Name: ______________________________ Date: ______________________________

A.P. CALCULUS AB MIDTERM REVIEW SHEET

Midterm Format:

SECTION I, PART A:	28 questions (55 minutes) NO CALCULATOR
SECTION I, PART A:	17 questions (50 minutes) CALCULATOR
SECTION II, PART B:	2 questions (30 minutes) CALCULATOR
SECTION II, PART B:	4 questions (60 minutes) NO CALCULATOR

Directions on Exam for NO CALCULATOR QUESTIONS:

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the provided scantron. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function *f* may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

Directions on Exam for CALCULATOR QUESTIONS:

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BE SURE YOU ARE USING PAGE 2 OF THE SCANTRON TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 1 OF THE SCANTRON.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function *f* may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

NO CALCULATOR QUESTIONS:

1) Evaluate the following limits:

a) $\lim_{x\to 0} \frac{4x^5+7x^3}{9x^5-21x^3}$

b) $\lim_{x\to -5} \frac{x^2-7x-60}{x^2-25}$

c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

d) $\lim_{x\to\infty} \frac{\sqrt{16x^6+9}}{x^3-x+2}$

e) $\lim_{x\to\infty} \frac{(6-x)(3x-1)}{(x-1)(9x+8)}$

f) $\lim_{h\to 0} \frac{\ln(5+x)-\ln 5}{h}$

g) $\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\left(\frac{\pi}{2}\right)}{h}$

h) $\lim_{x\to\pi} \tan(x-\sin x)$

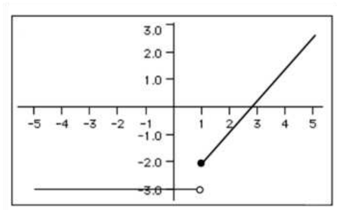
i) $\lim_{x\to\frac{\pi}{2}} \frac{\sin x+\cos(3x)-1}{2x-\pi}$

2) Determine the following limits from the graph of f(x).

a) $\lim_{x\to 1^-} f(x)$

b) $\lim_{x\to 1^+} f(x)$

c) $\lim_{x\to 1} f(x)$

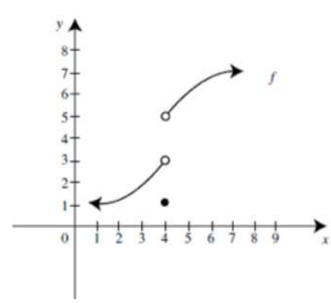


3) The graph of a function *f* is shown below. Which of the following statements is/are true?

I. $\lim_{x\to 4^-} f(x)=3$

II. x = 4 is not in the domain of *f*.

III. $\lim_{x\to 4} f(x)$ does not exist



4) Find the value(s) of c that satisfy the Mean Value Theorem for derivatives on the interval [-1, 1] for the function $f(x) = 3x^3 + 5x - 2$.

5) Let *f* be a differentiable function such that $f(3) = 7$ and $f(7) = 3$. Let *g* be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value *c* for $3 < c < 7$ such that $f'(c) = -1$.

(b) Show that $g'(3) = g'(7)$. Use this result to explain why there must be a value *k* for $3 < k < 7$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all *x*, then the graph of *g* does not have a point of inflection.

(d) Let $h(x) = x - f(x)$. Explain why there must be a value *r* for $3 < r < 7$ such that $h(r) = 0$.

6) What is the average rate of change of the given functions on the given closed interval?

a) $f(x) = x^2 - 4$ on $[0, 5]$

b) $f(x) = \sin 2x$ on $[0, \frac{\pi}{4}]$

7) Given the following functions, find $f'(x)$:

a) $f(x) = \dfrac{6}{\sqrt{x}} - 10\sqrt{x} + 8\sqrt{x^5}$

b) $f(x) = e^{(4/x^2)}$

c) $f(x) = (x+2)(x^2+7)^4$

8) If $f(x) = \sin(2x)$, find $f'\left(\dfrac{\pi}{12}\right)$.

9) If $f(x) = x^2 + 5x$, then

a) $\dfrac{d}{dx}(f(\ln x)) =$

b) $\dfrac{d}{dx}(f(\tan x)) =$

10) In the xy-plane, the line $2x + y = p$, where p is a constant, is tangent to the graph of $y = x^2 - 6x - 1$.

Determine the value of p.

11) If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1; x \le 1 \\ ax^4 - 4bx^2 - 3x; x > 1 \end{cases}$ is differentiable for all real values of x,

then determine the value of b.

$$f(x)=\begin{cases} x^2+x+1, x\le 0 \\ e^x, x>0 \end{cases}$$

12) Let *f* be the function given.
Which of the following statements are true about *f*?

I. *f* has a limit at $x = 0$
II. *f* is continuous at $x = 0$
III. *f* is differentiable at x = 0

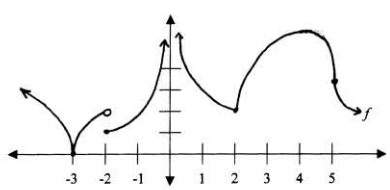
13) Let *f* be the function given.

$$f(x)=\begin{cases} \dfrac{x^2-9}{x-3}, x\ne 3 \\ 4, x=3 \end{cases}$$

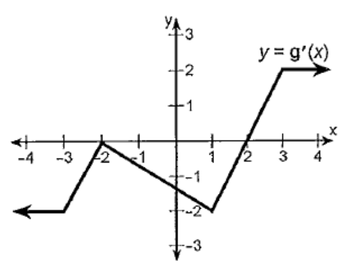
Which of the following statements are true about *f*?

I. $\lim_{x\to 3} f(x)$ exists
II. *f* is continuous at $x= 3$
III. *f* is differentiable at $x= 3$

14) The graph of the piecewise defined function *f* is shown in the given figure. The graph has a vertical tangent line at $x = 5$ and a horizontal tangent line at $x = 4$. What are all the values of *x*, at which *f* is continuous but not differentiable?

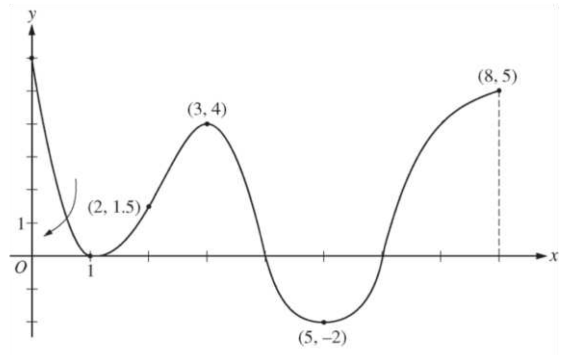


15) The graph of the derivative of the function g is shown below.
Which of the following is true about function *g*?



I. function *g* is increasing for x ≥ 2 only
II. function *g* has 4 points of non-differentiability
III. function *g* is not a continuous function
IV. *g* has a local maximum as x = 2

16) Let *f* be a function with a second derivative given by $f''(x) = x^2(x - 5)(x + 2)$.
What are the x-coordinates of the points of inflection of the graph of *f*?



Graph of f'

17) The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at x = 1, x = 3, and x = 5. The function f is defined for all real numbers.

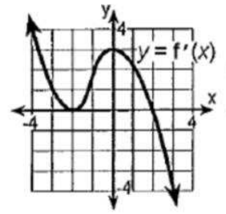
(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

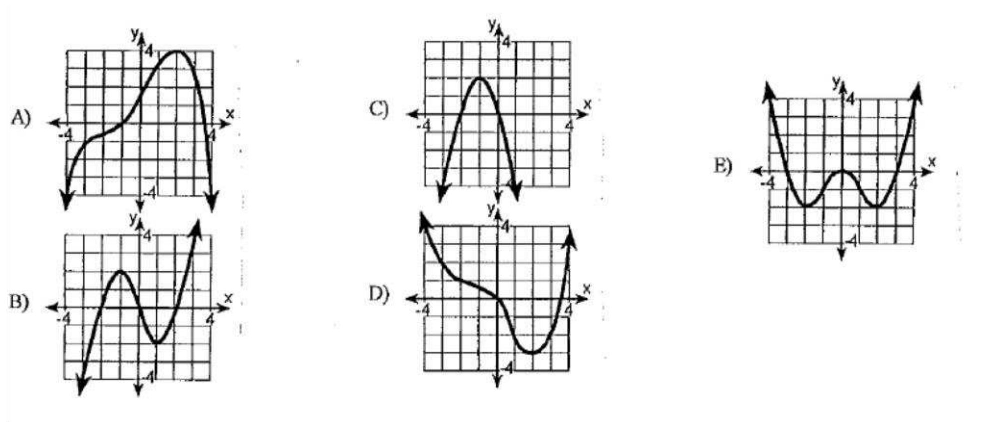
(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(5) = -\frac{3}{2}$, find the slope of the line tangent to the graph of g at x = 5.

18) The graph of the derivative of function f is shown.



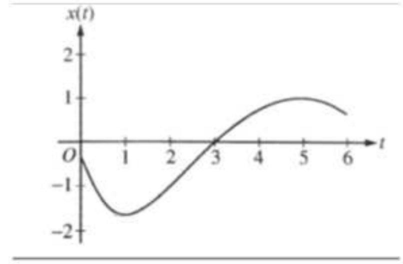
Which of the following could be a graph of function f?



19) The function f is twice differentiable with $f(1) = 3, f'(1) = 4$, and $f''(1) = 2$.
What is the value of the approximation of $f(0.8)$ using the tangent line to the graph of f at $x = 1$?

20) For the function f, $f'(x) = 4x + 2$ and $f(1) = 5$. What is the approximation for $f(1.3)$ found by using the line tangent to the graph of f at $x = 1$?

21) Let f be a differentiable function such that $f(5) = 9$, $f(7) = 5$, $f'(5) = -3$, and $f'(7) = -1$. The function g is differentiable and $g(x)$ is the inverse of $f(x)$ for all x. What is the value of $g'(5)$?



22) A particle moves along a straight line. The graph of the particle's position $x(t)$at time t is shown above for 0 < t < 6. The graph has horizontal tangents at t = 1 and t = 5 and a point of inflection at t= 2. For what values of t is the velocity of the particle decreasing?

23) Determine the slope of the line tangent to the following curves at the given value of x.

(a) $y = \arcsin(2x)$ at $x = \frac{1}{4}$

(b) $y = \arctan(x^2)$ at $x = \frac{1}{2}$

24) Consider the curve given by the equation $2y^3 = 4 - 3xy$. It can be shown that $\frac{dy}{dx} = \frac{-3y}{6y^2 + 3x}$.

(a) Write an equation for the line tangent to the curve at the point (-2, 2).

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where x = -2 and y =2.

25) What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3,2)?

26) A container has the shape of an open right circular cone. The height of the container is 16 in and the diameter of the opening is 16 in. Water in the container is evaporating so that its depth *h* is changing at the constant rate of $-\frac{5}{14}$ in/hr. (The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$)

(a) Find the volume V of water in the container when $h = 7$ in. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 7$ in. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

27) Integrate each of the following expressions.

a) $\int\left(\frac{1}{x^4}-\frac{2}{x^5}\right)dx$

b) $\int x^{\frac{1}{2}}(x-4)dx$

c) $\int(\sin(5x)+\cos(5x))dx$

d) $\int x^3\sqrt{5x^4+20}dx$

e) $\int\frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$

f) $\int\frac{x}{x^2+5}dx$

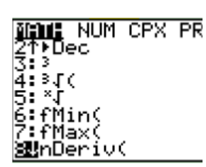
<u>Using a CALCULATOR on the AP EXAM</u>

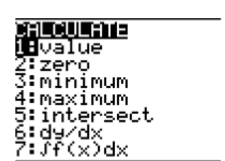
The AP CALCULUS Exams are written with the assumption that the test taker has access to a graphing calculator to use for the following:

1) Graph a function within a window
2) Find the roots of an equation (CALC ZERO)
3) Find the *<u>numerical value</u>* of a derivative at a point

   Math→ 8: nDeriv or CALC 6: dy/dx

   Fill into $\frac{d}{d\square}(\square)\Big|_{\square=\square}$ or nDeriv(expression, variable, value)

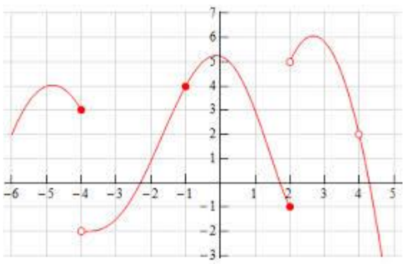




Example: Given $f(x)=e^x\cos(3x)$, find $f'(0.2)$ without finding $f'(x)$ first.

CALCULATOR QUESTIONS:

1) The graph of a function *f* is shown below.



a) If *f* is continuous at *b*, what is a possible value of *b*?

b) If $\lim_{x \to b} f(x)$ exists and *f* is not continuous at *b*, what is the value of *b*?

c) For what values of *b* does the $\lim_{x \to b} f(x)$ not exist?

2) Line l is tangent to the graph of $y = e^x$ at the point (k, e^k).

What is the negative value of *k* for which the *y*-intercept of l is $\frac{1}{4}$?

3) If *f* is a continuous function on the closed interval [a, b], which of the following must be true?

(A) There is a number c in the open interval (a,b) such that $f(c) = 0$
(B) There is a number c in the open interval (a,b) such that $f(a) < f(c) < f(b)$.
(C) There is a number c in the closed interval $[a,b]$ such that $f(c) \geq f(x)$ for all x in $[a,b]$.
(D) There is a number c in the open interval (a,b) such that $f'(c) = 0$
(E) There is a number c in the open interval (a,b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

4) The function *f* is continuous for $-3 \leq x \leq 3$ and $f(-3) = f(3) = 0$.

If there is no c, where $-3 < c < 3$, for which $f'(c) = 0$, then what must be true for $-3 < k < 3$?

5) For $f(x) = \cos^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of *f* is greater than the instantaneous rate of change of *g* for which value of *x*?

(A) 0.8 (B) 1.3 (C) 1.4 (D) 0 (E) -0.7

6) A man takes a cough medicine at 8 am, which starts to deplete as time goes on. For $0 \le t \le 24$, the amount *A(t)*, in milligrams, of the 10-milligram dose of the cough medicine remaining in the body after *t* hours is given by the formula $A(t) = 10.02(0.831)^t$.

(a) Find the average rate of change of *A(t)* over the interval $0 \le t \le 24$. Indicate units of measure.

(b) Find the value of *A' (3)*. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Another person takes a similar dose of another drug for a cold during the course of a day. The amount *B,* in milligrams, of this 10-milligram dose remaining in the body is modeled by $B(t) = -0.296\ln(0..31t)$, where *B(t)*is measured in milligrams and *t* is measured in hours. At the 3rd hour of the day, is the amount of the cough medicine or the cold medicine in the body depleting faster? Justify your answer.

(d) For *t* > 24, *L(t)*, the linear approximation to *A* at *t* = 24, is a better model for the amount of cough medicine remaining in the body. Use *L(t)*to predict the time at which there will be 0.2 milligrams of the cough medicine remaining in the body. Show the work that leads to your answer.

7). The function *f* is continuous on the closed interval [2, 4] and twice differentiable on the open interval $2 < x < 4$. If $f'(3) = 2$ and $f''(x) > 0$ on the open interval (2, 4), which of the following could be a table of values for *f*?

(A)

x	$f(x)$
2	3.5
3	5
4	7.5

(B)

x	$f(x)$
2	3
3	5
4	6.5

(C)

x	$f(x)$
2	3
3	5
4	7

(D)

x	$f(x)$
2	2.5
3	5
4	6.5

(E)

x	$f(x)$
2	2.5
3	5
4	7

8) Given $g(x) = 2x^4 - 6x^3 + 3$,

a) Determine the number of relative extrema for the function $g(x)$.

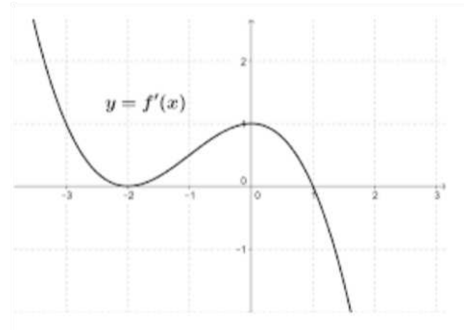
b) Determine the number of points of inflection for the graph of $g(x)$.

9) Let *f* be a function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of *x* does *f* attain its maximum value on the closed interval $0 \le x \le 2$?

10) The first derivative of the function *f* is given by $f'(x) = x - 4e^{-\sin 2x}$. How many points of inflection does the graph of *f* have on the interval $0 < x < 2\pi$?

11) Suppose the continuous function *f* is defined on the closed interval [0, 3] such that its derivative *f'* is defined by $f'(x) = e^x \sin(x^2) - 1$. On what interval(s) is *f* decreasing?

12) The graph of the derivative of function f is given on the right. The graph has horizontal tangent lines at x = -2 and x = 0. Find the values of x where f has a relative maximum. Justify your answer.



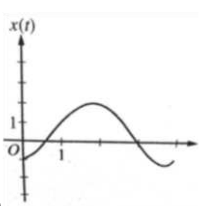
13) If $y = x^2 - 6x - 15$, what is the minimum value of the product xy?

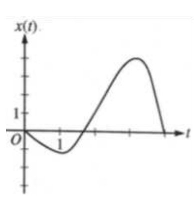
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

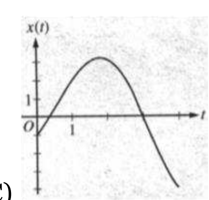
14) The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time t = 0, the particle is at the origin.

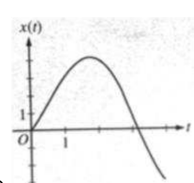
a) For $0 \leq t < 3$, determine between what two values of t does the particle change direction?

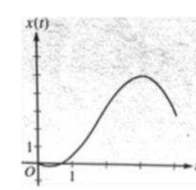
b) Which of the following could be the graph of the position, $x(t)$, of the particle?

(A) 

(B) 

(C) 

(D) 

(E) 

15) A particle moves along a straight line. For $0 \le t \le 6$, the velocity of the particle is given by $v(t) = -t^3 + (4t + t^2)^{8/7} + 5$, and the position of the particle is given by $s(t)$.

(a) Find all values of t in the interval $3 \le t \le 5$ for which the speed of the particle is 3.

(b) In what direction and how fast is the particle moving at $t = 3$ seconds?

(c) Find all times t in the interval $0 \le t \le 6$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 5$? Give a reason for your answer.

16) The position of a particle moving along a line is given by $v(t) = -2t^3 + 4t^2 + 3^{-t}$ for $t \ge 0$. For what values of t is the speed of the particle decreasing?

17) An object moves along the x – axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$. Consider the following statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

18) A particle moves along the x – axis so that at any time $t \geq 0$, its velocity is given by $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

19) The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(2) = 6$, and $f'(2) = 7$, what are the values of $g(6)$ and $g'(6)$?

20) The temperature of a cup of hot chocolate, in degrees Fahrenheit, is modeled by C, a differentiable function of the number of minutes after the hot chocolate is poured. Explain what $H'(6) = -4$ means in context.

21) The radius of a sphere is decreasing at a rate of 5 inches per second. At the instant when the radius of the sphere is 4 inches, what is the rate of change, in cubic inches per second, of the volume of the sphere? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)