

APPENDIX C

C:01 | HCF and LCM by Prime Factors (Extension)



C:01

- 1 List all factors of 30.
- 2 List all factors of 54.
- 3 What is the highest common factor of 30 and 54?
- 4 List all multiples of 4 that are less than 40.
- 5 List all multiples of 3 that are less than 40.
- 6 What is the lowest common multiple of 4 and 3?
- 7 Use a factor tree to write 48 as a product of its prime factors.

Write the following products of prime numbers in index notation.

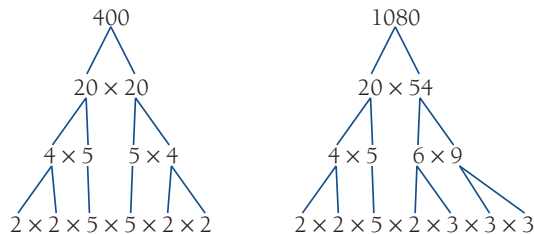
- 8 $2 \times 2 \times 2 \times 2 \times 2$
- 9 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
- 10 $3 \times 3 \times 5 \times 5 \times 5 \times 7$

When we express numbers as products of their prime factors, it is sometimes easier to find their highest common factor (HCF) and their lowest common multiple (LCM).

worked example 1

Find the highest common factor of 400 and 1080.

Solution



From the factor trees:

Number	Product of prime factors
400	$(2) \times (2) \times (2) \times 2 \times (5) \times 5$ or $2^4 \times 5^2$
1080	$(2) \times (2) \times (2) \times 3 \times 3 \times 3 \times (5)$ or $2^3 \times 3^3 \times 5$

The two numbers have $2 \times 2 \times 2 \times 5$ in common.

$2 \times 2 \times 2 \times 5 = 2^3 \times 5 = 40$, so the highest common factor of 400 and 1080 is 40.

worked example 2

Find the lowest common multiple of 400 and 1080.

Solution

We write each number as a product of its primes:

400	$2 \times 2 \times 2 \times 2 \times 5 \times 5$ or $2^4 \times 5^2$	1080	$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ or $2^3 \times 3^3 \times 5$
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$$\text{LCM} = (2 \times 2 \times 2 \times 2 \times 5 \times 5) \times (3 \times 3 \times 3)$$



This is 400 written as a product of its primes.



These are the prime factors of 1080 not already written in the first part.

$\text{LCM} = 2^4 \times 3^3 \times 5^2 = 10800$, so the lowest common multiple of 400 and 1080 is 10800.

worked example 3

Find the highest common factor and the lowest common multiple of 12 150 and 39 375, if

$$12\,150 = 2 \times 3^5 \times 5^2 \quad \text{and} \quad 39\,375 = 3^2 \times 5^4 \times 7.$$

Solution

$$12\,150 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$$

$3 \times 3 \times 5 \times 5$ is contained in both,

$$39\,375 = 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$$

\therefore the HCF is $3^2 \times 5^2$ or 225.

$2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 7$ contains both numbers,

\therefore the LCM is $2 \times 3^5 \times 5^4 \times 7$ or 2 126 250.

The highest common factor of 12 150 and 39 375 is 225 and the lowest common multiple of these two numbers is 2 126 250.

Exercise C:01

1 Use this table to answer the questions.

Number	Products of prime factors	
144	$2 \times 2 \times 2 \times 2 \times 3 \times 3$	$2^4 \times 3^2$
324	$2 \times 2 \times 3 \times 3 \times 3 \times 3$	$2^2 \times 3^4$
1890	$2 \times 3 \times 3 \times 3 \times 5 \times 7$	$2 \times 3^3 \times 5 \times 7$
4900	$2 \times 2 \times 5 \times 5 \times 7 \times 7$	$2^2 \times 5^2 \times 7^2$
1960	$2 \times 2 \times 2 \times 5 \times 7 \times 7$	$2^3 \times 5 \times 7^2$
3375	$3 \times 3 \times 3 \times 5 \times 5 \times 5$	$3^3 \times 5^3$
8232	$2 \times 2 \times 2 \times 3 \times 7 \times 7 \times 7$	$2^3 \times 3 \times 7^3$
1568	$2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$	$2^5 \times 7^2$

a Find the highest common factor of:

- i 144 and 324
- ii 324 and 1890
- iii 1890 and 4900
- iv 4900 and 1960
- v 1960 and 3375
- vi 3375 and 8232
- vii 8232 and 1568
- viii 1960 and 8232
- ix 1890 and 3375
- x 1568 and 3375

b Find the lowest common multiple of:

- i** 144 and 324 **ii** 324 and 1890 **iii** 1890 and 4900 **iv** 4900 and 1960
v 1960 and 3375 **vi** 3375 and 8232 **vii** 8232 and 1568 **viii** 1960 and 8232
ix 1899 and 3375 **x** 1568 and 3375

2 Complete the table below and then answer the questions.

Number	Products of prime factors	
18		
36		
24		
300		
1050		
1250		
2475		
2310		

a Find the highest common factor of:

- i** 18 and 36
ii 36 and 24
iii 24 and 300
iv 300 and 1050
v 1250 and 1050
vi 2475 and 1250
vii 2475 and 2310
viii 1050 and 2475
ix 1250 and 2310
x 24 and 2475

b Find the lowest common multiple of:

- i** 18 and 36 **ii** 36 and 24 **iii** 24 and 300 **iv** 300 and 1050
v 1250 and 1050 **vi** 2475 and 1250 **vii** 2475 and 2310 **viii** 1050 and 2475
ix 1250 and 2310 **x** 24 and 2475

3 a Find the highest common factor of 1155 and 2079.

b Find the lowest common multiple of 1155 and 2079.

c What is the highest common factor of 264 and 1386?

d What is the lowest common multiple of 264 and 1386?

4 a Two people are jogging around an oval.

They start together and one takes 168 seconds to complete exactly one lap while the other takes 189 seconds. How long after they start will it take before they again meet at the starting point?

b Two cannons are fired together, then one is fired every 72 minutes while the other is fired every 108 minutes. How long after the first shot will they again be fired together?

c Judy is told that she may purchase chairs for \$44 each and tables for \$231 each, as long as she pays exactly the same amount for chairs as for tables. What is the least amount she needs to spend to take advantage of these prices?



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