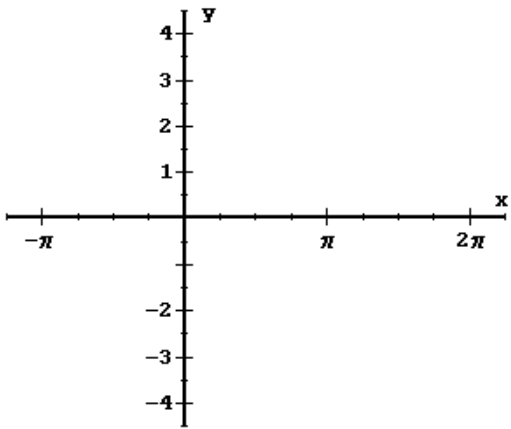
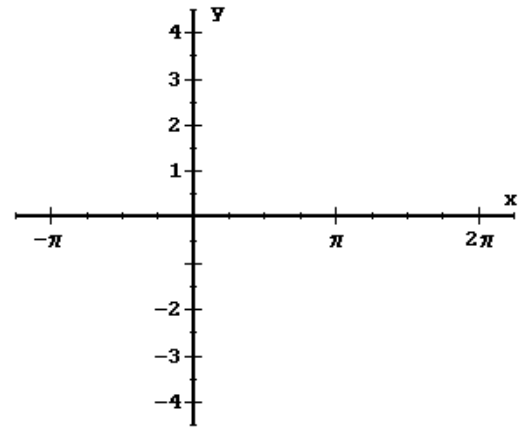
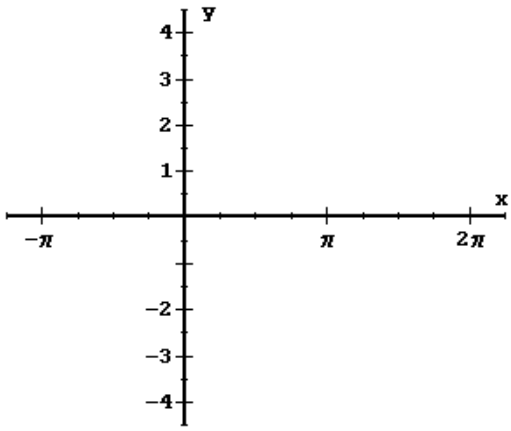
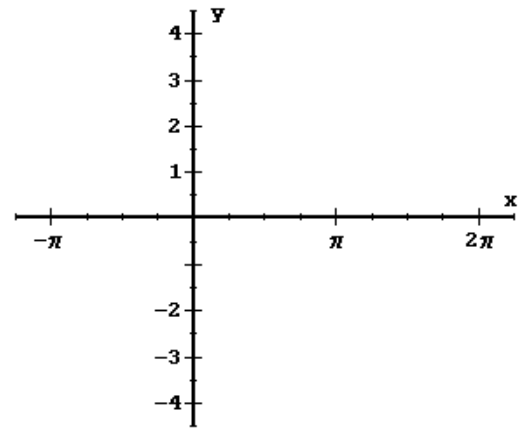
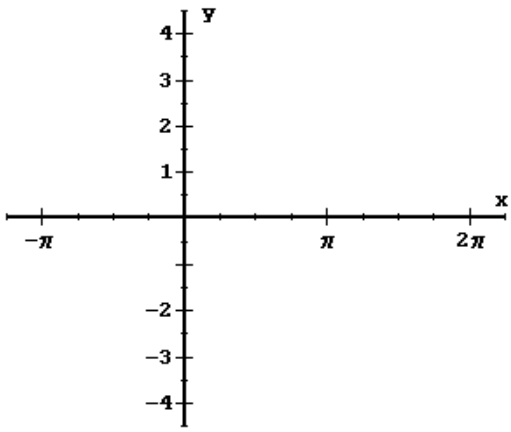


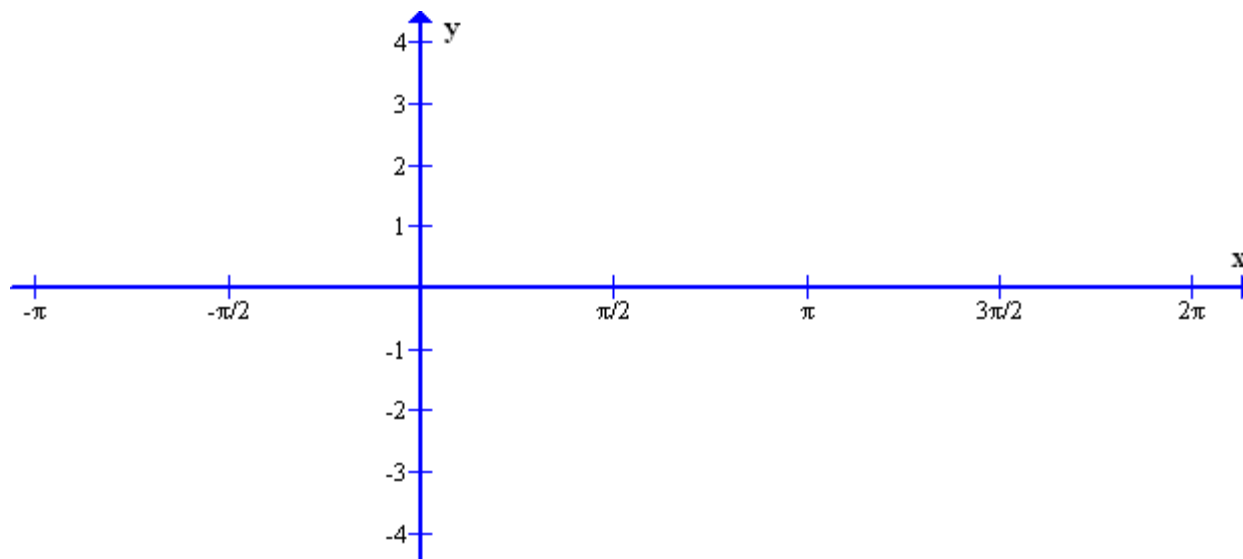
Horizontal Compression
 $y = f(4x) = \sin 4x$



Horizontal Stretch
 $y = f(\frac{1}{2} x) = \sin \frac{1}{2} x$







Graph: $y = 3 \sin \left(2x - \frac{\pi}{2} \right) - 1$

Domain:

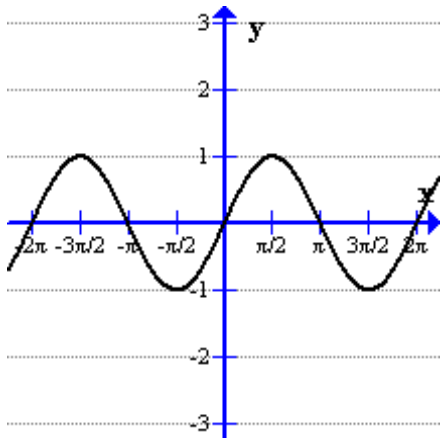
Range:

Amplitude:

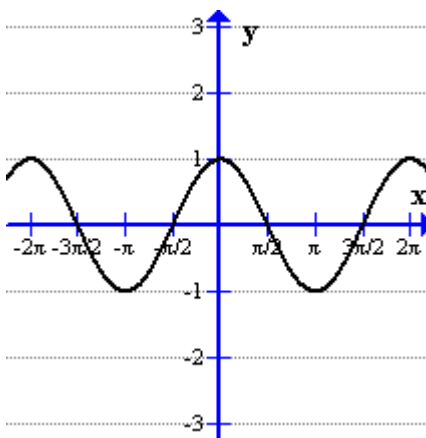
Period:

Transformations of Sine and Cosine

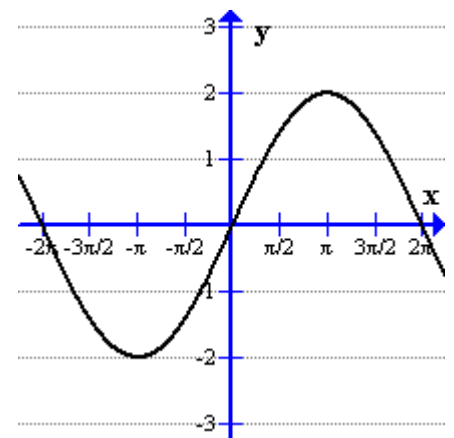
Find two equations for each graph. Use SINE for the 1st equation, then use COSINE for the 2nd equation.



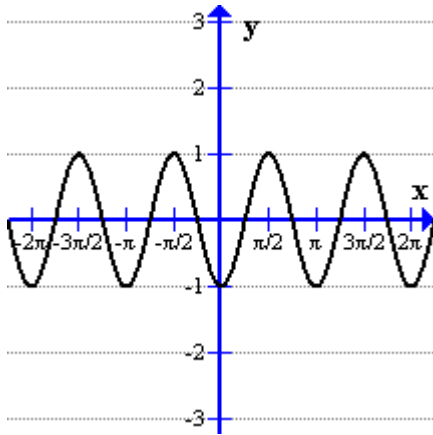
$y =$
 $y =$



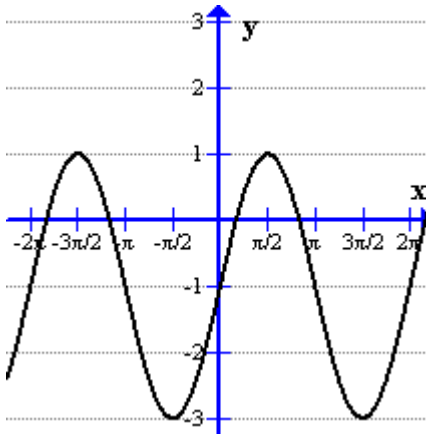
$y =$
 $y =$



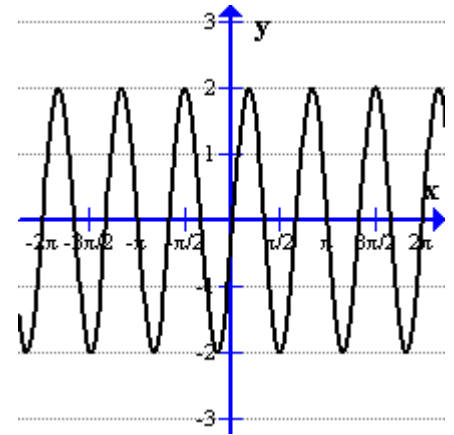
$y =$
 $y =$



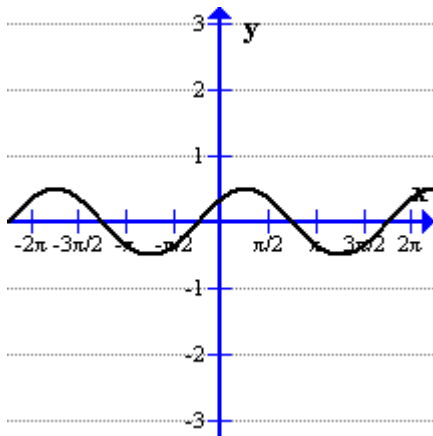
$y =$
 $y =$



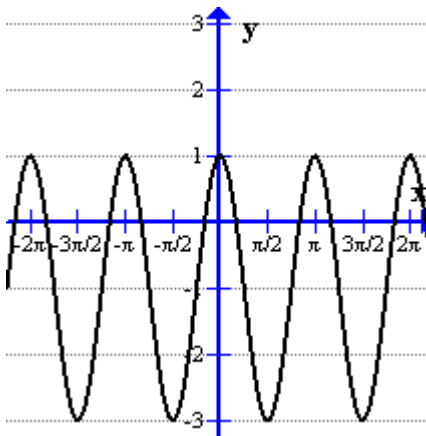
$y =$
 $y =$



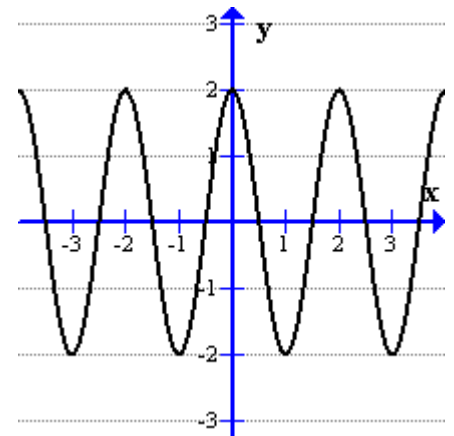
$y =$
 $y =$



$y =$
 $y =$



$y =$
 $y =$



$y =$
 $y =$

MAC1114 Test 2 Sample Part 1

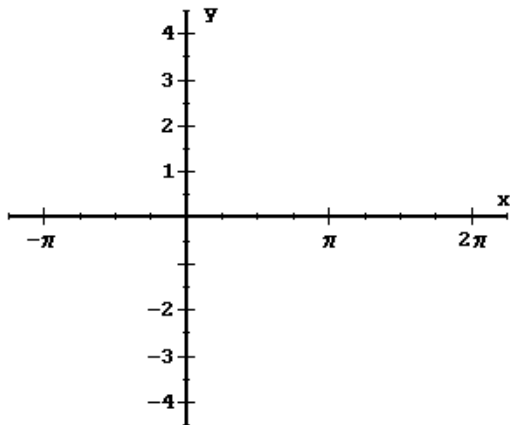
Name _____

Fill in the following table with the appropriate values on a Unit Circle. **No Calculators Allowed.**

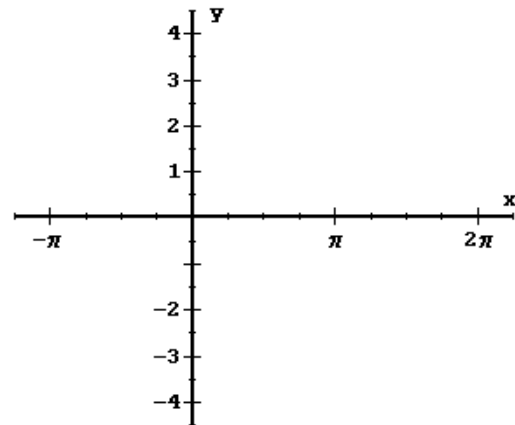
	θ in degrees	θ in radians	Reference Angle (in degrees)	Coordinates (x, y)	$\sin \theta$	$\cos \theta$	$\tan \theta$
1)	120°						
2)		$\frac{\pi}{4}$					
3)			XXX	(0,1)			
4)					$\sin \theta = \frac{1}{2}$ θ in QII		
5)						$\cos \theta = \frac{\sqrt{3}}{2}$ θ in QIV	
6)							$\tan \theta = -1$ θ in QII

7) A) Convert 200° to radian measure. B) Convert $\frac{\pi}{10}$ to degree measure.

8) Formulas:
 Arc Length: $s =$ _____
 Area of a Sector: $A =$ _____
 Linear Velocity: $V =$ _____ = _____ = _____
 Angular Velocity: $\mathbf{w} = \boldsymbol{\omega} =$ _____



9) Graph $y = 2\sin(x) + 1$



10) Graph $y = \cos(2x)$

MAC1114 Test 2 Sample Part 2

Name _____

Calculator is allowed / required on Part 2

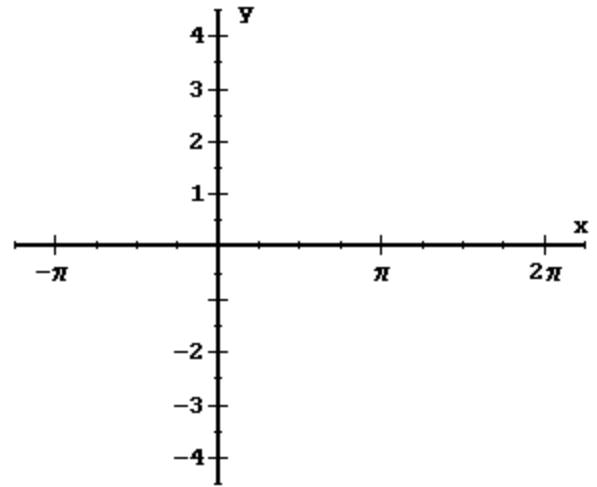
11) Graph $y = \sin\left(x + \frac{\pi}{4}\right)$

State the Domain

State the Range

State the Amplitude

State the Period



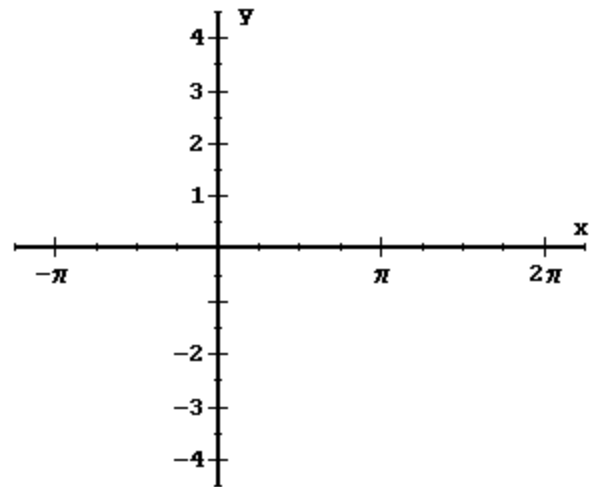
12) Graph $y = 3\cos\left(\frac{1}{2}x\right)$

State the Domain

State the Range

State the Amplitude

State the Period

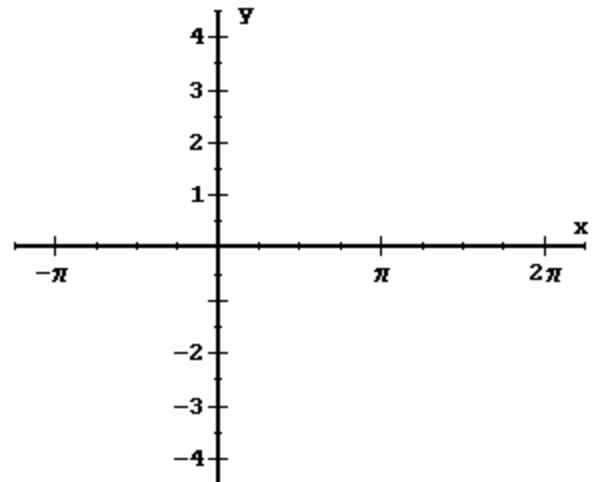
**Show all asymptotes with dotted lines.**

13) Graph $y = \cot x$

State the Domain

State the Range

State the Period



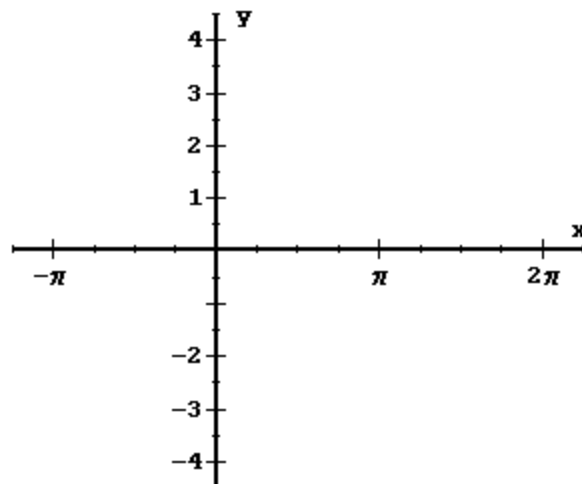
Show all asymptotes with dotted lines

14) Graph $y = \tan(2x)$

State the Domain

State the Range

State the Period



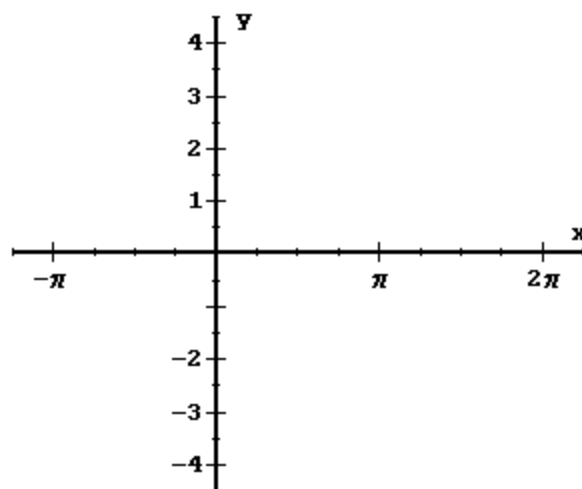
15) Graph $y = 2\sin\left(x + \frac{\pi}{4}\right) - 1$

State the Domain

State the Range

State the Amplitude

State the Period



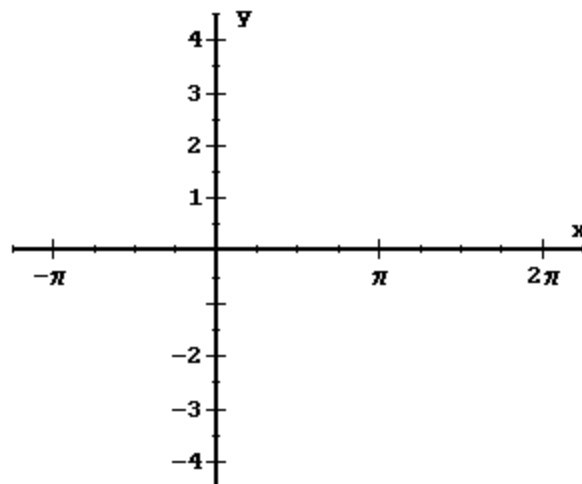
16) Graph $y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$

State the Domain

State the Range

State the Amplitude

State the Period



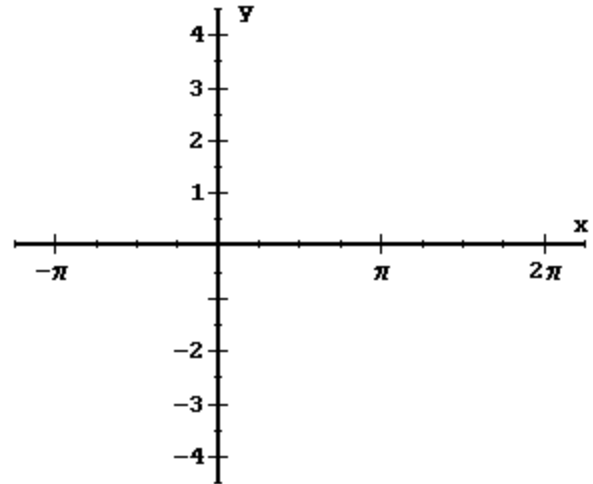
Show all asymptotes with dotted lines

17) Graph $y = \sec(x)$

State the Domain

State the Range

State the Period



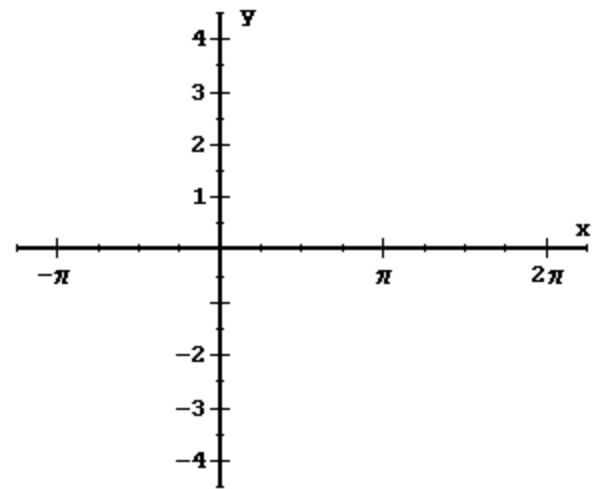
18) Graph $y = \csc\left(\frac{1}{2}x\right)$

State the Domain

State the Range

State the Amplitude

State the Period



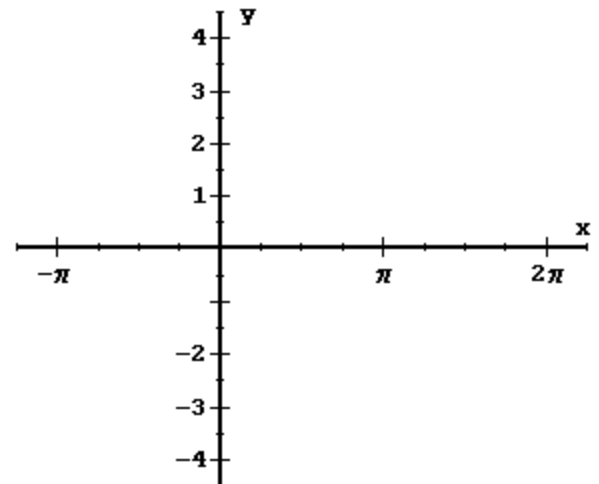
19) Graph $y = 3\sec(2x)$

State the Domain

State the Range

State the Amplitude

State the Period



Formulas: $s = r\theta$ $A = \frac{1}{2} r^2 \theta$

$$r \cdot t = d \quad \Rightarrow \quad r = \frac{d}{t} \quad \Rightarrow \quad v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

$$\omega = \frac{\theta}{t}$$

- 20) Find the length of the arc of a circle of diameter 10 feet, intercepted by a central angle of 150° .
- A) Exact Length
- B) Length rounded to 2 decimal places.
- 21) A young boy cuts a piece of apple pie (a sector). The diameter of the circular pie pan is 9 inches. He noticed that the angle of the pie cut in the center was exactly 60° . Find the area of the boy's piece of pie.
- 22) Suppose that a jogger is running around a circular track, 50 m in radius. The jogger runs $\frac{2}{3}$ of the way around the track in 60 seconds, then stops gasping for air.
- A) How fast was the jogger running (in meters per second m/s)?
- B) A coach standing in the center of the track area was videotaping the runner. How fast was the coach spinning (in radians per second)?

The following Unit Circle is given for your use and reference only.
No points are given for values filled out on the circle.

