EXPONENTS and LOGARITHMS and the GEOMETRIC SERIES
Diploma Exam Questions Categorized by Outcome

General Outcomes
Generate and analyze exponential patterns.

Solve exponential and logarithmic equations and verify identities.

Represent and analyze exponential and logarithmic functions, using technology as appropriate.

Specific Outcome
2.1 Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]

1. What is one-half of 2^{20} ?

   A. 2^{10}

   B. 1^{20}

   C. 2^{19}

   D. 1^{10}

2. If $\log_3 y = c - \log_3 x$, where $y > 0$ and $x > 0$, then y is equal to

   A. $c - x$

   B. $\frac{c}{x}$

   C. $\frac{c^3}{x}$

   D. $\frac{3^c}{x}$

3. A clothing store is going out of business. The owner reduces the cost of each item by 10% of the current price at the start of each week. A jacket costs \$120.00 during the 1st week of the sale. If this jacket is still in the store during the 5th week of the sale, then the price of the jacket, to the nearest cent, will be

A. \$70.00

B. \$70.86

C. \$78.73

D. \$80.00

4. The sum of the first 10 terms of the geometric sequence $-4, 6, -9, \ldots,$ to the nearest tenth, is

A. 153.8

B. 90.7

C. -61.5

D. -453.3

5. In the first stage of a letter-writing campaign, a person writes the same letter to each of 3 pen pals. In the second stage, each of these 3 pen pals copies the letter and sends it to 3 other pen pals, and so on, following this pattern. If it is assumed that no pen pal receives a letter twice, the least number of stages it would take until the minimum total sum of pen pals who received the letter is 9 800 is

A. 7

B. 8

C. 9

D. 10

6. For a geometric sequence, $t_7 = 5x + 2$ and $t_{10} = x - 23$. If the common ratio is 2, the numerical **value of t_{10}** is

A. –26

B. –24

C. –3

D. –1

7. A square picture measures 10 cm on each side. A photocopier is set to produce a copy with the length of each side reduced to 80% of the original. Each successive image is photocopied. After 8 reductions, the area of the last image of the picture, correct to the nearest tenth, is

A. 16.8 cm^2

B. 4.4 cm^2

C. 2.8 cm^2

D. 1.7 cm^2

8. When the terms of a series defined by $\sum_{k=3}^{100}\left[2(k-1)^2\right]$ are written in ascending order, the third term is

A. 8

B. 32

C. 50

D. 64

9. The **sum** of the first *n* terms of a sequence is defined by $S_n = n^2$, where $n \in N$. The first four terms of the **sequence** are

A. 1, 2, 3, 4

B. 1, 5, 14, 30

C. 1, 2, 4, 16

D. 1, 3, 5, 7

10. In a geometric sequence, $t_2 = -6$ and $t_6 = -486$. If $t_3 > 0$, then the common ratio is

A. –3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

11. If the sum of the first 5 terms of a geometric series is 410 and the common ratio is $-\frac{1}{4}$, then the first term is

A. 81.5

B. 512

C. 1 024

D. 104 960

12. The sum of the geometric series $\frac{1}{4} + \frac{1}{2} + 1 + \ldots + 2\,048$ is

A. 2 047.75

B. 2 049.75

C. 4 095.7

D. 4 096.75

13. Expressed in sigma notation, $\log_2(2)+\log_2(4)+\log_2(8)+\log_2(16)$ is equivalent to

A. $\sum_{n=1}^{4}(n)$

B. $\sum_{n=1}^{4}(2n)$

C. $\sum_{n=1}^{4}(n^2)$

D. $\sum_{n=1}^{4}(2^n)$

14. A geometric series with the common ratio $\sqrt{3}$ is described by

A. $\sum_{n=1}^{k}(\sqrt{3})^n$

B. $\sum_{n=1}^{k}(3+\sqrt{3})^n$

C. $\sum_{n=1}^{k}\sqrt{3}n$

D. $\sum_{n=1}^{k}(3+\sqrt{3})n$

Numerical Response

1. A certain pile driver pounds a steel column into the ground. On the first drive, the column is pounded 1.8 m into the ground, and on each successive drive it moves 92% as far as it did on its previous drive.

The **total distance** that the column moves in 60 drives, correct to the nearest tenth of a metre, is __________ m.

15. In a geometric sequence, the first term is $\frac{2}{81}$ and the sixth term is $\frac{3}{16}$. The common ratio for this geometric sequence is

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 2

D. 3

16. A chain letter begins when one person sends a letter to several people. Each recipient then sends a copy of the letter to a specified number of his or her friends.

In 1935, such a chain letter was received by almost every person in the city of Denver. This particular chain letter was started by a person who sent it to five people (“link” 1), each of whom sent it to five more people (“link” 2), and so on.

Assume that no person received the letter twice and that each person who received the letter carried out the instructions to send it to five more people in Denver. Knowing that the chain letter was received by almost everyone in the city of Denver after 8 links in the chain, the population of Denver could be estimated by calculating

A. $5(5)^7$

B. $1 + 7(5)$

C. $\frac{5(5^8 - 1)}{5 - 1}$

D. $\frac{8[2(1) + (7)(5)]}{2}$

17. Which of the following sums has the **largest** value?

A. $\sum_{k=1}^{100}(k + 2)$

B. $\sum_{k=1}^{100}2k$

C. $\sum_{k=1}^{100}2^k$

D. $\sum_{k=1}^{100}\left(\frac{1}{2}\right)^k$

18. If $t_3 = 144$ and $t_6 = 486$ in a geometric sequence, then t_8 is equal to

A. 648

B. 656

C. 546.75

D. 1 093.5

19. A tree grows 1.5 m during the first year of planting. During each subsequent year, the tree grows $\frac{9}{10}$ of the previous year's growth. When the growth of the tree since planting is 11.35 m, its age is

A. 4.8 years

B. 12.0 years

C. 13.4 years

D. 15.0 years

20. The value of $\sum_{n=2}^{5} \log_n n^2$ is

A. 8

B. 14

C. 28

D. 54

21. In a geometric sequence, the fourth term, t_4, is 108 and $t_6 = 972$. If the common ratio is positive, then the sum of the first ten terms of the sequence is

A. 1 080

B. 39 364

C. 78 732

D. 118 096

22. The sum of the first ten terms of the geometric sequence -4, 6, -9,…, to the nearest tenth, is

A. 153.8

B. 90.7

C. -61.5

D. -453.3

23.

A clothing store is going out of business. The owner reduces the cost of each item by 10% of the current price at the start of each week. A jacket costs \$120.00 during the 1st week of the sale. If this jacket is still in the store during the 5th week of the sale, then the price of the jacket, to the nearest cent, will be

A. \$70.00
B. \$70.86
C. \$78.73
D. \$80.00

Numerical Response

2. A quantity of water contains 200 g of pollutants. Each time this quantity of water passes through a filter, 15% of its pollutants are removed. Correct to the nearest tenth of a gram, the number of grams of pollutants still in the water after it passes through 6 filters is __________.

Numerical Response

3. What is the seventh term in the geometric series 300 + 60 + 12+…? (Round answer to the nearest hundredth.)

Numerical Respo

2.2 Connect geometric sequences to exponential functions over the set of natural numbers. [E, R, V]

1.

The value of i in the compound interest equation $5 = (1 + i)^6$ is

A. $i = \sqrt[6]{4}$

B. $i = \sqrt[6]{5} - 1$

C. $i = \dfrac{\log 5}{\log 6} - 1$

D. $i = \dfrac{\log 6}{\log 5} - 1$

2.3 Solve exponential equations that have numerical bases that are powers of one another. [E, R]

1. If $\left(\sqrt{2}\right)^{3x-1} = (4)^{x+3}$, then x is equal to

   A. –13

   B. – 4

   C. 2

   D. 7

2. If $W = 16^m \times 8^{2m-1}$, then $\log_2 W$ equal to

   A. $3m - 1$

   B. $10m - 1$

   C. $10m - 3$

   D. 10^{3m-1}

3. If $\frac{1}{3}(27)^{x-1} = \sqrt[4]{9^x}$, then the value of *x* is

A. -0.80

B. 0.86

C. 1.33

D. 1.60

4. Determine the solution to the nearest hundredth for the equation $\left(\frac{1}{8}\right)^{x-3}$ = 2(16) $^{2x+1}$.

2.4 Use the laws of exponents and logarithms to:

- solve and verify exponential equations and identities
- solve logarithmic equations
- simplify logarithmic expressions

1. A student graphed the following equations.

Equation I $y_1 = \log_{10} x$

Equation II $y_2 = 5^{x-3}$

Equation III $y_3 = x - 3$

Equation IV $y_4 = x$

The student could estimate the solution to the equation $\log_5 x = x - 3$ by using the graphs of equations

A. I and II
B. I and III
C. II and III
D. II and IV

2. The value of $\sum_{n=1}^{5} \log n$ is equal to the value of

A. $\log\left(n^5\right)$

B. $\log\left(5^n\right)$

C. $\log(5!)$

D. $\log(15)$

3. An investment of $1 000 is earning 4% interest per annum compounded annually. If the value, *V*, of the investment after *t* years is given by $V = 1000(1.04)^t$, then *t*, written as a function of *V*, is

A. $t = \dfrac{\log(V)}{3} - \log(1.04)$

B. $t = \dfrac{\log(V)}{3\log(1.04)}$

C. $t = \log(V) - 3 - \log(1.04)$

D. $t = \dfrac{\log(V) - 3}{\log(1.04)}$

4. The relationship between the length and mass of a particular species of snakes is

$$\log m = \log a + 3\log l$$

where a is a given constant
m is the mass of the snake in grams
l is the length of the snake in metres

This relationship can also be written as

A. $\log\left(\frac{m}{al^3}\right) = 0$

B. $\log\left(\frac{m}{3al}\right) = 0$

C. $\log\left(\frac{am}{l^3}\right) = 0$

D. $\log\left(\frac{am}{3l}\right) = 0$

5. The expression $\log_a\left(\frac{1}{a^b}\right)$, where $a > 0$, is equal to

A. $-b$

B. b

C. a^b

D. a^{-b}

6. The graphs of $y = \log_3(x-1)+1$ and of $y = \log_3(2x+1)$ intersect at a point. An equation that could be used to find this point of intersection is

A. $\log_3[(2x+1)(x-1)] = 1$

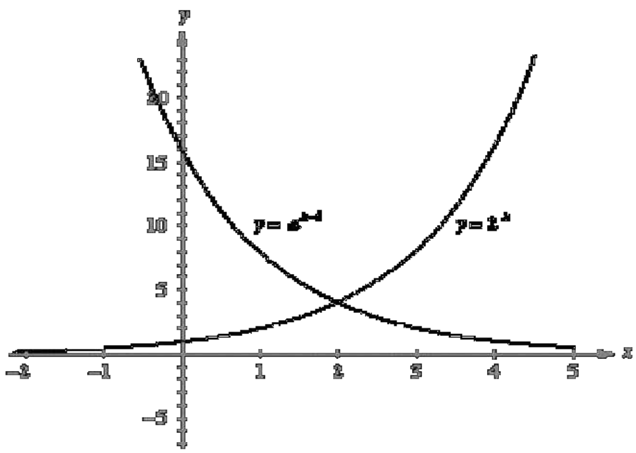
B. $\log_3 \frac{(2x+1)}{(x-1)} = 1$

C. $\log_3(2x+1)\log_3(x-1) = 1$

D. $\frac{\log_3(2x+1)}{\log_3(x-1)} = 1$

Numerical Response

5. The graphs of $y = 2^x$ and $y = a^{x-4}$ intersect when $x = 2$, as shown below.



Given that *a* is a positive number, the value of *a*, correct to the nearest tenth is
.

7. The expression $\log_5\left(\dfrac{25m^3}{n}\right)$, where $m > 0$ and $n > 0$, is equal to

A. $\dfrac{6\log_5(m)}{\log_5(n)}$

B. $6\log_5(m) - \log_5(n)$

C. $2 + 3\log_5(m) - \log_5(n)$

D. $25 + 3\log_5(m) - \log_5(n)$

8. The solution of the equation $4^x = 24$ is

A. $\log_{10} 6$

B. $4\log_{10} 24$

C. $\dfrac{\log_{10} 24}{\log_{10} 4}$

D. $\log_{10} 24 - \log_{10} 4$

9. The value of $\log_8(16) - \log_8(2) + \log_8(4)$ is

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 2

D. 4

10. The value of the expression $\log_{\sqrt{5}} 5 + 2\log_5 125$ is

A. 11

B. 9.5

C. 8

D. 6.5

Numerical Response

6. If $\log_b(a) = 3.8$ and $\log_b(c) = 2.1$, then correct to the nearest tenth, the value of $\log_b\left(\frac{a}{c}\right)$ is .

11. The expression $\log_a\left(\frac{y^3\sqrt{x}}{5}\right)$ is equivalent to

A. $\log_a(3y) + \log_a\left(\frac{1}{2}x\right) - \log_a 5$

B. $\log_a(3y) + 2\log_a x - \log_a 5$

C. $3\ \log_a y + \frac{1}{2}\ \log_a x - \log_a 5$

D. $3\ \log_a y + 2\ \log_a x - \log_a 5$

Numerical Response

7. Given the equation $\log_9 x = \log_4 64$, the value of x, to the nearest integer, is ____________ .

12. The expression $2\log_a 5 + \log_a 6 - \frac{1}{3}\log_a 8,\ a > 0,$ equals

A. $\log_a 29$

B. $\log_a 30$

C. $\log_a 75$

D. $\frac{8}{3}\log_a 3$

13. If $\log_2(x) + 8 = 0$, then the value of x is

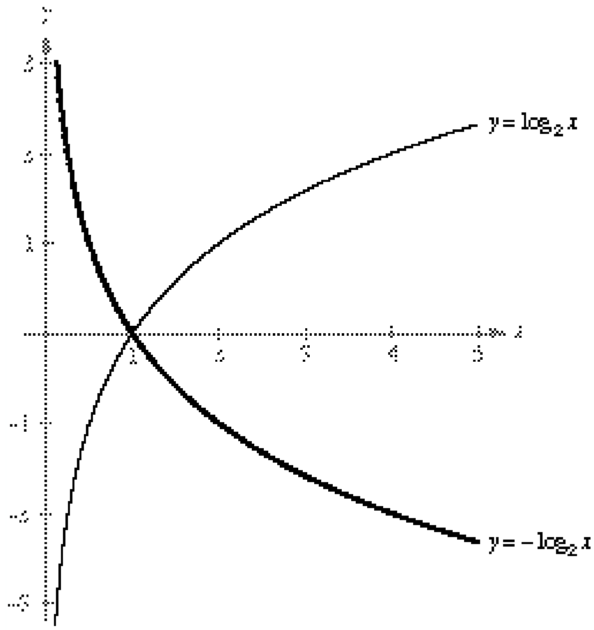
A. –3

B. $-\frac{1}{256}$

C. $\frac{1}{256}$

D. 3

14. A student uses a computer program to plot the partial graph of $y = \log_2 x$. The student then reflects this graph in the *x*-axis, as shown below.



The student realizes the equation of the new graph can be $y = -\log_2 x$ or

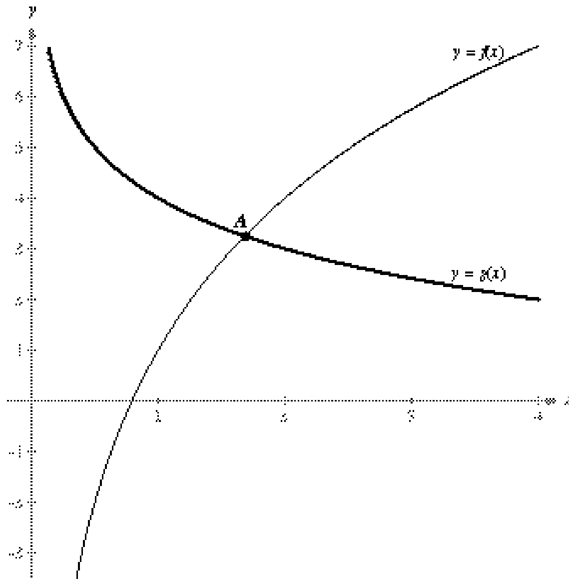
A. $y = \log_2(-x)$

B. $2^y = x$

C. $y = -2^x$

D. $y = \log_2\left(\frac{1}{x}\right)$

15. The partial graphs of $f(x)=1+3\log_2 x$ and $g(x)=4-\log_2 x$ are shown below. The graphs intersect at point *A*.



Based on the information above, the *x*-coordinate of point *A* can be determined by solving

A. $3\log_2 x+1=0$

B. $4-\log_2 x=0$

C. $\log_2 x=\frac{5}{2}$

D. $\log_2 x=\frac{3}{4}$

16. If $\log_2 b=c$ then $\log_4 b$ equals

A. $\frac{c}{2}$

B. c^2

C. 2c

D. $\sqrt{c}$

Numerical Response

8. An exponential function f is defined by $f(x) = 7(1.5)^{x-6}$. If $f(k) = 91$, then the value of *k*, correct to the nearest tenth, is __________.

17. If $B > 0$, $C > 0$, and $A = B^2C$, then $\log_{10}(C)$ is equal to

A. $2\log_{10}(B) - \log_{10}(A)$
B. $\log_{10}(A) - 2\log_{10}(B)$
C. $\log_{10}(2B) - \log_{10}(A)$
E. $\log_{10}(A) - \log_{10}(2B)$

18. The expression $3^{\log_2(8)}$ is equal to
A. 3
B. 12
C. 27
D. 81

19. If $2\log_2 x + \log_2 16 = \log_2 9$, then the value of x is

A. $\frac{7}{2}$
B. $\frac{9}{8}$
C. $\frac{3}{4}$
D. $\frac{9}{32}$

20. If $5\log_7 y = 2\log_7 m$, then m is equal to

A. $y^{\frac{5}{2}}$

B. $y^{\frac{2}{5}}$

C. y^5

D. y^{10}

21. The value of $\sum_{n=1}^{5} \log_{10} n$ is

A. $\log_{10} 5$

B. $\log_{10} 6$

C. $\log_{10} 15$

D. $\log_{10} 120$

22.

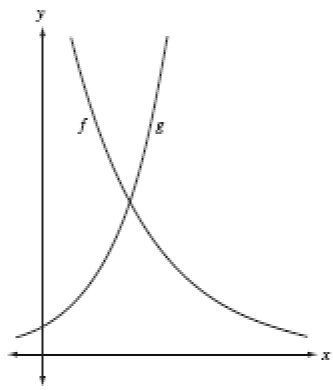
The expression $\log_a(a^4 b) - \log_a(ab)$ is equivalent to

A. 3
B. 4
C. $3a$
D. a^3

23

Use the following information to answer the next question.

The partial graphs of $f(x) = 4^{-x+2}$ and $g(x) = 3^{2x}$ are shown below.



The solution to the equation $4^{-x+2} = 3^{2x}$, correct to the nearest hundredth, is

A. 5.47
B. 2.55
C. 0.77
D. 0.43

9. Numerical Response

If $ab = 24$, then to the nearest hundredth, the value of $2\log_{10} a + 2\log_{10} b$, where $a, b > 0$, is __________.

(Record your answer in the numerical-response section on the answer sheet.)

24

The expression $\log_{\frac{1}{3}}\left(\frac{1}{r}\right)$, $r > 3$, is equal to

A. $\log_3 r$

B. $\log_r 3$

C. $\log(3r)$

D. $\log_3\left(\frac{1}{r}\right)$

25. Verify that $\left(3^{\log x}\right)\left(3^{\log x}\right)$ = $3^{\log x^2}$, where x > 0.

26.

Solve $\log_7(x+1) + \log_7(x-5) = 1$, and verify your solution.

Numerical Response 10

In the equation $3^{2x+1} = 7$, the value of x, to the nearest hundredth, is ______________.

27.

If $\log_3 x = 15$, then $\log_3\left(\frac{1}{3}x\right)$ is equal to

A. 14
B. 12
C. 5
D. -15

2.5 Graph and analyze an exponential function using technology.

1.Graph of $y = a^x$ and the graph of $y = \left(\frac{1}{a}\right)^x$, where $a > 0$, are reflections of each other about the

A. x-axis
B. y-axis
C. line $y = x$
D. line $y = -x$

Numerical Response

11. The solution to a particular problem is the intersection point of two exponential functions, $y = \left(\frac{1}{2}\right)^{x-1}$ and $y = 3^x$. Using a graphing calculator, a student found that the *y*-coordinate of this point, correct to the nearest hundredth, was __________.

2. A beam of light passes through a piece of opaque material. The intensity I of the beam of light as it exits the material is given by

$$I = \left(4^{-ct}\right) \times I_0,$$

where I_0 is the initial intensity, c is the absorption factor, and t is the thickness of the material in centimetres. If the intensity of the beam I, the initial intensity I_0, and the absorption factor c are known and $\frac{I}{I_0} = R$, then an expression for t is

A. $\frac{-\log R}{4c}$

B. $\frac{-\log R}{c \log 4}$

C. $\frac{-1}{c} \log\left(\frac{R}{4}\right)$

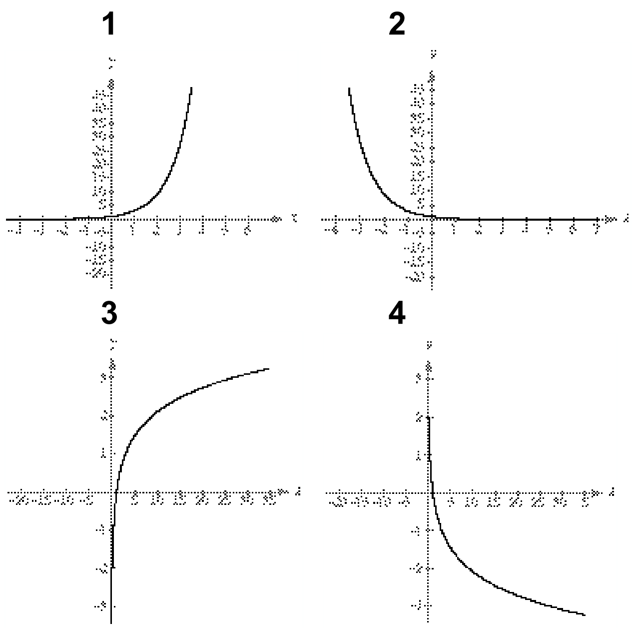
D. $\frac{-\log(R-4)}{c}$

3. The *y*-intercept of the graph of $y = a^x, a > 0$, is

A. 0
B. 1
C. undefined
D. dependent on the value of a

Numerical Response

12. Graphs of Four Functions



Match each of the graphs, as numbered above, to the statement below that describes it.

This graph represents an exponential function with a base between 0 and 1.
____________ (Record in the first column.)

This graph represents an exponential function with a base greater than 1.
____________ (Record in the second column.)

This graph represents a logarithmic function with a base between 0 and 1.
____________ (Record in the third column.)

This graph represents a logarithmic function with a base greater than 1.
____________ (Record in the fourth column.)

Use a graphing method to determine the value of x if $3^x = 4$.

4.

The graph of $y = f(x) = b^x$, where $b > 1$, is translated such that the equation of the new graph is expressed as $y - 2 = f(x - 1)$. The range of the new function is

A. $y > 2$
B. $y > 3$
C. $y > -1$
D. $y > -2$

2.6 Model, graph, and apply exponential functions to solve problems. [R, T, V]

1. The population of a city was 173 500 on January 1, 1978, and it was 294 000 on January 1, 1992. If the growth rate of the city can be modeled as an exponential function, then the average annual growth rate of the city, expressed to the nearest tenth of a percentage, was

   A. 1.0%

   B. 3.8%

   C. 6.9%

   D. 12.1%

2. The price of a particular product doubles every 35 years. If the price of the product was $16.40 on January 1, 1996, then the price of the product will be $36.50 in the year

   A. 2028

   B. 2031

   C. 2036

   D. 2040

3. The number of cellular phone customers of a particular telephone company increased by an average of 40%/a from 1994 to 1998. If there were 963 300 customers on January 2, 1998, then the number of customers on January 2, 1994, to the nearest hundred, would have been

   A. 3 700 600

   B. 688 100

   C. 250 800

   D. 172 100

4. In 1996, a particular car was valued at \$27 500 and its value decreased exponentially each year afterward. For each of the first 7 years, the value of the car decreased by 24% of the previous year's value. If t is the number of years and v is the value of the car, then the equation for the car's value when $t \le 7$ is

A. $v = 27500(1.76)^t$

B. $v = 27500(1.24)^t$

C. $v = 27500(0.76)^t$

D. $v = 27500(0.24)^t$

5. **Radioactive Decay**

$$m(t) = m_o \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where $m(t)$= mass of radioactive material at time t (in minutes)

m_0 = original mass

t = time in minutes

h = half-life in minutes

After 204 minutes, a sample of Plutonium-232 decayed to $\frac{1}{64}$ of its original mass. The half-life of Plutonium-232 is approximately

A. 0.3 minutes

B. 6 minutes

C. 34 minutes

D. 1 224 minutes

6. **Radioactive Decay**

$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{h}},$$

where $N(t)$= the mass present at time t

$N_0 = N(0)$

t = time in days

h = half-life of the material in days

The half-life of Iodine-126 is 13 days. The length of time, to the nearest day, that it will take 10 g of Iodine-126 to decay to 0.1 g is

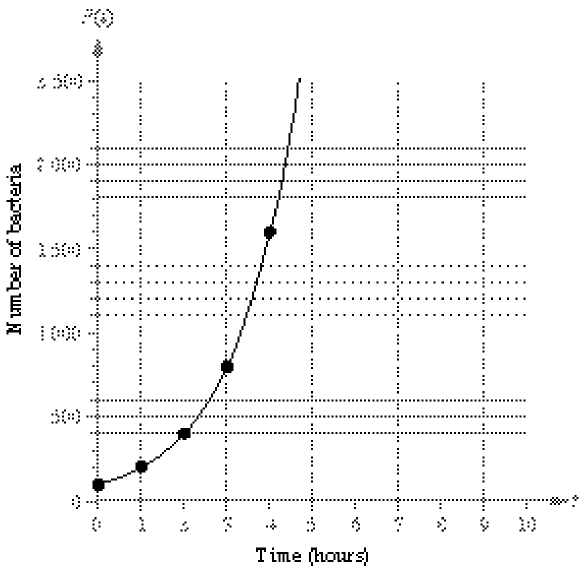
A. 130 days

B. 86 days

C. 79 days

D. 30 days

7. In an experiment, the observed number of bacteria at time *t*, in hours, is graphed. A smooth curve is drawn through the points, as shown below.



If the approximate number of bacteria $P(t)$ at any time t is modelled by exponential growth, then $P(t)$ is given by

A. $P(t) = 2(100)^t$

B. $P(t) = 100^{\frac{t}{2}}$

C. $P(t) = 2(100)^{2t}$

D. $P(t) = 100(2)^t$

8. The population of a small village is now 807, which is 3 times what it was 5 years ago. If the population continues to increase exponentially at this rate, then the population 7 ears from now will be

A. 1 560

B. 2 905

C. 3 757

D. 4 842

9. Atmospheric air pressure is halved for every 5.5 km increase in altitude above the surface of the Earth. The approximate air pressure at an altitude of 25 km, as a percentage of the surface air pressure, is

A. 1.2 %

B. 4.3 %

C. 23 %

D. 86 %

10. The number of E. coli bacteria at time t in hours is given by $N(t) = N_o(8)^t$, where N_o is the initial number of bacteria at $t = 0$. If the initial number of bacteria is 4 000, then the expected number of bacteria 1 h 15 min later is

A. 4 600

B. 5 000

C. 43 713

D. 53 817

11. In a nuclear disaster at Chernobyl in April 1986, approximately 12 600 kg of radioactive iodine-131 was released into the atmosphere. The half-life of iodine-131 is 8.04 days; therefore, after 8.04 days, half of the iodine-131 had decayed. The amount, $N(t)$, of iodine-131, in kg, remaining after t days is given by

$$N(t) = 12600\left(\frac{1}{2}\right)^{\frac{t}{8.04}}$$

The approximate mass of iodine-131 remaining after 30 days was

A. 70 kg

B. 131 kg

C. 420 kg

D. 949 kg

12. Half-life of Phosphorus-32

$$A(t) = A_{\text{o}}\left(\frac{1}{2}\right)^{\frac{t}{h}}, \text{ where}$$

$A(t)$ = the mass present at time t
A_o = $A(0)$
t = time
h = half-life of phosphorus-32

The half-life of phosphorus-32 is 14.3 days. The length of time that it will take 96.2 g of phosphorus-32 to decay to 12.5 g, to the nearest day, is

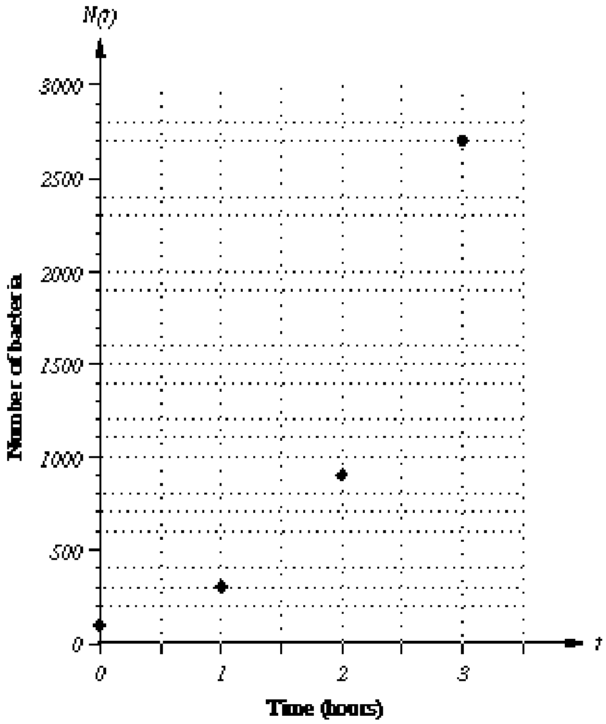
A. 8 days

B. 26 days

C. 42 days

D. 52 days

13. The graph of the number of bacteria in a culture counted at t = 0, 1, 2, 3 is shown below.



An exponential function of the form $N(t) = N_o(k)^{at}$, where N_o, k, and a are natural numbers, that fits the experimental data is

A. $N(t) = 100(2)^t$

B. $N(t) = 100(3)^t$

C. $N(t) = 100(3)^{2t}$

D. $N(t) = 100(2)^{3t}$

14. A strain of bacteria grows exponentially. Immediately after 2 h there were 1 000 bacteria, and immediately after 3 h there were 5 000 bacteria. If no bacteria die, then the time to the nearest hundredth of an hour for the population to double is

A. 0.20 h

B. 0.43 h

C. 1.29 h

D. 2.32 h

15.

Use the following information to answer the next question.

The population, P, of an insect colony at 3-month intervals is shown in the table below.

Date	Jan 1 2003	Apr 1 2003	July 1 2003	Oct 1 2003	Jan 1 2004
Population	500	1 000	2 000	4 000	8 000

If it is assumed that the growth rate remains constant, then the exponential function that describes the population of insects in terms of n, where n is the number of **years** after Jan 1, 2003, is

A. $P = 500(2)^{4n}$

B. $P = 500(2)^{3n}$

C. $P = 500(2)^{n}$

D. $P = 500(2)^{\frac{n}{4}}$

16.

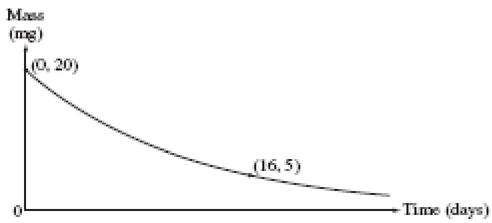
The population of a city was 173 500 on January 1, 1988, and it was 294 000 on January 1, 2002. If the growth rate of the city can be modelled as an exponential function, then the average annual growth ra' . of the city, expressed to the nearest tenth of a percent, was

A. 1.0%
B. 3.8%
C. 6.9%
D. 12.1%

17.

Use the following information to answer the next question.

The graph below represents the mass of a particular isotope of iodine in a person's body as a function of time. The initial mass of 20 mg decays to 5 mg after 16 complete days.



If the half-life of an isotope is the time it takes to decay to half its mass, then the half-life of this isotope of iodine, in hours, is approximately

A. 120 h
B. 192 h
C. 240 h
D. 384 h

13. Numerical Response

Earthquake intensity is given by $I = I_0(10)^m$, where I_0 is the reference intensity and m is magnitude. A particular major earthquake of magnitude 7.9 is 120 times as intense as a particular minor earthquake. The magnitude, to the nearest tenth, of the minor earthquake is ____________.

2.7 Change functions from exponential form to logarithmic form and vice versa. [CN, E]

1. If $\log_x\left(\frac{1}{64}\right) = -\frac{3}{2}$, then x is equal to

A. 16
B. 8
C. $\frac{1}{8}$
D. $\frac{1}{16}$

2. If $\log_3 x = 15$, then $\log_3\left(\frac{1}{3}x\right)$ is equal to

A. 14
B. 12
C. 5
D. -15

3. The equation $y = 4^{3x}$ can also be written as

A. $y = \frac{\log_3 x}{4}$

B. $y = \frac{\log_4 x}{3}$

C. $x = \frac{\log_3 y}{4}$

D. $x = \frac{\log_4 y}{3}$

Numerical Response

14. If $\log_7 m = -\frac{2}{3}$, then the value of m, correct to the nearest hundredth, is __________.

4. If $\log_a 16 = 2$ and $\log_8 b = 2$, then $\log_2(ab)$ equals

A. 256

B. 128

C. 10

D. 8

5. If $\log_4(2y+x) = 2$, $(2y+x) > 0$, then y in terms of x is

A. $\frac{16-x}{2}$

B. $\frac{8-x}{2}$

C. $\frac{8+x}{2}$

D. $\frac{16+x}{2}$

6. If $\log_b(x+d) = f$, where $b > 0$ and $(x+d) > 0$, then *x*, in terms of *b, d,* and *f*, is

A. $b^f - d$

B. $f^b - d$

C. $d^f - b$

D. $f^d - b$

7. The inverse, f^{-1}, of the function $f(x) = \log_5(x), x > 0$, is

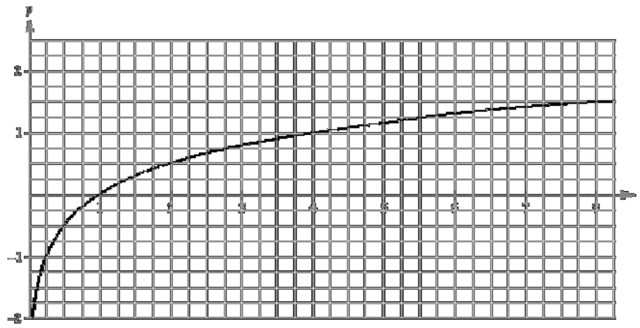
A. $f^{-1}(x) = 5^x, x \in R$

B. $f^{-1}(x) = x^5, x \in R$

C. $f^{-1}(x) = \log_x(5), x > 0$

D. $f^{-1}(x) = \log_5\left(\frac{1}{x}\right), x > 0$

A partial graph of $y = \log_a x$ passes through the point $\left(8, \frac{3}{2}\right)$, as shown below.



Numerical Response

15. Correct to the nearest tenth, the value of *a* is _______ .

8. An exponential form of $n\log_a b = c, a > 1, b > 0$, and $n \neq 0$, is

A. $a^c = b^n$

B. $a^b = c^n$

C. $b^a = c^n$

D. $b^c = a^n$

9. The point $(9,4)$ lies on the graph of $y = 2\log_b x$. Correct to the nearest tenth, the value of b is

A. 0.5

B. 1.4

C. 2.0

D. 3.0

10. If $14 = 7^{2x}$, then the value of x is

A. $\frac{\log 7}{2}$

B. $\frac{\log 2}{2}$

C. $\frac{\log 14}{\log 49}$

D. $\frac{\log 14}{\log \sqrt{7}}$

11. If $\log_x \left(\frac{1}{9}\right) = -2$, then the value of x is

A. 3

B. –3

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

12. If $\log x = 2.4$ and $\log y = -1.6$, then the value of $\dfrac{x}{y}$ is

A. -1.5

B. 4

C. $10^{-1.5}$

D. 10^{4}

13. If $\log_x \dfrac{125}{27} = -\dfrac{3}{2}$, then the value of x is

A. $\dfrac{9}{25}$

B. $\dfrac{25}{9}$

C. $-\dfrac{9}{25}$

D. $-\dfrac{25}{9}$

14. A logarithmic form of $81^{\frac{3}{4}} = 27$ is

A. $\log_{27}\left(\dfrac{3}{4}\right) = 81$

B. $\log_{\frac{3}{4}}(27) = 81$

C. $\log_{27}(81) = \dfrac{3}{4}$

D. $\log_{81}(27) = \dfrac{3}{4}$

15. Given that $\log_b 64 = \dfrac{3}{2}$, the value of b is

A. 16

B. $42\dfrac{2}{3}$

C. 96

D. 512

16. If $\log_5(125x) = 25$, then the value of x is

A. 5^{28}

B. 5^{22}

C. $5^{\frac{25}{3}}$

D. 5

17. If $3\log_2 x = 12$, then x is equal to
A. 16
B. 8
C. 4
D. 2

18. If $W = 16^m \times 8^{2m-1}$, then $\log_2 W$ equal to
A. $3m-1$
B. $10m-1$
C. $10m-3$
D. 10^{3m-1}

19. If $\log_3 x = 2$ and $\log_2 y = x$, then y is equal to

A. 8

B. 9

C. 256

D. 512

Numerical Response

16. If $\log_2(2x-3) = 3$, then correct to the nearest tenth the value of x is __________.

Numerical Response

17. If $10^a = 4$, then 10^{1+2a}, correct to the nearest whole number, is equal to __________

20.

The equation $\log_k B = \frac{m}{n}$ is equivalent to

A. $\frac{m}{n} = \sqrt[k]{B}$

B. $B = \sqrt[k]{\frac{m}{n}}$

C. $B = \sqrt[n]{k^m}$

D. $k = \sqrt[n]{B^m}$

21.

If $\log_x\left(\frac{1}{64}\right) = -\frac{3}{2}$, then x is equal to

A. 16

B. 8

C. $\frac{1}{8}$

D. $\frac{1}{16}$

22.

The equation $y = 4^{3x}$ can also be written as

A. $y = \frac{\log_3 x}{4}$

B. $y = \frac{\log_4 x}{3}$

C. $x = \frac{\log_3 y}{4}$

D. $x = \frac{\log_4 y}{3}$

23.

If $\log_3 x = 15$, then $\log_3\left(\frac{1}{3}x\right)$ is equal to

A. 14
B. 12
C. 5
D. -15

2.8 Use logarithms to model practical problems. [CN, PS, V]

1. The equation that defines the decibel level for any sound is

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where L = loudness in decibels
I = intensity of sound being measured
I_0 = intensity of sound at the threshold of hearing

Given that normal conversation is 1 000 000 times as intense as I_0, then the loudness of normal conversation is

A. 5 decibels

B. 6 decibels

C. 16 decibels

D. 60 decibels

2. The lead concentration of human blood, C_B, measured in μg/100 mL, increases with the mean lead concentration of the environmental air to which a person is exposed, C_L, measured in μg/m^3, according to the formula $C_B = 60 \log C_L - 20$. If the lead concentration of samples of environmental air has values between 5.0 and 100.0 μg/m^3, then the range of lead content in human blood is

A. 21.9 to 100.0 μg/100 mL
B. 20.0 to 60.0 μg/100 mL
C. 2.6 to 100.0 μg/100 mL
D. 2.1 to 6.0 μg/100 mL

3.

Use the following information to answer the next question.

The gain, G, measured in decibels, of an amplifier is defined by the equation

$$G = 10\log\left(\frac{P}{P_i}\right)$$

where P is the output power of the amplifier, in watts, and P_i is the input power of the amplifier, in watts.

If the gain of the amplifier is 23 decibels, then the ratio $\frac{P}{P_i}$ is

A. $10\log(23)$
B. $\log(2.3)$
C. 10^{23}
D. $10^{2.3}$

4.

Use the following information to answer the next question.

An equation that defines the decibel level for any sound is

$L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where L = loudness in decibels
I = intensity of sound being measured
I_0 = intensity of sound at the threshold of hearing

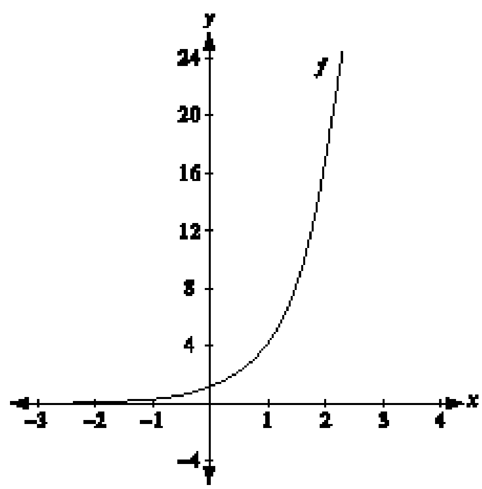
Given that normal conversation is 1 000 000 times as intense as I_0, the loudness of normal conversation is

A. 5 decibels
B. 6 decibels
C. 16 decibels
D. 60 decibels

2.9 Explain the relationship between the laws of logarithms and the laws of exponents. [C, T]

2.10 Graph and analyze logarithmic functions with and without technology. [R, T, V]

1. The partial graph of the exponential function $f(x) = 4^x$ is shown below.



The domain of the inverse function f^{-1} is

A. $x > 0, x \in R$

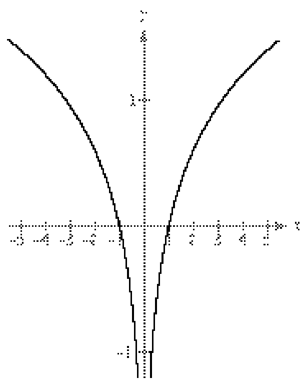
B. $x < 0, x \in R$

C. $x \geq 0, x \in R$

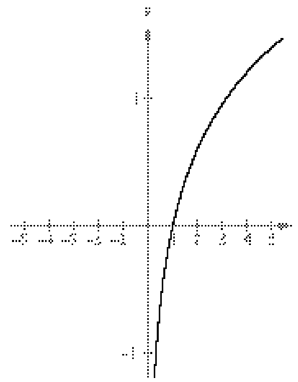
D. $x \in R$

2. A student used a graphing calculator to illustrate identities. The student assumed that $\log\left(x^2\right) = 2\log x$ because $\log_a\left(M^n\right) = n\log_a M$. The student graphed $y = \log\left(x^2\right)$ and obtained the graph shown below on the left. The student then graphed $y = 2\log x$ and obtained the graph shown below on the right.

Graph of $y = \log\left(x^2\right)$



Graph of $y = 2\log x$



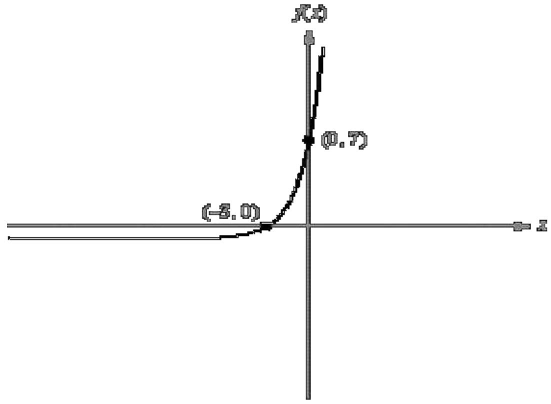
The student realized that the reason why the graphs are not identical is that

A. $\log 0$ is not defined
B. where $x < 0$, $\log\left(x^2\right)$ is defined and $\log x$ is not defined
C. where $x < 0$, $\log x$ is defined and $\log\left(x^2\right)$ is not defined
D. the range of $y = \log\left(x^2\right)$ is different from the range of $y = 2\log x$

3. The *x*-intercept of the graph of $y = \log_b x$, where $b > 0$ and $b \neq 1$, is

A. 0
B. 1
C. undefined
D. dependent on the value of *b*

4. The partial graph of the exponential function $f(x)=2^{x+3}-1, x \in R$, is shown below.



If $g(x)$ is the inverse of $f(x)$, then the range of $g(x)$ is

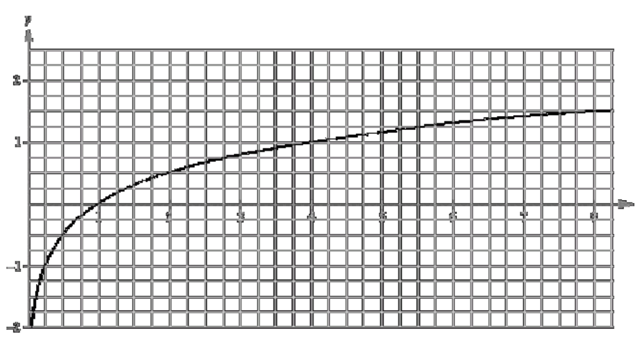
A. $g(x) \geq -1$

B. $g(x) > 0$

C. $g(x) > 1$

D. $g(x) \in R$

5. A partial graph of $y = \log_a x$ passes through the point $\left(8, \frac{3}{2}\right)$, as shown below.



Which of the following statements is true for the given information?

A. The range of $y = \log_a x$ is $y \leq 2$.
B. The domain of $y = \log_a x$ is $x \in R$.
C. The graph of $y = \log_a x$ has no *y*-intercept.
D. The *y*-intercept of the graph of $y = \log_a x$ is –2.5.

6. A student wishes to graph $y = \log_3 x$ on a calculator, but her calculator can only graph logarithmic functions if the equations are expressed in common logarithms. She could obtain the graph of $y = \log_3 x$ by graphing

A. $y = \dfrac{\log x}{3}$

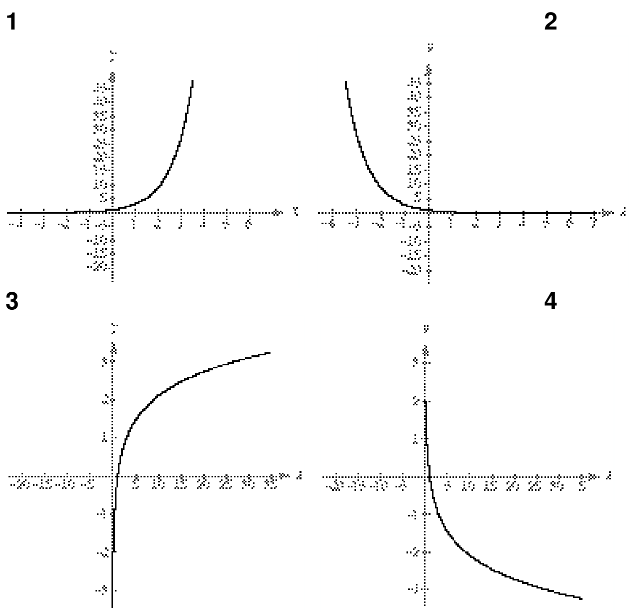
B. $y = \dfrac{\log x}{\log 3}$

C. $y = \log(x - 3)$

D. $y = \log\left(\dfrac{x}{3}\right)$

Numerical Response

18. Graphs of Four Functions



Match each of the graphs, as numbered above, to the statement below that describes it.

This graph represents an exponential function with a base between 0 and 1.
___________ (Record in the first column.)
This graph represents an exponential function with a base greater than 1.
___________ (Record in the second column.)
This graph represents a logarithmic function with a base between 0 and 1.
___________ (Record in the third column.)
This graph represents a logarithmic function with a base greater than 1.
___________ (Record in the fourth column.)

7. A student wants to use a graphing calculator to graph $y = \log_5 x$. If the calculator accepts only common logarithms, then an equivalent equation that could be used to obtain the graph is

A. $y = \dfrac{\log x}{\log 5}$

B. $y = \log x - \log 5$

C. $y = 5 \log x$

D. $y = \dfrac{\log x}{5}$

8. The *y*-intercept of the graph $y = \log_3(x+2)$ is

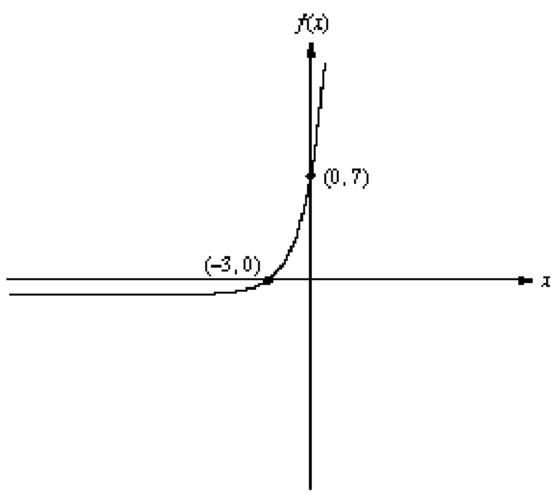
A. 2

B. 0.631

C. –1

D. -1.76

9. The graph of the exponential function $f(x) = 2^{x+3} - 1$ is shown below.



If $g(x)$ is the inverse of $f(x)$, then the domain of $g(x)$ is

A. $x > -1$

B. $x > 0$

C. $x > 1$

D. $x \in R$

Exponents, Logarithms and Geometric Series Answer Key

2.1

1C	10A	19C	
2D	11B	20A	
3C	12C	21D	
4B	13A	22B	
5B	14A	23C	
6B	15B		
7C	16C		
8B	17C		
9D	18D		

2.2

1B			

2.3

1A			
2C			
3D			

2.4

1D	10C	19C	
2C	11C	20A	
3D	12C	21D	
4A	13C	22D	
5A	14D	23C	
6B	15D	24A	
7C	16A	25	
8C	17B	26	
9B	18C	27A	

2.5

1B			
2B			
3B			
4A			

2.6

1B	10D		
2C	11D		
3C	12C		
4C	13B		
5C	14B		
6B	15A		
7D	16B		
8C	17B		
9B			

2.7

1A	10C	19D	
2A	11A	20C	
3D	12D	21A	
4D	13A	22D	
5A	14D	23A	
6A	15A		
7A	16B		
8A	17A		
9D	18C		

2.8

1D			
2A			
3D			
4D			

2.9

2.10

1A			
2B			
3B			
4D			
5C			
6B			
7A			
8B			
9A			

Numerical Response

1) 22.3	10) 0.39		
2) 75.4	11) 1.53		
3) 0.02	12) 2.7		
4) 0.36	13) 5.8		
5) 0.5	14) 0.27		
6) 1.7	15) 4.0		
7) 729	16) 5.5		
8) 12.3	17) 160		
9) 2.76	18) 2143		

WRITTEN RESPONSE #1

Legend has it that the game of chess was invented for a Persian king by one of his servants. The king asked the servant how he would like to be paid for the game. The servant stated that he would like one grain of rice to be placed on the first square of the chessboard, two grains of rice to be placed on the second square, four grains of rice on the third square, eight grains of rice on the fourth square, and so on. Each subsequent square was to have twice as many grains of rice than the previous square, as shown in the chart below.

Square	1	2	3	4	5	6	7
Number of grains of rice in the square	1	2	4	8			

- Complete the chart for squares 5, 6, and 7.

- Write an expression that represents the numbers of grains of rice, R, on the n^{th} square of the chessboard.

- If the king only had 1 000 000 grains of rice, which square would he be in the process of filling when he ran out of rice?

- If the servant had asked for the payment of rice to be placed on only the black squares of the chessboard, with 1 grain on the first black square, 2 grains on the second black square, 4 grains on the third black square, and so on, then only 32 squares would have rice on them. What would be the **total** number of grains of rice on the chessboard if the king filled every black square?

WRITTEN RESPONSE #1 SOLUTION

1. •

Square	1	2	3	4	5	6	7
# of grains	1	2	4	8	16	32	64

• From the table we see that the number of grains doubles in each successive square.

So, the expression that represents the number of grains in square *n* can be found using the general term formula $t_n = a \cdot r^{n-1}$ where t_n would be given as *R* (the number of grains) a, the first term is one and r, the common ratio is 2 so the expression is $R = 1 \cdot 2^{n-1}$ or, more simply, $R = 2^{n-1}$.

▪ We'll use the formula for sum of a geometric series.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Plugging in *a* = 1, *r* = 2, S_n = 1,000,000, we get

$$1000000 = \frac{1(2^n - 1)}{2 - 1}$$
$$1000000 = 2^n - 1$$
$$1000001 = 2^n$$

Take logs of both sides of the equation

$$\log 1000001 = n \log 2$$
$$\frac{\log 1000001}{\log 2} = n$$
$$19.9 \approx n$$

So, we know that the king can fill 19.9 squares with 1,000,000 grains of rice. The king ran out of rice while filling square 20.

▪ Again we use the formula for sum of a geometric series.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Plugging in *a* = 1, *r* = 2, *n* = 32, we get

$$S_{32} = \frac{1(2^{32} - 1)}{2 - 1}$$
$$= 4,294,967,295$$

If the king filled every black square, there would be 4,294,967,295 grains.

WRITTEN RESPONSE #2

☐ A student places a spherical jawbreaker in a cup of water. As the surface of the jawbreaker dissolves, the jawbreaker maintains a spherical shape. The volume of the jawbreaker, V, as a function of time, *t*, is shown below.

Time *t* (min)	Volume *V* (mm^3)
0	905.000
1	814.500
2	733.050
3	659.745
4	

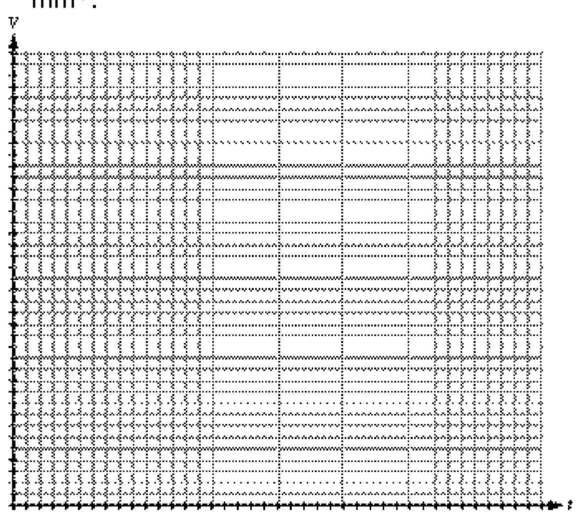
- If it is assumed that this geometric pattern continues, what is the volume of the jawbreaker, to the nearest thousandth of a mm^3, when t = 4 min?

- Write an equation for volume, *V*, as a function of time, *t*, in the form $V = a(b^t)$.

- Using your equation from the previous bullet, determine, to the nearest tenth of a minute, the time it takes the jawbreaker to reach 100 mm^3.

- Create a graph of the equation that you found for volume in the second bullet. State an appropriate viewing window for the first 40 minutes of the dissolving time. Use the form $x:[x_{\min}, x_{mxa}, x_{scl}]$ and $y:[y_{\min}, y_{\max}, y_{scl}]$

  On the grid below, sketch your graph and indicate the point where the volume equals 100 mm^3.



- A particular machine produces many different coloured jawbreakers. The probability that a black jawbreaker is produced is 0.12. If the jawbreakers are packaged in boxes of 60, then what is the probability, to the nearest hundredth, that a box will contain at least 5 black jawbreakers?

WRITTEN RESPONSE #2 SOLUTION

1. • The volumes of the jawbreaker at time *t* form a geometric sequence.

   In this sequence, the first term is *a* = 905.000.

   To find the common ratio, we find the quotient of two consecutive terms in the sequence:

   $$r = \frac{t_1}{t_0} = \frac{814.500}{905.000} = 0.9$$

   The volume of the jawbreaker at *t* = 4 min is:

   $t_4 = t_3 \times 0.9 = 659.745 \times 0.9 = 593.771$ mm^3.

   • $a = 905$, the common ratio is $b = 0.9$ so the equation is $V = 905(0.9)^t$, $t \geq 0$

   • We use the equation $V = 905(0.9)^t$ to determine the time it takes the jawbreaker to reach a volume of 100 mm^3.

   Plugging in *V* = 100, we get

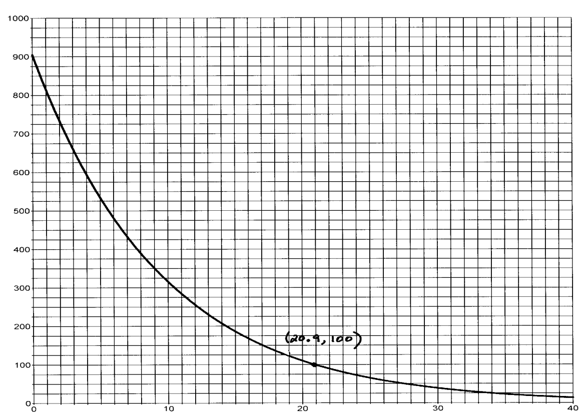
   $$100 = 905(0.9)^t$$
   $$0.1105 = (0.9)^t$$

   We take logs on both sides to get

   $$\log 0.1105 = \log(0.9)^t$$

   $$\log 0.1105 = t \log 0.9$$
   $$\frac{\log 0.1105}{\log 0.9} = t$$
   $$20.9 = t$$

   It takes 20.9 minutes for the jawbreaker to reach a volume of 100 mm^3.



   An appropriate viewing window is:

   $x : [0,40,5]$ $y : [1,1000,100]$

We want to find the probability that a box of 60 jawbreakers contains at least 5 black ones.

   The probability that exactly *k* jawbreakers out of 60 are black is a binomial probability. It is

given by the expression
$P(k) = {}_{60}C_k\,(0.12)^k(0.88)^{60-k}$.

Find the probabilities that 0, 1, 2, 3 and 4 jawbreakers are black and subtract them from 1.

Using the expression for binomial probability
$P(k) = {}_{60}C_k\,(0.12)^k(0.88)^{60-k}$, and plugging in $k = 0, 1, 2, 3, 4$, we get

$P(0) = {}_{60}C_0\,(0.12)^0(0.88)^{60} = 0.0005$

$P(1) = {}_{60}C_1\,(0.12)^1(0.88)^{59} = 0.0038$

$P(2) = {}_{60}C_2\,(0.12)^2(0.88)^{58} = 0.0154$

$P(3) = {}_{60}C_3\,(0.12)^3(0.88)^{57} = 0.0405$

$P(4) = {}_{60}C_4\,(0.12)^4(0.88)^{56} = 0.0787$

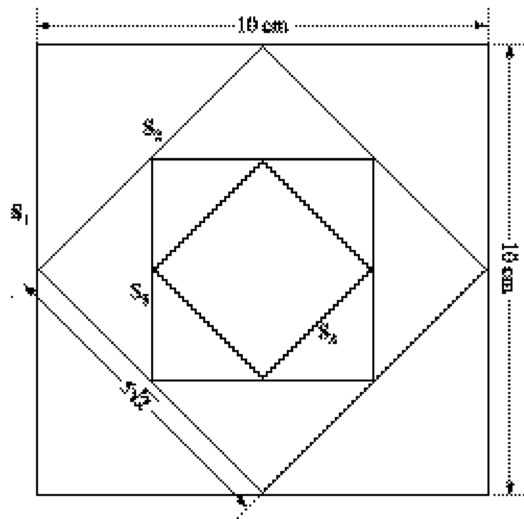
The probability that at least 5 jawbreakers are black is

$$\begin{aligned} &1 - P(0) - P(1) - P(2) - P(3) - P(4) \\ &\quad = 1 - 0.0005 - 0.0038 - 0.0154 - 0.0405 - 0.0787 \\ &\quad = 0.8611 \end{aligned}$$

Correct to the nearest hundredth, the probability is 0.86

WRITTEN RESPONSE #3

☐ A square, S_1, with sides of 10 cm is modeled below. A second square, S_2, with sides of $5\sqrt{2}$ cm is inscribed in S_1 so that the vertices of S_2 lie at the midpoints of the sides of S_1. A third square, S_3, is inscribed in S_2 so that the vertices of S_3 lie at the midpoints of the sides of S_2. This process is continued indefinitely.



- The lengths of the sides of S_1, S_2, S_3, … form a geometric sequence. Determine the exact length of a side of S_3.

- The **perimeter** of each square forms a term in a geometric sequence. Determine the **exact** value of the **perimeter** of S_4.

- Show how the **areas** of consecutive squares form a sequence that is geometric.

- Determine an expression that represents the sum of the first k terms of the geometric sequence calculated in the previous bullet.

WRITTEN RESPONES #3 SOLUTIONS

- We are told that the lengths of the sides: 10, $5\sqrt{2}$, S_3, … form a geometric sequence.

  To find the common ratio, we divide $r = \frac{t_2}{t_1} = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2}$.

  To find the 3rd term of the sequence we multiply the 2nd term by the common ratio.
  $S_3 = 5\sqrt{2} \times \frac{\sqrt{2}}{2} = \frac{10}{2} = 5$

  So the length of S_3 is 5 cm.

- To find the perimeter of S_4, we first find the length of the side S_4 and then multiply it by 4.

  To find the length of side S_4, we multiply S_3 by the common ratio.
  $S_4 = 5 \times \frac{\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$

  The perimeter of the fourth square is 4 times this length
  $P_4 = 4\left(\frac{5\sqrt{2}}{2}\right) = 10\sqrt{2} \doteq 14.14$

  So the perimeter of S_4 is ~14.14cm.

- The area of square S_1 is (10)(10) = 100cm^2
  The area of square S_2 is $(5\sqrt{2})(5\sqrt{2})$ = 50cm^2
  The area of square S_3 is (5)(5) = 25cm^2

  100,50,25… is a geometric sequence with common ratio $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{1}{2}$

- To find the sum of the first k terms of the geometric sequence 100,50,25, … we use the formula $S_n = \frac{a(r^n - 1)}{r - 1}$

  Plugging in a = 100,r = 0.5, n = k, we get
  $S_k = \frac{100(0.5^k - 1)}{0.5 - 1} = \frac{100(0.5^k - 1}{-0.5}$
  $= -200(0.5^k - 1)$

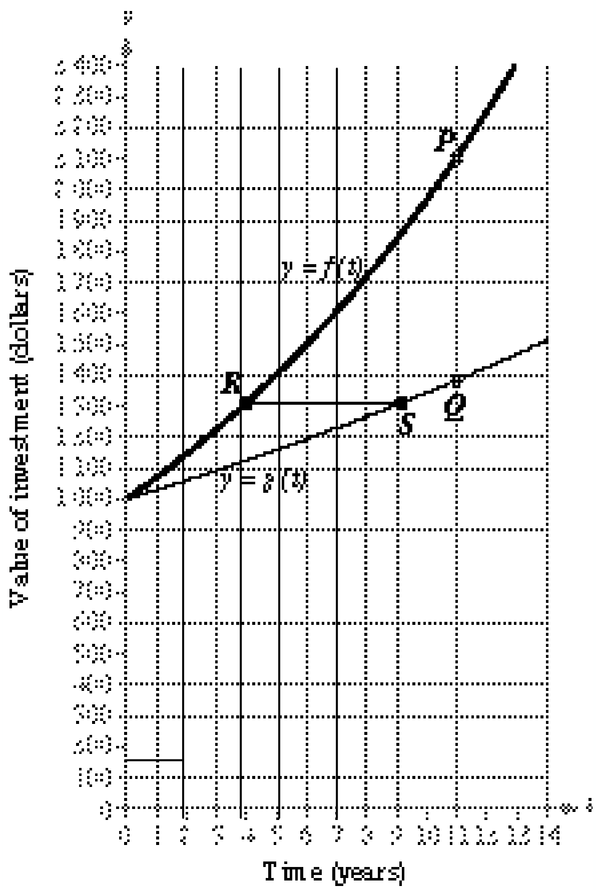
WRITTEN RESPONSE #4

Student A and student B each deposited $1 000 in different investment plans

The partial graph of the exponential function, *f*, approximates the value of the $1 000 that student A deposited into a plan earning 7%/a interest compounded annually.

The partial graph of the exponential function, *g*, approximates the value of the $1 000 that student B deposited into a plan earning *i*%/a interest compounded annually.

Both graphs are shown below



The function $f(t) = 1000(1.07)^t, t \in N$, represents the value in dollars, after *t* years, of the $1 000 that student A invested at 7%/a compounded annually

- Determine algebraically the minimum amount of time it will take for student A’s investment to be worth at least $2 100.

  Explain how the graph supports or does not support your answer.

- The point $(2,1060.90)$ lies on the graph of $y = g(t)$. Algebraically determine the interest rate.
- By referring to the **two** investments described in the problem, explain what the length of the vertical line segment $\overline{PQ}$ and the length of the horizontal line segment $\overline{RS}$ represent.

WRITTEN RESPONSE #4 SOLUTION

$$f(t) = 1000(1.07)^t$$
$$2100 = 1000(1.07)^t$$
$$\frac{\log 2.1}{\log 1.07} = t$$

It will take 11 years. The graph of $y = f(t)$ appears to pass through $(11, 2100)$; therefore it supports my answer.

• $g(t) = 1000\left(1 + \frac{i}{100}\right)^t$, where i = the interest rate expressed as a percent

$$1060.90 = 1000\left(1 + \frac{i}{100}\right)^2$$
$$1.0609 = \left(1 + \frac{i}{100}\right)^2$$
$$3 = i$$

Therefore the interest rate is 3%

• The length of line segment $\overline{PQ}$ represents the difference in the amounts of the two investments at a particular point in time.

The length of line segment $\overline{RS}$ represents the difference in time required for the two investments to reach the same value.

WRITTEN RESPONSE #5

A mathematics class was asked to solve the equation

$$3^{x+2} = 6^x$$

The attempts of two students to solve the equation are shown below. Each student made one error that led to an incorrect solution.

Student A

$$\begin{aligned} 3^{x+2} &= 6^x \\ 3^{x+2} &= 3^{2x} \\ x+2 &= 2x \\ 2 &= 2x - x \\ 2 &= x \end{aligned}$$

Student B

$$\begin{aligned} 3^{x+2} &= 6^x \\ \log 3^{x+2} &= \log 6^x \\ x + 2\log 3 &= x\log 6 \\ 2\log 3 &= -x + x\log 6 \\ 2\log 3 &= x(-1 + \log 6) \\ \frac{2\log 3}{-1+\log 6} &= x \end{aligned}$$

x is approximately –4.3

- Show that neither $x = 2$ nor $x = -4.3$ satisfies the equation $3^{x+2} = 6^x$.

- Identify the error that was made by each student and state why each error leads to an incorrect answer.

- Algebraically solve the equation $3^{x+2} = 6^x$ and give the solution, correct to the nearest hundredth.

WRITTEN RESPONSE #5 SOLUTION

- **Student A** **Student B**

Student A	Student B
$3^{x+2} = 6^x$	$3^{x+2} = 6^x$
$3^{2+2} = 6^2$	$3^{-4.3+2} = 6^{-4.3}$
$81 = 36$	$0.0799 = 0.00045$

Both of these equations are false; the left hand side does not equal the right hand side, therefore, neither solution satisfies the equation.

- **Student A**
  Student A stated that $6^x = 3^{2x}$ which would indicate that $6 = 3^2$ which is obviously not correct.
  Student B
  In the third line of their work the student did not realize that log 3 was multiplied by (x + 2), so that in line 4 they incorrectly subtracted *x*.

- $$3^{x+2} = 6^x$$
$$\log 3^{x+2} = \log 6^x$$
$$(x+2)\log 3 = x\log 6$$
$$x\log 3 + 2\log 3 = x\log 6$$
$$2\log 3 = x\log 6 - x\log 3$$
$$2\log 3 = x(\log 6 - \log 3)$$
$$\frac{2\log 3}{\log 6 - \log 3} = x$$
$$3.17 = x$$

WRITTEN RESPONSE #6

☐ The table below shows the number of new phone numbers assigned and the total number of phone numbers in use in a small Alberta community, over a six-year period.

Year	Number of New Phone Numbers Assigned	Total Number of Phone Numbers In Use	Growth Rate
1989	—	2261	—
1990	172	2433	7.6%
1991	136	2569	
1992	172	2741	
1993	154	2895	
1994	155	3050	
1995	111	3161	

Complete the chart above by determining the annual growth rates, correct to the nearest tenth of a percentage.

- Assume that the mean growth rate is 6%. Using this growth rate, determine an exponential function, $P(t)$, that approximates the total number of phone numbers in use where t is the number of years after 1989.

- All the telephone numbers in this community belong to the 598-exchange. This means that they all begin with the digits 598 followed by any 4 digits. What is the total possible number of telephone numbers beginning with the digits 598? What assumptions did you make to obtain your answer?

- Use the mean growth rate of 6% to predict the year in which a new exchange will be required for the community.

WRITTEN RESPONSE #6 SOLUTIONS

Year	Number of New Phone Numbers Assigned	Total Number of Phone Numbers In Use	Growth Rate
1989	—	2261	—
1990	172	2433	7.6%
1991	136	2569	**5.6%**
1992	172	2741	**6.7%**
1993	154	2895	**5.6%**
1994	155	3050	**5.4%**
1995	111	3161	**3.6%**

- $P(t) = 2261(1.06)^t$

  PERMUTATIONS AND COMBINATIONS:
  ANY FOUR DIGITS – 10 000 total phone numbers beginning with the digits 598. The assumption made is that the digits can be repeated.
- $P(t) = 2261(1.06)^t$

  $10000 = 2261(1.06)^t$

  $4.24 = 1.06^t$

  $t = \log_{1.06} 4.42$

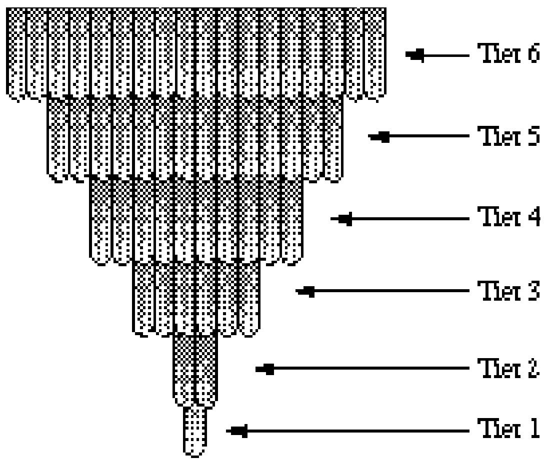
  $t = \dfrac{\log 4.42}{\log 1.06}$

  $t = 25.5$

  Therefore, 25 and a half years after 1989 (i.e. the year 2014), a new exchange will be required for this community.

WRITTEN RESPONSE #7

A lighting store specializes in custom-made chandeliers built in tiers. Each tier consists of a number of rectangular glass pieces arranged in a circular formation.



The chandelier shown is not the exact one described in this written-response question.

The numbers of pieces of glass in each successive tier of a chandelier form a geometric sequence. Every chandelier uses 1 glass piece for tier 1. In a chandelier with 5 or more tiers, there are 81 glass pieces in tier 5.

- Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.

Tier Number (n)	Number of glass pieces in n^{th} tier	Total number of glass pieces in a chandelier with n tiers
1	1	1
2		
3		
4		
5	81	

- The cost for each glass piece is $1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.

- Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.

- The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?

WRITTEN RESPONSE #7 SOLUTIONS

-

Tier Numb er (n)	Number of glass pieces in n^{th} tier	Total number of glass pieces in a chandelier with n tiers
1	1	1
2	3	4
3	9	13
4	27	40
5	81	121

- **GEOMETRIC SERIES:** $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$$S_7 = \frac{1(3^7 - 1)}{3 - 1} = 1093$$

$1093 \times \$1.75 = \$1912.75$. Therefore, the cost is $\$1912.75$

- $C(n) = 1.75 \times \left(\dfrac{3^n - 1}{2}\right) = 0.875(3^n - 1)$

- **STATISTICS:**
  Find the probability that a piece of glass measures between 3.9 and 4.1 using z-scores:

  Since $z = \dfrac{x - \mu}{\sigma}$, $z_{4.1} = \dfrac{4.1 - 4}{0.05} = 2$ and $z_{3.9} = \dfrac{3.9 - 4}{0.05} = -2$

  $ShadeNorm(-2,2)$ gives an area of 0.9545. This means that 95.45% are acceptable and, as a result 4.56% are rejected. Within 100 000 glass pieces, 4550 are going to be rejected.

WRITTEN RESPONSE #8

Use the following information to answer the next question

Chantal is saving to go on a trip to New Zealand. Beginning in 1998, every January 1, she purchases an investment certificate for $1000. The five certificates that she has now purchased each earn 6%/a compounded annually. Chantal sets up the table shown to keep track of her investments.

	Purchase Year				
	2002	**2001**	**2000**	**1999**	**1998**
Current Value as of January 1, 2002	$1000	$1060	$1123.60		

- Complete the chart by determining the current value of the investment certificates that she purchased in 1999 and 1998.

- Use the geometric sum formula to verify that the total amount Chantel has saved until now is equal to the sum of the 5 **current** values.

- Chantel has determined that she needs $14 500 for her trip to New Zealand. She plans to continue purchasing $1000 certificates on January 1 of each year and investing them at 6%/a compounded annually. Determine algebraically the minimum number of certificates that she will need to purchase in order to save at least $14 500.

- The total number of certificates that Chantel will need to purchase in order to save a total of at least $14 500 can also be determined graphically. Write an equation or equations that could be graphed in order to determine the total number of certificates that she will need to purchase, and describe how you would use the graph or graphs to find the answer.

WRITTEN RESPONSE #8 SOLUTIONS

-

	Purchase Year				
	2002	**2001**	**2000**	**1999**	**1998**
Current Value as of January 1, 2002	$1000	$1060	$1123.60	**$1191.02**	**$1262.48**

- Sum of the five current values is 1000 + 1060 + 1123.60 + 1191.02 + 1262.48 = $5637.10
  Geometric Sum Formula:

  Common ratio is $\frac{1060}{1000} = 1.06$, $a = 1000$ and $n = 5$

  If using $S_n = \frac{a(r^n - 1)}{r - 1}$

  $$S_5 = \frac{1000(1.06^5 - 1)}{1.06 - 1} = \$5637.09$$

  If using $S_n = \frac{r \times t_n - a}{r - 1}$

  $$S_5 = \frac{1.06(1262.48) - 1000}{1.06 - 1} = \$5637.15$$

- $$14500 = \frac{1000(1.06^n - 1)}{1.06 - 1}$$
  $$870 = 1000(1.06^n - 1)$$
  $$0.87 = 1.06^n - 1$$
  $$1.87 = 1.06^n$$
  $$n = \log_{1.06} 1.87$$
  $$n = \frac{\log 1.87}{\log 1.06}$$
  $$n = 10.7$$

  Therefore, she will need to purchase 11 certificates to have saved at least $14500.

- Possibility with one equation: $y_1 = \frac{1000(1.06^n - 1)}{1.06 - 1}$, which can be simplified to $y = 16666.66667(1.06^n - 1)$. Use the window $x : [0,15,1]$, $y : [0,15000,1000]$ and trace along the graph until $y > 14500$. The x-coordinate, rounded to the nearest whole number, will be the number of certificates from the start.

Possibility with two equations: $y_1 = \dfrac{1000(1.06^n - 1)}{1.06 - 1}$, which can be simplified to $y = 16666.66667(1.06^n - 1)$ and $y_2 = 14500$. Use the window $x:[0,15,1], y:[0,15000,1000]$ and enter the equations for y_1 and y_2 into the calculator. Calculate the point of intersection between y_1 and y_2. The x-coordinate of this point will be the number of certificates when rounded up to the nearest whole number.

WRITTEN RESPONSE #9

Use the following information to answer the first question.

A teacher provided two students, Peggy and Susan, with the data in the table below. The data shows the temperature of juice over time when an insulated glass of juice at a temperature of 22.10°C was placed in a refrigerator.

Time (h)	0	1	2	3	4
Juice temperature (°C)	22.10	18.70	15.90	13.51	11.48

- Peggy thinks that the data representing the temperature of the juice over time could be modelled using an exponential regression equation $y = ab^x$, where y is the temperature in degrees Celsius and x is the time in hours. Find the exponential regression equation that Peggy would obtain if the value of a and the value of b are expressed to four decimal places.

- Susan thinks that the data representing the temperature of the juice over time could be modelled using a geometric sequence. **Determine** the general term, $t_n = ar^{n-1}$, $n \in N$, that Susan would obtain if the value of a and the value of r are expressed to the nearest hundredth.

- In the context of the question, **explain** which of the two models, Peggy's or Susan's, is more appropriate for the data.

- A glass of water was placed in a different refrigerator. The temperature of the water as a function of time can be modelled by the formula $C = 19.2(0.91)^t$, where C is the temperature in degrees Celsius and t is the time in hours. How many hours, to the nearest tenth, would the water have to remain in this refrigerator to reach a temperature of $11.0°C$? **Justify** how you arrived at your answer.

WRITTEN RESPONSE #10

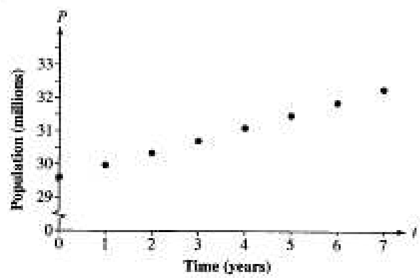
Write the first four terms in the series defined as $\sum_{i=2}^{20}[(-1)^{i-1}(2)^{-2i+5}]$.

WRITTEN RESPONSE #11

Use the following information to answer the next question.

In 1995, Canada's population was 29.60 million and was growing at about 1.24% per year. Based on this information, a teacher calculated the predicted population for Canada for seven subsequent years. The teacher then presented his students with the table and graph shown below.

Year	Time (t) in Years	Population (P) in Millions
1995	0	29.60
1996	1	29.97
1997	2	30.34
1998	3	30.71
1999	4	31.10
2000	5	31.48
2001	6	31.87
2002	7	32.27



a. Student A wanted to represent the population data as an exponential regression equation in the form $P = a(b)^t$, where t = time in years and P = population in millions. Enter the data in Time(t) as list L_1 and Population(P) as list L_2, and determine the exponential regression values of a, to the nearest hundredth, and b, to the nearest ten thousand.

b. Student B realized that the data could be modelled as a geometric sequence. The student listed the first three terms of the sequence as $t_1 = 29.60$, $t_2 = 29.97$ and $t_3 = 30.34$, and then wrote the general geometric sequence formula

$$t_n = 29.60(1.0124)^{n-1}, n \in N$$

- Predict the population of Canada, in millions, in the year 2010 using both students' equation models. Explain what the values of t and n represent.
- Which students' equation model is more appropriate to describe Canada's population growth? Support your answer with appropriate explanations.

WRITTEN RESPONSE #12

Graph $y = \log_2 x$, and identify the domain, range, x- and y-intercepts, and asymptotes.

The following data represent the cooling of a cup of hot chocolate over time. Use exponential regression to find an equation in the form $T = a(b)^t$, where t = time (min) and T = temperature (°C).

Time (min)	Temperature (°C)
0	60
5	54
10	48
15	44
20	41