(x,y) Marks the Spot

Blue Ribbon Algebra Workshop
Math 693

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Introduction:

Students will use a graphing calculator and a Calculator Based Ranger (CBR unit or Ranger) to collect data from a walking activity designed to allow students to physically model a system of linear equations. The data collected will be analyzed both graphically and algebraically. Students will complete an activity sheet with their collected data. They will calculate slope and y-intercept and formulate equations using these quantities. Individual data will then be combined for pairs of walkers and the resulting plot studied and discussed. This activity has been adapted from an activity presented in Real-World Math with the CBL System (1994, Texas Instrument). Adaptations have been included to assist in implementation of this lesson in a collaborative setting. However, this activity could be incorporated in any Applied Math or Algebra classroom.

West Virginia Content Standards and Objectives:

Standard 2: Algebra (MA.S.2)

Students will:

- demonstrate understanding of patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships; and
- analyze change in various contexts through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics.

Objectives:

Students will:

AM2.2.2	determine the slope of a line given an equation of a line, the graph of a line, and two points to be identified.
AM2.2.4	write an equation of a line given the graph of a line, two points on the line, the slope and a point, and the slope and y-intercept.
AM2.2.5	solve systems of linear equations numerically and graphically, by the elimination method, and by the substitution method.

TEC.9-12.1.2	demonstrate knowledge and appropriate use of hardware components, software programs, and their connections.
TEC.9-12.3.2	select and use appropriate technology tools to efficiently collect, analyze, and display data that is relevant to class assignments.

Materials needed:

CBR units (one for each pair of walkers)
TI-83 plus silver edition calculators (will be referred to as TI-83 Plus SE) (one for each student)
meter sticks (one per group of six)
stop watches (one per group of six)
masking tape (one roll per group of six)
student activity sheets (three per group of six)

Time:

The activity could be completed in one ninety minute block period.

Prior knowledge:

Students should understand the principles of slope and y-intercept and be able to use them to write the equation for a line in slope-intercept form.
Students should be able to follow a set of instructions involving the use of a graphing calculator.
Students should have some understanding of linear systems and the different forms that their solutions can take.

Procedures:

1. Clear an area in the classroom large enough to allow each group a working space of approximately two meters wide by six meters long. Another location will need to be reserved if your classroom space is not large enough for the activity to occur.
2. Place CBR units and calculators on tables or desks with units facing out to the open activity area and separated by approximately one meter. If the Ranger program has not been transferred to each calculator, this should be done before the day of the activity.
3. Discuss with students the basic function of the CBR unit, the purpose for linking to the TI-83 Plus SE, and the activity to be completed. In a collaborative setting, the teachers could model the walking portion of

the activity for the students. A review of the possibilities for the solution of a linear system could also occur here.

4. Students will divide into groups of six to collect data. In each group, there will be two walkers, two Ranger operators, and two recorders. Each pair of students will assume all group roles by rotating through three trials of the activity. Each pair of students will take the data that they collect as walkers to complete the activity sheet together, so that each initial group of six students will turn in three activity sheets.
5. Walker one should stand directly in front of CBR unit one with the unit positioned behind him/her. Walker two should stand approximately six meters in front of CBR unit two and face the unit. Ranger operators one and two should stand beside their respective CBR units. The time recorder and the measurement recorder should position themselves to the sides of the activity area. The Ranger operators will ready the units for data collection as discussed in the activity sheet and start data collection by stating "one, two, three, go" and pressing enter on the calculator. At that same instant, the time recorder starts the stopwatch, and the walkers start walking straight ahead. At the moment when the walkers pass each other, the time recorder stops the stopwatch and the measurement recorder marks the floor with a piece of masking tape to denote the distance the walkers were from the CBR units when they passed each other. The walkers continue walking to the end of their available space. The CBR units should have finished recording data by this time, or the collection time should be adjusted.
6. Group members should now look at the data displayed on the screens of the TI-83 Plus SE calculators. They should see a straight line on the calculator screen. If they both do not have a straight line, the data collection should be repeated until straight lines are obtained. If additional trials are required, the group should discuss what might have caused the problem and what they might change to do a better job.
7. Group members rotate group roles so that the walkers are now recorders or Ranger operators, and so on. A new set of calculators must now be linked to the CBR units since the first set of walkers needs the data on the first set of calculators to complete their activity sheets.

8. Data collection should now proceed as discussed in steps five and six above for the new walkers. After this pair of walkers completes their data collection, the group members should rotate again to obtain data for the third set of walkers using a third set of graphing calculators.
9. After all data is collected, each pair of walkers completes an activity sheet using the data they collected as walkers. When all pairs have completed the activity sheet, the entire class could come back together to discuss the results of the activity.

Evaluation:

Walker pairs will receive a grade based on their completion of the activity sheet.

Performance Descriptors:

A teacher's activity sheet has been attached that categorizes each of the questions on the student activity sheet according to the levels of Bloom's Taxonomy. These levels make up the basis of our performance descriptors.

Distinguished

- The student demonstrates exceptional and exemplary performance with distinctive and sophisticated synthesis and evaluation of knowledge and skills that exceeds the standard in algebra. This will be evidenced by answering all questions on the activity sheet completely and correctly.

Above Mastery

- The student demonstrates competent and proficient performance and shows a thorough and effective application and analysis of knowledge and skills that exceeds the standard in algebra. This will be evidenced by answering all questions on the activity sheet completely and correctly with the exception of numbers twelve and fourteen. Answers to twelve and fourteen must reflect some understanding, but the explanation may be incomplete.

Mastery

- The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate performance that meets the standard in algebra. This will be evidenced by correct answers for all of the eleven knowledge and comprehension questions on the activity sheet.

Partial Mastery

- The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in algebra. This will be evidenced by correct answers for eight of the eleven knowledge and comprehension questions on the activity sheet.

Novice

- The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in algebra. This will be evidenced by correct answers for less than eight of the knowledge and comprehension questions on the activity sheet.

Extensions:

1. Students could attempt to model a system of linear equations that do not intersect.
2. Students could attempt to model a system of linear equations that turns out to be one line.
3. Students could perform a linear regression with the TI-83 Plus SE using the data they collect as walkers and compare the regression equation obtained with the calculator to the equation they formulated on the activity sheet.

Adaptations:

1. Assign six people to a group so that the demands on each member are fewer.
2. Provide oral and written instructions.
3. Put key words on activity sheet in bold print to assist in comprehension of what the problems are asking for.
4. Teachers model walking at a constant pace.
5. Provide accommodations as needed so that student's particular limitations do not hamper the completion of their duties as group members.

Resources used:

1. Real-World Math with the CBL System by Chris Brueningsen, Bill Bower, Linda Antinone, and Elisa Brueningsen. (1994, Texas Instrument)
2. Getting Started with CBR (1997, Texas Instrument).

Websites:

1. http://www.curriculum.org/occ/profiles/
   This site contains sample courses of study written for the Ontario Public School System. Included are a large variety of activities for utilizing technology.
2. http://goals2000mathematics.truman.edu/modexam.html
   This site uses graphing calculators and internet research to investigate linear relations.
3. http://www.visi.com/~dethier/activities.htm
   This site contains many activities related to algebra. Manipulatives and technology are included throughout.

Bloom's Taxonomy

TEACHER ACTIVITY SHEET

(x,y) Marks the Spot Activity Sheet Names________________________

Knowledge

1. For your trial as a walker, please **record** each group member's role.

   walker one ___________________________
   walker two ___________________________
   time recorder ___________________________
   measurement recorder ___________________________
   Ranger operator one ___________________________
   Ranger operator two ___________________________

Knowledge

2. **time** when walkers passed each other = ____________
   distance from CBR units when walkers passed each other = __________

Knowledge

3. Please make a separate **sketch** below of the plot showing on each of your TI-83 Plus SE calculators. **Label** the axes appropriately on each sketch.

Comprehension

4. The **slope** of the line for walker one is positive or negative. (circle one)
   The **slope** of the line for walker two is positive or negative. (circle one)

Knowledge

5. Each walker should now press **TRACE** on their calculator. Press the left or right arrow to position the cursor over the y-intercept on each plot.

   y-intercept for walker one = ______________________
   y-intercept for walker two = ______________________

Comprehension

6. Think back to your data collection trials as walkers. What **physical quantity** does the **y-intercept** represent from your walking trial? Please respond with one or two complete sentences.

Knowledge

7. Trace along each plot to obtain any **two points** (x_1,y_1) and (x_2,y_2).

walker one: (x_1,y_1) = ______________ (x_2,y_2) = ________________
walker two: (x_1,y_1) = ______________ (x_2,y_2) = ________________

Comprehension

8. **Calculate the slope** of each plot using the formula below:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

walker one slope: walker two slope:

Comprehension

9. Use the y-intercepts and slopes obtained for each walker to **write an equation** in slope-intercept form ($y = mx + b$) for each walker.

walker one equation: _________________________
walker two equation: _________________________

Press Y= and **enter** both of the equations obtained above into y_1 and y_2. Press **GRAPH**. The entered equations should appear on the screen with your original data.

Knowledge

10. **Make sketches** below showing the original data and the new graphed system of equations.

Comprehension

11. Does the equation you entered for your data match the data that you collected?

    walker one: yes or no
    walker two: yes or no

Evaluation

12. How could you **adjust** your equation to better fit your data? Explain your decision in one or two complete sentences.

Press TRACE and use the arrow keys to position the cursor over the **intersection point** of the graphs. Press ZOOM and select ZOOM IN. Position the cursor again so it appears to be over the intersection point.

Knowledge

13. What are the **coordinates** of the intersection point?

14. Are these the same as the time and distance recorded as you and the other walker passed each other? Should they be the same? **Explain.**

Knowledge | Comprehension | Synthesis

(x,y) Marks the Spot Group Instructions Name_________________

You will be using a TI-83 Plus Silver Edition (TI-Plus SE) calculator and a Calculator Based Ranger (CBR unit or Ranger) to model a linear system of equations. Students in your group will rotate through the roles of walkers, Ranger operators, and recorders. You will be asked to record data on this activity sheet and perform calculations based on the data collection trial in which you assumed the role of a walker. You will work with the other student who assumed the role of walker at the same time that you did to combine both of your sets of data and complete the activity sheet.

For the first data collection trial, fill the following six roles with your group members: time recorder, measurement recorder, Ranger operator one, Ranger operator two, walker one, and walker two. Follow the directions below to proceed with data collection.

Ranger operators set up the Ranger program on the TI-83 Plus SE as follows:

1. Connect calculator and CBR unit with link cable.
2. Turn on the TI-83 Plus SE.
3. Press Program (PRGM) and select RANGER by pressing enter.
4. Press enter twice. Select 1 (SETUP/SAMPLE) by pressing enter again.
5. Press enter to select no for REALTIME.
6. Use down arrow to get to TIME. Type in 5 and press enter.
7. Check UNITS to make sure both of you will be measuring in meters.
8. Use up arrow to get to START NOW. Press enter to select.
9. CBR unit is now ready. Press enter when data collection is to begin.

Walker one should stand directly in front of CBR unit one with the unit positioned behind him/her. Walker two should stand approximately six meters in front of CBR unit two and face the unit. Ranger operators one and two should stand beside their respective CBR units. The time recorder and the measurement recorder should position themselves to the sides of the activity area. After the Ranger operators have prepared the CBR units for data collection and all students are in place, the Ranger operators will signal the beginning of the trial by stating “one, two, three, go” and pressing enter on the calculator. At the same instant, the time recorder starts the stopwatch and the walkers start walking straight ahead at a constant speed. At the moment when the walkers pass each other, the time recorder stops the stopwatch and the measurement recorder marks the floor with a piece of masking tape to denote the distance the walkers were from the CBR units when they passed each other. The walkers should not stop or pause then, but should continue walking to the end of their available space. The CBR units should have finished recording data by this time, or the collection time should be adjusted.

Group members should now look at the data displayed on the screens of the TI-83 Plus SE calculators. Both screens should show linear relationships. If one or both screens do not display lines, additional data collection trials should be completed until both walkers obtain linear results. If additional trials are required, your group should discuss what might have caused the problem and what each of you can do to help achieve better results. To complete new trial, the Ranger operators should press enter to return to the menu, type in 5 to REPEAT SAMPLE, and press enter when data collection is to begin again.

All group members will now rotate into new group roles so that the walkers are now recorders or Ranger operators, and so on. The first set of calculators will be used by walkers one and two in the completion of their activity sheet, so these should be unlinked from the CBR units now. The new Ranger operators should link a new set of calculators to the CBR unit using the instructions above.

Data collection should proceed as above for the new set of walkers. After the second set of walkers obtains a good set of data, group members rotate again to allow the remaining two members to serve as walkers. Data for these walkers will then be collected using another new set of calculators. After all data collection has been completed, each pair of walkers completes the attached activity sheet using the data they collected as walkers.

(x,y) Marks the Spot Activity Sheet Names______________________

1. For your trial as a walker, please **record** each group member's role.

   walker one ___________________________
   walker two ___________________________
   time recorder ___________________________
   measurement recorder ___________________________
   Ranger operator one ___________________________
   Ranger operator two ___________________________

2. **time** when walkers passed each other = ____________
   distance from CBR units when walkers passed each other = __________

3. Please make a separate **sketch** below of the plot showing on each of your TI-83 Plus SE calculators. **Label** the axes appropriately on each sketch.

4. The **slope** of the line for walker one is positive or negative. (circle one)
   The **slope** of the line for walker two is positive or negative. (circle one)

5. Each walker should now press **TRACE** on their calculator. Left or right arrow over to position the cursor over the y-intercept on each plot.

   y-intercept for walker one = ______________________
   y-intercept for walker two = ______________________

6. Think back to your data collection trials as walkers. What **physical quantity** does the **y-intercept** represent from your walking trial? Please respond with one or two complete sentences.

7. Trace along each plot to obtain **two points** (x_1,y_1) and (x_2,y_2).

   walker one: (x_1,y_1) = ______________ (x_2,y_2) = ________________
   walker two: (x_1,y_1) = ______________ (x_2,y_2) = ________________

8. **Calculate the slope** of each plot using the formula below:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

   walker one slope: walker two slope:

9. Use the y-intercepts and slopes obtained for each walker to **write an equation** in slope-intercept form $(y = mx + b)$ for each walker.

   walker one equation: ________________________
   walker two equation: ________________________

Press Y= and **enter** both of the equations obtained above into y_1 and y_2. Press **GRAPH**. The entered equations should appear on the screen with your original data.

10. **Make sketches** below showing the original data and the new graphed system of equations.

11. Does the equation you entered for your data match the data that you collected?

    walker one: yes or no
    walker two: yes or no

12. How could you **adjust** your equation to better fit your data?

Press TRACE and arrow over to the **intersection point** of the graphs. Press ZOOM and select ZOOM IN. Position the cursor again so it appears to be over the intersection point.

13. What are the **coordinates** of the intersection point?

14. Are these the same as the time and distance recorded as you and the other walker passed each other? Should they be the same? **Explain.**