

Math 3201 Notes

Chapter 8: Sinusoidal Functions

Learning Goals: See p. 481 text.

§8.1 Understanding Angles (1 class)

Read *GOAL* p. 484 text.

Outcomes:

1. Demonstrate an understanding of angles expressed in degrees and radians. pp.484-489
 2. Define and illustrate the **unit circle**. See notes
 3. Convert an angle measure from degree to **radian** measure. p. 486
 4. Convert an angle measure from radian to degree measure. p. 487
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Measuring Angles

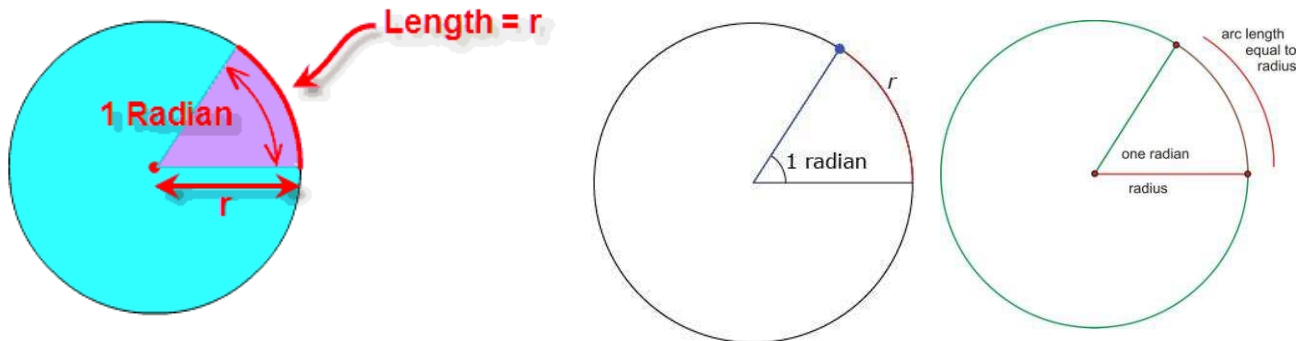
We often measure quantities using different units.

E.g.: We can measure distance in meters (m), kilometers (km), inches (in.), yards (yds.), or miles (mi.)

E.g.: We can measure temperature in Celsius ($^{\circ}C$) or Fahrenheit ($^{\circ}F$)

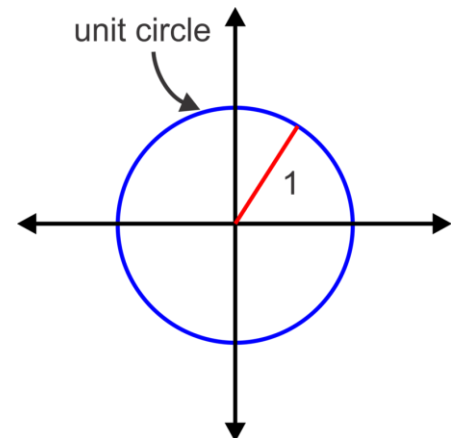
Up to now, we have always measured angles in degrees. However, we could measure angles in **radians**.

Defⁿ: One **radian** is the measure of the angle made when we take the radius of the circle and wrap it in an arc along the circumference of the circle.



The measure of the purple angle above is 1 radian or just 1.

Defⁿ: The **unit circle** is the circle with radius 1.



Let's consider the unit circle ($r = 1$).

1. What is the formula for the circumference of any circle? _____
2. What is the circumference of the unit circle? _____
3. How many degrees are there in the unit circle? _____
4. Why must the circumference of the unit circle and the number of degrees in the unit circle be equal?

5. State the equation that shows that the circumference of the unit circle and the number of degrees in the unit circle are equal. _____
6. Divide both sides of the equation by 2π to determine how many degrees are in 1 radian.

Summary

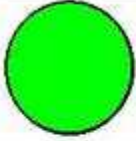
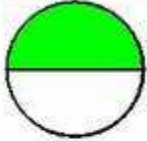
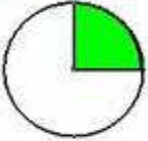
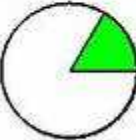
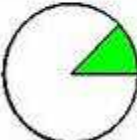
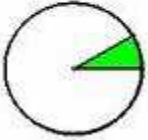
- $1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$
- To convert from radians to degrees, multiply by $\frac{180^\circ}{\pi}$
- To convert from degrees to radians, divide by $\frac{180^\circ}{\pi}$ or multiply by $\frac{\pi}{180^\circ}$

E.g.: Complete the table below for some special angles. Check your answers using the diagram that follows the table.

Angle Measure in Radians	Angle Measure in Degrees
2π	$2\pi \times \frac{180^\circ}{\pi} = \frac{360^\circ \pi}{\pi} = 360^\circ$
π	
$\frac{\pi}{2}$	

$\frac{\pi}{3}$	$\frac{\pi}{3} \times \frac{180^\circ}{\pi} = \frac{180^\circ \pi}{3\pi} = 60^\circ$
$\frac{\pi}{4}$	
$\frac{\pi}{6}$	

Some Common Angles

		
360°	180°	90°
2π	π	$\frac{\pi}{2}$
		
60°	45°	30°
$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

Sample Exam Questions

1. Convert $\frac{\pi}{8}$ to degrees.

$$\frac{\pi}{8} \times \frac{180^\circ}{\pi} = \frac{180^\circ \pi}{8\pi} = 22.5^\circ$$

2. Convert 1.25 rad to degrees.

$$1.25 \times \frac{180^\circ}{\pi} = \frac{225^\circ}{\pi} \approx 71.6^\circ$$

3. Convert 72° to radians.

$$72^\circ \times \frac{\pi}{180^\circ} = \frac{72^\circ \pi}{180^\circ} \approx 1.26$$

Complete the following table.

Angle Measure in Radians	Angle Measure in Degrees
$\frac{3\pi}{4}$	
$\frac{13\pi}{6}$	
$\frac{4\pi}{9}$	
5.3 rad	

Complete the following table.

Angle Measure in Radians	Angle Measure in Degrees
	120°
	240°
	450°
	660°

E.g.: For each pair of angles, determine which is larger. The first one is done for you.

a) 150° & 2

$$2 \times \frac{180^\circ}{\pi} = \frac{360^\circ}{\pi} \approx 114.6^\circ \text{ so } 150^\circ \text{ is larger.}$$

b) 450° & 7

c) 3π & 8

Read “Key Ideas” p. 489

Read “Need to Know” p. 489

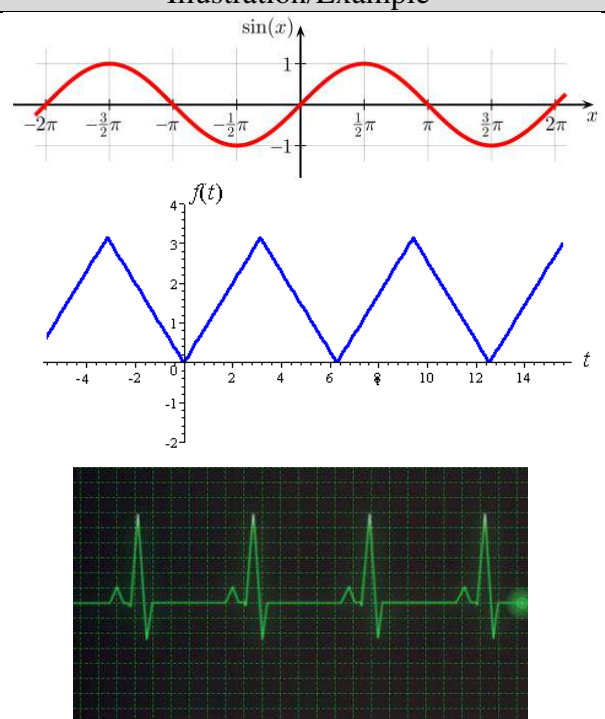
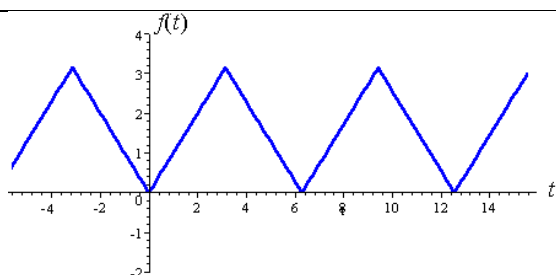
Do #'s 1, 2, 4, 5, 8, pp. 489-490 text in your homework booklet.

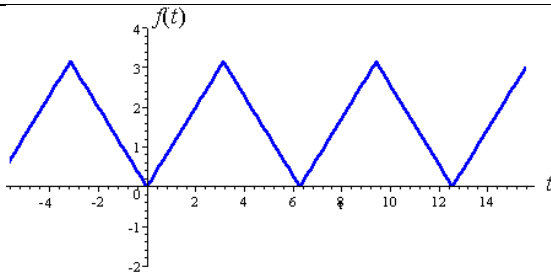
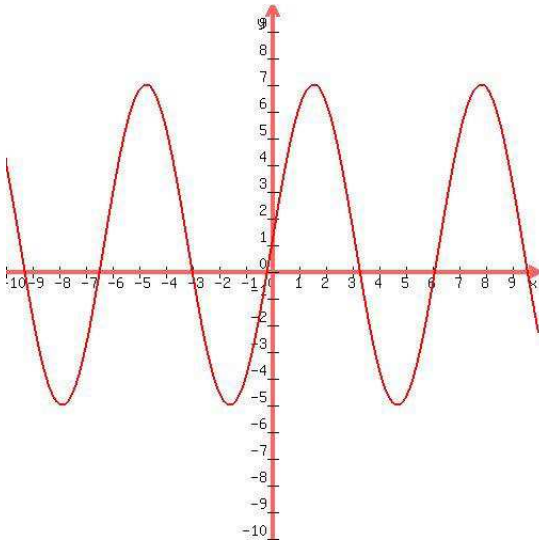
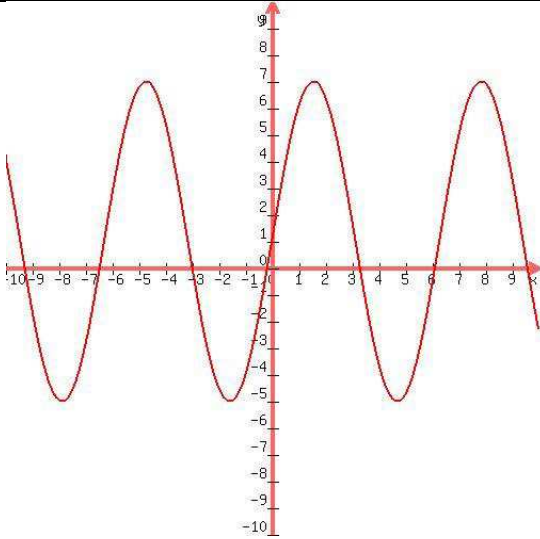
§8.2 Exploring Graphs of Periodic Functions (1 class)

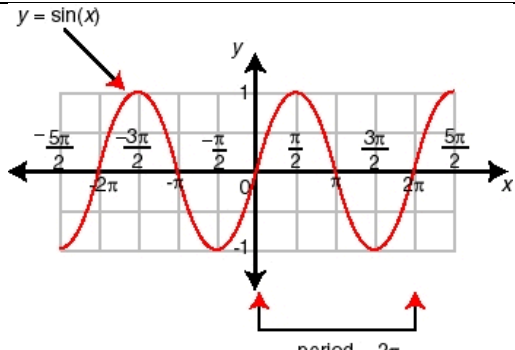
Read *GOAL* p. 491 text.

Outcomes:

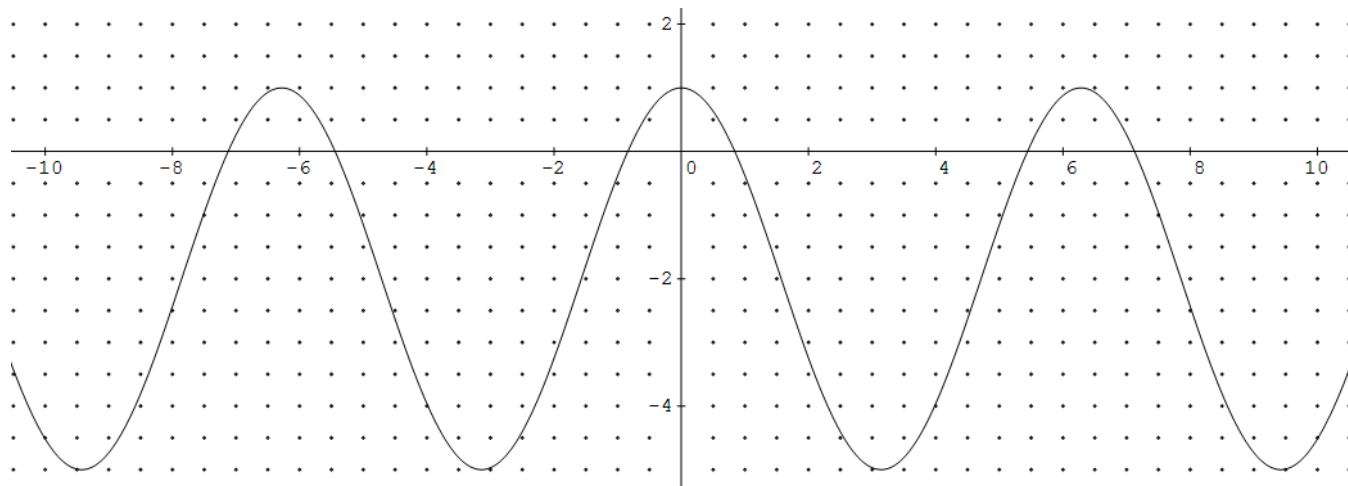
1. Define and illustrate a **periodic function**. p. 492
2. Describe, orally and in written form, the **characteristics** of a periodic function by analyzing its graph.
3. Identify the **maximum value** and the **minimum value** of a periodic function. p. 492
4. Define and identify the **midline** of a periodic function. p. 492
5. Define and identify the **amplitude** of a periodic function. p. 492
6. Define and identify the **period** of a periodic function. p. 492
7. Define and identify a **sinusoidal function**. p. 497 **Section 8.3**
8. Explain and illustrate how a periodic function and a sinusoidal function can differ. **Section 8.3**

Term	Definition/Description	Illustration/Example
Periodic Function	A function whose graph repeats in regular intervals or cycles.	 The illustration shows three examples of periodic functions. The top graph is a sine wave, $\sin(x)$, plotted on a coordinate plane with x ranging from -2π to 2π and y ranging from -1 to 1. The middle graph is a triangular wave, $f(t)$, plotted on a coordinate plane with t ranging from -4 to 14 and $f(t)$ ranging from -2 to 4. The bottom graph is an ECG signal, which is a periodic waveform with sharp peaks and troughs, plotted on a grid.
Maximum Value	The largest value of the dependent variable	 The illustration shows a triangular wave, $f(t)$, plotted on a coordinate plane with t ranging from -4 to 14 and $f(t)$ ranging from -2 to 4. The wave has a maximum value of 3. The maximum value is 3.

Minimum Value	The smallest value of the dependent variable	 The minimum value is 0.
Midline	The horizontal line halfway between the maximum and minimum values.	 The equation of the midline is $y = 1$.
Amplitude	The distance from the midline to the maximum value OR the distance from the midline to the minimum value.	 The amplitude is 6.

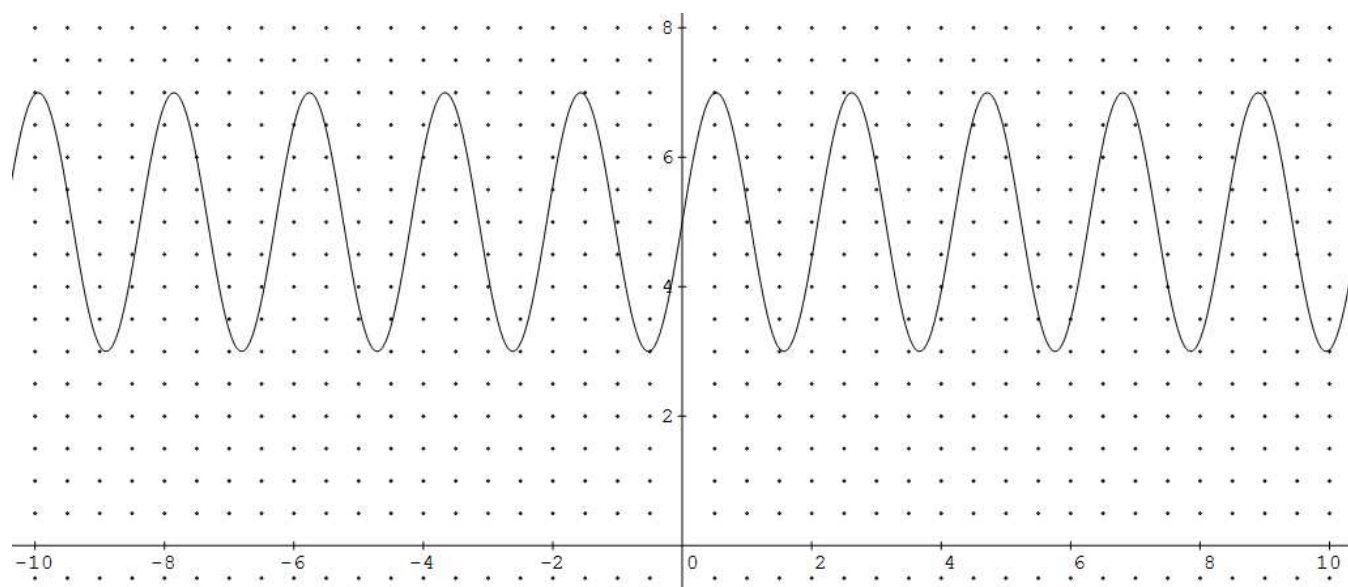
Period	The length of the interval of the domain to complete one cycle.	
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E.g.: Complete the table for the graph below.



Periodic Function? (Y or N)	
Maximum Value	
Minimum Value	
Equation of the Midline	
Amplitude	
Period	

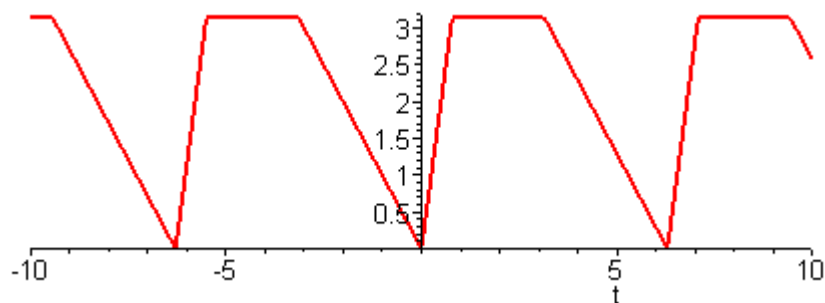
E.g.: Complete the table for the graph below.



Periodic Function? (Y or N)	
Maximum Value	
Minimum Value	
Midline	
Amplitude	
Period	

E.g.: Draw a periodic function with the following characteristics:

- Maximum value 3.
- Minimum value 0.
- Period 6.25



Write the equation of the midline. _____

Give the amplitude. _____

§8.3 The Graphs of Sinusoidal Functions (2 classes)

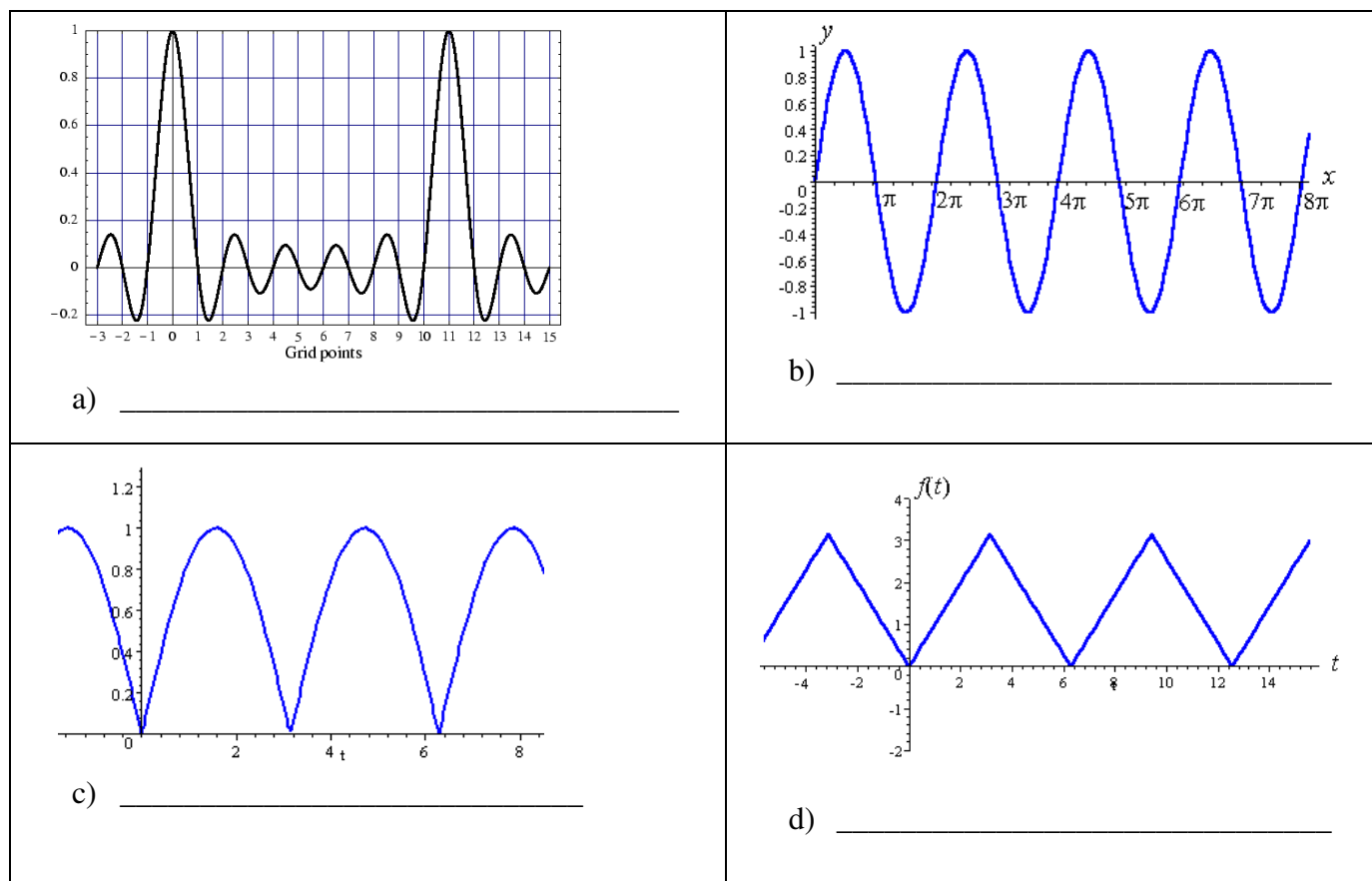
Read *GOAL* p. 497 text.

Outcomes:

1. Define and illustrate a **sinusoidal function**. p. 497
2. Graph sinusoidal functions using both degrees and radians. pp. 497-505
3. Describe, orally and in written form, the **characteristics** of a sinusoidal function by analyzing its graph. pp. 497-505
4. Identify the **maximum value** and the **minimum value** of a sinusoidal function. pp. 497-505
5. Define and identify the **midline** of a sinusoidal function. pp. 497-505
6. Define and identify the **amplitude** of a sinusoidal function. pp. 497-505
7. Define and identify the **period** of a sinusoidal function. pp. 497-505
8. Determine the **domain** of a sinusoidal function. pp. 497-505
9. Determine the **range** of a sinusoidal function. pp. 497-505
10. Explain and illustrate how a periodic function and a sinusoidal function can differ. pp. 497-505
11. Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning. pp. 502-504

Defⁿ: A **sinusoidal function** is a periodic function that has the same shape as $y = \sin x$. How is a **sinusoidal** graph different from a **periodic** graph?

E.g.: Label each of the following graphs as periodic, sinusoidal, or both.



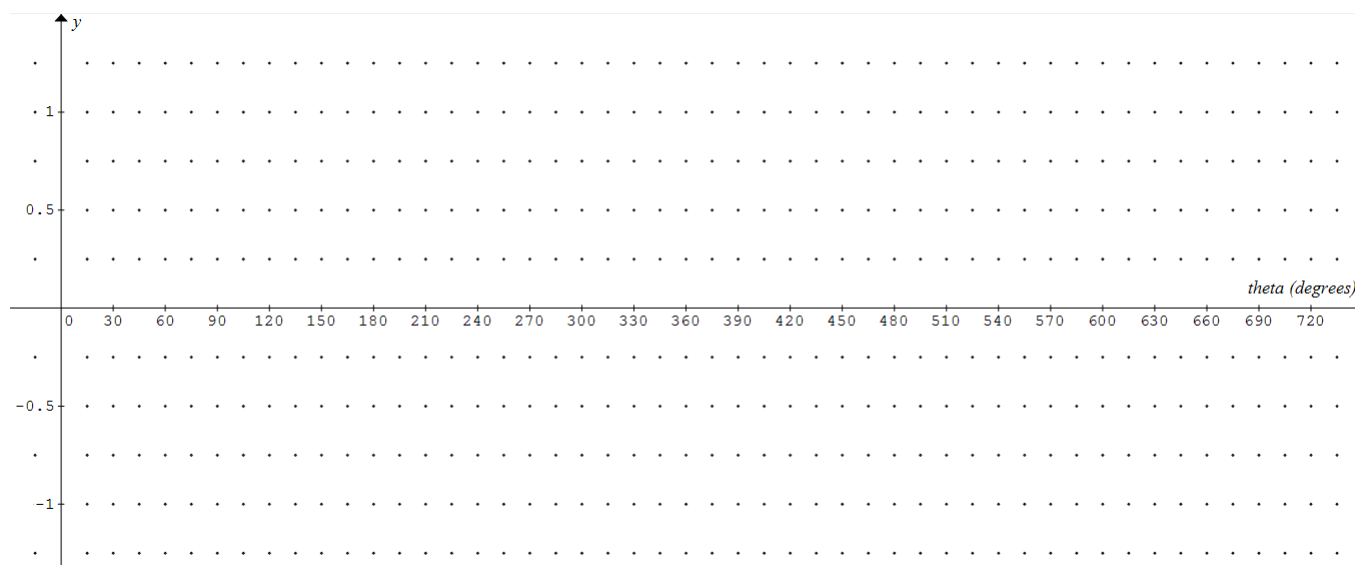
Graphing the Sine Curve ($y = \sin \theta$) using Degrees

Use your calculator to complete the following table and carefully draw the graph of the sine curve ($y = \sin \theta$) on the grid on the next page. Use at least 2 decimal places for the value of y . **Don't forget to put your calculator in degree mode.**

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°
y													

θ	300°	315°	330°	360°	390°	405°	420°	450°	480°	495°	510°	540°	570°
y													

θ	585°	600°	630°	660°	675°	690°	720°
y							



Complete the table to give the characteristics of the Sine Curve ($y = \sin \theta$).

Periodic Function? (Y or N)	
Sinusoidal Function? (Y or N)	
Maximum Value	
Minimum Value	
Equation of the Midline	
Amplitude	
Period	
Domain	
Range	
x-intercepts	
y-intercept	

The FIVE KEY POINTS below will help you graph one period of $y = \sin \theta$ quickly.

θ	y
0°	0
90°	1
180°	0
270°	-1
360°	0

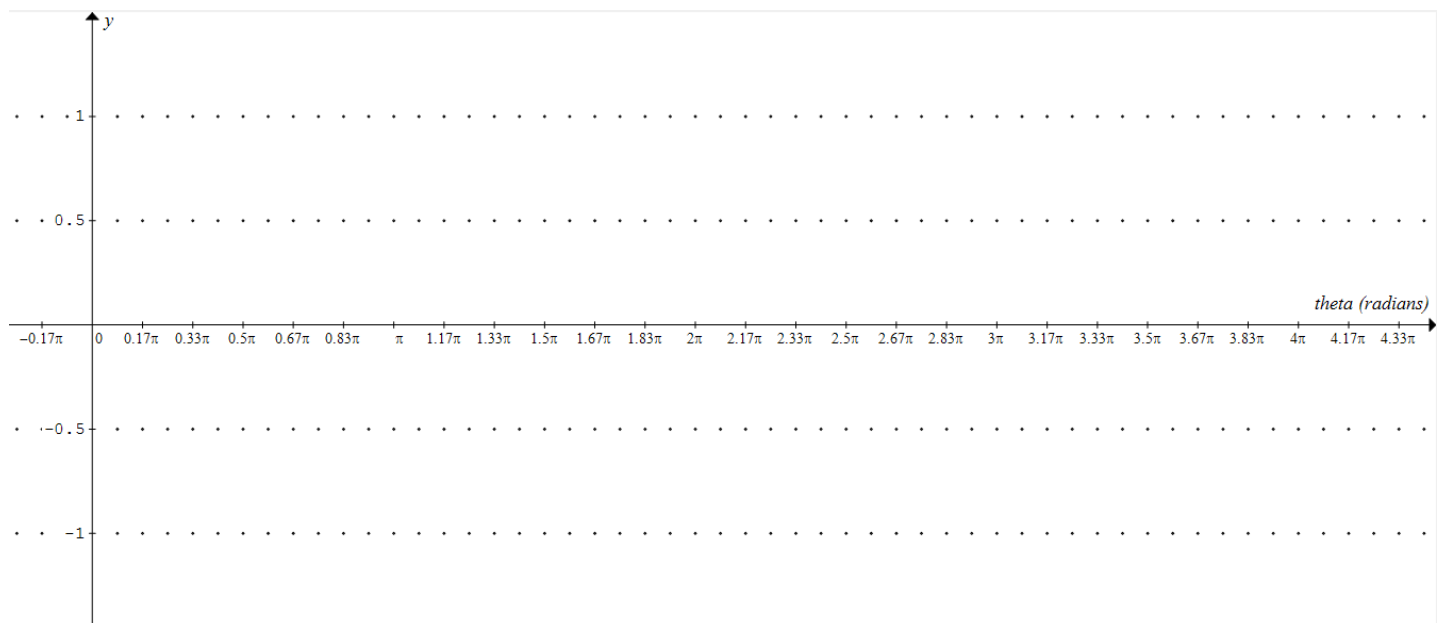
Graphing the Sine Curve ($y = \sin \theta$) using Radians

Use your calculator to complete the following table and carefully draw the graph of the sine curve ($y = \sin \theta$) on the grid on the next page. Use at least 2 decimal places for the value of y . **Don't forget to put your calculator in radian mode.**

θ	0	$\frac{\pi}{6}$ 0.52	$\frac{\pi}{4}$ 0.79	$\frac{\pi}{3}$ 1.05	$\frac{\pi}{2}$ 1.57	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
y													

θ	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π	$\frac{13\pi}{6}$	$\frac{9\pi}{4}$	$\frac{7\pi}{3}$	$\frac{5\pi}{2}$	$\frac{8\pi}{3}$	$\frac{11\pi}{4}$	$\frac{17\pi}{6}$	3π	$\frac{19\pi}{6}$
y													

θ	$\frac{13\pi}{4}$	$\frac{10\pi}{3}$	$\frac{7\pi}{2}$	$\frac{11\pi}{3}$	$\frac{15\pi}{4}$	$\frac{23\pi}{6}$	4π
y							



Complete the table to give the characteristics of the Sine Curve ($y = \sin \theta$).

Periodic Function? (Y or N)	
Sinusoidal Function? (Y or N)	
Maximum Value	
Minimum Value	
Equation of the Midline	
Amplitude	
Period	
Domain	
Range	
x-intercepts	
y-intercept	

The FIVE KEY POINTS below will help you graph one period of $y = \sin \theta$ quickly.

θ	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

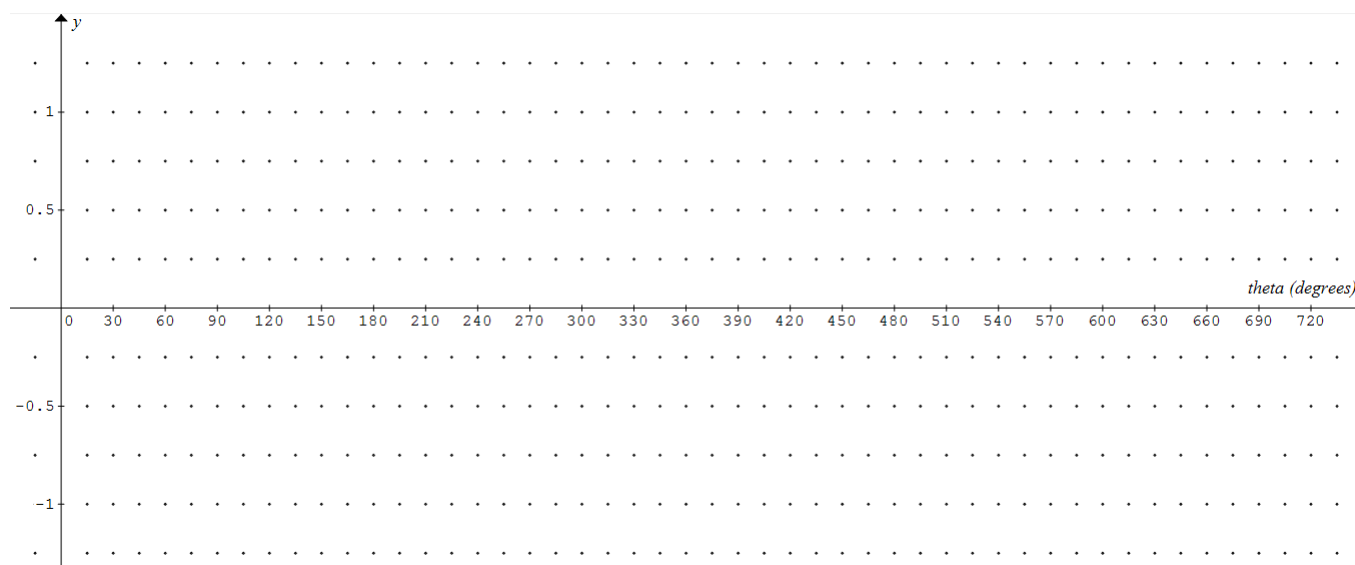
Graphing the Cosine Curve ($y = \cos \theta$) using Degrees

Use your calculator to complete the following table and carefully draw the graph of the cosine curve ($y = \cos \theta$) on the grid on the next page. Use at least 2 decimal places for the value of y . **Don't forget to put your calculator in degree mode.**

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°
y													

θ	300°	315°	330°	360°	390°	405°	420°	450°	480°	495°	510°	540°	570°
y													

θ	585°	600°	630°	660°	675°	690°	720°
y							



Complete the table to give the characteristics of the cosine curve ($y = \cos \theta$).

Periodic Function? (Y or N)	
Sinusoidal Function? (Y or N)	
Maximum Value	
Minimum Value	

Equation of the Midline	
Amplitude	
Period	
Domain	
Range	
x-intercepts	
y-intercept	

The FIVE KEY POINTS below will help you graph one period of $y = \cos \theta$ quickly.

θ	y
0°	1
90°	0
180°	-1
270°	0
360°	1

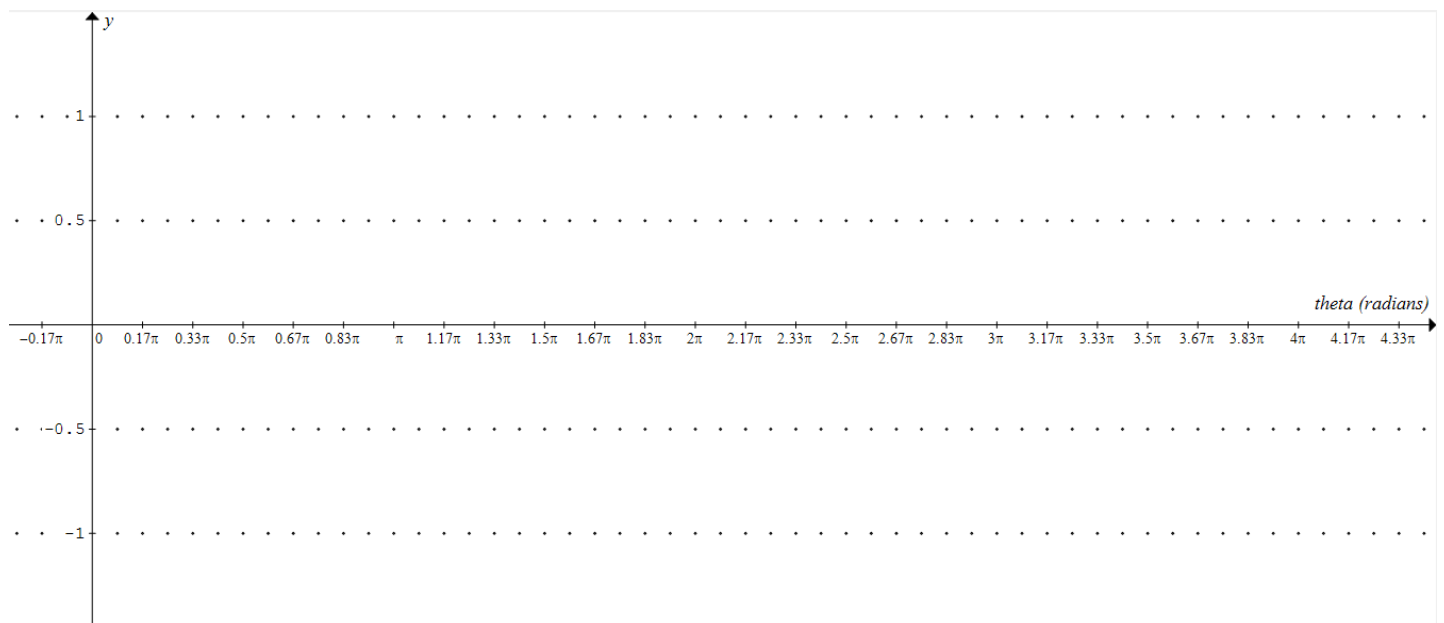
Graphing the Cosine Curve ($y = \cos \theta$) using Radians

Use your calculator to complete the following table and carefully draw the graph of the cosine curve ($y = \cos \theta$) on the grid on the next page. Use at least 2 decimal places for the value of y . **Don't forget to put your calculator in radian mode.**

θ	0	$\frac{\pi}{6}$ 0.52	$\frac{\pi}{4}$ 0.79	$\frac{\pi}{3}$ 1.05	$\frac{\pi}{2}$ 1.57	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
y													

θ	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π	$\frac{13\pi}{6}$	$\frac{9\pi}{4}$	$\frac{7\pi}{3}$	$\frac{5\pi}{2}$	$\frac{8\pi}{3}$	$\frac{11\pi}{4}$	$\frac{17\pi}{6}$	3π	$\frac{19\pi}{6}$
y													

θ	$\frac{13\pi}{4}$	$\frac{10\pi}{3}$	$\frac{7\pi}{2}$	$\frac{11\pi}{3}$	$\frac{15\pi}{4}$	$\frac{23\pi}{6}$	4π
y							



Complete the table to give the characteristics of the cosine curve ($y = \cos \theta$).

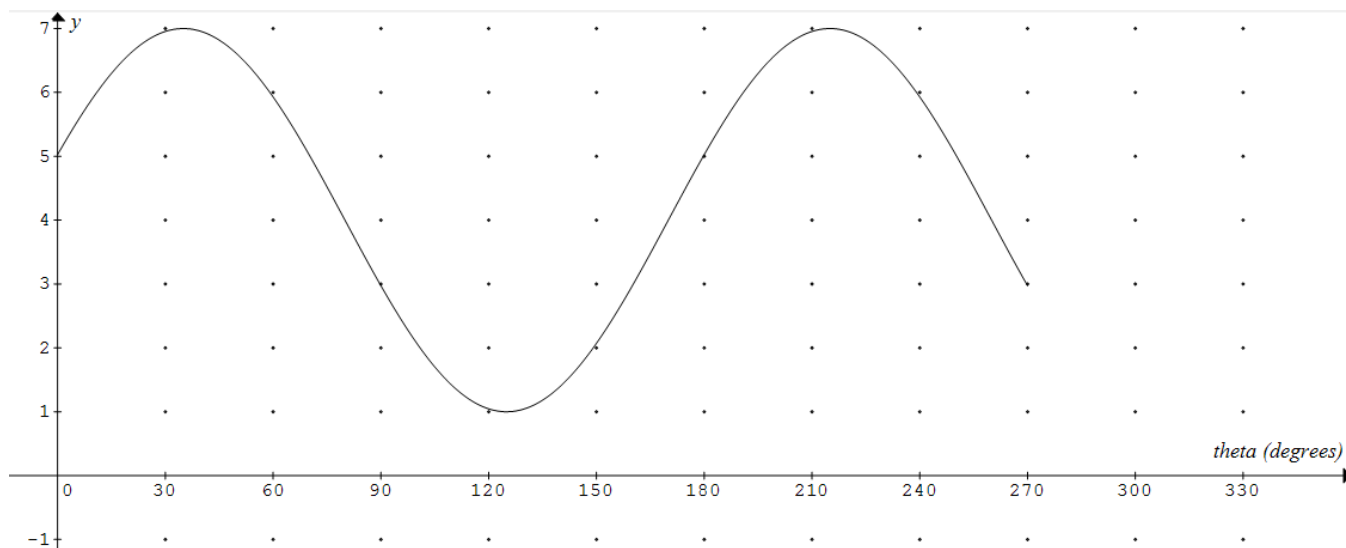
Periodic Function? (Y or N)	
Sinusoidal Function? (Y or N)	
Maximum Value	
Minimum Value	
Equation of the Midline	
Amplitude	
Period	
Domain	
Range	
x-intercepts	
y-intercept	

The FIVE KEY POINTS below will help you graph one period of $y = \cos \theta$ quickly.

θ	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

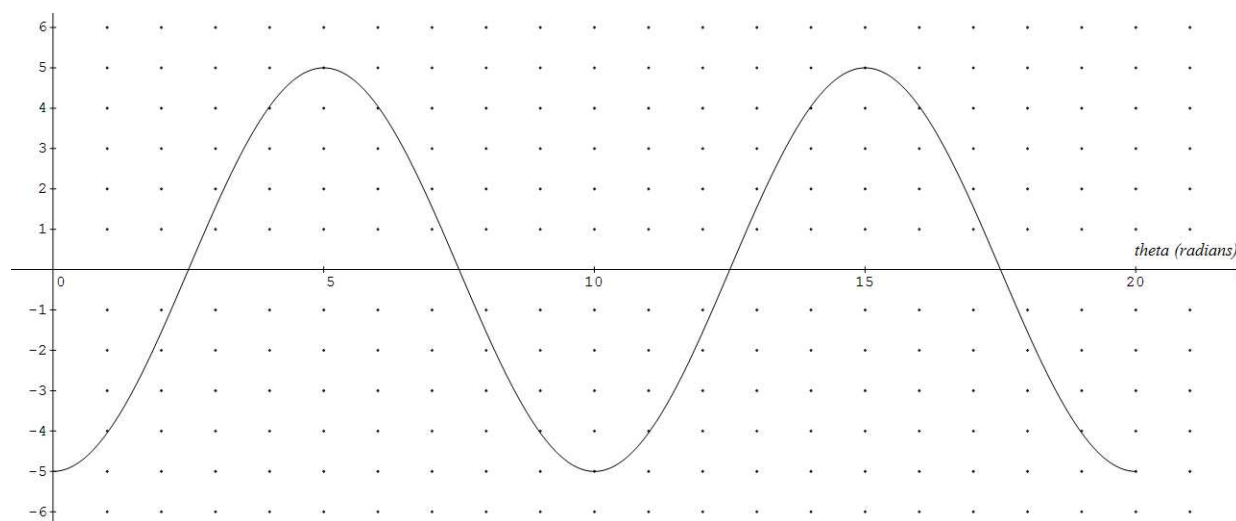
E.g.: Sketch a sinusoidal graph with the following characteristics.

- a) Domain: $\{x | 0^\circ \leq x \leq 270^\circ\}$
- b) Range: $\{y | 1 \leq y \leq 7\}$
- c) Period: 180°
- d) y-intercept: 5



E.g.: Sketch a sinusoidal graph with the following characteristics.

- a) Domain: $\{x | 0 \leq x \leq 20\}$
- b) Maximum value: 5
- c) Minimum value: -5
- d) Period: 10
- e) y-intercept: -5



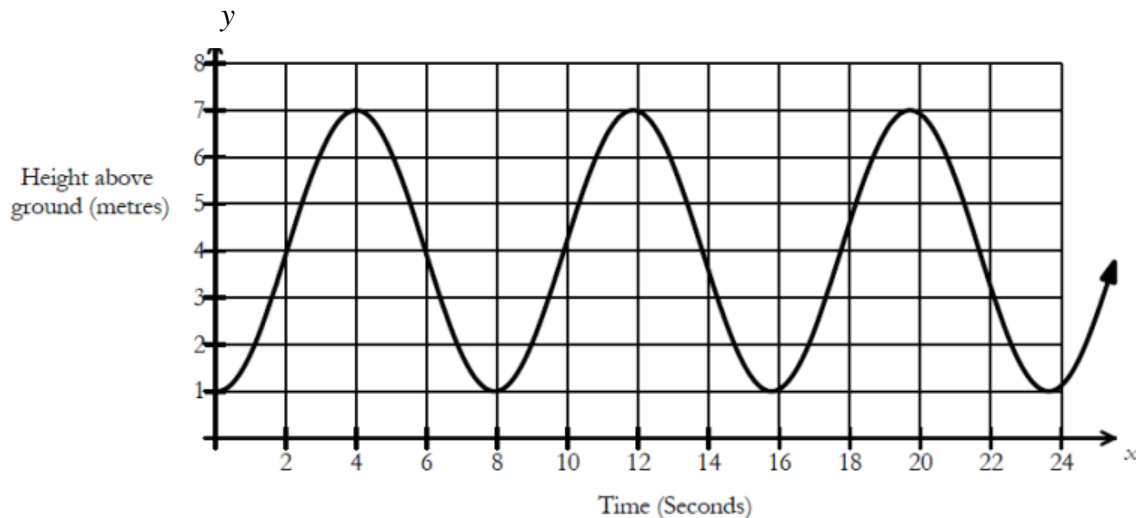
Read “Key Ideas”, bullet #2, p. 505

Read “Need to Know”, p. 505

Do #'s 1-6, 12, pp. 506-510 text in your homework booklet.

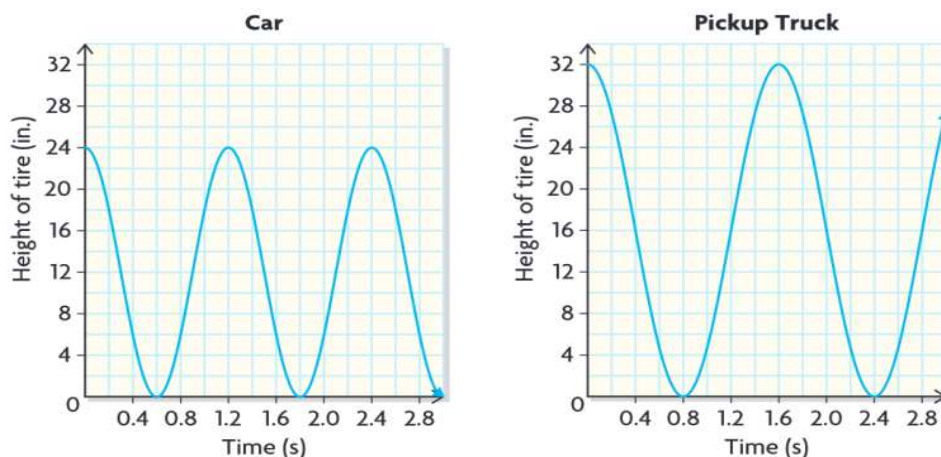
Applications of Sinusoidal Functions

E.g.: While riding a Ferris wheel, Mason’s height above the ground in terms of time can be represented by the following graph.



- How far is the bottom of the Ferris wheel off the ground? _____
 - What is the range of the function? _____
 - What does the range represent? _____
 - What is the diameter of the Ferris wheel? _____
 - What is the equation of the midline? _____
 - What does the equation of the midline represent? _____
 - How long does it take for the Ferris wheel to make one complete revolution? _____
 - What characteristic does the time in g. correspond to? _____
-

E.g.: Alexis and Colin own a car and a pickup truck. They put a chalk mark on the top outer edge of a tire on each vehicle. The following graphs show the height of the chalk mark on the tires above the ground as they rotated while the vehicles were driven at the same slow, constant speed. What can you determine about the characteristics of the tires and the vehicles from these graphs?



- The diameter of the car's tires is 24in, so the car uses 12 inch tires. The diameter of the truck's tires is 32in, so the truck uses 16 inch tires.
- The circumference of the car's tires is $2\pi(12) \approx 75.4\text{in}$. The circumference of the truck's tires is $2\pi(16) \approx 100.5\text{in}$.
- The car's tires make one complete rotation every 1.2s. The truck's tires make one complete rotation every 1.6s.
- The speed of the car is $\frac{75.4\text{in}}{1.2\text{s}} \approx 62.8\text{in/s}$. The speed of the truck is $\frac{100.5\text{in}}{1.6\text{s}} \approx 62.8\text{in/s}$.
- The center of the car's axle is 12 inches above the ground. The center of the truck's axle is 16 inches above the ground.

Read "Key Ideas", bullet #1, p. 505

Do # 8, p. 508 text in your homework booklet.

§8.4 The Equations of Sinusoidal Functions (3 classes)

Read *GOAL* p. 516 text.

Outcomes:

1. Describe, orally and in written form, the characteristics of a sinusoidal function by analyzing its equation. pp. 516-526
2. Match the equation of a sinusoidal function to its corresponding graph. p. 516
3. Solve problems involving sinusoidal functions. p. 522

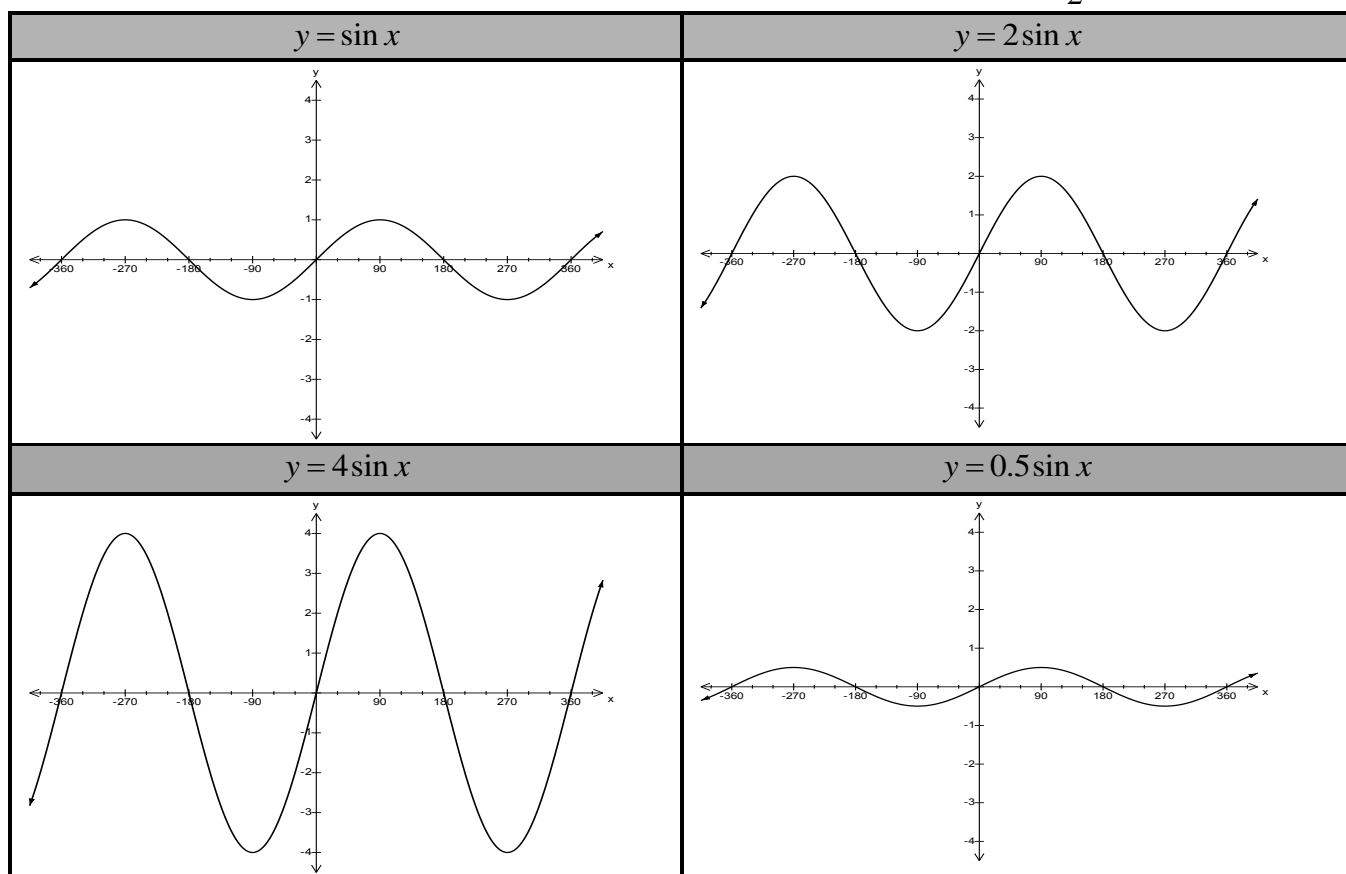
In this section we will investigate how the parameters a , b , c , and d affect the graph of a sinusoidal function written in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

It is important to remember the following characteristics of the base graphs $y = \sin x$ and $y = \cos x$.

$y = \sin x$			$y = \cos x$	
Amplitude (A)	1		Amplitude (A)	1
Period	360° or 2π		Period	360° or 2π
Equation of Midline	$y = 0$		Equation of Midline	$y = 0$

The Effect of Changing the Value of a in $y = a \sin x$ on the Graph of $y = \sin x$ if $a > 0$

Let's sketch and compare the graphs of $y = \sin x$, $y = 2 \sin x$, $y = 4 \sin x$ and $y = \frac{1}{2} \sin x = 0.5 \sin x$.



Complete the following table.

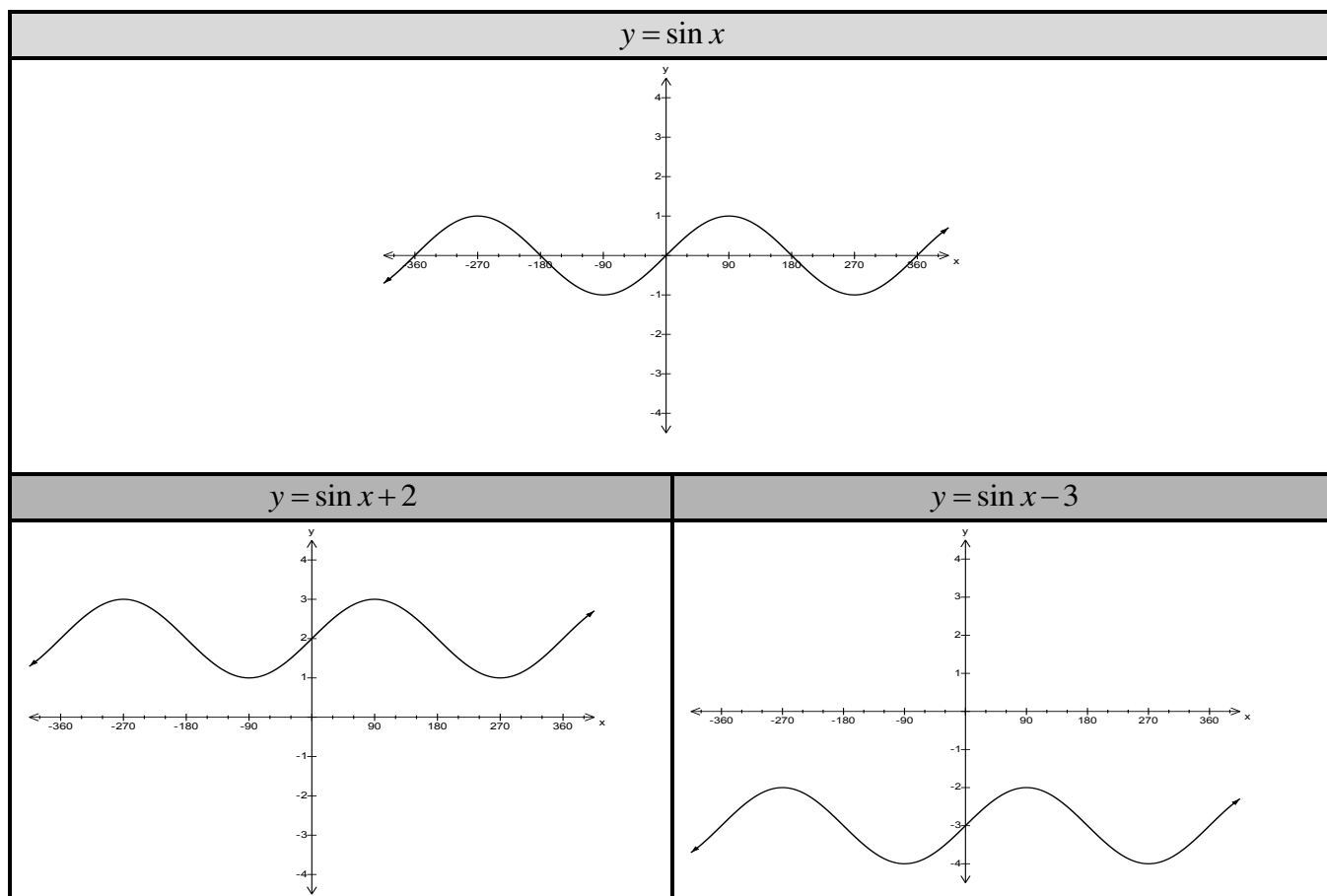
Equation	Amplitude	Period	Equation of Midline
$y = 1 \sin x = \sin x$	1	360° or 2π	$y = 0$
$y = 2 \sin x$			
$y = 4 \sin x$			
$y = \frac{1}{2} \sin x = 0.5 \sin x$			

Describe the affect that changing the value of ' a ' has on the graph of $y = \sin x$.

Do you think that changing the value of ' a ' will affect the cosine graph in the same way that it affects the sine graph?

The Effect of Changing the Value of d in $y = \sin x + d$ on the Graph of $y = \sin x$

Let's sketch and compare the graphs of $y = \sin x$, $y = \sin x + 2$, and $y = \sin x - 3$.



Complete the following table.

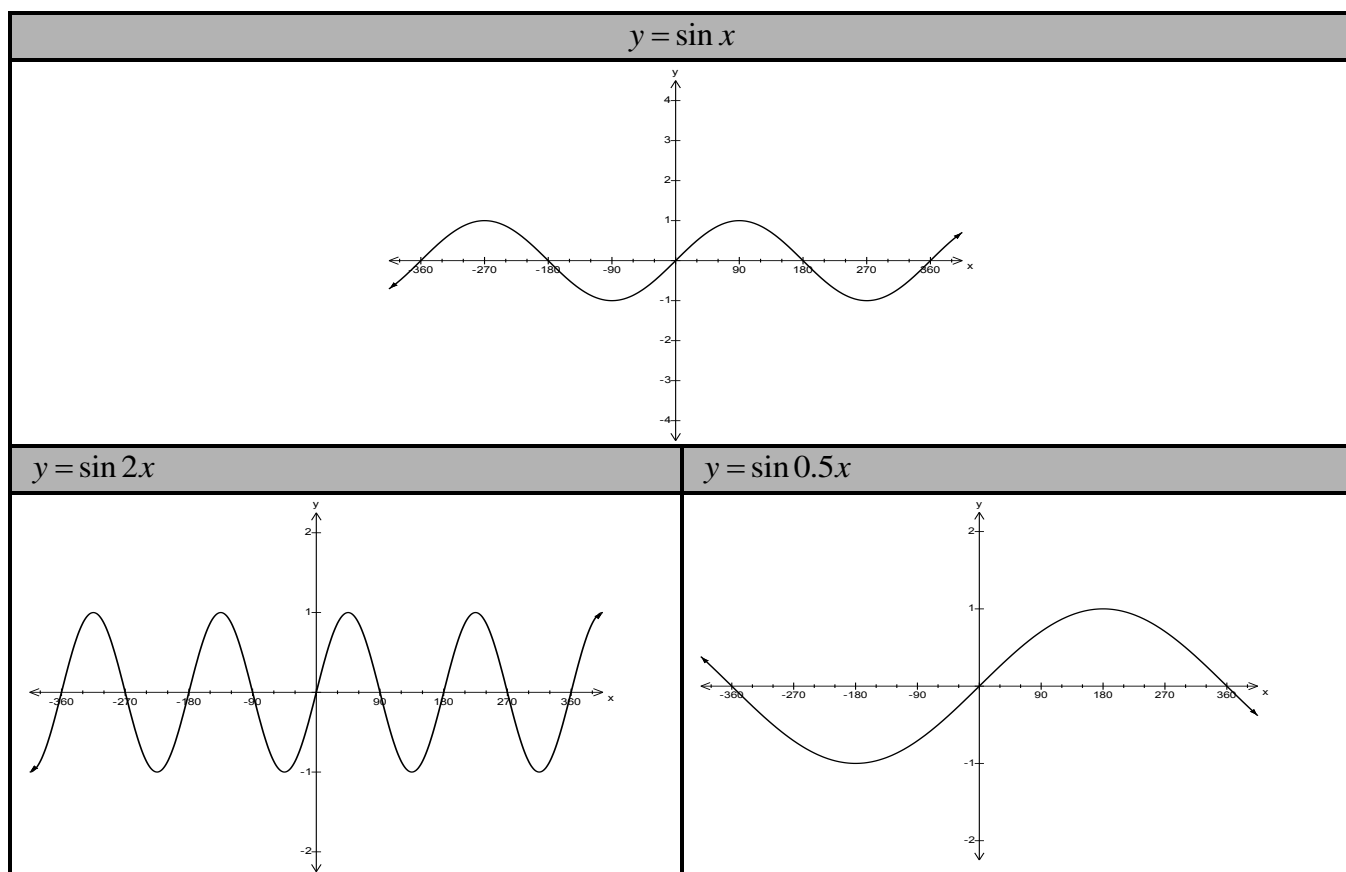
Equation	Amplitude	Period	Equation of Midline
$y = \sin x$	1	360° or 2π	$y = 0$
$y = \sin x + 2$			
$y = \sin x - 3$			

Describe the affect that changing the value of ' d ' has on the graph of $y = \sin x$.

Do you think that changing the value of ' d ' will affect the cosine graph in the same way that it affects the sine graph?

The Effect of Changing the Value of b in $y = \sin bx$ on the Graph of $y = \sin x$ if $b > 0$

Let's sketch and compare the graphs of $y = \sin x$, $y = \sin 2x$, and $y = \sin \frac{1}{2}x = \sin 0.5x$.



Complete the following table.

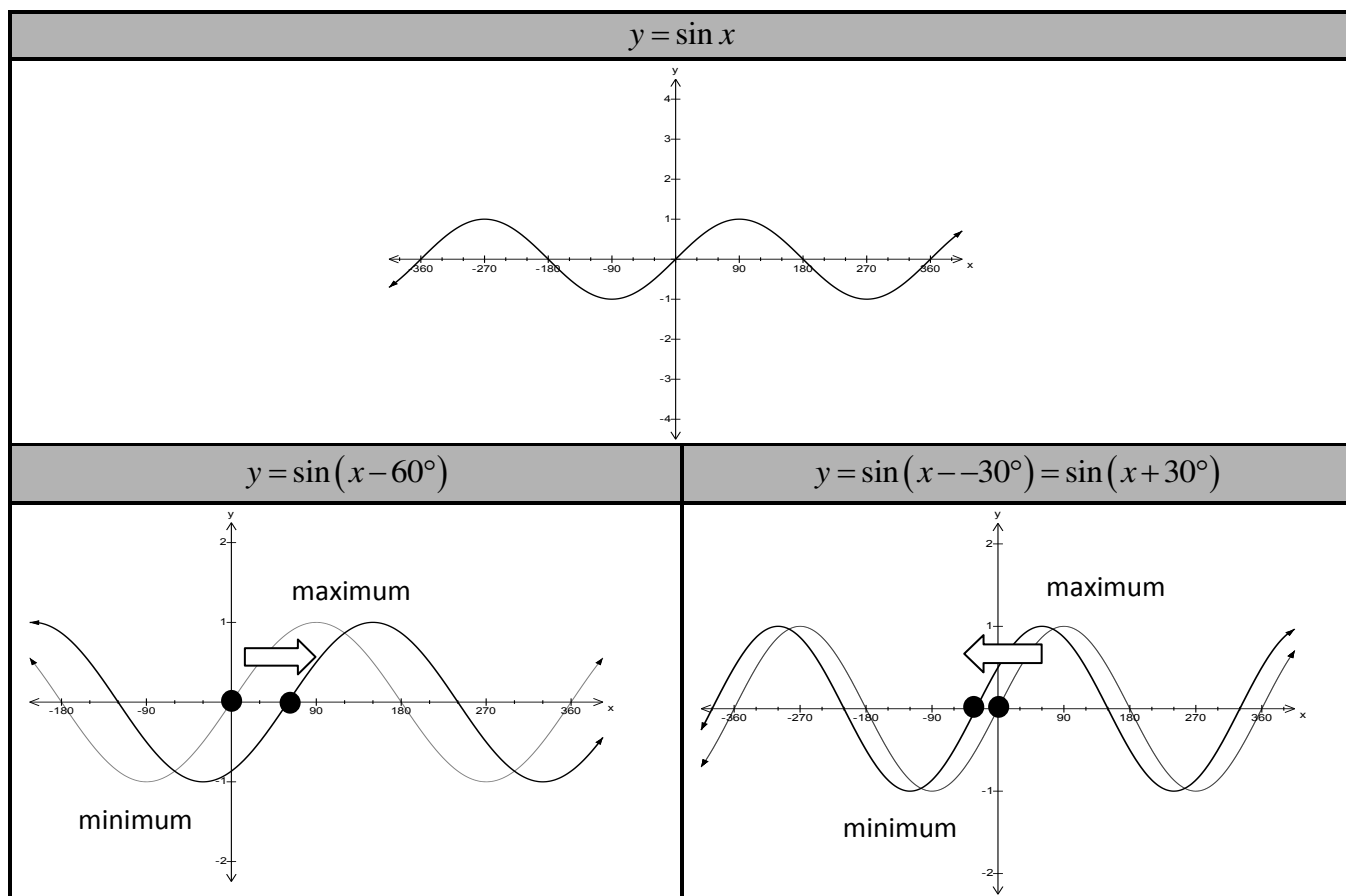
Equation	Amplitude	Period	Equation of Midline
$y = \sin x$	1	360° or 2π	$y = 0$
$y = \sin 2x$			
$y = \sin \frac{1}{2}x = \sin 0.5x$			

Describe the affect that changing the value of ' b ' has on the graph of $y = \sin x$.

Do you think that changing the value of ' b ' will affect the cosine graph in the same way that it affects the sine graph?

The Effect of Changing the Value of c in $y = \sin(x - c)$ on the Graph of $y = \sin x$

Let's sketch and compare the graphs of $y = \sin x$, $y = \sin(x - 60^\circ)$, and $y = \sin(x - -30^\circ) = \sin(x + 30^\circ)$.



Complete the following table.

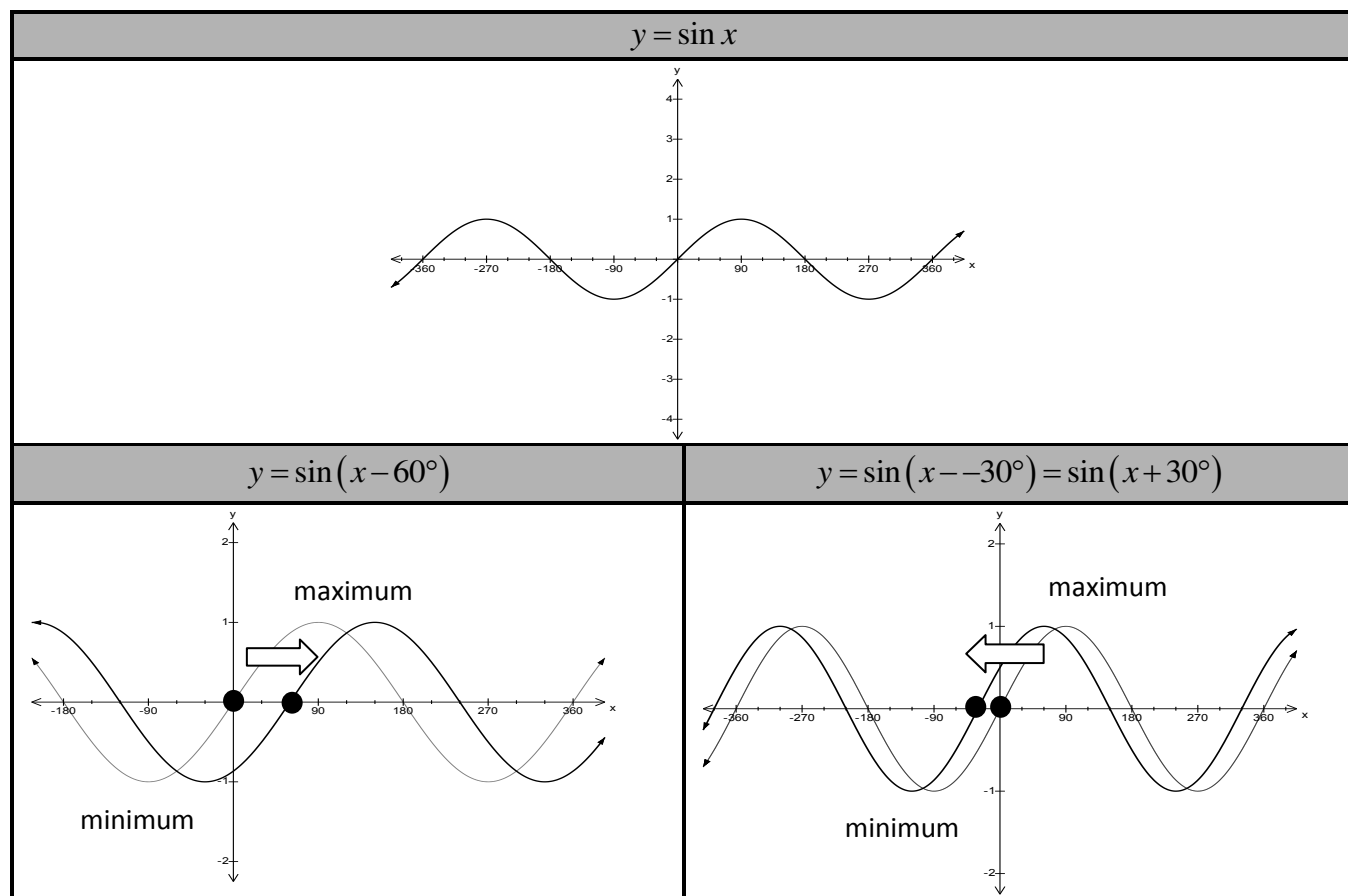
Equation	Amplitude	Period	Equation of Midline	Affect on Graph of $y = \sin x$
$y = \sin x$	1	360° or 2π	$y = 0$	None
$y = \sin(x - 60^\circ)$				
$y = \sin(x - -30^\circ) = \sin(x + 30^\circ)$				

Describe the affect that changing the value of ' c ' has on the graph of $y = \sin x$.

Do you think that changing the value of ' c ' will affect the cosine graph in the same way that it affects the sine graph?

The Effect of Changing the Value of c in $y = \sin(x - c)$ on the Graph of $y = \sin x$

Let's sketch and compare the graphs of $y = \sin x$, $y = \sin(x - 60^\circ)$, and $y = \sin(x - -30^\circ) = \sin(x + 30^\circ)$.



Complete the following table.

Equation	Amplitude	Period	Equation of Midline	Affect on Graph of $y = \sin x$
$y = \sin x$	1	360° or 2π	$y = 0$	None
$y = \sin(x - 60^\circ)$				
$y = \sin(x - -30^\circ) = \sin(x + 30^\circ)$				

Describe the affect that changing the value of ' c ' has on the graph of $y = \sin x$.

Do you think that changing the value of ' c ' will affect the cosine graph in the same way that it affects the sine graph?

Summary

$$y = a \sin b(x - c) + d$$

Affects amplitude (points to a)

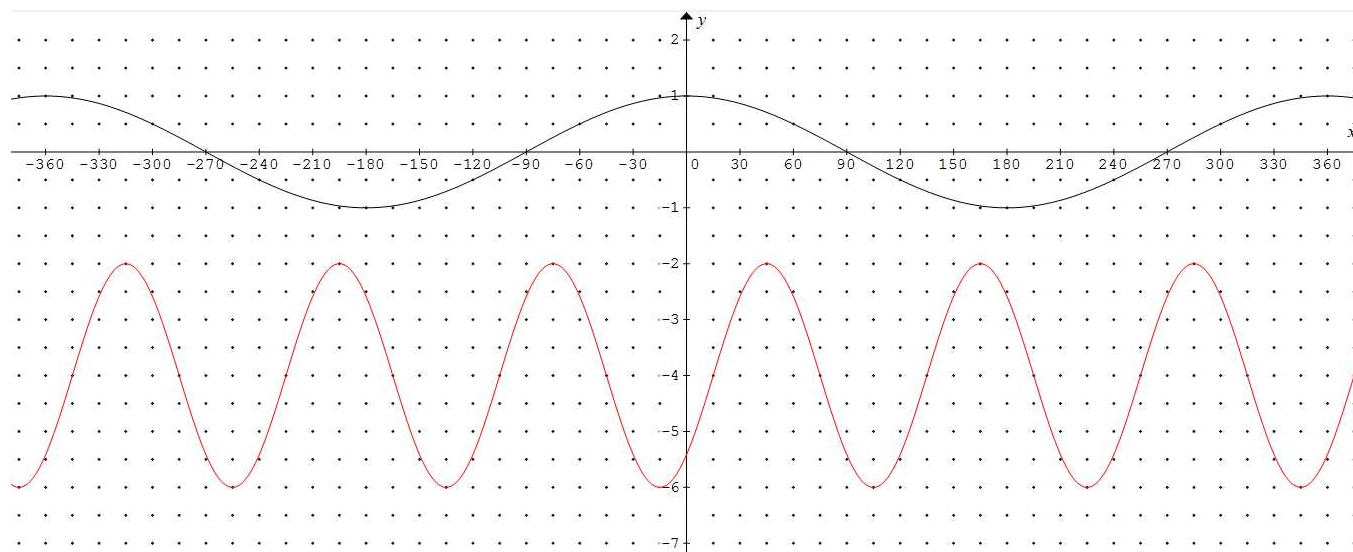
Affects period (points to b)

Shifts graph horizontally (points to c)

Shifts graph vertically (points to d)

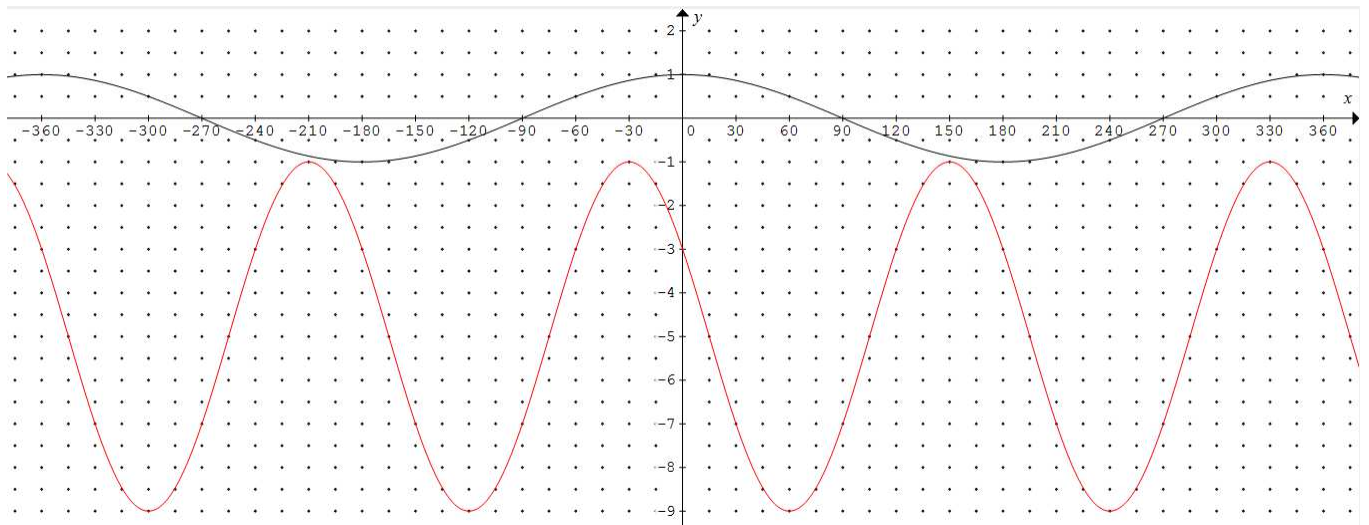
E.g.: Describe the characteristics of the graph of $y = 2 \cos 3(x - 45^\circ) - 4$. Sketch the graph of $y = \cos x$ and $y = 2 \cos 3(x - 45^\circ) - 4$ to check your description. $a = 2$, $b = 3$, $c = 45^\circ$, $d = -4$

Equation	Amplitude	Period	Equation of Midline (Vertical Translation)	Horizontal Translation
$y = \cos x$	1	360° or 2π	$y = 0$ (none)	None
$y = 2 \cos 3(x - 45^\circ) - 4$	$2(1) = 2$	$\frac{360^\circ}{3} = 120^\circ$ or $\frac{2\pi}{3}$	$y = -4$ (down 4)	right 45°



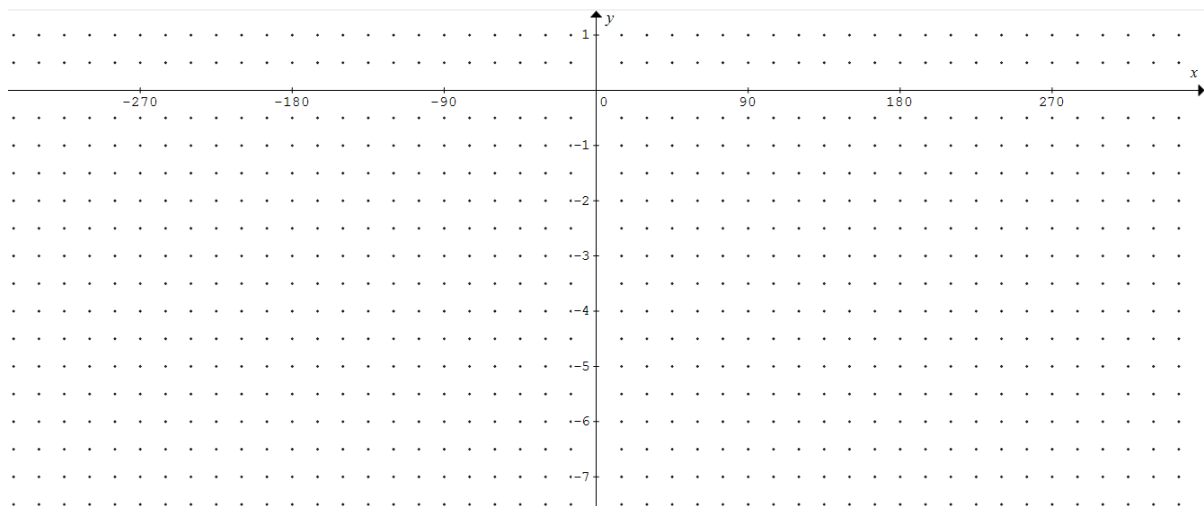
E.g.: Describe the characteristics of the graph of $y = 4\cos 2(x + 30^\circ) - 5$. Sketch the graph of $y = \cos x$ and $y = 4\cos 2(x + 30^\circ) - 5$ to check your description. $a = 4$, $b = 2$, $c = -30^\circ$, $d = -5$

Equation	Amplitude	Period	Equation of Midline (Vertical Translation)	Horizontal Translation
$y = \cos x$	1	360° or 2π	$y = 0$ (none)	None
$y = 4\cos 2(x + 30^\circ) - 5$	$4(1) = 4$	$\frac{360^\circ}{2} = 180^\circ$ or $\frac{2\pi}{2} = \pi$	$y = -5$ (down 5)	left 30°



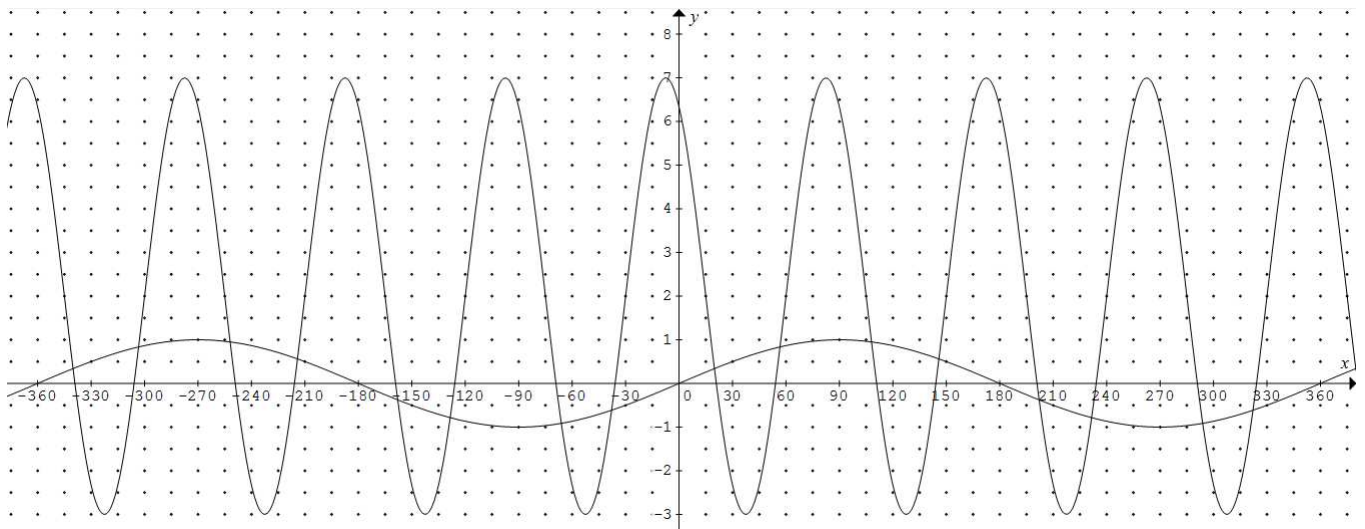
Sample Exam Question

On the grid provided, sketch the graph of a sinusoidal function given its domain is $\{x \mid -180^\circ \leq x \leq 180^\circ\}$ its range is $\{y \mid -7 \leq y \leq -1\}$, it's period is 120° , and it's y-intercept is -7.



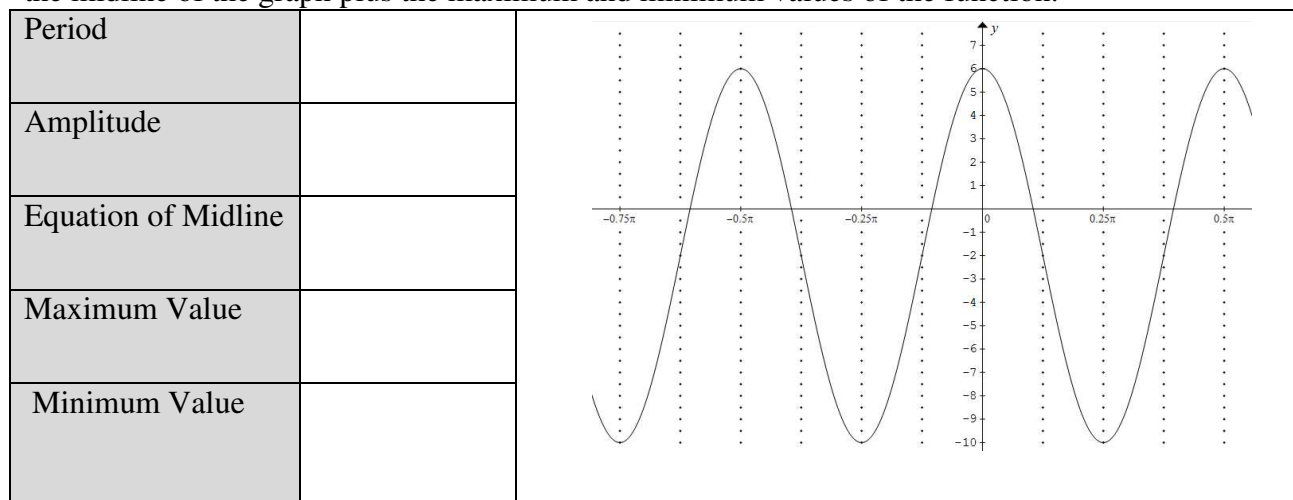
E.g.: Describe the characteristics of the graph of $y = 5 \sin 4(x - 60^\circ) + 2$. Sketch the graph of $y = \sin x$ and $y = 5 \sin 4(x - 60^\circ) + 2$ to check your description. $a = 5$, $b = 4$, $c = 60^\circ$, $d = 2$

Equation	Amplitude	Period	Equation of Midline (Vertical Translation)	Horizontal Translation
$y = \sin x$	1	360° or 2π	$y = 0$ (none)	None
$y = 5 \sin 4(x - 60^\circ) + 2$	$5(1) = 5$	$\frac{360^\circ}{4} = 90^\circ$ or $\frac{2\pi}{4} = \frac{\pi}{2}$	$y = 2$ (up 2)	right 60°



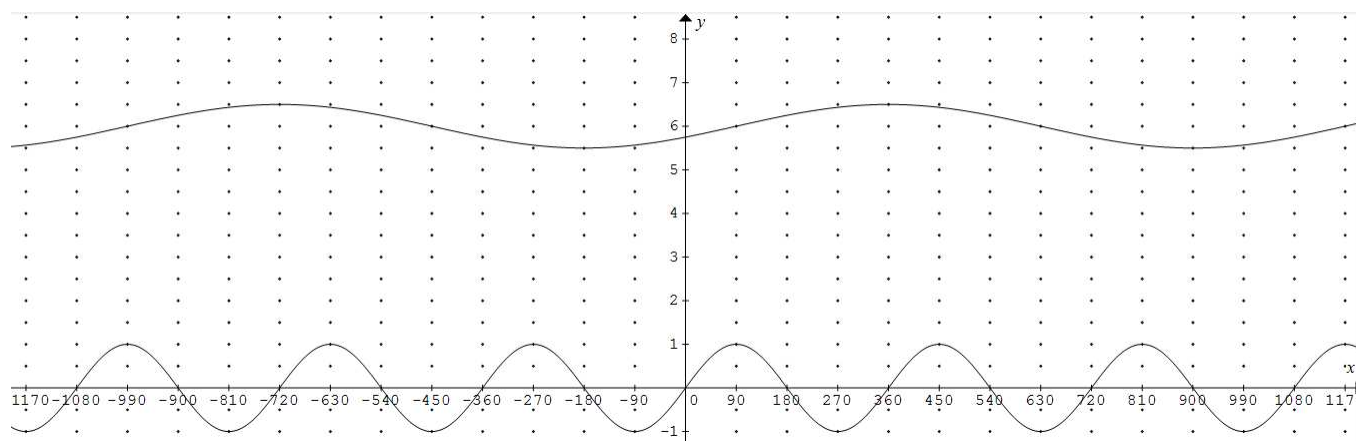
Sample Exam Question

Given the graph of the sinusoidal function below, determine the period, amplitude, and the equation of the midline of the graph plus the maximum and minimum values of the function.



E.g.: Describe the characteristics of the graph of $y = \frac{1}{2} \sin \frac{1}{3}(x - 90^\circ) + 6$. Sketch the graph of $y = \sin x$ and $y = \frac{1}{2} \sin \frac{1}{3}(x - 90^\circ) + 6$ to check your description. $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = 90^\circ$, $d = 6$

Equation	Amplitude	Period	Equation of Midline (Vertical Translation)	Horizontal Translation
$y = \sin x$	1	360° or 2π	$y = 0$ (none)	None
$y = \frac{1}{2} \sin \frac{1}{3}(x - 90^\circ) + 6$	$\frac{1}{2}(1) = \frac{1}{2}$	$\frac{360^\circ}{\frac{1}{3}} = 1080^\circ$ or $\frac{2\pi}{\frac{1}{3}} = 6\pi$	$y = 6$ (up 6)	right 90°



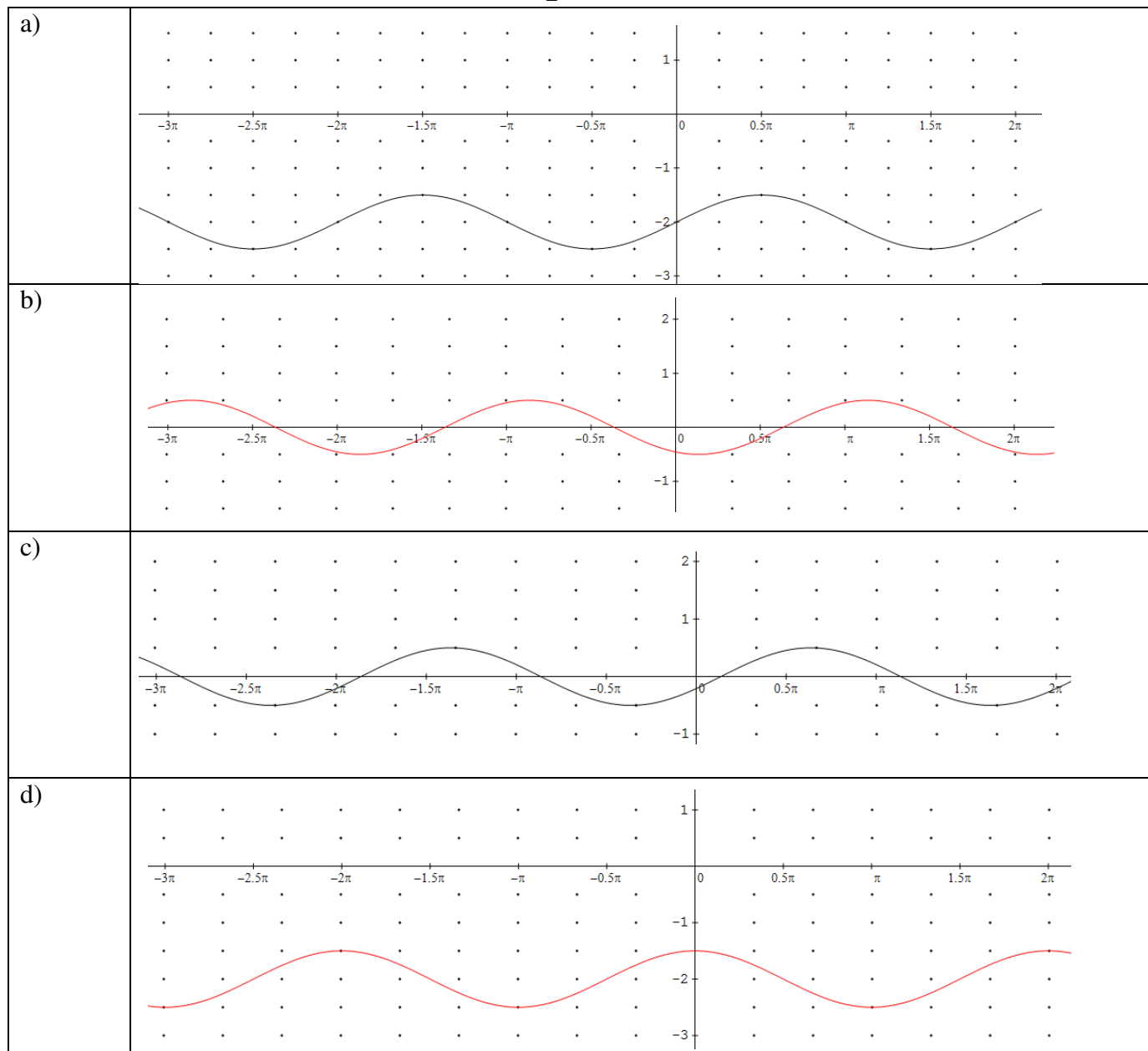
E.g.: Explain how the graph of $y = \sin(x + 2)$ differs from $y = \sin x + 2$.

The graph of $y = \sin(x + 2)$ is the graph of $y = \sin x$ shifted 2 units **left**.

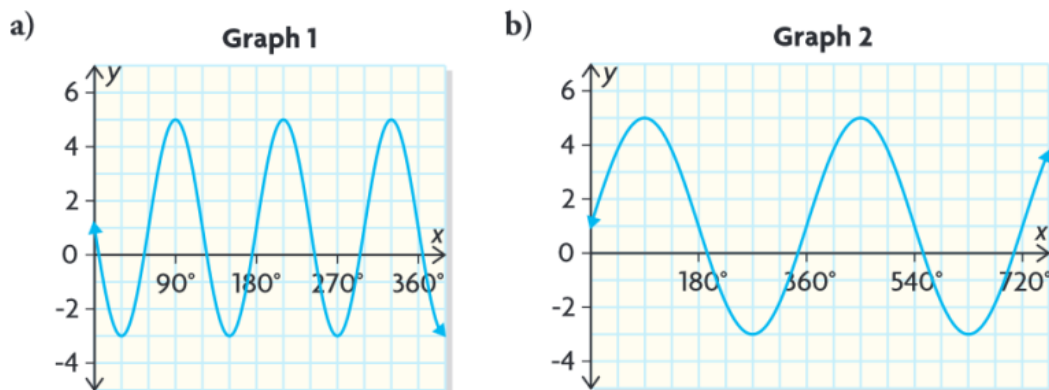
The graph of $y = \sin x + 2$ is the graph of $y = \sin x$ shifted 2 units **up**.

Sample Exam Question

Identify the graph of the sinusoidal function $y = \frac{1}{2}\cos x - 2$.



E.g.: Match each graph with the corresponding equation below. (Example 3, p. 522)



Graph 1 (Choose $(60^\circ, 1)$ as the key point.)	Graph 2 (Choose $(90^\circ, 5)$ as the key point.)
Amplitude = 4, $\therefore a = 4$	Amplitude = 4, $\therefore a = 4$
Period = $120^\circ \therefore \frac{360^\circ}{b} = 120^\circ \Rightarrow b = 3$	Period = $360^\circ \therefore b = 1$
Up 1 $\therefore d = 1$	Up 1 $\therefore d = 1$
Right $60^\circ \therefore c = 60^\circ$	Right $90^\circ \therefore c = 90^\circ$
$y = 4 \sin 3(x - 60^\circ) + 1$ (ii)	$y = 4 \cos(x - 90^\circ) + 1$ (i)

i. $y = 4 \cos(x - 90^\circ) + 1$

iii. $y = 5 \sin 3(x - 60^\circ)$

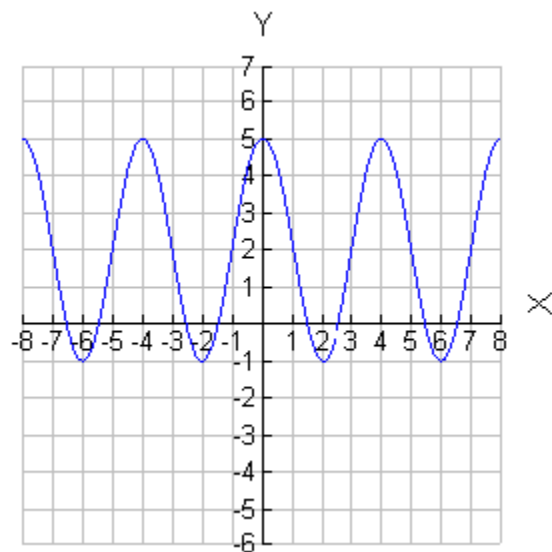
ii. $y = 4 \sin 3(x - 60^\circ) + 1$

iv. $y = 4 \cos 3(x - 60^\circ) + 1$

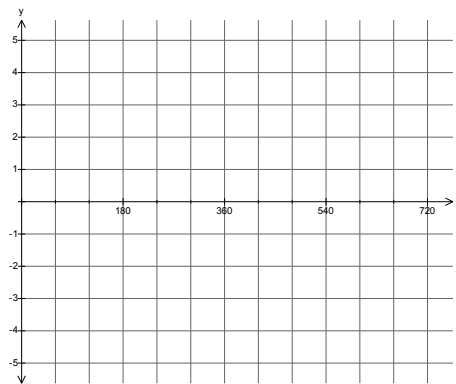
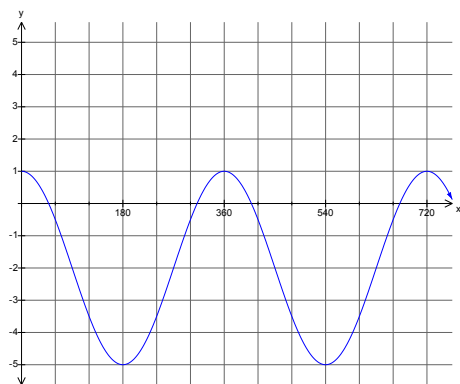
Sample Exam Question

Determine the amplitude of the sinusoidal graph below.

- a) 5
- b) 4
- c) 3
- d) -1



E.g.: Ashley created the following graph for the equation $y = 3\sin(x - 90^\circ) + 2$. Identify her error(s) and construct the correct graph.



Equation	Amplitude	Period	Equation of Midline (Vertical Translation)	Horizontal Translation
$y = \sin x$	1	360° or 2π	$y = 0$ (none)	None
$y = 3\sin(x - 90^\circ) + 2$	$3(1) = 3$	360° or 2π	$y = 2$ (up 2)	right 90°

E.g.: The temperature of an air-conditioned home on a hot day can be modeled using the function $t(x) = 1.5\cos(15^\circ x) + 20$, where x is the time in minutes after the air conditioner turns on and $t(x)$ is the temperature in degrees Celsius.

- What are the maximum and minimum temperatures in the home? $[21.5^\circ\text{C}, 18.5^\circ\text{C}]$
- What is the temperature 10 minutes after the air conditioner has been turned on?
 $[18.7^\circ\text{C}]$
- What is the period of the function? $[24\text{ min}]$ Interpret this value in the context of this situation.

Read “Key Ideas” p. 527

Do #'s 5-9, 12-15, 17, pp. 528-529 text in your homework booklet.

Note 13 iii) should be $y = 3\sin(x - 120^\circ) + 2$.

§8.5 Modeling Data Using Sinusoidal Functions (1 class)

Read *GOAL* p. 533 text.

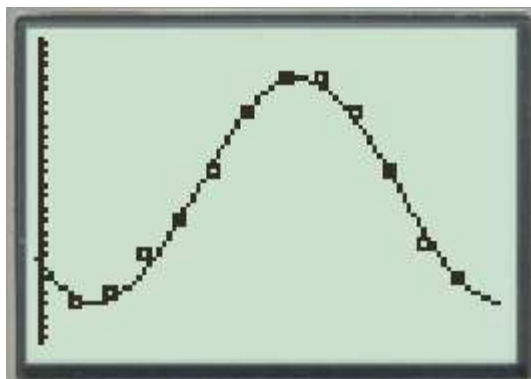
Outcomes:

1. Graph data, and determine the sinusoidal function that best approximates the data. pp. 533-540
2. Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning. pp. 533-540

E.g.: The table below shows the maximum temperature in Rome, Italy over a 12 month period. Find the sinusoidal equation that best models this data.

Months	Max Temperature
January	12
February	13
March	16
April	19
May	23
June	28
July	31
August	31
September	28
October	23
November	17
December	14

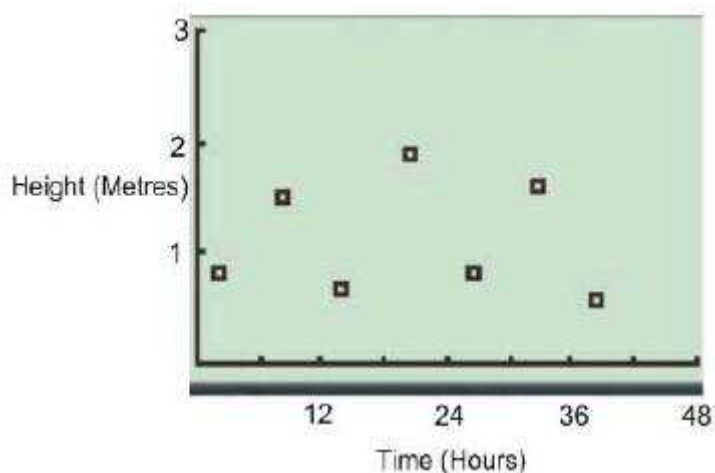
We let January be month 1, February month 2, and so on. Put the month numbers in L1 and the maximum temperature in L2 and do a **SinReg**. Don't forget radian mode!!!! Your scatter plot and equation should be the same as the screens below.



What would be the maximum temperature May of the following year (month 17)?

$$y = 9.39051754 \sin(0.5420298073(17) - 2.429187634) + 21.53661812 = 26.1^{\circ}\text{C}$$

E.g.: A marine biologist recorded the tide times in the Greater St. Lawrence area for July 17th and 18th , 2012 in the chart below. He recorded the data in his TI-83 calculator below. Estimate the water height after 15 hours of observation



Time (Hour:Minutes)	Height (Metres)
01:50	0.8
07:55	1.5
13:35	0.7
20:38	1.9
26:25	0.8
32:40	1.6
38:15	0.6

The water height after 15 hours would be about _____m.

Read “Key Idea” & “Need to Know” p. 541

Do #'s 4, 8, pp. 542-545 text in your homework booklet.

Do #'s 1 c, d, 2 c, d, 4, 5, 8, 9, 11, pp. 551-552 text in your homework booklet.
