NUMB3RS Activity: Stick or Switch?
Episode: "Man Hunt"

Topic: The "Monty Hall" or Three Door Problem **Grade Level:** 8 - 12
Objective: to simulate a very counterintuitive probability problem
Materials: a set of three index cards for each pair of students, a TI-83 Plus/TI-84 Plus graphing calculator for each student with the Prob Sim App
Time: 30 minutes

Introduction
In this episode, Charlie teaches a class called "Math for Non-mathematicians." In it, he suggests that in mathematics, instinct is not always to be trusted. To illustrate his point, he recreates what has become a classic probability problem, the "Monty Hall" problem. It is named after the host of a game show called "Let's Make a Deal," which was extremely popular several years ago. The problem itself is a variation of one of the games played on the show. In this activity, students will have the opportunity to test their intuition regarding its solution, and then simulate playing the game many times to collect data on long-term results. In the episode, Charlie gives a very brief summary of the solution, but more detailed investigation will aid in understanding it.

Discuss with Students
The Monty Hall problem is actually a variation of the final game played on each show. A contestant is shown three doors – behind two of the doors is a goat, and behind the third is a new car. The contestant is then asked to select one of the three doors. The host then opens one of the other two doors, revealing a goat. At this point, the contestant can decide whether to stick with original choice, or switch to the other door. Keep in mind that only the host knows which door hides the car. After deciding, the chosen door is opened.

The reason that this problem is a variation of the game show is that in reality, the host was not required to offer the choice. According to the New York Times (see a link to the article is in the "Extensions"), if the contestant picked a goat, the door was opened and the game was over. In other cases, once the contestant opted to stick or switch, the host would offer them money to change their mind, in order to build excitement and encourage the contestant to ultimately end up with a goat.

When students do the simulation, combine all of the data into a single table like the one that accompanies Activity 1. This should make it more evident that switching wins about twice as often as sticking.

Student Page Answers:
1. *On the show, and in most cases, the answer will be "it doesn't matter," with the reasoning that since there are now two doors and you do not know what is behind either of them, each one has a probability of 1/2 of hiding the car. For any student who has never seen this before and says "switch" see how closely the reasoning follows one of the two approaches in the rest of the activity.* **2.** *Answers will vary, but the contestant should probably win in about 1/3 of the trials.*
3. *Again, answers will vary, but the contestant should probably win in about 2/3 of the trials.*
4. *A summary should probably indicate that the contestant wins about twice as often when switching.* **5.** *1/3* **6.** *2/3; This can be rather subtle. If students have difficulty, proceed to #7 to see the results graphically.*

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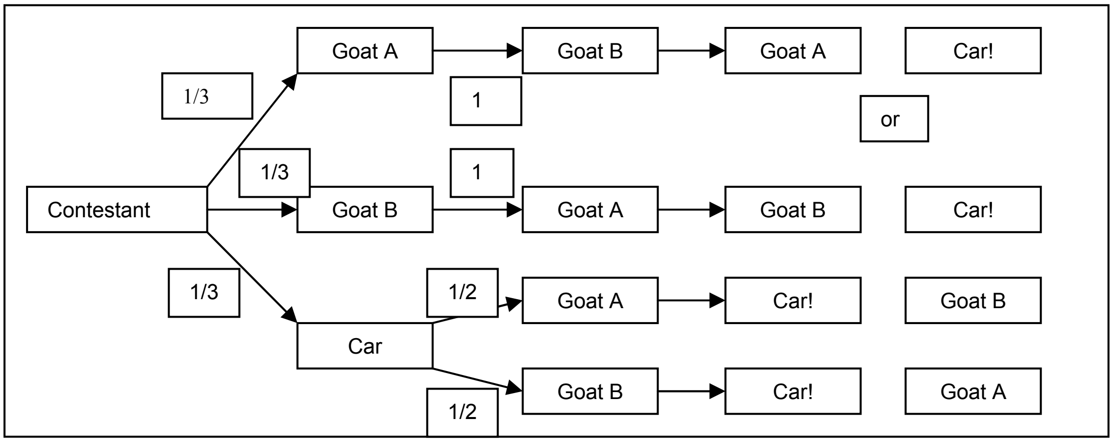
7. You pick: Host shows: Stick: Switch:



Summary of results:

	Goat A	***Goat B***	***Car***
Stick	*1/3*	*1/3*	*1/3*
Switch	*1/6*	*1/6*	*2/3*

Answer to "Extensions" question #5:
Stick, don't switch. The only way to win by switching is if both contestants pick a goat. There are only three possible options (each with probability 1/3):
1. Contestant A picks the car. The host eliminates contestant B, and A can only win by sticking.
2. Contestant B picks the car. The host eliminates contestant A, and B can only win by sticking.
3. Both contestants pick goats. The host eliminates one of them randomly and the remaining contestant can only win by switching.

So in this case, sticking wins 2/3 of the time, and switching only 1/3 of the time.

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Name: ________________________________ Date: _______________

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Suppose you are the contestant. On the stage are three large curtains (called "doors" from now on). Behind one of the doors is a new car; the other two doors each hide a goat. You are asked to select a door. The host then opens one of the doors you did not select, revealing a goat. You are asked whether you want to stick with your original choice or switch to the other door. After you decide, your chosen door is opened, and you win whatever is behind it.

1. How would *you* answer Charlie Eppes' question: "Should you switch doors, or does it not matter?" Explain your reasoning.

To simulate this game, work with a partner. Decide who wants to be the "contestant" and who wants to be the "host" (switch roles for the second activity.)

Host: Mark one of the three index cards to be the car, and the other two to be goats. On your calculator, you will use the Prob Sim App to toss a coin. Set up your calculator by pressing [APPS], scroll down to "Prob Sim," and press [ENTER]. Press any key to start the simulation. Select **1.Toss Coins**. From now on, each time you press [ENTER], a coin will be tossed and show the result "H" or "T."

Contestant: Start the Prob Sim App the same way, but scroll down to select **6.Random Numbers**. Press [ENTER]. Press the [ZOOM] key for settings. For "Numbers," select 1, for "Range," enter 1 – 3, and for "Repeat" select "Yes." Press the [GRAPH] key to select "OK." From now on, each time you press [ENTER], 1, 2, or 3 will show in the box on the screen.

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Activity 1

In this round, the contestant will always "stick."
To play the game:

- **a.** The host places the cards face down on the desk and lined up randomly, so the contestant does not know what is behind the "doors."
- **b.** The contestant uses the calculator to randomly select a door.
- **c.** (This one is critical!) The host mentally numbers the other two doors 1 and 2, and then uses the calculator to select the one to reveal. If the contestant selected one with a goat, do the coin toss anyway, then reveal the other goat. The host cannot reveal to the contestant if the coin toss was actually used or not.
- **d.** Tally the results in the table above. Repeat a total of 30 times, and total the results.

	Win (Car)	**Lose (Goat)**
Stick		
Switch		

2. Using the "stick" strategy, how many times does the contestant win and lose?

Activity 2

The host and contestant should switch roles. Repeat Activity 1 a total of 30 times, but this time the contestant should *switch doors* every time. Record your results on the same table.

3. Using the "switch" strategy, how many times does the contestant win and lose?

4. Combine your results for both activities with the rest of the class, so you can compare them to your answer to question #1.

Here's one way to determine the theoretical probability of winning:

5. When you first select a door, what is the probability that you will select the car?

This means that the probability is $\frac{2}{3}$ that the car is behind one of the other two doors. When the host shows you the goat behind one of the doors you did not select, the probability of that door hiding the car is now 0.

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6. Based on your answer to question #5, and the fact that the door you just saw has a probability of 0 to contain the car, what does that make the probability that the car is behind the other door (the one you would get if you switch)?

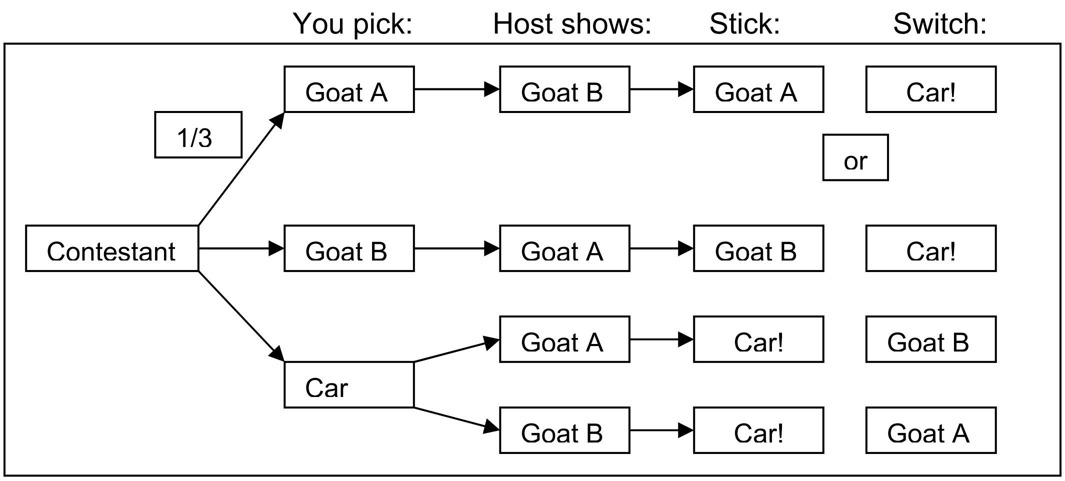
Here is a tree diagram for the whole game. Label the arrows as follows (one is done):

a. The probability of selecting each of the three doors is $\frac{1}{3}$.

b. If you select a goat, the host *must* show you the other goat (probability is 1).

c. If you select the car, the host randomly selects which goat to show you $\left(\frac{1}{2}\right)$.

d. Multiply the probabilities along each branch of the tree to get the final results.



7. Summarize the results in this table:

	Goat A	Goat B	Car
Stick			
Switch			

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

The Monty Hall problem is also called the "Three Door" problem. It has been studied with many variations and under many names since the 1880's. A major controversy erupted in 1990 in a column in *Parade* magazine called "Ask Marilyn." A summary of the events is included in the New York Times article listed below.

For the Student

For those interested in exploring this problem further:

1. What changes if there are four doors, and you get to see what is behind two of them, but you are asked to stick or switch after each one is opened?

2. What if there are four doors and you only get to see what's behind one of them?

3. What happens if there are n doors and you get to see what's behind (n – 1) of them, again given that you are asked to stick or switch after each one is opened?

4. What happens if there are n doors and you only get to see behind k of them, sticking or switching each time?

5. Suppose there are two contestants. If you both pick goats, the host eliminates one of you randomly. If one of you picks the car, the other is eliminated (but as always, only the host knows where the car is). If you are not eliminated and you are shown the goat behind the other contestant's door, do you stick or switch? (Your teacher has the answer to this one.)

Additional Resources

- For an excellent computer simulation of this problem, that includes the option of using more than three doors, see:
  http://www.shodor.org/interactivate/activities/montynew/index.html

- To read the article that appeared in the July 21, 1991 New York Times, see:
  http://www.mathe.tu-freiberg.de/~ernst/Lehre/AD/Monty-NYTimes.pdf

- For another activity that uses probability trees, see the *NUMB3RS* activity from Season 2, called, "How Reliable is the Test?" from the episode "Calculated Risk." To download this activity, go to **http://education.ti.com/exchange** and search for "6017."

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