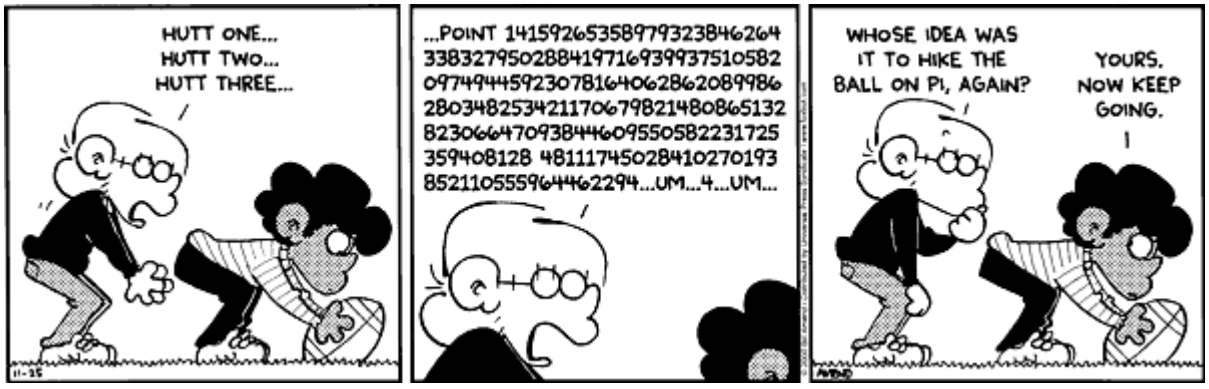
ACCELERATED MATHEMATICS

CHAPTER 12

CIRCLES AND 3D SHAPES

TOPICS COVERED:

- Circle vocabulary: radius, diameter, chord, tangent, secant
- Discovering pi
- Circumference of circles
- Area of circles
- Naming 3D shapes
- Faces, Edges, Vertices
- Nets
- Sketching solids
- Volume of prisms and cylinders



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Activity 11-1: **Accelerated Mathematics Formula Chart** Name:

Perimeter Rectangle $P = 2(l + w)$

Circumference Circle $C = 2\pi r$ or $C = \pi d$

Area

Rectangle or Parallelogram	$A = bh$	Circle	$A = \pi r^2$
Triangle	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$

Surface Area (8^{th} grade only)

	Lateral	**Total**
Prism	$S = Ph$	$S = Ph + 2B$
Pyramid	$S = \frac{1}{2}Pl$	$S = \frac{1}{2}Pl + B$
Cylinder	$S = 2\pi rh$	$S = 2\pi rh + 2\pi r^2$
Cone	$S = \pi rl$	$S = \pi rl + \pi r^2$

Volume

Triangular prism or Rectangular prism	$V = Bh$	Pyramid or Cone	$V = \frac{1}{3}Bh$
Cylinder	$V = \pi r^2 h$ or $V = Bh$	Sphere	$V = \frac{4}{3}\pi r^3$

(8^{th} grade only)

Pi $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$

Pythagorean Theorem $a^2 + b^2 = c^2$ (8^{th} grade only)

Customary – Length 1 mile = 1760 yards 1 yard = 3 feet 1 foot = 12 inches	***Metric – Length*** 1 kilometer = 1000 meters 1 meter = 100 centimeters 1 centimeter = 10 millimeters
Customary – Volume/Capacity 1 pint = 2 cups 1 cup = 8 fluid ounces 1 quart = 2 pints 1 gallon = 4 quarts	***Metric – Volume/Capacity*** 1 liter = 1000 milliliters
Customary – Mass/Weight 1 ton = 2,000 pounds 1 pound = 16 ounces	***Metric – Mass/Weight*** 1 kilogram = 1000 grams 1 gram = 1000 milligrams
Time 1 year = 12 months 1 year = 52 weeks 1 week = 7 days 1 day = 24 hours 1 hour = 60 minutes 1 minute = 60 seconds	

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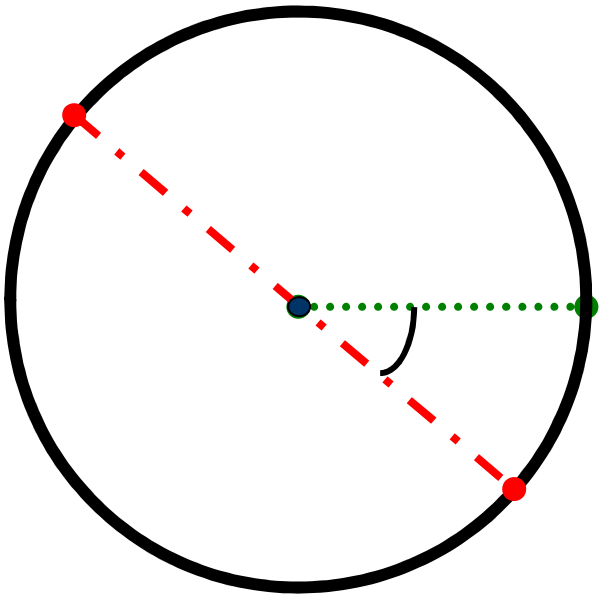
Activity 12-1: **Circle Vocabulary** Name:



Diameter: a segment that passes through the center of the circle and has both endpoints on the circle

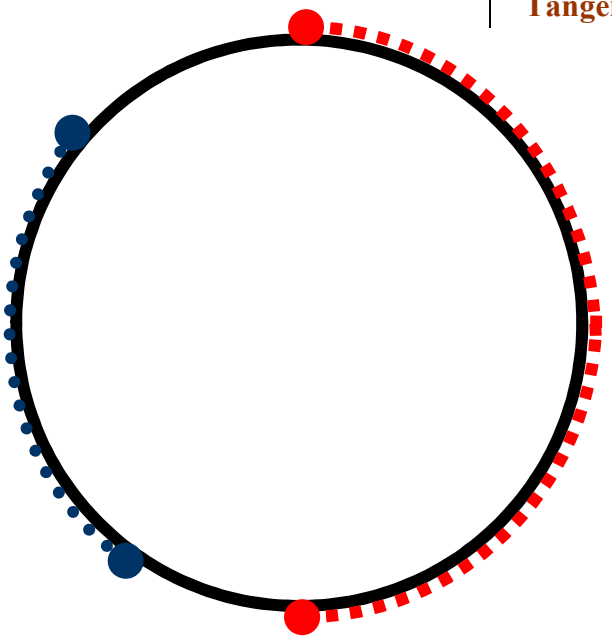
Radius: a segment that has one endpoint at the center of the circle and the other endpoint on the circle

Central angle: an angle whose vertex is the center of the circle

Center: the middle point of a circle

Semicircle: half of a circle

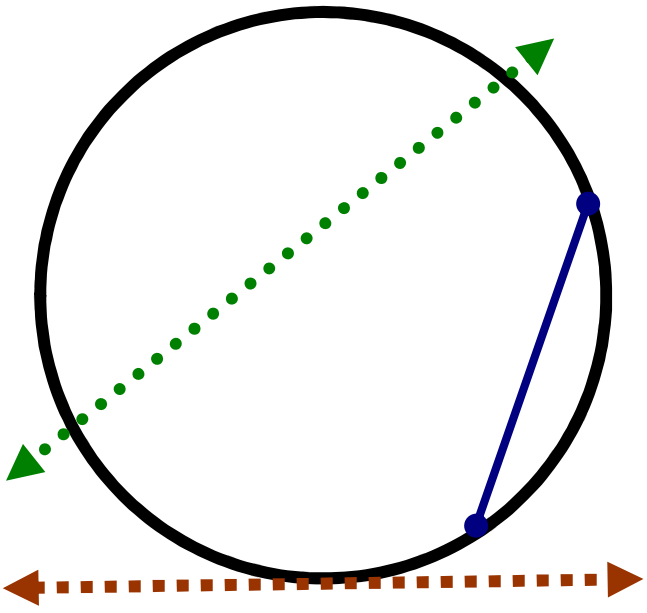
Arc: part of a circle



Chord: a segment that has both endpoints on the circle

Secant: a line that intersects the circle in exactly two points

Tangent: a line that intersects the circle in exactly one point



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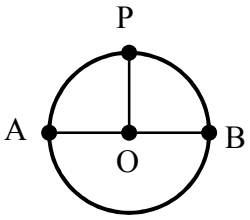
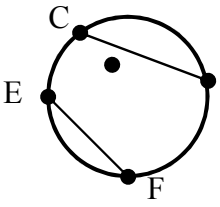
Activity 12-2: **Circles** Name:

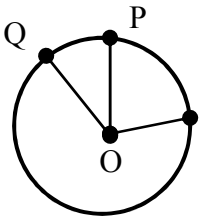
Symbols			
Angle BAC	$\angle BAC$	Ray AB	$\overrightarrow{AB}$
Line Segment AB	$\overline{AB}$	Arc AB	$\overset{\frown}{AB}$
Line AB	$\overleftrightarrow{AB}$		



Point O is the center of the circle.
$\overline{AB}$ is the diameter.
$\overline{OA}$ is the radius. $\overline{OP}$ is also a radius.



CD and EF are chords.
A diameter of a circle is the longest chord.

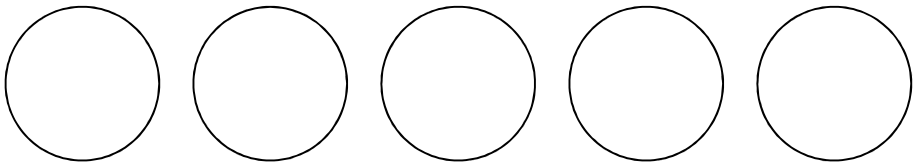


$\angle AOP$ and $\angle AOQ$ are central angles.

The diameter is twice the length of the radius.	The radius is half the diameter.
$d = 2r$	$r = \frac{d}{2}$

Draw and label the following parts (one per circle).

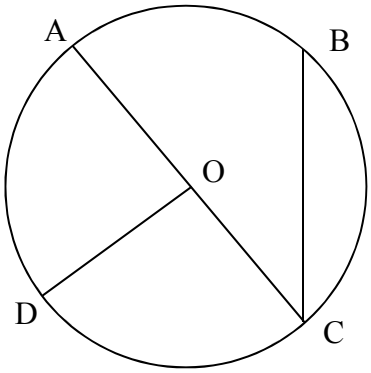
Chord $\overline{QR}$
Diameter $\overline{KL}$
Center O
Radius $\overline{OT}$
Central angle $\angle WOK$



Find the unknown length of each circle.

	Radius	**Diameter**		**Radius**	**Diameter**
6.	8 cm.		7.		110 in.
8.		48 ft.	9.	23 mm.	
10.		55 ft.	11.	18 in.	
12.	$6\frac{5}{8}$ yd.		13.		$7\frac{3}{4}$ in.
14.	12.3 mi.		15.		16.4 m.
16.	3.3 cm.		17.		1.25 yd.
18.		2.6 m.	19.	9.1 in.	

How can I remember the difference between diameter and radius?
Radius comes from the same root word as "ray". A ray of sunshine starts at the center of the sun and goes out from that point. The radius of a circle starts at the center of the circle!



Solve the following problems using the circle above.

1.	The points on a circle are all the same distance from the...	
2.	A line segment from the center to any point on the circle is a…	
3.	A line segment with both points on the circle is a…	
4.	A chord the passes through the center of a circle is a…	
5.	A diameter of the circle in the drawing above is the segment…	
6.	Which of the following is not a radius: $\overline{OA}, \overline{OD},$ or $\overline{BC}$?	
7.	Which of the following is not a chord: $\overline{BC}, \overline{OA},$ or $\overline{AC}$?	
8.	Part of a circle, such as between points B and C, is an…	
9.	An angle whose vertex is at the center of a circle is a…	
10.	Which of the following is not a central angle: $\angle AOD, \angle COD,$ or $\angle BCA$?	
11.	Points A, B, C, and D are all the same _____ from point O.	
12.	If the length of $\overline{AC}$ is 20 cm, then the length of $\overline{OC}$ is…	
13.	If the length of $\overline{OA}$ is 20 cm, then the length of $\overline{OD}$ is…	
14.	If the length of $\overline{OD}$ is 20 cm, then the length of $\overline{AC}$ is…	
15.	The length of a radius is _____ the length of a diameter.	
16.	The set of points in a plane at a fixed distance from a given point is a…	

Activity 12-4: **Measuring Circles** Name:

Measure at least 6 of the objects provided by your teacher. For each object record the information below. ***You may use a calculator as needed.***

Item Name	**Diameter** (measure in mm)	**Circumference** (measure in mm)	**Radius** (calculate in mm)	$\frac{Circumference}{Diameter}$ (round to the nearest ten-thousandth)
******* MEAN *******				
******* MEDIAN *******				
******* RANGE *******				
******* CLASS MEAN *******				
******* CLASS MEDIAN *******				

- Pi is the number of times a circle's diameter will fit around its circumference.

- Here is pi to 64 places:
  3.1415926535897932384626433832795028841971693993751058209749445923

- Pi occurs in hundreds of equations in many sciences including describing DNA, a rainbow, ripples where a raindrop fell into water, distribution of prime numbers, geometry problems, waves, navigation, etc.

- Half the circumference of a circle with radius 1 is exactly Pi. The area inside that circle is also exactly Pi!

- Taking the first 6,000,000,000 decimal places of Pi, this is the distribution:

0 occurs 599,963,005 times	1 occurs 600,033,260 times
2 occurs 599,999,169 times	3 occurs 600,000,243 times
4 occurs 599,957,439 times	5 occurs 600,017,176 times
6 occurs 600,016,588 times	7 occurs 600,009,044 times
8 occurs 599,987,038 times	9 occurs 600,017,038 times

- Pi is irrational. An irrational number is a number that cannot be expressed in the form (a / b) where a and b are integers.

- The Babylonians found the first known value for Pi in around 2000BC; they used (25/8) The Egyptians used Pi = 3 but improved this to (22 / 7). They also used (256/81).

- In around 200 BC Archimedes found that Pi was between (223 / 71) and (22 / 7). His error was no more than 0.008227%. He did this by approximating a circle as a 96 sided polygon.

- Ludolph Van Ceulen (1540 - 1610) spent most of his life working out Pi to 35 decimal places. Pi is sometimes known as Ludolph's Constant.

- The first person to use the Greek letter was Welshman William Jones in 1706. He used it as an abbreviation for the 'periphery' of a circle with unit diameter.

- The Pi memory champion is Hiroyoki Gotu (21 years old) who memorized an amazing 42,000 digits.

- Pi was calculated to 2,260,321,363 decimal places in 1991 by the Chudnovsky brothers in New York.

- Most people would say that a circle has no corners - but it is more accurate to say that it has an infinite number of corners.

- At position 762 there are six nines in a row. This is known as the Feynman Point.

Activity 12-6: **Circumference and Area** Name:

Circum means around or bend around. *Ferre* means to bring or carry. The circumference of a circle is the measure you need to get around the entire circle. Circumnavigate means to sail around the globe.

$$\pi \approx 3.14 \approx \frac{22}{7}$$

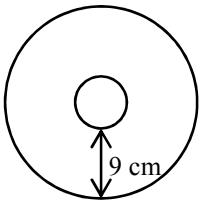
Important formulas for circles:

Radius to diameter: $r = \frac{d}{2}$	Diameter to radius: $d = 2r$
Circumference: $C = 2\pi r$ or $C = \pi d$	Area: $A = \pi r^2$

Find the circumference and area of each circle given the diameter or radius. Use 3.14 or $\frac{22}{7}$ for π.

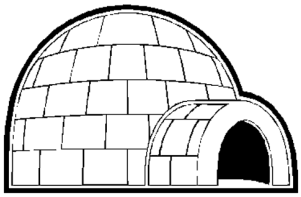
Show all work on separate paper including three steps for each problem: write the correct formula, fill in the numbers for the variables, and then solve the equation.

	Radius	**Circumference**	**Area**		**Diameter**	**Circumference**	**Area**
1.	3 in			2.	9 yd		
3.	$\frac{1}{2}$ yd			4.	1.2 ft		
5.	6 mi			6.	24 yd		
7.	55 m			8.			400π ft
9.	$\frac{3}{4}$ cm			10.		20π cm	
11.	7.1 cm			12.	50 in		
13.	$2\frac{1}{2}$ cm			14.	16 in		
15.		60π m		16.	32 in		
17.	x cm			18.	y in		

19-20.		Write an expression to find the circumference of the outside of the donut. The radius of the inner circle is 2 cm. Do not solve.	
		Write an expression to find the area of the donut. The radius of the inner circle is 2 cm. Do not solve.	
21.	If a 14 inch pizza has a 1 inch crust, what is the area of the crust? To the nearest percent, what percent of the pizza is crust?		

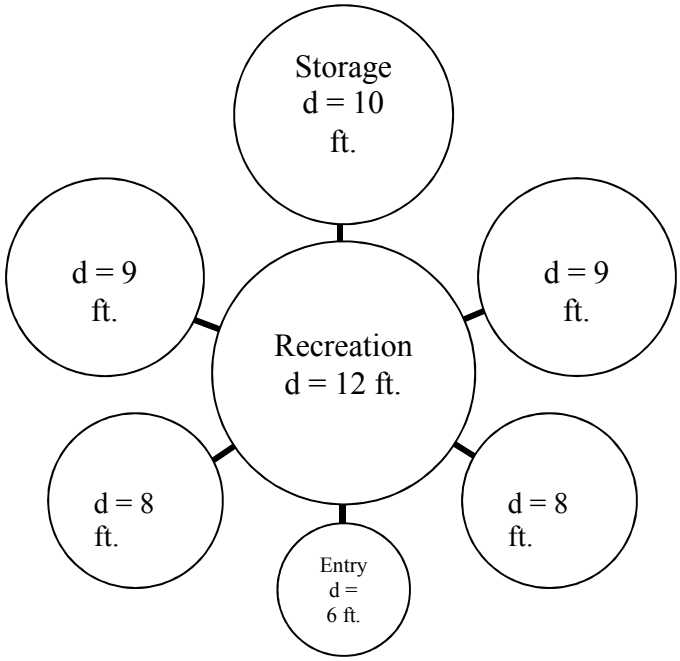
Activity 12-7: **Inuit Architecture** Name:

The Inuit are Native Americans who live primarily in the arctic regions of Alaska and Canada. The Inuit word *iglu* means "winter house". Later the term came to mean a domed structure built of snow blocks, as shown in the figure at the right.



Several families would build a cluster of iglus that were connected by passageways and shared storage and recreation chambers. Use the drawing to answer the following questions. Use 3.14 for pi.

1.	List the radius of each of the chambers.
2.	Determine the circumference of each chamber.
3.	Determine the area of each chamber. Then calculate the total area of all chambers.
4.	Estimate the distance for the front of the entry chamber to the back of the storage chamber.
5.	If Mr. Mangham runs around the storage iglu 5 times, how far will he run?



6. A person wishes to place a covering on the floor in the entry, recreation, and storage rooms. If coverings are sold in 40 square feet pieces, how many pieces will she need?

7. If the radius of one of the iglus is tripled, then the area of the iglu is multiplied by….

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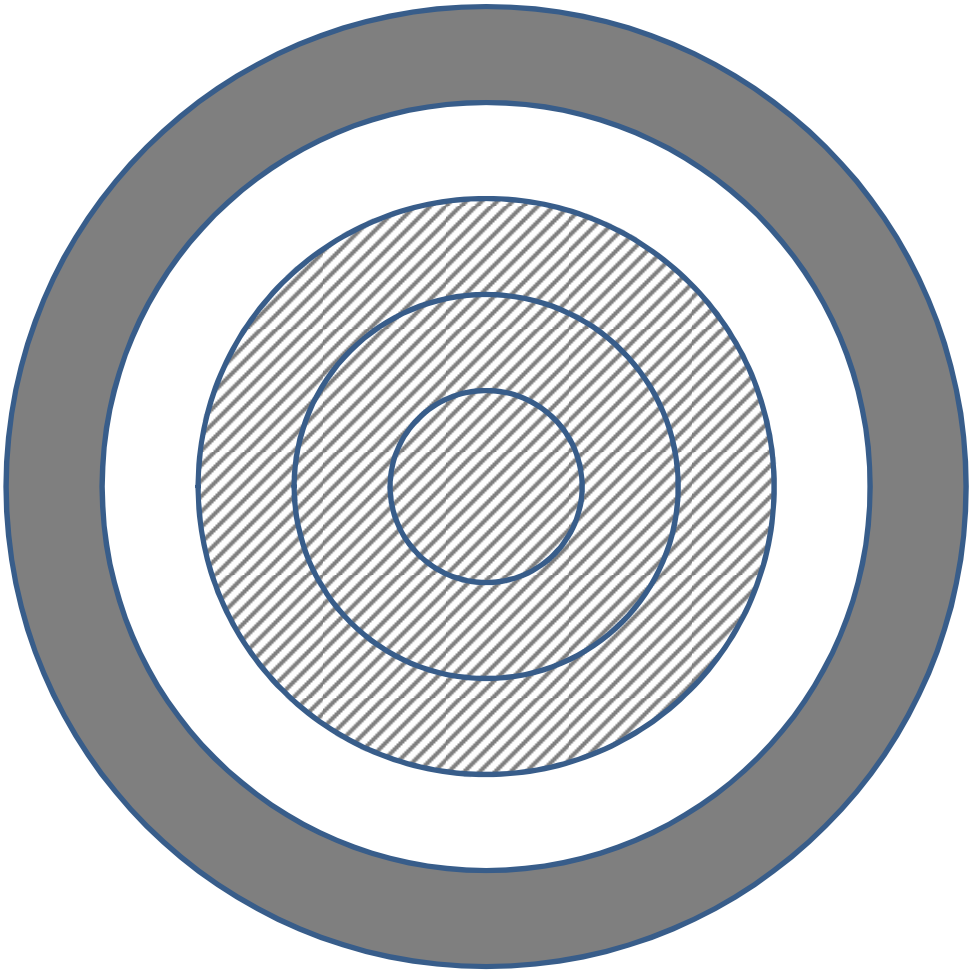
1. The radius of the inner circle is one inch. Each successive circle has a radius increase of one inch, so the outside circle has a radius of five inches (note that they are not drawn in actual size).

Which has a greater area – the shaded region or the striped region?



2. Let's determine your hat size! Have a friend measure around your head at approximately the place a hat would rest. Measure to the nearest one-eighth of an inch. Divide this by the fraction form of pi, round up to the nearest eighth, and you have your hat size!

Activity 12-9: **The Margaret Hunt Hill Bridge** Name:

The Margaret Hunt Bridge was completed near downtown Dallas in March 2012, almost five years after construction began. The bridge cost $182 million to build and it made out of 11,643,674 pounds of structural steel. It is a cable stayed bridge, not a suspension bridge.

The bridge cables are two different sizes. There are 44 large cables that have a diameter of 6.3 inches and are made up of 31 individual strands of wire. The 14 smaller cables have a diameter of 4.9 inches and are made up of 12 individual strands of wire.

1. What is the circumference of each of the large cables?
2. What is the area of a cross-section of one of the large cables?
3. What is the circumference of each of the smaller cables?
4. What is the area of a cross-section of one of the smaller cables?
5. Assuming the strands of wire inside the cables take up 90% of the area, what is your best guess at the diameter and circumference of a single strand of wire?

Activity 12-10: **Diameter, Circumference, and Area** Name:

You may use a calculator for this activity. Do not use the pi button – use 3.14 for pi.
For all problems involving an equation (circumference, area, etc.) show all steps.
For all problems asking "how many turns" your answer should be a whole number.

Fresh snow has just fallen and Ashley decides to make a snowman.
Ashley makes a snowball 6 inches in diameter. She then rolls the snowball one full turn.

1.	Draw a picture of the original snowball and draw and label the diameter.	
2.	How far does Ashley roll the snowball? **Show all equations and all steps in the box at the right.**	
3.	Ashley finds that 3 inches of new snow sticks as she rolls the snowball (3 inches added to every part on the original circle). Draw a picture of the new snowball.	
4.	What is the new diameter of the snowball after Ashley rolls it one full turn? (Hint: It is not 9 inches.)	
5.	How many total turns will Ashley have to make so that the diameter of the snowball becomes 72 inches? (Hint: The first 6 inches did not require any turns.)	
6.	How many total turns will it take for the circumference to become greater than 300 inches? (Hint: Start with the circumference formula to find *d*.) **Show your original equation and all steps in the box at the right.**	

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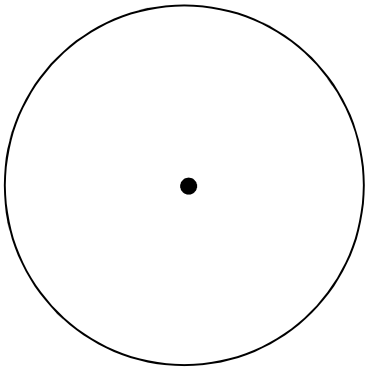
7.	How many total turns will it take for the area to become greater than 1000 in.^{2}? (Hint: Start with the area formula to find *r*.) **Show all equations and all steps in the box at the right.**	

8.	Ashley makes a second snowball and then rolls it one full turn. Again 3 inches of snow stick to it. She measures the new circumference (C) to be about 44 inches. To the nearest inch, what is the diameter of Ashley's rolled snowball? **Show all equations and all steps in the box at the right.**	
9.	Using the diameter from #8, what is the area of the rolled snowball? **Show all equations and all steps in the box at the right.**	
10.	What was the diameter of the snowball before Ashley rolled it?	
11.	What is the area of the snow that stuck to the original snowball in #8? **Show all equations and all steps in the box at the right.**	

12.	Ashley rolls a huge snowball 4 feet in diameter. She decides to make another snowball $\frac{2}{3}$ of its diameter to put on top. In **inches**, what is the diameter of the snowball Ashley wants to put on the top?	
13.	In square inches, what is the area of both snowballs combined? **Show all equations and all steps in the box at the right.**	
14.	Ashley started with a snowball 8 inches in diameter. How many complete turns will Ashley need to roll it before she has a snowball the size she wants to put on top?	

Activity 12-11: **More Circles** Name:

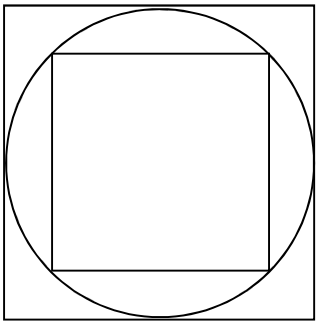
1. Draw a circle similar to the one below with a dot in the middle without lifting your pencil from the paper. Can it be done?



2. A pizza parlor has 3 size pizzas. Would you get more if you had one small pizza, three-fifths of a medium pizza, or one-third of a large pizza?

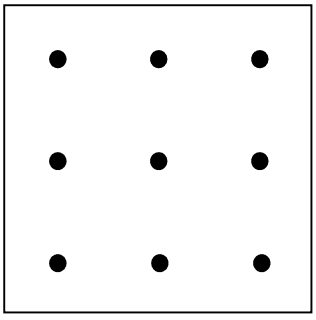
SIZE	DIAMETER
Small	10 inch diameter
Medium	14 inch diameter
Large	18 inch diameter

3. In the picture below, the circle's area is what percent of the larger squares area? If the larger square has an area of 10 square inches, what is the area of the smaller square?



4. Queen Dido's Cutting Trick: Take a 3x5 index card. Fold the card in half hamburger style. Start your first cut on the fold just barely in from the side. Cut almost all the way up to the top. Next cut from the top almost all the way back to the bottom. Continue until you end up with an odd number of cuts. Unfold the card and cut all the folds except for the ones at the end of the card. You will have a circle big enough to step through!

5. Draw two squares so that each dot is completely contained in a region by itself.



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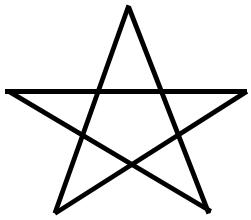
Activity 12-12: **3D Shapes/Volume/Surface Area** Name:

Math Book

Chapter 10.3 (pg. 534): Three-dimensional Figures Chapter 10.3 (pg. 539): Sketching Solids
Volume model: Centimeter Cubes
Chapter 10.6 (pg. 554): Volume of Prisms and Cylinders

Lateral face: A face that joins the bases of a solid. It is any edge or face that is not part of the base.

1.	A shipping company sells two types of cartons that are shaped like rectangular prisms. The larger carton has a volume of 720 cubic inches. The smaller carton has dimensions that are half the size of the larger carton. What is the volume, in cubic inches, of the smaller carton?	
2.	An ice cream carton has a volume of 64 fluid ounces. A second ice cream carton has dimensions that are three-fourths the size of the larger carton. What is the volume of the smaller carton?	
3.	How many 2 by 2 by 2 inch cubes will fit into a 4 by 8 by 12 inch box?	
4.	For a regular pentagonal prism, what is the ratio of the number of vertices to the number of edges? 2:3 3:2 3:5 5:3 (43% of all 11th graders answered this question correctly on TAKS.)	
5.	Name the solid that has 2 bases that are each 5 sided shapes and the vertices of each base are joined together forming 5 edges.	
6.	Identify the three-dimensional figure that can be formed by this net: 	
7.	You have two cubes. The smaller cube has dimensions that are x long and the larger cube has dimensions that are 4x long. If the smaller cube has a volume of 64 cubic feet, find the volume of the larger cube.	
8.	Identify the following solids: A. I have six flat faces. I have twelve edges. I have eight vertices. B. I have five flat faces. I have nine edges. I have six vertices. C. I have two flat faces. I can roll. D. I have five flat faces. I have eight edges. I have five vertices.	

Activity 12-13: **Volume and Capacity** Name:

Taken from TexTEAMS Rethinking Elementary Math, Part II

- If the first cube is 1 unit on an edge, it takes 1 unit cube to build it.
- The next larger cube is 2 units on an edge. It takes 8 units cubes to build it. Notice that there is one cube hidden by the other cubes no matter how hard you look for it.
- Continuing on with the next cubes, record the number of unit cubes it takes to build each and the number of hidden cubes.

Number of cubes on each edge	Total number of cubes	Hidden Cubes
1	1	0
2	8	1
3		
4		
5		
6		

1. If you know the number of units on the edge of a cube, explain how you could find out the number of cubes it would take to build.
2. If you know the number of unit cubes it takes to build a larger cube, can you figure out the units on an edge?
3. Do you notice any pattern about the hidden number of cubes? If so, describe it.

- Use the cubes to build a variety of rectangular prisms.
- For each one record the dimensions of your prism and the number of cubes it took to build it.

Dimensions of prism	Total number of cubes
2 by 3 by 4	12

4. Describe the patterns you see in your data.
5. Can you figure out the dimensions of your prism if you know the number of cubes it took to build it?
6. Can you figure out how many cubes it took to build your prism if you know its dimensions?

Activity 12-14: **The Perfectionism Scale** Name:

(Taken from The Perfectionism Scale, David D. Burns, M.D.)

Read each statement, then rate each one according to whether you:

+2 strongly agree

+1 agree

0 can't decide

-1 disagree

-2 strongly disagree

Answer with the first thought that comes to mind and answer honestly. No one else will see your answers unless you decide to share them.

	If I don't set high standards for myself, I am likely to end up a second-rate person.
	People will probably think less of me if I make a mistake.
	If I cannot do something really well, there is little point in doing it at all.
	I should be upset if I make a mistake.
	If I try hard enough, I should be able to excel at anything I attempt.
	It is shameful for me to display weakness or foolish behavior.
	I shouldn't have to repeat the same mistake many times.
	An average performance is bound to be unsatisfying to me.
	Failing at something important means I'm less of a person.
	If I get mad at myself for failing to live up to my expectations, it would help me to do better in the future.
	TOTAL

TOTALS

+15 to +20 – You're too good to be true. Maybe you're exaggerating your own capabilities and skills. And maybe you're used to exaggerating them because people have always expected you to be perfect.
+10 to +14 – You're too good for your own good. You're trying too hard – and it's time to ask why.
+5 to +9 – You're a borderline perfectionist. Certain events in your life may push you over the line into full-fledged perfectionism, but you usually manage to roll with the punches without going to extremes.
+1 to +4 – You're a healthy pursuer of excellence. You enjoy doing well, but you can turn your pursuit of excellence on and off at will. You probably spread your talents and abilities into several areas of life: academics, friendships, you health and appearance, hobbies, and play.
0 to -5 – You're used to hanging loose. Maybe you've made a conscious effort to be less perfectionist, or maybe you were born knowing how to relax and take it easy.
-6 to -10 – You're a little TOO relaxed. You favorite song is "Que sera, sera" (What will be, will be) and your favorite activity is lying in a hammock feeling the earth turn. A slight exaggeration, perhaps, but there is such a thing as overdoing underdoing.
-11 to -20 – You're barely breathing. Maybe you're exaggerating your own coolness. Read through the statements again, and this time respond to them honestly. You can't be apathetic about everything.

Tips for helping people who always have to be a perfectionist

1. Be average for a day. Allow yourself to be messy, late, incomplete, and imperfect.
2. Get involved in activities that are not graded or judged – activities that focus on process, not product.
3. Take a risk. Sign up for a course with a reputation for being challenging. Start a conversation with someone you don't know. Do an assignment or study without overdoing it.
4. Give yourself permission to make at least three mistakes a day.
5. Stop using the word "should" in your self-talk. Remove "I have to…"

Tips for helping people who always procrastinate

1. Allow more time that you think a project will take. For example, if you think a writing assignment will take 2 hours, give yourself three hours to do it.
2. Break down big and intimidating projects into smaller, more doable ones.
3. Reward yourself after each accomplishment, large or small.
4. Begin your day with your most difficult task, or the one you enjoy least. The rest of the day will seem easy by comparison.
5. Remove distractions from your workplace. Keep food, TV, magazines, games, and other temptations out of your way.