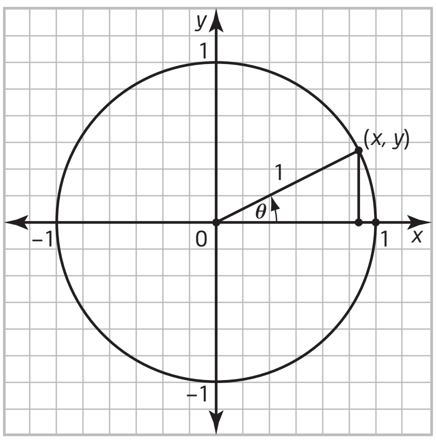
Chapter 4 Trigonometry

4.1 Special Angles

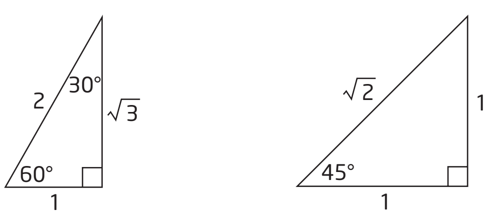
KEY CONCEPTS

- Using a unit circle is one way to find the trigonometric ratios for angles greater than 90°.
- Any point on a unit circle can be joined to the origin to form the terminal arm of an angle. The angle θ is measured starting from the initial arm along the positive x-axis, proceeding counterclockwise to the terminal arm.



The coordinates of the point (x, y) on a unit circle are related to θ such that $x = \cos\theta$ and $y = \sin\theta$.

- $\tan\theta = \dfrac{y}{x}$
- Exact trigonometric ratios for special angles can be determined using special triangles.



- The exact trigonometric ratios for 45° are $\sin 45° = \dfrac{1}{\sqrt{2}}$, $\cos 45° = \dfrac{1}{\sqrt{2}}$, and $\tan 45° = 1$.

Example

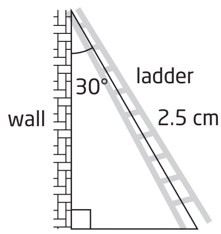
A ladder that is 2.5 m long is placed against a vertical wall so that the top of the ladder makes an angle of 30° with the wall.

a) Draw a diagram to represent this situation.

b) How far up the wall is the top of the ladder? Give an exact answer.

Solution

a)



b) Let h represent the height, in metres, from the top of the ladder to the ground. We know the length of the hypotenuse and we want to find the length of the side adjacent to 30°. Use the cosine ratio.

$\cos 30° = \dfrac{h}{2.5}$

Substitute $\cos 30° = \dfrac{\sqrt{3}}{2}$.

$\dfrac{\sqrt{3}}{2} = \dfrac{h}{2.5}$

Solve for h.

$h = \dfrac{2.5\sqrt{3}}{2}$

$= 1.25\sqrt{3}$

Substitute $1.25 = \dfrac{5}{4}$.

$h = \dfrac{5\sqrt{3}}{4}$

The top of the ladder is $\dfrac{5\sqrt{3}}{4}$ m up the wall.

A Practise

1. Complete the table. Use exact values only.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°			
30°			
45°			
60°			
90°			
180°			
270°			
360°			

2. **Use Technology** Use a calculator to complete the table. Express answers to four decimal places.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°			
30°			
45°			
60°			
90°			
180°			
270°			
360°			

3. **a)** What reference angle on the unit circle should you use to find the trigonometric ratios for 225°?
   b) State two other angles that have the same reference angle on the unit circle.
   c) Use a unit circle to find exact values for the three trigonometric ratios for 225°.
   d) State the exact values for the three trigonometric ratios for the angles in part b).

4. **a)** What reference angle on the unit circle would you use to find the trigonometric ratios for 150°?
   b) State two other angles that have the same reference angle on the unit circle.
   c) Use a unit circle to find exact values for the three trigonometric ratios for 150°.
   d) State the exact values for the three trigonometric ratios for the angles in part b).

5. **a)** What reference angle on the unit circle would you use to find the trigonometric ratios for 300°?
   b) State two other angles that have the same reference angle on the unit circle.
   c) Use a unit circle to find exact values for the three trigonometric ratios for 300°.
   d) State the exact values for the three trigonometric ratios for the angles in part b).

6. Use a unit circle to find the primary trigonometric ratios for 70°. Measure any side lengths needed. Compare your answers to those generated by a calculator.

7. Use a unit circle to find the primary trigonometric ratios for 220°. Measure any side lengths needed. Compare your answers to those generated by a calculator.

B Connect and Apply

☆8. **a)** Describe the CAST rule.
   b) Use the CAST rule to identify the two quadrants where each trigonometric ratio is positive and the two quadrants where each ratio is negative. Copy and complete the table to organize your results.

Trigonometric Ratio	Positive Quadrants	Negative Quadrants
Sine		
Cosine		
Tangent		

9. A ladder that is 3 m long is placed against a vertical wall so that the top of the ladder makes an angle of 60° with the wall.

a) Draw a diagram to represent this situation.

b) Find an exact expression for the distance between the bottom of the ladder and the wall.

c) How far up the wall is the top of the ladder?

10. A sports car is 14 km south of an intersection. A van is 14 km west of the same intersection.

a) Use trigonometry to find an exact expression for the distance between the two vehicles.

b) Describe an alternate method that can be used to solve this problem. Check your answer using that method.

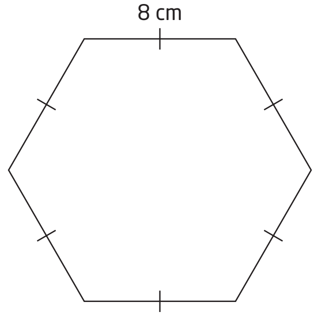
☆**11.** A hydro pole is stabilized at its top by two guy wires of equal length, each of which makes an angle of 60° with the ground. The wires are secured to the ground at points that are 10 m apart and on opposite sides of the pole.

a) Draw a diagram to represent this situation.

b) How tall is the hydro pole? Express your answer using exact values.

c) What is the length of each wire? Express your answer using exact values.

12. A floor tile in the shape of a regular hexagon has side lengths of 8 cm. Determine the area of the tile.



13. Determine the exact value of each expression.

a) $\cos 45° \times \sin 225° + \cos 210°$

b) $\tan 330° \times \cos 240° - 2 \cos 270°$

c) $\tan 60° \times 3 \sin 90° - \sin 315°$

14. Prove that $(\sin 30°)^2 + (\cos 30°)^2 = (\sin 315°)^2 + (\cos 315°)^2$.

C Extend

15. Determine all the possible measures of θ, where $0° \leq \theta \leq 360°$, that satisfy each equation.

a) $\sin \theta = \frac{\sqrt{3}}{2}$

b) $(\cos \theta)^2 = \frac{1}{2}$

c) $\sqrt{3} \tan \theta + 1 = 0$

16. The angle of elevation from point A on the ground to the top of a water tower is 30°. From point B, which is 10 m closer to the tower than point A, the angle of elevation is 45°. Determine the height of the water tower.

17. Each trigonometric ratio has a reciprocal ratio. The reciprocal of the tangent ratio is the cotangent ratio.

$\cot \theta = \frac{1}{\tan \theta}$

a) Show that the formula for the area, A, of a regular polygon with n sides in terms of its side length, s, is $A = \frac{ns^2}{4} \cot\left(\frac{180°}{n}\right)$.

b) Use the trigonometric ratios of special triangles and the formula in part a) to derive a formula for the area of each regular polygon.

i) square

ii) hexagon

iii) equilateral triangle