Mth 112 Pre Exam 2 *Franz Helfenstein* **NAME**

To receive full credit: You must **show your work**, it must be **neat** with answers **simplified**, **boxed** in, accurate to **two decimal places** and including **units**. Ask if you need any clarification. (100 points)

1) Convert S 40° E to its equivalent

(a) Mark S 40° E on the compass in Fig 1.

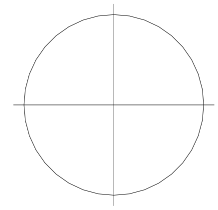


Fig 1

(b) θ-angle (DMS)_____________

(c) radian angle________________(keep π in your answer)

(d) Circle the quadrant(s) where **both** $\sin\theta > 0$ and $\cos\theta < 0$: **I II III IV**

(e) Circle the quadrant(s) where **both** $\tan\theta < 0$ and $\cos < 0$: **I II III IV**

(f) Convert $\frac{7\pi}{20}$ to degrees

2) Through how many radians will each of the following hands of a clock rotate in a 12-hour period? Leave your answer in terms of π.

(a) H**our hand** _________ (b) **Minute hand** _________ (c) **Second hand** _________

(d) A ratchet turns a bolt 52° 20' with each pull. How many radians will it turn if it is pulled 40 times?

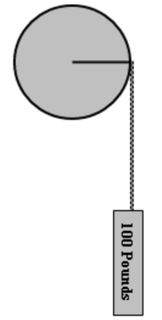
(e) How many revolution will the bolt turn?

3) (a) Find the equation of the two lines in slope-intercept form.

Line 1 passes through (-5, 7) to (4, -8), Line 2 passes through (-5, -8) to (4, 10),

(b) Find the angle between the two lines. Hint: Find the angles with respect to the x-axis first.

4) The circular pulley shown has a radius of 9 inches, and the belt is attached as shown. A 100-pound weight is attached to one end of the belt, which is pulled tight. Through which positive angle, **<u>in radians</u>**, must the pulley be rotated to cause the weight to be raised 20 inches? Include π in your answer.



5) Jen's favorite bike is her 1963 Schwinn Fleet. It has a beefy drive train: the front gear has a diameter of 9 inches, and the back gear has a diameter of 3 inches (see picture at right). Additionally, her rear wheel is 28" in diameter. She's pedaling down the road at 60 RPM

(a) How fast is she going in miles per hour? Round to mph.

(b) If a rock that was stuck in the tire comes loose and shoots out straight ahead how fast is it going? *Hint: It's moving at the same speed as the surface of the wheel.*

6) Given $\sin\theta = \dfrac{2}{\sqrt{6}}$ Find (exactly) (a) $\cos\theta$ (b) $\tan\theta$

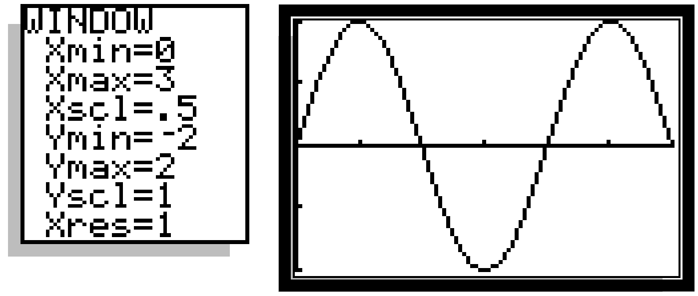
(c) When $\tan^{-1} t = \theta$ find $\sin\theta$

7) (a) Simplify to just one trig function: $\dfrac{\sin t}{\tan t}$

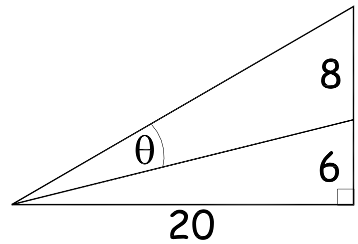
(b) Simplify to one term: $\tan^2(3t)\cos^2(3t) + \cos^2(3t)$

(c) Multiply and simplify to one term: $1 - (\sin t + \cos t)^2$

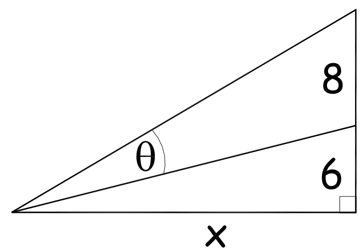
8) (a) Give the equation of this function in the form y = A sin (bt + c) Use exact values.



9) Find θ.



10) Express θ as a function of ***x***.

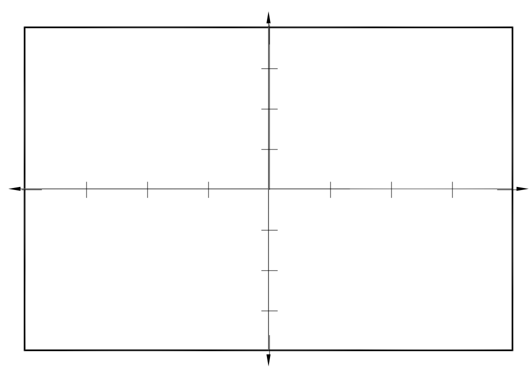


11) (a) Graph $y_1 = \dfrac{\sin t}{1+\cos t} + \dfrac{1+\cos t}{\sin t}$

(b) Graph $y_2 = \dfrac{2}{\sin t}$

(c) What can you conclude about y_1 & y_2?

Prove your conjecture algebraically.



Sketch the graph in [-2π, 2π] × [-8, 8]

12) Suppose that a weight hanging from a spring is pulled downward from its equilibrium point 10 centimeters and let go. In 12 seconds for the weight completes 50 up-down cycles. Express the height y(t) in terms of a sine (or cosine) function.

BONUS

Assume the spring above undergoes dampening so that its amplitude is given by Ae^{-at}. It takes 90 seconds for the spring to reach a state where it is only moving up and down 1mm from the equilibrium position. What is the model for this spring under these circumstances?

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1) Convert S 40° E to its equivalent

(a) Mark <u>S 40° E</u> on the compass in Fig 1. $\frac{-50\pi}{180}$

Fig 1 S 40° E

(b) θ-angle (DMS) -50°, 310°

(c) radian angle $\frac{-5\pi}{18}, \frac{31\pi}{18}$ (keep π in your answer)

(d) Circle the quadrant(s) where **<u>both</u>** $\sin\theta > 0$ and $\cos\theta < 0$: **I II III IV**

(e) Circle the quadrant(s) where **<u>both</u>** $\tan\theta < 0$ and cos < 0: **I II III IV**

(f) Convert $\frac{7\pi}{20}$ to degrees $\frac{7\pi}{20}\cdot\frac{180}{\pi} = 63°$

2) Through how many radians will each of the following hands of a clock rotate in a 12-hour period? Leave your answer in terms of π.

(a) **Hour hand** 2π (b) **Minute hand** 24π (c) **Second hand** 1440π

(d) A ratchet turns a bolt 52° 20' with each pull. How many radians will it turn if it is pulled 40 times?

36.5 rad

(e) How many revolution will the bolt turn?

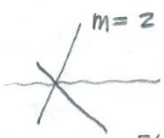
5.8 rev

3) (a) Find the equation of the two lines in slope-intercept form.

Line 1 passes through (-5, 7) to (4, -8), Line 2 passes through (-5, -8) to (4, 10),

$y_1 = -5/3\,x + -4/3$

$y_2 = 2x + 2$



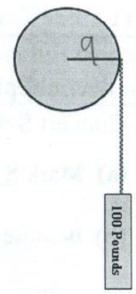
$\theta_1 = \tan^{-1}(-5/3) = -59.04°$

$\theta_2 = \tan^{-1}(2) = 63.43°$

(b) Find the angle between the two lines. Hint: Find the angles with respect to the x-axis first.

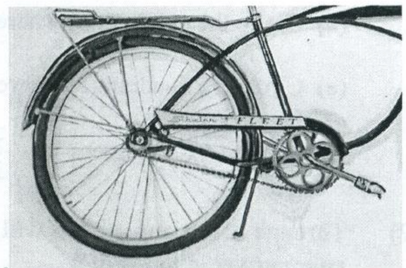
122.47°

4) The circular pulley shown has a radius of 9 inches, and the belt is attached as shown. A 100-pound weight is attached to one end of the belt, which is pulled tight. Through which positive angle, **in radians**, must the pulley be rotated to cause the weight to be raised 20 inches? Include π in your answer.



$$20 \text{ in} \; \frac{2\pi \text{ rad}}{18\pi \text{ in}} = \frac{20}{9} \text{ rad}$$

5) Jen's favorite bike is her 1963 Schwinn Fleet. It has a beefy drive train: the front gear has a diameter of 9 inches, and the back gear has a diameter of 3 inches (see picture at right). Additionally, her rear wheel is 28" in diameter. She's pedaling down the road at 60 RPM



(a) How fast is she going in miles per hour? Round to mph.

$$\frac{180 \text{ rev}}{\text{min}} \; \frac{60 \text{ min}}{1 \text{ hr}} \; \frac{28\pi \text{ in}}{1 \text{ rev}} \; \frac{1 \text{ ft}}{12 \text{ in}} \; \frac{1 \text{ mi}}{5280 \text{ ft}} = 14.99 \text{ mph}$$

(b) If a rock that was stuck in the tire comes loose and shoots out straight ahead how fast is it going? *Hint: It's moving at the same speed as the surface of the wheel.*

14.99 mph

6) Given $\sin\theta = \frac{2}{\sqrt{6}}$ Find (exactly) (a) $\cos\theta = \sqrt{2/6} = \sqrt{1/3}$ (b) $\tan\theta = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\sqrt{6}$ θ 2 $\sqrt{2}$

(c) When $\tan^{-1}\frac{t}{1} = \theta$ find $\sin\theta = \frac{t}{t^2+1}$

t^2+1 t θ 1

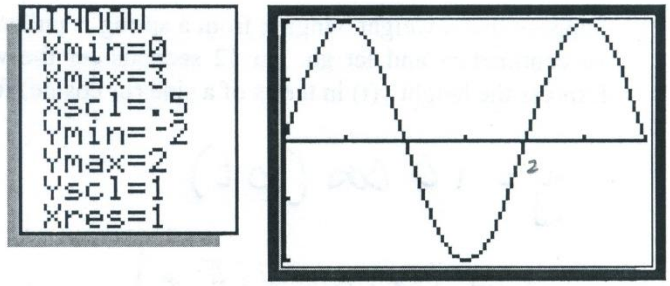
7) (a) Simplify to just one trig function: $\frac{\sin t}{\tan t} = \frac{\sin t}{\frac{\sin t}{\cos t}} = \cos t$

(b) Simplify to one term: $\tan^2(3t)\cos^2(3t) + \cos^2(3t) = [\tan^2(3t)+1]\cos^2(3t) = \sec^2(3t)\cdot\cos^2(3t) = 1$

(c) Multiply and simplify to one term: $1 - (\sin t + \cos t)^2$

$$1 - (\sin^2 t + 2\sin t\cos t + \cos^2 t) = -2\sin t\cos t = -\sin(2t)$$

8) (a) Give the equation of this function in the form y = A sin (bt + c) Use exact values.

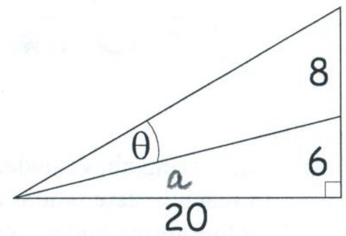


$y = 2\sin(\pi t)$

$b \cdot 2 = 2\pi$

$b = \pi$

9) Find θ.



$\tan a = 6/20$

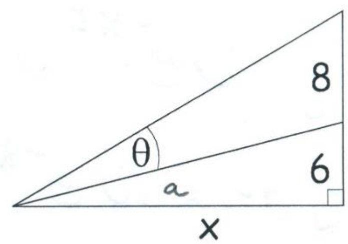
$\tan(a+\theta) = \frac{14}{20}$

$a+\theta = \tan^{-1} 14/20$

$\theta = \tan^{-1}(14/20) - \tan^{-1}(6/20)$

$\sim 18.29^{\circ}$

10) Express θ as a function of x.



$\tan a = 6/x$

$\tan(a+\theta) = \frac{14}{x}$

$\theta = \tan^{-1}(14/x) - \tan^{-1}(6/x)$

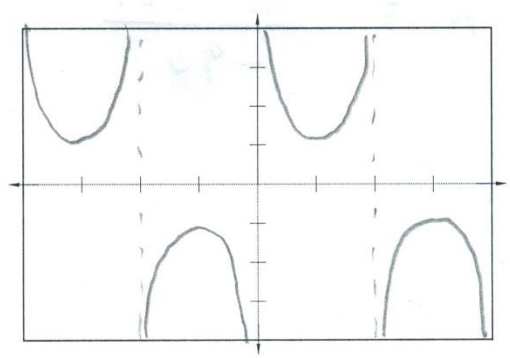
11) (a) Graph $y_1 = \frac{\sin t}{1+\cos t} + \frac{1+\cos t}{\sin t}$

(b) Graph $y_2 = \frac{2}{\sin t}$

(c) What can you conclude about y_1 & y_2?

They're equivalent

Prove your conjecture algebraically.

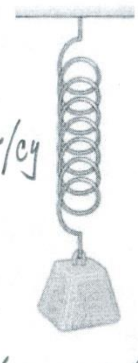


Sketch the graph in [-2π, 2π] × [-8, 8]

$\frac{\sin t}{1+\cos t}\cdot\frac{\sin t}{\sin t} + \frac{1+\cos t}{\sin t}\cdot\frac{1+\cos t}{1+\cos t}$

$\frac{\cancel{\sin^2 t} + 1 + 2\cos t + \cancel{\cos^2 t}}{\sin t(1+\cos t)} = \frac{2+2\cos t}{(1+\cos t)\sin t} = \frac{2\cancel{(1+\cos t)}}{\sin t\cancel{(1+\cos t)}} = \frac{2}{\sin t}$

12) Suppose that a weight hanging from a spring is pulled downward from its equilibrium point 10 centimeters and let go. In 12 seconds ~~for~~ the weight completes 50 up-down cycles. Express the height y(t) in terms of a sine (or cosine) function.



$y = 10\cos(bt)$

$bt = 2\pi \quad t = \frac{12}{50}\ \text{sec/cy}$

$y = 10\cos\left(\frac{25\pi}{3}t\right)$

$b = \frac{2\pi \cdot \cancel{50}^{25}}{\cancel{12}_{3\cancel{6}}} = \frac{25\pi}{3}$

$y = 10\sin\left(\frac{25\pi}{3}\left(t + \frac{3}{50}\right)\right)$ other possible phase shifts

BONUS

Assume the spring above undergoes dampening so that its amplitude is given by Ae^{-at}. It takes 90 seconds for the spring to reach a state where it is only moving up and down 1mm from the equilibrium position. What is the model for this spring under these circumstances?

$y = 10e^{-kt} \cdot \cos\left(\frac{25\pi}{3}t\right)$

$y(90) = 10e^{-k90} = 0.1\ \text{cm}$

$k = \frac{\ln 0.01}{-90}$