

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Without using your calculator, write down the sign of the following trigonometric ratios:

(a) $\sec 300^\circ$

(b) $\operatorname{cosec} 190^\circ$

(c) $\cot 110^\circ$

(d) $\cot 200^\circ$

(e) $\sec 95^\circ$

Solution:

(a) 300° is in the 4th quadrant

$$\sec 300^\circ = \frac{1}{\cos 300^\circ}$$

In 4th quadrant \cos is +ve, so $\sec 300^\circ$ is +ve.

(b) 190° is in the 3rd quadrant

$$\operatorname{cosec} 190^\circ = \frac{1}{\sin 190^\circ}$$

In 3rd quadrant \sin is -ve, so $\operatorname{cosec} 190^\circ$ is -ve.

(c) 110° is in the 2nd quadrant

$$\cot 110^\circ = \frac{1}{\tan 110^\circ}$$

In the 2nd quadrant \tan is -ve, so $\cot 110^\circ$ is -ve.

(d) 200° is in the 3rd quadrant.

\tan is +ve in the 3rd quadrant, so $\cot 200^\circ$ is +ve.

(e) 95° is in the 2nd quadrant

\cos is -ve in the 2nd quadrant, so $\sec 95^\circ$ is -ve.

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Exercise A, Question 2

Question:

Use your calculator to find, to 3 significant figures, the values of

(a) $\sec 100^\circ$

(b) $\operatorname{cosec} 260^\circ$

(c) $\operatorname{cosec} 280^\circ$

(d) $\cot 550^\circ$

(e) $\cot \frac{4\pi}{3}$

(f) $\sec 2.4^c$

(g) $\operatorname{cosec} \frac{11\pi}{10}$

(h) $\sec 6^c$

Solution:

(a) $\sec 100^\circ = \frac{1}{\cos 100^\circ} = -5.76$

(b) $\operatorname{cosec} 260^\circ = \frac{1}{\sin 260^\circ} = -1.02$

(c) $\operatorname{cosec} 280^\circ = \frac{1}{\sin 280^\circ} = -1.02$

(d) $\cot 550^\circ = \frac{1}{\tan 550^\circ} = 5.67$

(e) $\cot \frac{4\pi}{3} = \frac{1}{\tan \frac{4\pi}{3}} = 0.577$

$$(f) \sec 2.4^{\circ} = \frac{1}{\cos 2.4^{\circ}} = -1.36$$

$$(g) \operatorname{cosec} \frac{11\pi}{10} = \frac{1}{\sin \frac{11\pi}{10}} = -3.24$$

$$(h) \sec 6^{\circ} = \frac{1}{\cos 6^{\circ}} = 1.04$$

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Exercise A, Question 3

Question:

Find the exact value (in surd form where appropriate) of the following:

(a) $\operatorname{cosec} 90^\circ$

(b) $\cot 135^\circ$

(c) $\sec 180^\circ$

(d) $\sec 240^\circ$

(e) $\operatorname{cosec} 300^\circ$

(f) $\cot (-45^\circ)$

(g) $\sec 60^\circ$

(h) $\operatorname{cosec} (-210^\circ)$

(i) $\sec 225^\circ$

(j) $\cot \frac{4\pi}{3}$

(k) $\sec \frac{11\pi}{6}$

(l) $\operatorname{cosec} \left(-\frac{3\pi}{4} \right)$

Solution:

(a) $\operatorname{cosec} 90^\circ = \frac{1}{\sin 90^\circ} = \frac{1}{1} = 1$ (refer to graph of $y = \sin \theta$)

(b) $\cot 135^\circ = \frac{1}{\tan 135^\circ} = \frac{1}{-\tan 45^\circ} = \frac{1}{-1} = -1$

(c) $\sec 180^\circ = \frac{1}{\cos 180^\circ} = \frac{1}{-1} = -1$ (refer to graph of $y = \cos \theta$)

(d) 240° is in 3rd quadrant

$$\sec 240^\circ = \frac{1}{\cos 240^\circ} = \frac{1}{-\cos 60^\circ} = \frac{1}{-\frac{1}{2}} = -2$$

$$(e) \operatorname{cosec} 300^\circ = \frac{1}{\sin 300^\circ} = \frac{1}{-\sin 60^\circ} = -\frac{1}{\frac{1}{2}\sqrt{3}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$(f) \cot(-45^\circ) = \frac{1}{\tan(-45^\circ)} = \frac{1}{-\tan 45^\circ} = \frac{1}{-1} = -1$$

$$(g) \sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2$$

(h) -210° is in 2nd quadrant

$$\operatorname{cosec}(-210^\circ) = \frac{1}{\sin(-210^\circ)} = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

(i) 225° is in 3rd quadrant

$$\begin{aligned} \sec 225^\circ &= \frac{1}{\cos 225^\circ} = \frac{1}{-\cos 45^\circ} \\ &= \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2} \end{aligned}$$

(j) $\frac{4\pi}{3}$ is in 3rd quadrant

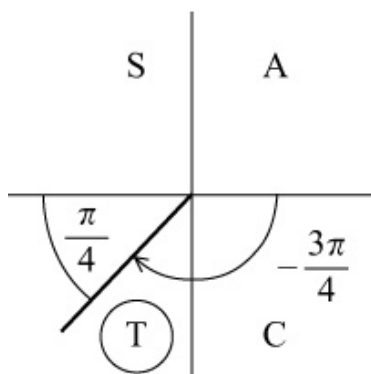
$$\cot \frac{4\pi}{3} = \frac{1}{\tan \frac{4\pi}{3}} = \frac{1}{+\tan \frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(k) $\frac{11\pi}{6} = 2\pi - \frac{\pi}{6}$ (in 4th quadrant)

$$\sec \frac{11\pi}{6} = \frac{1}{\cos \frac{11\pi}{6}} = \frac{1}{\cos \frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

(1)

$$\operatorname{cosec} = \left(-\frac{3\pi}{4}\right) = \frac{1}{\sin\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\sin\frac{\pi}{4}} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$



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Exercise A, Question 4

Question:

(a) Copy and complete the table, showing values (to 2 decimal places) of $\sec \theta$ for selected values of θ .

θ	0°	30°	45°	60°	70°	80°	85°	95°
$\sec \theta$	1		1.41			5.76	11.47	

θ	100°	110°	120°	135°	150°	180°	210°
$\sec \theta$		-2.92		-1.41			-1.15

(b) Copy and complete the table, showing values (to 2 decimal places) of $\operatorname{cosec} \theta$ for selected values of θ .

θ	10°	20°	30°	45°	60°	80°	90°	100°	120°	135°	150°	160°	170°
$\operatorname{cosec} \theta$				1.41			1		1.15	1.41			

θ	190°	200°	210°	225°	240°	270°	300°	315°	330°	340°	350°	390°
$\operatorname{cosec} \theta$					-1.15				-2			

(c) Copy and complete the table, showing values (to 2 decimal places) of $\cot \theta$ for selected values of θ .

θ	-90°	-60°	-45°	-30°	-10°	10°	30°	45°	60°
$\cot \theta$	0	-0.58					1.73	1	0.58

θ	90°	120°	135°	150°	170°	210°	225°	240°	270°
$\cot \theta$			-1					0.58	

Solution:

(a) Change $\sec \theta$ into $\frac{1}{\cos \theta}$ and use your calculator.

θ	0°	30°	45°	60°	70°	80°	85°	95°
$\sec \theta$	1	1.15	1.41	2	2.92	5.76	11.47	-11.47

θ	100°	110°	120°	135°	150°	180°	210°
$\sec \theta$	-5.76	-2.92	-2	-1.41	-1.15	-1	-1.15

(b) Change $\operatorname{cosec} \theta$ to $\frac{1}{\sin \theta}$ and use your calculator.

θ	10°	20°	30°	45°	60°	80°	90°	100°	120°
$\operatorname{cosec} \theta$	5.76	2.92	2	1.41	1.15	1.02	1	1.02	1.15

θ	135°	150°	160°	170°	190°	200°	210°	225°	240°
$\operatorname{cosec} \theta$	1.41	2	2.92	5.76	-5.76	-2.92	-2	-1.41	-1.15

θ	270°	300°	315°	330°	340°	350°	390°
$\operatorname{cosec} \theta$	-1	-1.15	-1.41	-2	-2.92	-5.76	2

(c) Change $\cot \theta$ to $\frac{1}{\tan \theta}$ and use your calculator.

θ	-90°	-60°	-45°	-30°	-10°	10°	30°	45°	60°
$\cot \theta$	0	-0.58	-1	-1.73	-5.67	5.67	1.73	1	0.58

θ	90°	120°	135°	150°	170°	210°	225°	240°	270°
$\cot \theta$	0	-0.58	-1	-1.73	-5.67	1.73	1	0.58	0

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Exercise B, Question 1

Question:

(a) Sketch, in the interval $-540^\circ \leq \theta \leq 540^\circ$, the graphs of:

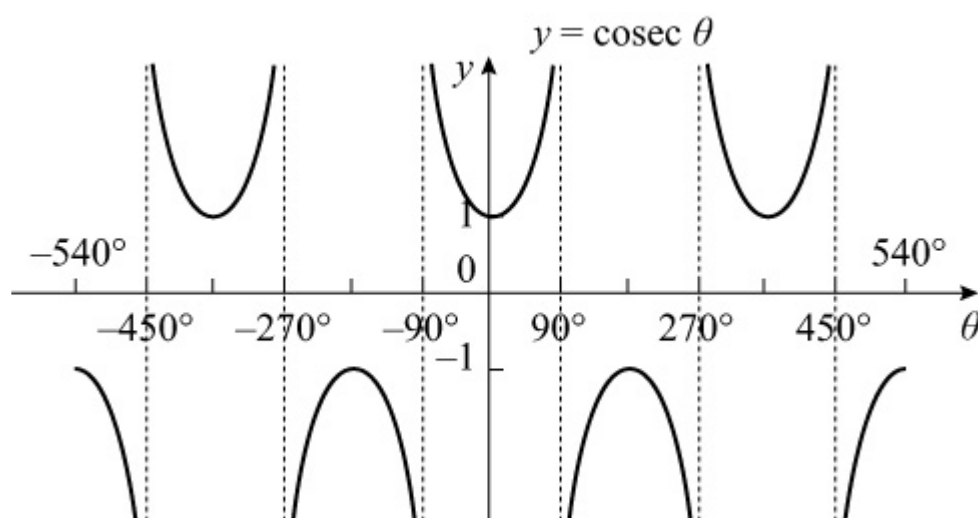
(i) $\sec \theta$ (ii) $\operatorname{cosec} \theta$ (iii) $\cot \theta$

(b) Write down the range of

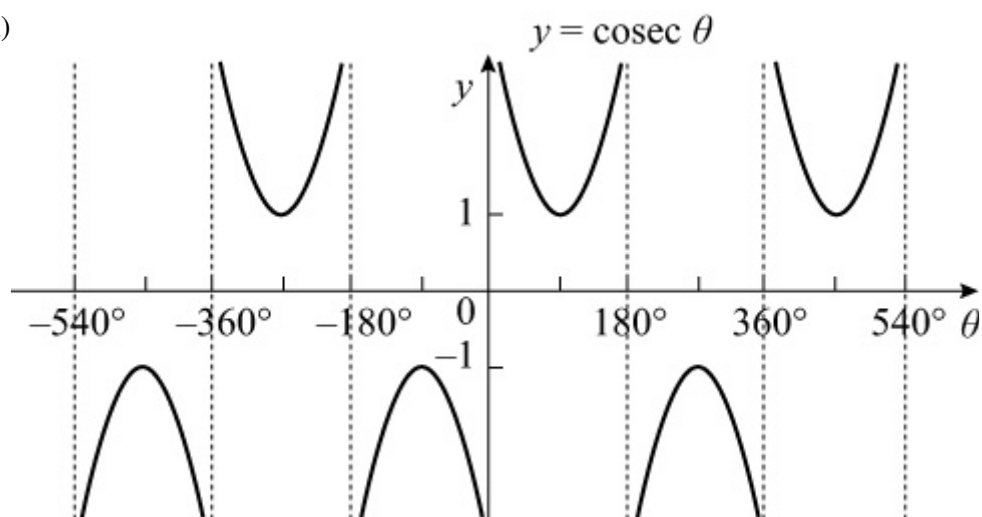
(i) $\sec \theta$ (ii) $\operatorname{cosec} \theta$ (iii) $\cot \theta$

Solution:

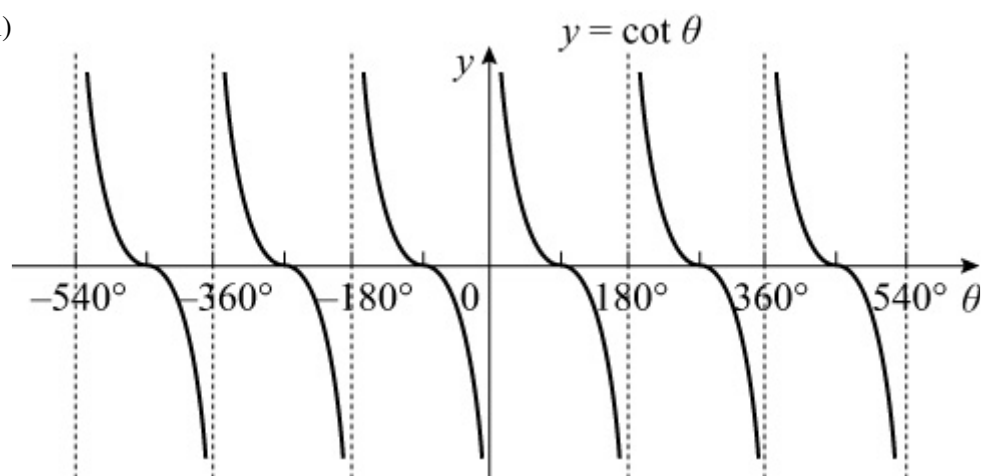
(a)(i)



(ii)



(iii)



(b)(i) (Note the gap in the range) $\sec \theta \leq -1, \sec \theta \geq 1$

(ii) (cosec θ also has a gap in the range) $\operatorname{cosec} \theta \leq -1, \operatorname{cosec} \theta \geq 1$

(iii) $\cot \theta$ takes all real values, i.e. $\cot \theta \in \mathbb{R}$.

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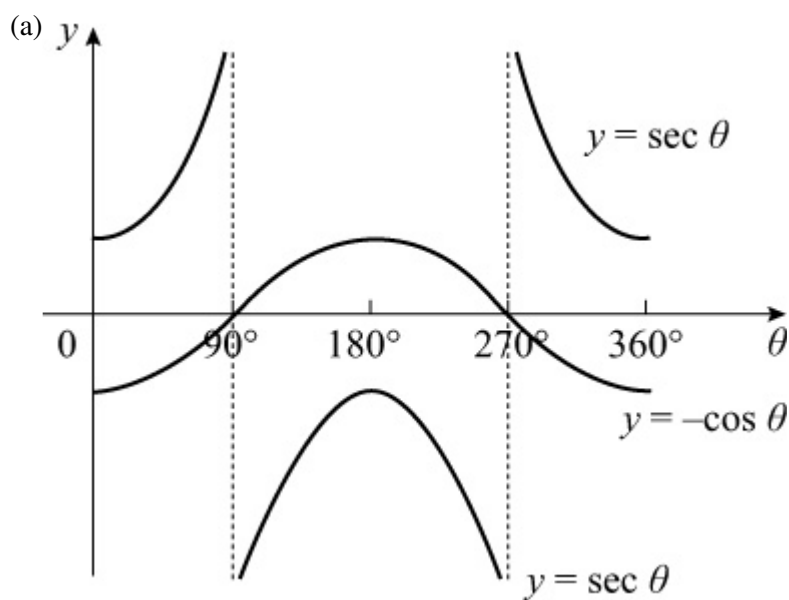
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Exercise B, Question 2

Question:

- (a) Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360^\circ$, the graphs of $y = \sec \theta$ and $y = -\cos \theta$.
- (b) Explain how your graphs show that $\sec \theta = -\cos \theta$ has no solutions.

Solution:



- (b) You can see that the graphs of $\sec \theta$ and $-\cos \theta$ do not meet, so $\sec \theta = -\cos \theta$ has no solutions.

Algebraically, the solutions of $\sec \theta = -\cos \theta$

are those of $\frac{1}{\cos \theta} = -\cos \theta$

This requires $\cos^2 \theta = -1$, which is not possible for real θ .

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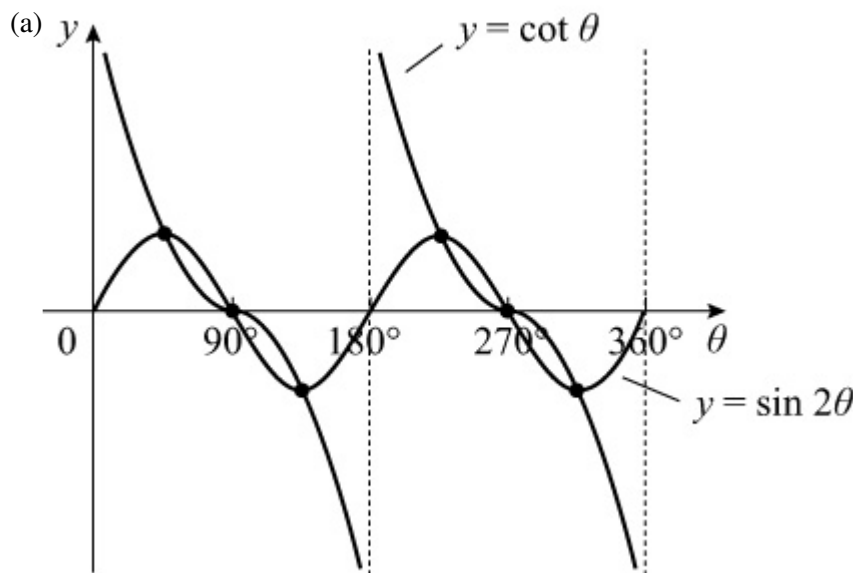
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Exercise B, Question 3

Question:

- (a) Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360^\circ$, the graphs of $y = \cot \theta$ and $y = \sin 2\theta$.
- (b) Deduce the number of solutions of the equation $\cot \theta = \sin 2\theta$ in the interval $0 \leq \theta \leq 360^\circ$.

Solution:



- (b) The curves meet at the maxima and minima of $y = \sin 2\theta$, and on the θ -axis at odd integer multiples of 90° .

In the interval $0 \leq \theta \leq 360^\circ$ there are 6 intersections.

So there are 6 solutions of $\cot \theta = \sin 2\theta$, is $0 \leq \theta \leq 360^\circ$.

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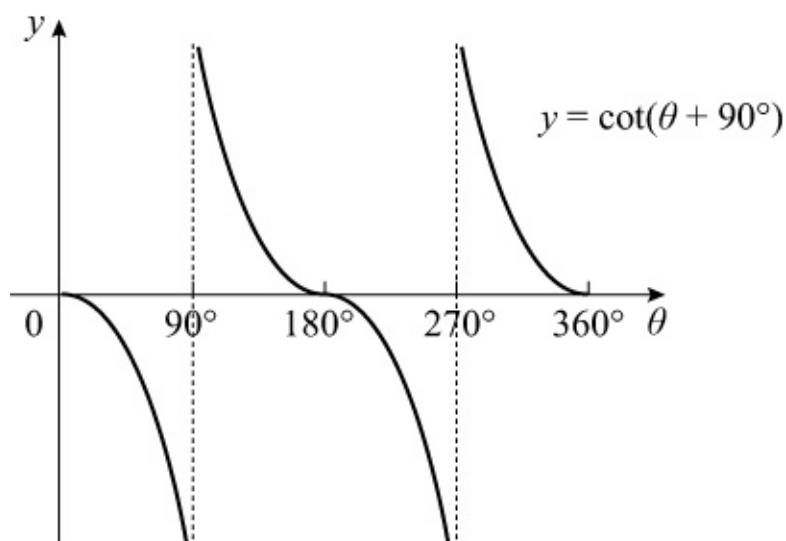
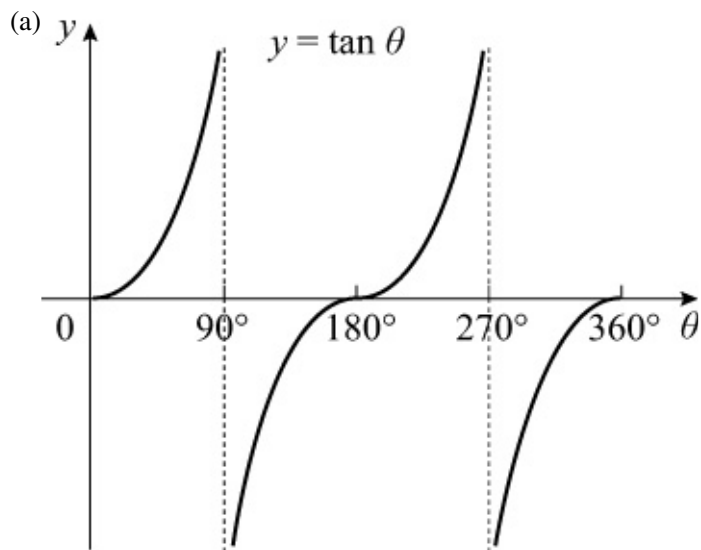
Exercise B, Question 4

Question:

(a) Sketch on separate axes, in the interval $0 \leq \theta \leq 360^\circ$, the graphs of $y = \tan \theta$ and $y = \cot(\theta + 90^\circ)$.

(b) Hence, state a relationship between $\tan \theta$ and $\cot(\theta + 90^\circ)$.

Solution:



(b) $y = \cot(\theta + 90^\circ)$ is a reflection in the θ -axis of $y = \tan \theta$, so $\cot(\theta + 90^\circ) = -\tan \theta$

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Exercise B, Question 5

Question:

(a) Describe the relationships between the graphs of

(i) $\tan \left(\theta + \frac{\pi}{2} \right)$ and $\tan \theta$

(ii) $\cot (- \theta)$ and $\cot \theta$

(iii) $\operatorname{cosec} \left(\theta + \frac{\pi}{4} \right)$ and $\operatorname{cosec} \theta$

(iv) $\sec \left(\theta - \frac{\pi}{4} \right)$ and $\sec \theta$

(b) By considering the graphs of $\tan \left(\theta + \frac{\pi}{2} \right)$, $\cot (- \theta)$, $\operatorname{cosec} \left(\theta + \frac{\pi}{4} \right)$ and $\sec \left(\theta - \frac{\pi}{4} \right)$, state which pairs of functions are equal.

Solution:

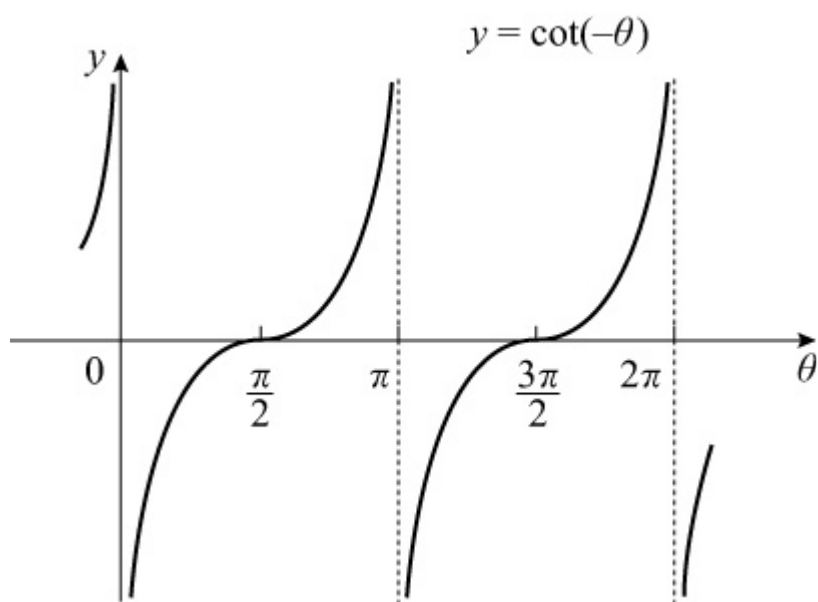
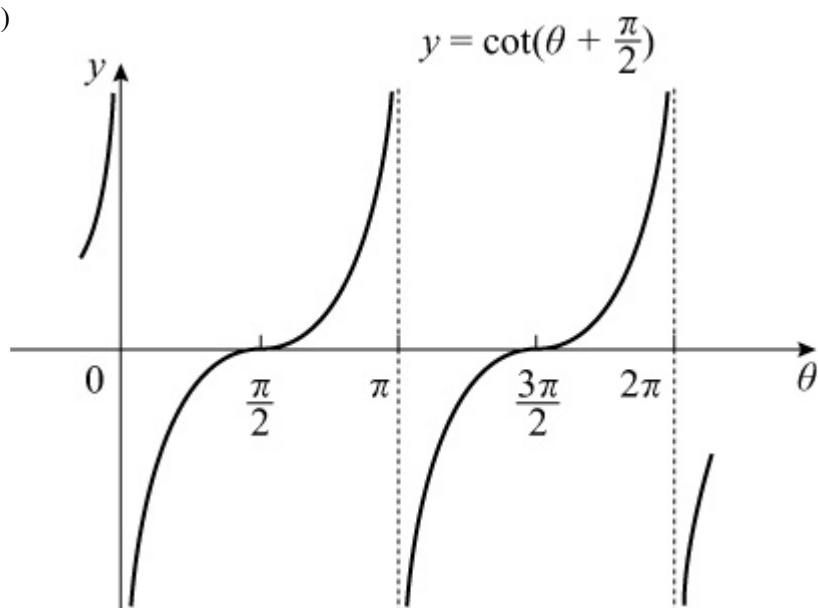
(a) (i) The graph of $\tan \left(\theta + \frac{\pi}{2} \right)$ is the same as that of $\tan \theta$ translated by $\frac{\pi}{2}$ to the left.

(ii) The graph of $\cot (- \theta)$ is the same as that of $\cot \theta$ reflected in the y-axis.

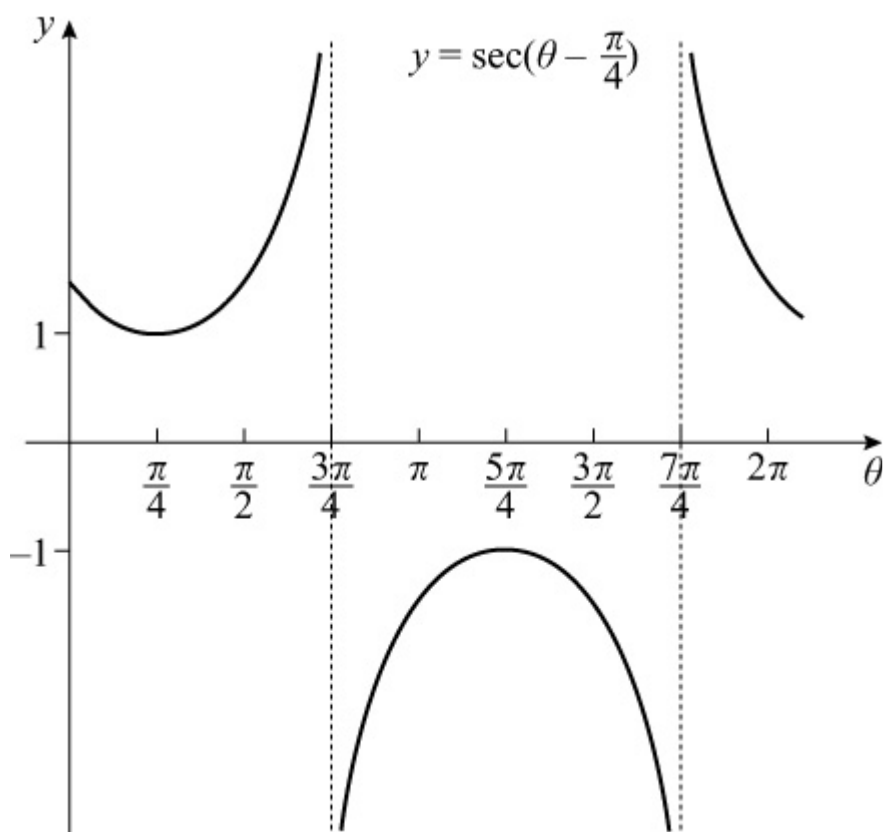
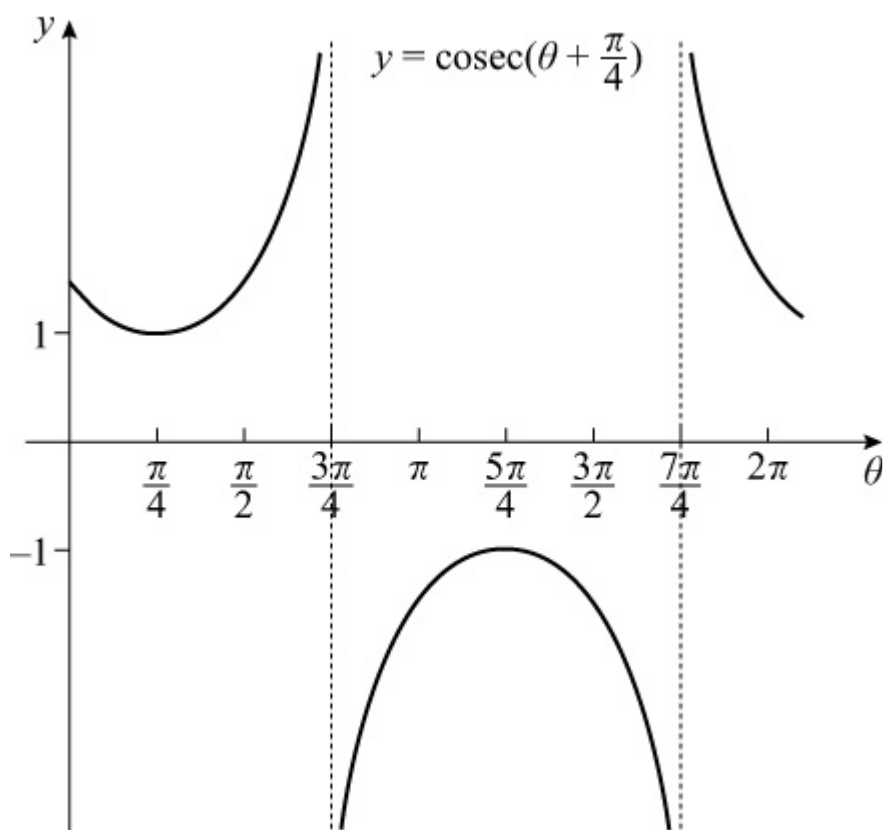
(iii) The graph of $\operatorname{cosec} \left(\theta + \frac{\pi}{4} \right)$ is the same as that of $\operatorname{cosec} \theta$ translated by $\frac{\pi}{4}$ to the left.

(iv) The graph of $\sec \left(\theta - \frac{\pi}{4} \right)$ is the same as that of $\sec \theta$ translated by $\frac{\pi}{4}$ to the right.

(b)

(reflect $y = \cot \theta$ in the y -axis)

$$\tan \left(\theta + \frac{\pi}{2} \right) = \cot (- \theta)$$



$$\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right) = \sec\left(\theta - \frac{\pi}{4}\right)$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

Question:

Sketch on separate axes, in the interval $0 \leq \theta \leq 360^\circ$, the graphs of:

(a) $y = \sec 2\theta$

(b) $y = -\operatorname{cosec} \theta$

(c) $y = 1 + \sec \theta$

(d) $y = \operatorname{cosec} (\theta - 30^\circ)$

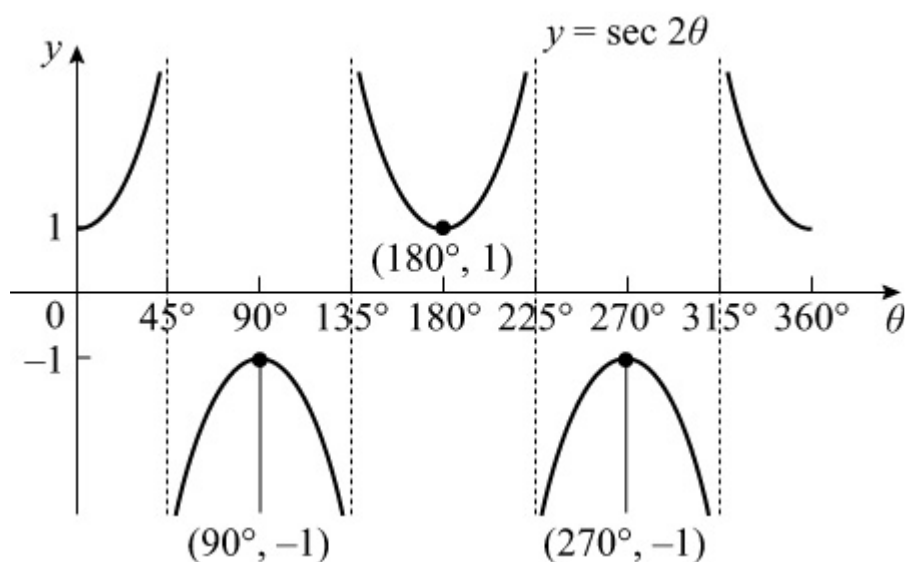
In each case show the coordinates of any maximum and minimum points, and of any points at which the curve meets the axes.

Solution:

(a) A stretch of $y = \sec \theta$ in the θ direction with scale factor $\frac{1}{2}$.

Minimum at $(180^\circ, 1)$

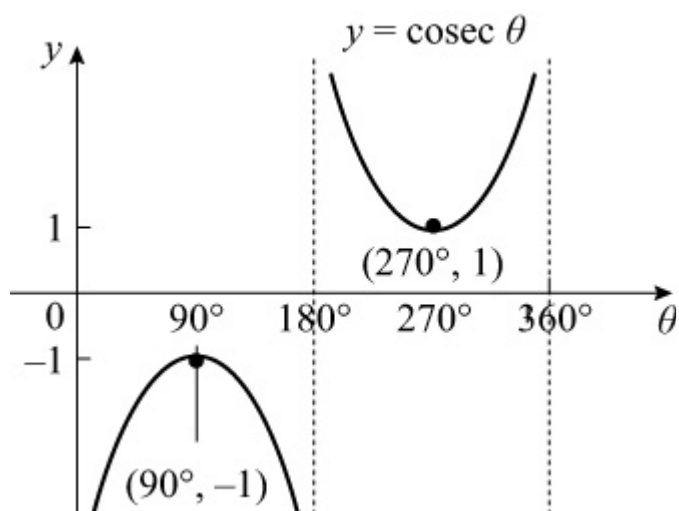
Maxima at $(90^\circ, -1)$ and $(270^\circ, -1)$



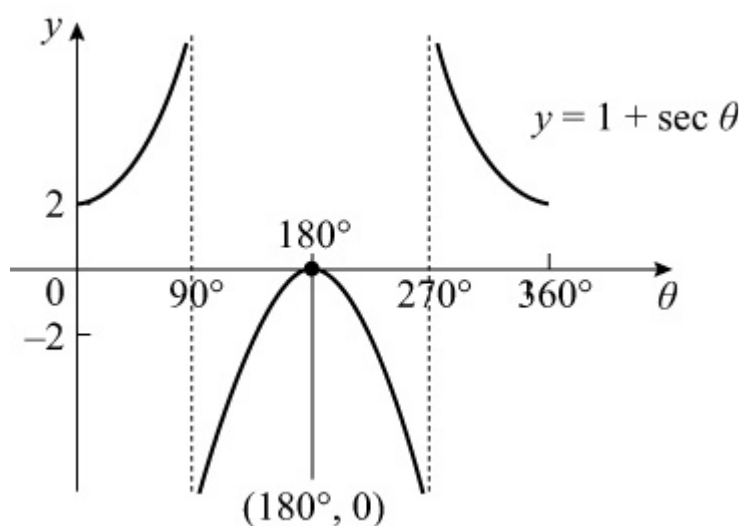
(b) Reflection in θ -axis of $y = \operatorname{cosec} \theta$.

Minimum at $(270^\circ, 1)$

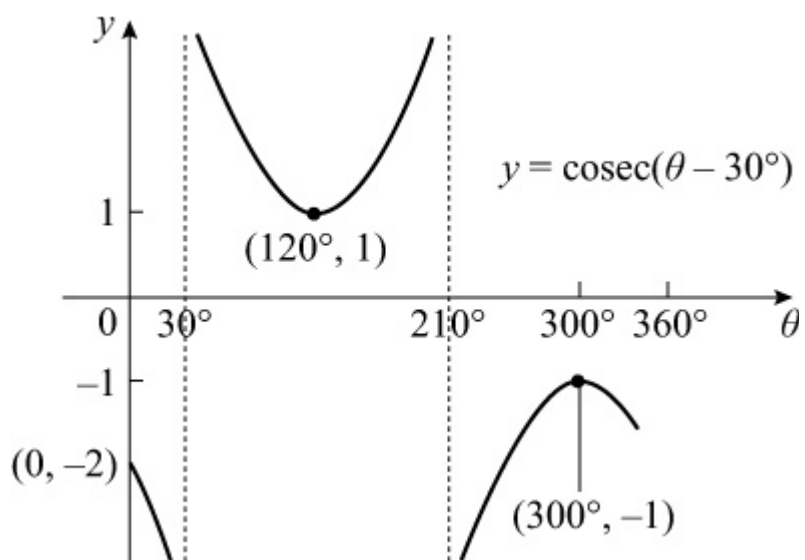
Maximum at $(90^\circ, -1)$



(c) Translation of $y = \sec \theta$ by $+1$ in the y direction.
Maximum at $(180^\circ, 0)$



(d) Translation of $y = \operatorname{cosec} \theta$ by 30° to the right.
Minimum at $(120^\circ, 1)$
Maximum at $(300^\circ, -1)$



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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

Write down the periods of the following functions. Give your answer in terms of π .

(a) $\sec 3\theta$

(b) $\operatorname{cosec} \frac{1}{2}\theta$

(c) $2 \cot \theta$

(d) $\sec (-\theta)$

Solution:

(a) The period of $\sec \theta$ is 2π radians.

$y = \sec 3\theta$ is a stretch of $y = \sec \theta$ with scale factor $\frac{1}{3}$ in the θ direction.

So period of $\sec 3\theta$ is $\frac{2\pi}{3}$.

(b) $\operatorname{cosec} \theta$ has a period of 2π .

$\operatorname{cosec} \frac{1}{2}\theta$ is a stretch of $\operatorname{cosec} \theta$ in the θ direction with scale factor 2.

So period of $\operatorname{cosec} \frac{1}{2}\theta$ is 4π .

(c) $\cot \theta$ has a period of π .

$2 \cot \theta$ is a stretch in the y direction by scale factor 2.

So the periodicity is not affected.

Period of $2 \cot \theta$ is π .

(d) $\sec \theta$ has a period of 2π .

$\sec (-\theta)$ is a reflection of $\sec \theta$ in y -axis, so periodicity is unchanged.

Period of $\sec (-\theta)$ is 2π .

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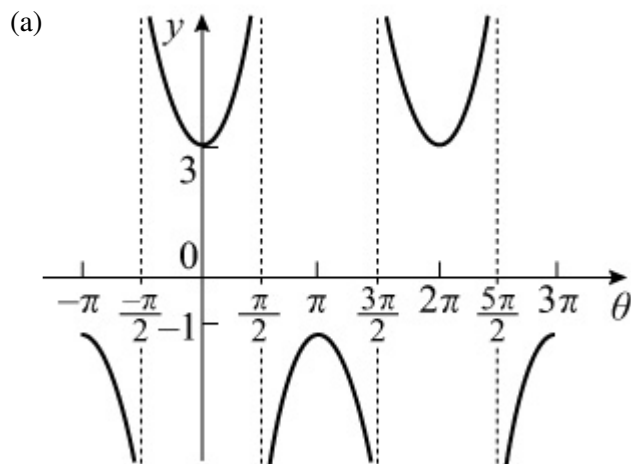
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 8

Question:

- (a) Sketch the graph of $y = 1 + 2 \sec \theta$ in the interval $-\pi \leq \theta \leq 2\pi$.
- (b) Write down the y-coordinate of points at which the gradient is zero.
- (c) Deduce the maximum and minimum values of $\frac{1}{1 + 2 \sec \theta}$, and give the smallest positive values of θ at which they occur.

Solution:



- (b) The y coordinates at stationary points are -1 and $\frac{1}{3}$.

- (c) Minimum value of $\frac{1}{1 + 2 \sec \theta}$ is where $1 + 2 \sec \theta$ is a maximum.

So minimum value of $\frac{1}{1 + 2 \sec \theta}$ is $\frac{1}{-1} = -1$

It occurs when $\theta = \pi$ (see diagram) (1st +ve value)

Maximum value of $\frac{1}{1 + 2 \sec \theta}$ is where $1 + 2 \sec \theta$ is a minimum.

So maximum value of $\frac{1}{1 + 2 \sec \theta}$ is $\frac{1}{3}$

It occurs when $\theta = 2\pi$ (1st +ve value)

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

Give solutions to these equations correct to 1 decimal place.

Rewrite the following as powers of $\sec \theta$, $\operatorname{cosec} \theta$ or $\cot \theta$:

$$(a) \frac{1}{\sin^3 \theta}$$

$$(b) \sqrt{\frac{4}{\tan^6 \theta}}$$

$$(c) \frac{1}{2 \cos^2 \theta}$$

$$(d) \frac{1 - \sin^2 \theta}{\sin^2 \theta}$$

$$(e) \frac{\sec \theta}{\cos^4 \theta}$$

$$(f) \sqrt{\operatorname{cosec}^3 \theta \cot \theta \sec \theta}$$

$$(g) \sqrt{\tan \theta}$$

$$(h) \frac{\operatorname{cosec}^2 \theta \tan^2 \theta}{\cos \theta}$$

Solution:

$$(a) \frac{1}{\sin^3 \theta} = \left(\frac{1}{\sin \theta} \right)^3 = \operatorname{cosec}^3 \theta$$

$$(b) \sqrt{\frac{4}{\tan^6 \theta}} = \frac{2}{\tan^3 \theta} = 2 \times \left(\frac{1}{\tan \theta} \right)^3 = 2 \cot^3 \theta$$

$$(c) \frac{1}{2 \cos^2 \theta} = \frac{1}{2} \times \left(\frac{1}{\cos \theta} \right)^2 = \frac{1}{2} \sec^2 \theta$$

$$(d) \frac{1 - \sin^2 \theta}{\sin^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} \quad (\text{using } \sin^2 \theta + \cos^2 \theta \equiv 1)$$

$$\text{So } \frac{1 - \sin^2 \theta}{\sin^2 \theta} = \left(\frac{\cos \theta}{\sin \theta} \right)^2 = \cot^2 \theta$$

$$(e) \frac{\sec \theta}{\cos^4 \theta} = \frac{1}{\cos \theta} \times \frac{1}{\cos^4 \theta} = \frac{1}{\cos^5 \theta} = \left(\frac{1}{\cos \theta} \right)^5 = \sec^5 \theta$$

$$(f) \sqrt{\operatorname{cosec}^3 \theta \cot \theta \sec \theta} = \sqrt{\frac{1}{\sin^3 \theta} \times \frac{\cos \theta}{\sin \theta} \times \frac{1}{\cos \theta}} = \sqrt{\frac{1}{\sin^4 \theta}} =$$

$$\frac{1}{\sin^2 \theta} = \operatorname{cosec}^2 \theta$$

$$(g) \frac{2}{\sqrt{\tan \theta}} = 2 \times \frac{1}{(\tan \theta)^{\frac{1}{2}}} = 2 \cot^{\frac{1}{2}} \theta$$

$$(h) \frac{\operatorname{cosec}^2 \theta \tan^2 \theta}{\cos \theta} = \frac{1}{\sin^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{1}{\cos \theta} = \left(\frac{1}{\cos \theta} \right)^3 = \sec^3 \theta$$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

Give solutions to these equations correct to 1 decimal place.

Write down the value(s) of $\cot x$ in each of the following equations:

(a) $5 \sin x = 4 \cos x$

(b) $\tan x = -2$

(c) $3 \frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

Solution:

(a) $5 \sin x = 4 \cos x$

$$\Rightarrow 5 = 4 \frac{\cos x}{\sin x} \quad (\text{divide by } \sin x)$$

$$\Rightarrow \frac{5}{4} = \cot x \quad (\text{divide by } 4)$$

(b) $\tan x = -2$

$$\Rightarrow \frac{1}{\tan x} = \frac{1}{-2}$$

$$\Rightarrow \cot x = -\frac{1}{2}$$

(c) $3 \frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

$$\Rightarrow 3 \sin^2 x = \cos^2 x \quad (\text{multiply by } \sin x \cos x)$$

$$\Rightarrow 3 = \frac{\cos^2 x}{\sin^2 x} \quad (\text{divide by } \sin^2 x)$$

$$\Rightarrow \left(\frac{\cos x}{\sin x} \right)^2 = 3$$

$$\Rightarrow \cot^2 x = 3$$

$$\Rightarrow \cot x = \pm \sqrt{3}$$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

Give solutions to these equations correct to 1 decimal place.

Using the definitions of **sec**, **cosec**, **cot** and **tan** simplify the following expressions:

(a) $\sin \theta \cot \theta$

(b) $\tan \theta \cot \theta$

(c) $\tan 2\theta \operatorname{cosec} 2\theta$

(d) $\cos \theta \sin \theta (\cot \theta + \tan \theta)$

(e) $\sin^3 x \operatorname{cosec} x + \cos^3 x \sec x$

(f) $\sec A - \sec A \sin^2 A$

(g) $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

Solution:

(a) $\sin \theta \cot \theta = \sin \theta \times \frac{\cos \theta}{\sin \theta} = \cos \theta$

(b) $\tan \theta \cot \theta = \tan \theta \times \frac{1}{\tan \theta} = 1$

(c) $\tan 2\theta \operatorname{cosec} 2\theta = \frac{\sin 2\theta}{\cos 2\theta} \times \frac{1}{\sin 2\theta} = \frac{1}{\cos 2\theta} = \sec 2\theta$

(d) $\cos \theta \sin \theta (\cot \theta + \tan \theta) = \cos \theta \sin \theta \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right)$
 $= \cos^2 \theta + \sin^2 \theta = 1$

(e) $\sin^3 x \operatorname{cosec} x + \cos^3 x \sec x = \sin^3 x \times \frac{1}{\sin x} + \cos^3 x \times$

$\frac{1}{\cos x} = \sin^2 x + \cos^2 x = 1$

$$\begin{aligned} \text{(f) } & \sec A - \sec A \sin^2 A \\ &= \sec A (1 - \sin^2 A) \quad (\text{factorise}) \\ &= \frac{1}{\cos A} \times \cos^2 A \quad (\text{using } \sin^2 A + \cos^2 A \equiv 1) \\ &= \cos A \end{aligned}$$

$$\begin{aligned} \text{(g) } & \sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x \\ &= \frac{1}{\cos^2 x} \times \cos^5 x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \times \sin^4 x \\ &= \cos^3 x + \sin^2 x \cos x \\ &= \cos x (\cos^2 x + \sin^2 x) \\ &= \cos x \quad (\text{since } \cos^2 x + \sin^2 x \equiv 1) \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

Show that

$$(a) \cos \theta + \sin \theta \tan \theta \equiv \sec \theta$$

$$(b) \cot \theta + \tan \theta \equiv \operatorname{cosec} \theta \sec \theta$$

$$(c) \operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta$$

$$(d) (1 - \cos x) (1 + \sec x) \equiv \sin x \tan x$$

$$(e) \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} \equiv 2 \sec x$$

$$(f) \frac{\cos \theta}{1 + \cot \theta} \equiv \frac{\sin \theta}{1 + \tan \theta}$$

Solution:

$$\begin{aligned} (a) \text{ L.H.S.} &\equiv \cos \theta + \sin \theta \tan \theta \\ &\equiv \cos \theta + \sin \theta \frac{\sin \theta}{\cos \theta} \\ &\equiv \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} \\ &\equiv \frac{1}{\cos \theta} \quad (\text{using } \sin^2 \theta + \cos^2 \theta \equiv 1) \\ &\equiv \sec \theta \equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{ L.H.S.} &\equiv \cot \theta + \tan \theta \\ &\equiv \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\ &\equiv \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} \\ &\equiv \frac{1}{\sin \theta \cos \theta} \\ &\equiv \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} \\ &\equiv \operatorname{cosec} \theta \sec \theta \equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (c) \text{ L.H.S.} &\equiv \operatorname{cosec} \theta - \sin \theta \\ &\equiv \frac{1}{\sin \theta} - \sin \theta \end{aligned}$$

$$\begin{aligned}
&\equiv \frac{1 - \sin^2 \theta}{\sin \theta} \\
&\equiv \frac{\cos^2 \theta}{\sin \theta} \\
&\equiv \cos \theta \times \frac{\cos \theta}{\sin \theta} \\
&\equiv \cos \theta \cot \theta \equiv \text{R.H.S.}
\end{aligned}$$

(d) L.H.S. $\equiv (1 - \cos x)(1 + \sec x)$

$$\begin{aligned}
&\equiv 1 - \cos x + \sec x - \cos x \sec x \quad (\text{multiplying out}) \\
&\equiv \sec x - \cos x \\
&\equiv \frac{1}{\cos x} - \cos x \\
&\equiv \frac{1 - \cos^2 x}{\cos x} \\
&\equiv \frac{\sin^2 x}{\cos x} \\
&\equiv \sin x \times \frac{\sin x}{\cos x} \\
&\equiv \sin x \tan x \equiv \text{R.H.S.}
\end{aligned}$$

(e) L.H.S. $\equiv \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$

$$\begin{aligned}
&\equiv \frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x) \cos x} \\
&\equiv \frac{\cos^2 x + (1 - 2 \sin x + \sin^2 x)}{(1 - \sin x) \cos x} \\
&\equiv \frac{2 - 2 \sin x}{(1 - \sin x) \cos x} \quad (\text{using } \sin^2 x + \cos^2 x \equiv 1) \\
&\equiv \frac{2(1 - \sin x)}{(1 - \sin x) \cos x} \quad (\text{factorising}) \\
&= \frac{2}{\cos x} \\
&\equiv 2 \sec x \equiv \text{R.H.S.}
\end{aligned}$$

(f)

$$\begin{aligned}\text{L.H.S.} &\equiv \frac{\cos \theta}{1 + \cot \theta} \\ &\equiv \frac{\cos \theta}{1 + \frac{1}{\tan \theta}} \\ &\equiv \frac{\cos \theta}{\frac{\tan \theta + 1}{\tan \theta}} \\ &\equiv \frac{\cos \theta \tan \theta}{1 + \tan \theta} \\ &\equiv \frac{\cos \theta \times \frac{\sin \theta}{\cos \theta}}{1 + \tan \theta} \\ &\equiv \frac{\sin \theta}{1 + \tan \theta} \equiv \text{R.H.S.}\end{aligned}$$

Solutionbank

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Exercise C, Question 5

Question:

Solve, for values of θ in the interval $0 \leq \theta \leq 360^\circ$, the following equations. Give your answers to 3 significant figures where necessary.

(a) $\sec \theta = \sqrt{2}$

(b) $\operatorname{cosec} \theta = -3$

(c) $5 \cot \theta = -2$

(d) $\operatorname{cosec} \theta = 2$

(e) $3 \sec^2 \theta - 4 = 0$

(f) $5 \cos \theta = 3 \cot \theta$

(g) $\cot^2 \theta - 8 \tan \theta = 0$

(h) $2 \sin \theta = \operatorname{cosec} \theta$

Solution:

(a) $\sec \theta = \sqrt{2}$

$$\Rightarrow \frac{1}{\cos \theta} = \sqrt{2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

Calculator value is $\theta = 45^\circ$

$\cos \theta$ is +ve $\Rightarrow \theta$ in 1st and 4th quadrants

Solutions are $45^\circ, 315^\circ$

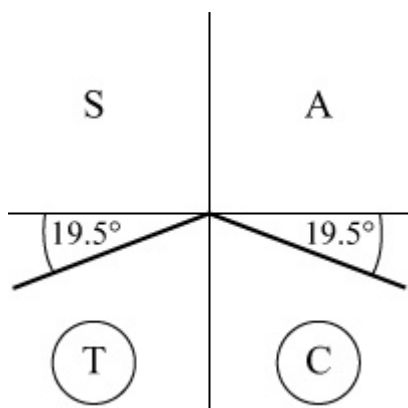
(b) $\operatorname{cosec} \theta = -3$

$$\Rightarrow \frac{1}{\sin \theta} = -3$$

$$\Rightarrow \sin \theta = -\frac{1}{3}$$

Calculator value is -19.5°

$\sin \theta$ is -ve $\Rightarrow \theta$ is in 3rd and 4th quadrants



Solutions are 199° , 341° (3 s.f.)

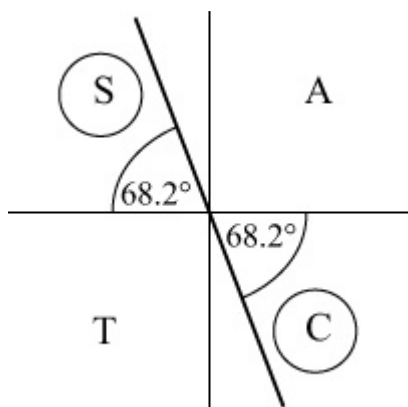
(c) $5 \cot \theta = -2$

$$\Rightarrow \cot \theta = -\frac{2}{5}$$

$$\Rightarrow \tan \theta = -\frac{5}{2}$$

Calculator value is -68.2°

$\tan \theta$ is $-ve \Rightarrow \theta$ is in 2nd and 4th quadrants



Solutions are 112° , 292° (3 s.f.)

(d) $\operatorname{cosec} \theta = 2$

$$\Rightarrow \frac{1}{\sin \theta} = 2$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$\sin \theta$ is $+ve \Rightarrow \theta$ is in 1st and 2nd quadrants

Solutions are 30° , 150°

(e) $3 \sec^2 \theta = 4$

$$\Rightarrow \sec^2 \theta = \frac{4}{3}$$

$$\Rightarrow \cos^2 \theta = \frac{3}{4}$$

$$\Rightarrow \cos \theta = \pm \frac{\sqrt{3}}{2}$$

Calculator value for $\cos \theta = \frac{\sqrt{3}}{2}$ is 30°

As $\cos \theta$ is \pm , θ is in all four quadrants

Solutions are $30^\circ, 150^\circ, 210^\circ, 330^\circ$

(f) $5 \cos \theta = 3 \cot \theta$

$$\Rightarrow 5 \cos \theta = 3 \frac{\cos \theta}{\sin \theta}$$

Note Do not cancel $\cos \theta$ on each side. Multiply through by $\sin \theta$.

$$\Rightarrow 5 \cos \theta \sin \theta = 3 \cos \theta$$

$$\Rightarrow 5 \cos \theta \sin \theta - 3 \cos \theta = 0$$

$$\Rightarrow \cos \theta (5 \sin \theta - 3) = 0 \quad (\text{factorise})$$

So $\cos \theta = 0$ or $\sin \theta = \frac{3}{5}$

Solutions are $(90^\circ, 270^\circ), (36.9^\circ, 143^\circ) = 36.9^\circ, 90^\circ, 143^\circ, 270^\circ$.

(g) $\cot^2 \theta - 8 \tan \theta = 0$

$$\Rightarrow \frac{1}{\tan^2 \theta} - 8 \tan \theta = 0$$

$$\Rightarrow 1 - 8 \tan^3 \theta = 0$$

$$\Rightarrow 8 \tan^3 \theta = 1$$

$$\Rightarrow \tan^3 \theta = \frac{1}{8}$$

$$\Rightarrow \tan \theta = \frac{1}{2}$$

$\tan \theta$ is +ve $\Rightarrow \theta$ is in 1st and 3rd quadrants

Calculator value is 26.6°

Solutions are 26.6° and $(180^\circ + 26.6^\circ) = 206.6^\circ$ and 207° (3 s.f.).

(h) $2 \sin \theta = \operatorname{cosec} \theta$

$$\Rightarrow 2 \sin \theta = \frac{1}{\sin \theta}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \pm \frac{1}{\sqrt{2}}$$

Calculator value for $\sin^{-1} \frac{1}{\sqrt{2}}$ is 45°

Solution are in all four quadrants

Solutions are $45^\circ, 135^\circ, 225^\circ, 315^\circ$

Solutionbank

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Exercise C, Question 6

Question:

Solve, for values of θ in the interval $-180^\circ \leq \theta \leq 180^\circ$, the following equations:

(a) $\operatorname{cosec} \theta = 1$

(b) $\sec \theta = -3$

(c) $\cot \theta = 3.45$

(d) $2 \operatorname{cosec}^2 \theta - 3 \operatorname{cosec} \theta = 0$

(e) $\sec \theta = 2 \cos \theta$

(f) $3 \cot \theta = 2 \sin \theta$

(g) $\operatorname{cosec} 2\theta = 4$

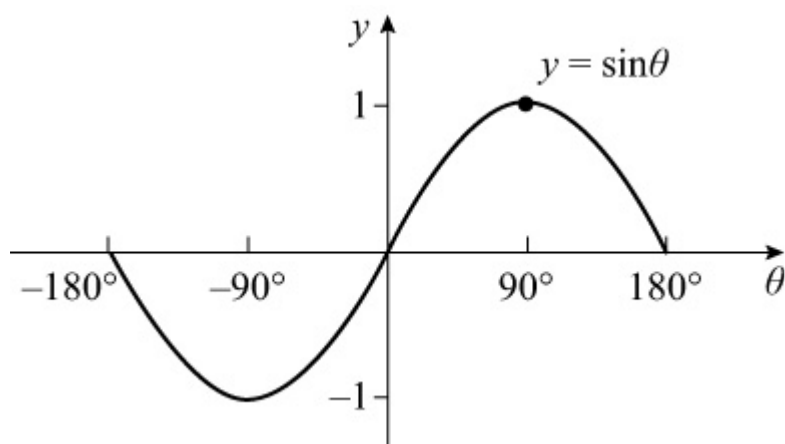
(h) $2 \cot^2 \theta - \cot \theta - 5 = 0$

Solution:

(a) $\operatorname{cosec} \theta = 1$

$$\Rightarrow \sin \theta = 1$$

$$\Rightarrow \theta = 90^\circ$$

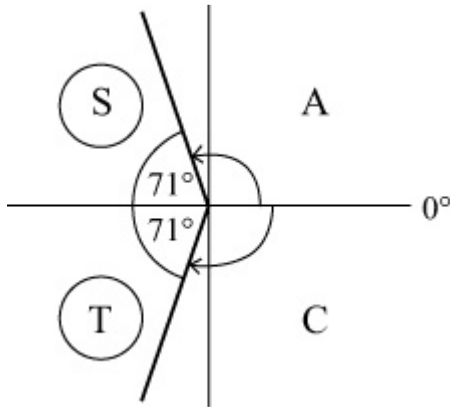


(b) $\sec \theta = -3$

$$\Rightarrow \cos \theta = -\frac{1}{3}$$

Calculator value for $\cos^{-1} \left(-\frac{1}{3} \right)$ is 109° (3 s.f.)

$\cos \theta$ is -ve $\Rightarrow \theta$ is in 2nd and 3rd quadrants



Solutions are 109° and -109°

[If you are not using the quadrant diagram, answer in this case would be $\cos^{-1} \left(-\frac{1}{3} \right)$

and $-360^\circ + \cos^{-1} \left(-\frac{1}{3} \right)$. See key point on page 84.]

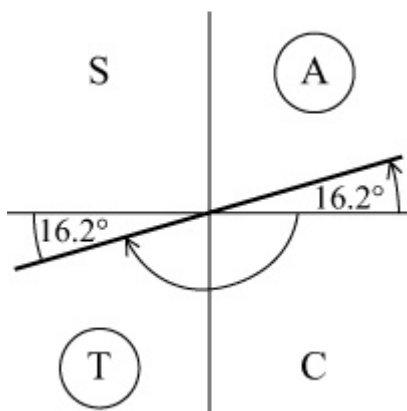
(c) $\cot \theta = 3.45$

$$\Rightarrow \frac{1}{\tan \theta} = 3.45$$

$$\Rightarrow \tan \theta = \frac{1}{3.45} = 0.28985\dots$$

Calculator value for $\tan^{-1} (0.28985\dots)$ is 16.16°

$\tan \theta$ is +ve $\Rightarrow \theta$ is in 1st and 3rd quadrants



Solutions are 16.2° , $-180^\circ + 16.2^\circ = 16.2^\circ$, -164° (3 s.f.)

$$\begin{aligned}
 \text{(d) } 2 \operatorname{cosec}^2 \theta - 3 \operatorname{cosec} \theta &= 0 \\
 \Rightarrow \operatorname{cosec} \theta (2 \operatorname{cosec} \theta - 3) &= 0 \quad (\text{factorise}) \\
 \Rightarrow \operatorname{cosec} \theta = 0 \text{ or } \operatorname{cosec} \theta &= \frac{3}{2} \\
 \Rightarrow \sin \theta = \frac{2}{3} \quad \operatorname{cosec} \theta = 0 &\text{ has no solutions}
 \end{aligned}$$

Calculator value for $\sin^{-1} \frac{2}{3}$ is 41.8°

θ is in 1st and 2nd quadrants

Solutions are 41.8° , $(180 - 41.8)^\circ = 138.2^\circ$ (3 s.f.)

$$\begin{aligned}
 \text{(e) } \sec \theta &= 2 \cos \theta \\
 \Rightarrow \frac{1}{\cos \theta} &= 2 \cos \theta \\
 \Rightarrow \cos^2 \theta &= \frac{1}{2} \\
 \Rightarrow \cos \theta &= \pm \frac{1}{\sqrt{2}}
 \end{aligned}$$

Calculator value for $\cos^{-1} \frac{1}{\sqrt{2}}$ is 45°

θ is in all quadrants, but remember that $-180^\circ \leq \theta \leq 180^\circ$

Solutions are $\pm 45^\circ$, $\pm 135^\circ$

$$\begin{aligned}
 \text{(f) } 3 \cot \theta &= 2 \sin \theta \\
 \Rightarrow 3 \frac{\cos \theta}{\sin \theta} &= 2 \sin \theta \\
 \Rightarrow 3 \cos \theta &= 2 \sin^2 \theta \\
 \Rightarrow 3 \cos \theta &= 2(1 - \cos^2 \theta) \quad (\text{use } \sin^2 \theta + \cos^2 \theta \equiv 1) \\
 \Rightarrow 2 \cos^2 \theta + 3 \cos \theta - 2 &= 0 \\
 \Rightarrow (2 \cos \theta - 1)(\cos \theta + 2) &= 0 \\
 \Rightarrow \cos \theta = \frac{1}{2} \text{ or } \cos \theta = -2
 \end{aligned}$$

As $\cos \theta = -2$ has no solutions, $\cos \theta = \frac{1}{2}$

Solutions are $\pm 60^\circ$

$$\text{(g) } \operatorname{cosec} 2\theta = 4$$

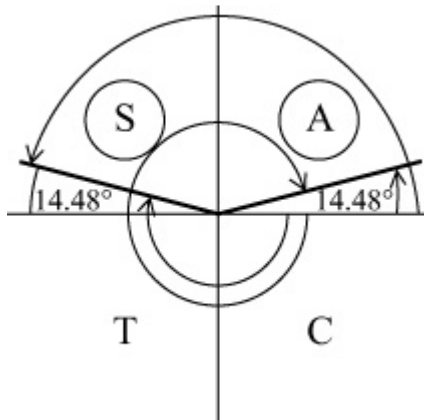
$$\Rightarrow \sin 2\theta = \frac{1}{4}$$

Remember that $-180^\circ \leq \theta \leq 180^\circ$

$$\text{So } -360^\circ \leq 2\theta \leq 360^\circ$$

$$\text{Calculator solution for } 2\theta \text{ is } \sin^{-1} \frac{1}{4} = 14.48^\circ$$

$\sin 2\theta$ is +ve $\Rightarrow 2\theta$ is in 1st and 2nd quadrants



$$2\theta = -194.48^\circ, -345.52^\circ, 14.48^\circ, 165.52^\circ$$

$$\theta = -97.2^\circ, -172.8^\circ, 7.24^\circ, 82.76^\circ = -173^\circ, -97.2^\circ, 7.24^\circ, 82.8^\circ$$

(3 s.f.)

$$(h) 2 \cot^2 \theta - \cot \theta - 5 = 0$$

As this quadratic in $\cot \theta$ does not factorise, use the quadratic formula

$$\cot \theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

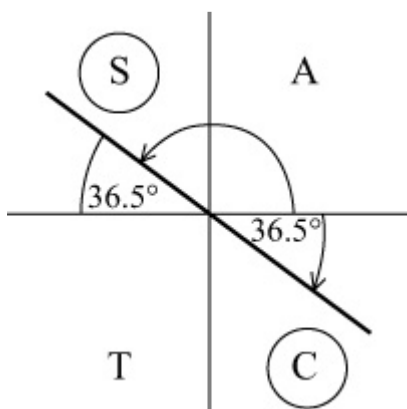
(You could change $\cot \theta$ to $\frac{1}{\tan \theta}$ and work with the quadratic

$$5 \tan^2 \theta + \tan \theta - 2 = 0)$$

$$\text{So } \cot \theta = \frac{1 \pm \sqrt{41}}{4} = -1.3507... \text{ or } 1.8507 \dots$$

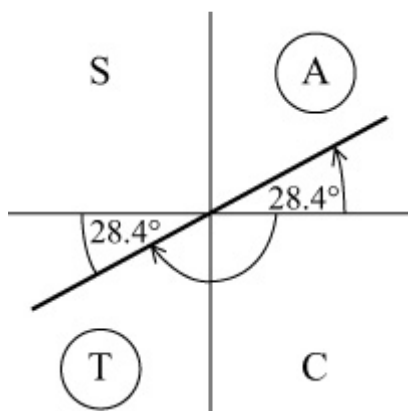
$$\text{So } \tan \theta = -0.7403... \text{ or } 0.5403 \dots$$

The calculator value for $\tan \theta = -0.7403...$ is $\theta = -36.51^\circ$



Solution are $-36.5^\circ, +143^\circ$ (3 s.f.).

The calculator value for $\tan \theta = 0.5403\dots$ is $\theta = 28.38^\circ$



Solution are 28.4° , $(-180 + 28.4)^\circ$

Total set of solutions is -152° , -36.5° , 28.4° , 143° (3 s.f.)

Solutionbank

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Exercise C, Question 7

Question:

Solve the following equations for values of θ in the interval $0 \leq \theta \leq 2\pi$.
Give your answers in terms of π .

(a) $\sec \theta = -1$

(b) $\cot \theta = -\sqrt{3}$

(c) $\operatorname{cosec} \frac{1}{2}\theta = \frac{2\sqrt{3}}{3}$

(d) $\sec \theta = \sqrt{2} \tan \theta \quad \left(\theta \neq \frac{\pi}{2}, \theta \neq \frac{3\pi}{2} \right)$

Solution:

(a) $\sec \theta = -1$

$$\Rightarrow \cos \theta = -1$$

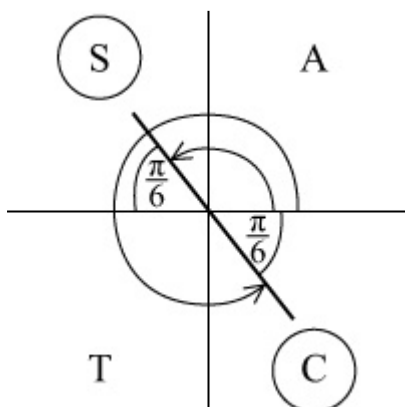
$$\Rightarrow \theta = \pi \quad (\text{refer to graph of } y = \cos \theta)$$

(b) $\cot \theta = -\sqrt{3}$

$$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$$

Calculator solution is $-\frac{\pi}{6}$ $\left(\text{you should know that } \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \right)$

$-\frac{\pi}{6}$ is not in the interval



Solution are $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{5\pi}{6}, \frac{11\pi}{6}$

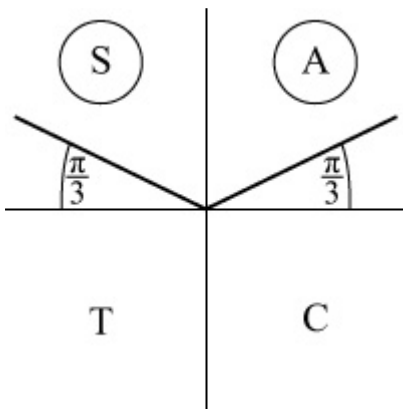
$$(c) \operatorname{cosec} \frac{1}{2}\theta = \frac{2\sqrt{3}}{3}$$

$$\Rightarrow \sin \frac{1}{2}\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Remember that $0 \leq \theta \leq 2\pi$

so $0 \leq \frac{1}{2}\theta \leq \pi$

First solution for $\sin \frac{1}{2}\theta = \frac{\sqrt{3}}{2}$ is $\frac{1}{2}\theta = \frac{\pi}{3}$



$$\text{So } \frac{1}{2}\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$(d) \sec \theta = \sqrt{2} \tan \theta$$

$$\Rightarrow \frac{1}{\cos \theta} = \sqrt{2} \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow 1 = \sqrt{2} \sin \theta \quad (\cos \theta \neq 0)$$

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$$

Solutions are $\frac{\pi}{4}, \frac{3\pi}{4}$

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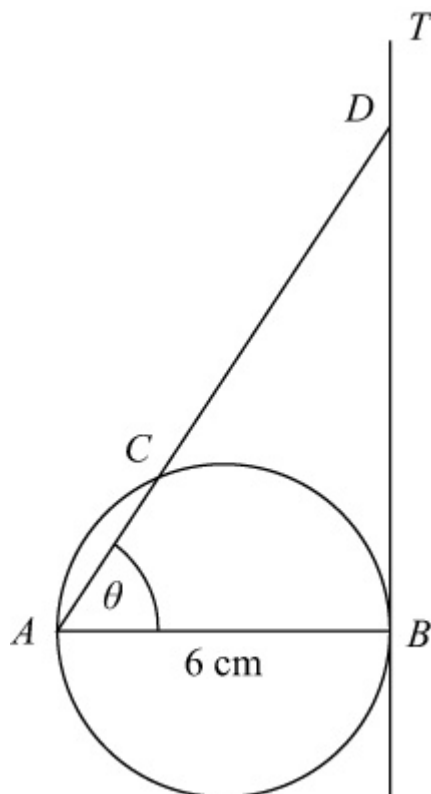
Exercise C, Question 8

Question:

In the diagram $AB = 6$ cm is the diameter of the circle and BT is the tangent to the circle at B . The chord AC is extended to meet this tangent at D and $\angle DAB = \theta$.

(a) Show that $CD = 6 (\sec \theta - \cos \theta)$.

(b) Given that $CD = 16$ cm, calculate the length of the chord AC .



Solution:

(a) In right-angled triangle ABD

$$\frac{AB}{AD} = \cos \theta$$

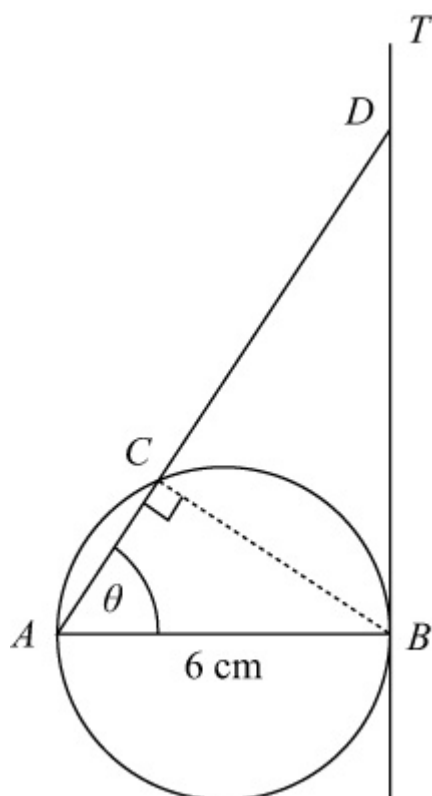
$$\Rightarrow AD = \frac{6}{\cos \theta} = 6 \sec \theta$$

In right-angled triangle ACB

$$\frac{AC}{AB} = \cos \theta$$

$$\Rightarrow AC = 6 \cos \theta$$

$$DC = AD - AC = 6 \sec \theta - 6 \cos \theta = 6 (\sec \theta - \cos \theta)$$



(b) As $16 = 6 \sec \theta - 6 \cos \theta$

$$\Rightarrow 8 = \frac{3}{\cos \theta} - 3 \cos \theta$$

$$\Rightarrow 8 \cos \theta = 3 - 3 \cos^2 \theta$$

$$\Rightarrow 3 \cos^2 \theta + 8 \cos \theta - 3 = 0$$

$$\Rightarrow (3 \cos \theta - 1) (\cos \theta + 3) = 0$$

$$\Rightarrow \cos \theta = \frac{1}{3} \quad \text{as } \cos \theta \neq -3$$

From (a) $AC = 6 \cos \theta = 6 \times \frac{1}{3} = 2 \text{ cm}$

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Exercise D, Question 1

Question:

Simplify each of the following expressions:

(a) $1 + \tan^2 \frac{1}{2}\theta$

(b) $(\sec \theta - 1)(\sec \theta + 1)$

(c) $\tan^2 \theta (\operatorname{cosec}^2 \theta - 1)$

(d) $(\sec^2 \theta - 1) \cot \theta$

(e) $(\operatorname{cosec}^2 \theta - \cot^2 \theta)^2$

(f) $2 - \tan^2 \theta + \sec^2 \theta$

(g) $\frac{\tan \theta \sec \theta}{1 + \tan^2 \theta}$

(h) $(1 - \sin^2 \theta)(1 + \tan^2 \theta)$

(i) $\frac{\operatorname{cosec} \theta \cot \theta}{1 + \cot^2 \theta}$

(j) $(\sec^4 \theta - 2 \sec^2 \theta \tan^2 \theta + \tan^4 \theta)$

(k) $4 \operatorname{cosec}^2 2\theta + 4 \operatorname{cosec}^2 2\theta \cot^2 2\theta$

Solution:

(a) Use $1 + \tan^2 \theta = \sec^2 \theta$ with θ replaced with $\frac{1}{2}\theta$.

$$1 + \tan^2 \left(\frac{1}{2}\theta \right) = \sec^2 \left(\frac{1}{2}\theta \right)$$

(b) $(\sec \theta - 1)(\sec \theta + 1)$ (multiply out)
 $= \sec^2 \theta - 1$
 $= (1 + \tan^2 \theta) - 1$

$$= \tan^2 \theta$$

$$\begin{aligned} \text{(c)} \quad & \tan^2 \theta (\operatorname{cosec}^2 \theta - 1) \\ &= \tan^2 \theta [(1 + \cot^2 \theta) - 1] \\ &= \tan^2 \theta \cot^2 \theta \\ &= \tan^2 \theta \times \frac{1}{\tan^2 \theta} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & (\sec^2 \theta - 1) \cot \theta \\ &= \tan^2 \theta \cot \theta \\ &= \tan^2 \theta \times \frac{1}{\tan \theta} \\ &= \tan \theta \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & (\operatorname{cosec}^2 \theta - \cot^2 \theta)^2 \\ &= [(1 + \cot^2 \theta) - \cot^2 \theta]^2 \\ &= 1^2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & 2 - \tan^2 \theta + \sec^2 \theta \\ &= 2 - \tan^2 \theta + (1 + \tan^2 \theta) \\ &= 2 - \tan^2 \theta + 1 + \tan^2 \theta \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{(g)} \quad & \frac{\tan \theta \sec \theta}{1 + \tan^2 \theta} \\ &= \frac{\tan \theta \sec \theta}{\sec^2 \theta} \\ &= \frac{\tan \theta}{\sec \theta} \\ &= \tan \theta \cos \theta \\ &= \frac{\sin \theta}{\cos \theta} \times \cos \theta \\ &= \sin \theta \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad & (1 - \sin^2 \theta) (1 + \tan^2 \theta) \\ &= \cos^2 \theta \times \sec^2 \theta \end{aligned}$$

$$\begin{aligned} &= \cos^2 \theta \times \frac{1}{\cos^2 \theta} \\ &= 1 \end{aligned}$$

$$(i) \frac{\operatorname{cosec} \theta \cot \theta}{1 + \cot^2 \theta}$$

$$= \frac{\operatorname{cosec} \theta \cot \theta}{\operatorname{cosec}^2 \theta}$$

$$= \frac{1}{\operatorname{cosec} \theta} \times \cot \theta$$

$$= \frac{\sin \theta}{1} \times \frac{\cos \theta}{\sin \theta}$$

$$= \cos \theta$$

$$\begin{aligned} (j) & \sec^4 \theta - 2 \sec^2 \theta \tan^2 \theta + \tan^4 \theta \\ &= (\sec^2 \theta - \tan^2 \theta)^2 \quad (\text{factorise}) \\ &= [(1 + \tan^2 \theta) - \tan^2 \theta]^2 \\ &= 1^2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} (k) & 4 \operatorname{cosec}^2 2\theta + 4 \operatorname{cosec}^2 2\theta \cot^2 2\theta \\ &= 4 \operatorname{cosec}^2 2\theta (1 + \cot^2 2\theta) \\ &= 4 \operatorname{cosec}^2 2\theta \operatorname{cosec}^2 2\theta \\ &= 4 \operatorname{cosec}^4 2\theta \end{aligned}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

Given that $\operatorname{cosec} x = \frac{k}{\operatorname{cosec} x}$, where $k > 1$, find, in terms of k , possible values of $\cot x$.

Solution:

$$\operatorname{cosec} x = \frac{k}{\operatorname{cosec} x}$$

$$\Rightarrow \operatorname{cosec}^2 x = k$$

$$\Rightarrow 1 + \cot^2 x = k$$

$$\Rightarrow \cot^2 x = k - 1$$

$$\Rightarrow \cot x = \pm \sqrt{k - 1}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 3

Question:

Given that $\cot \theta = -\sqrt{3}$, and that $90^\circ < \theta < 180^\circ$, find the exact value of

(a) $\sin \theta$

(b) $\cos \theta$

Solution:

(a) $\cot \theta = -\sqrt{3}$ $90^\circ < \theta < 180^\circ$

$$\Rightarrow 1 + \cot^2 \theta = 1 + 3 = 4$$

$$\Rightarrow \operatorname{cosec}^2 \theta = 4$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2} \quad (\text{as } \theta \text{ is in 2nd quadrant, } \sin \theta \text{ is +ve)}$$

(b) Using $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \cos \theta = -\frac{\sqrt{3}}{2} \quad (\text{as } \theta \text{ is in 2nd quadrant, } \cos \theta \text{ is -ve)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 4

Question:

Given that $\tan \theta = \frac{3}{4}$, and that $180^\circ < \theta < 270^\circ$, find the exact value of

(a) $\sec \theta$

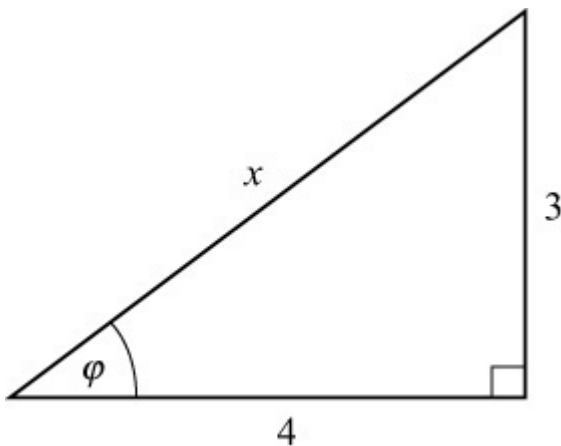
(b) $\cos \theta$

(c) $\sin \theta$

Solution:

$$\tan \theta = \frac{3}{4} \quad 180^\circ < \theta < 270^\circ$$

Draw right-angled triangle where $\tan \theta = \frac{3}{4}$



Using Pythagoras' theorem, $x = 5$

$$\text{So } \cos \theta = \frac{4}{5} \text{ and } \sin \theta = \frac{3}{5}$$

As θ is in 3rd quadrant, both $\sin \theta$ and $\cos \theta$ are $-ve$.

$$\text{So } \sin \theta = -\frac{3}{5}, \cos \theta = -\frac{4}{5}$$

(a) $\sec \theta = \frac{1}{\cos \theta} = -\frac{5}{4}$

(b) $\cos \theta = -\frac{4}{5}$

$$(c) \sin \theta = -\frac{3}{5}$$

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Exercise D, Question 5

Question:

Given that $\cos \theta = \frac{24}{25}$, and that θ is a reflex angle, find the exact value of

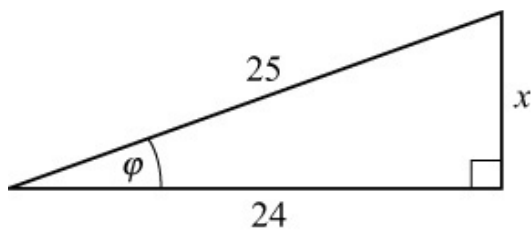
- (a) $\tan \theta$
 (b) $\operatorname{cosec} \theta$

Solution:

$$\cos \theta = \frac{24}{25}, \theta \text{ reflex}$$

As $\cos \theta$ is +ve and θ reflex, θ is in the 4th quadrant.

Use right-angled triangle where $\cos \theta = \frac{24}{25}$



Using Pythagoras' theorem,

$$25^2 = x^2 + 24^2$$

$$\Rightarrow x^2 = 25^2 - 24^2 = 49$$

$$\Rightarrow x = 7$$

So $\tan \phi = \frac{7}{24}$ and $\sin \phi = \frac{7}{25}$

As θ is in 4th quadrant,

$$(a) \tan \theta = -\frac{7}{24}$$

$$(b) \operatorname{cosec} \theta = \frac{1}{\sin \theta} = -\frac{1}{\frac{7}{25}} = -\frac{25}{7}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 6

Question:

Prove the following identities:

$$(a) \sec^4 \theta - \tan^4 \theta \equiv \sec^2 \theta + \tan^2 \theta$$

$$(b) \operatorname{cosec}^2 x - \sin^2 x \equiv \cot^2 x + \cos^2 x$$

$$(c) \sec^2 A (\cot^2 A - \cos^2 A) \equiv \cot^2 A$$

$$(d) 1 - \cos^2 \theta \equiv (\sec^2 \theta - 1) (1 - \sin^2 \theta)$$

$$(e) \frac{1 - \tan^2 A}{1 + \tan^2 A} \equiv 1 - 2 \sin^2 A$$

$$(f) \sec^2 \theta + \operatorname{cosec}^2 \theta \equiv \sec^2 \theta \operatorname{cosec}^2 \theta$$

$$(g) \operatorname{cosec} A \sec^2 A \equiv \operatorname{cosec} A + \tan A \sec A$$

$$(h) (\sec \theta - \sin \theta) (\sec \theta + \sin \theta) \equiv \tan^2 \theta + \cos^2 \theta$$

Solution:

$$\begin{aligned} (a) \text{ L.H.S.} &\equiv \sec^4 \theta - \tan^4 \theta \\ &\equiv (\sec^2 \theta - \tan^2 \theta) (\sec^2 \theta + \tan^2 \theta) && \text{(difference of two} \\ &\text{squares)} \\ &\equiv (1) (\sec^2 \theta + \tan^2 \theta) && \text{(as} \\ 1 + \tan^2 \theta &\equiv \sec^2 \theta \Rightarrow \sec^2 \theta - \tan^2 \theta \equiv 1) \\ &\equiv \sec^2 \theta + \tan^2 \theta \equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{ L.H.S.} &\equiv \operatorname{cosec}^2 x - \sin^2 x \\ &\equiv (1 + \cot^2 x) - (1 - \cos^2 x) \\ &\equiv 1 + \cot^2 x - 1 + \cos^2 x \\ &\equiv \cot^2 x + \cos^2 x \equiv \text{R.H.S.} \end{aligned}$$

$$(c) \text{ L.H.S.} \equiv \sec^2 A (\cot^2 A - \cos^2 A)$$

$$\begin{aligned}
&\equiv \frac{1}{\cos^2 A} \left(\frac{\cos^2 A}{\sin^2 A} - \cos^2 A \right) \\
&\equiv \frac{1}{\sin^2 A} - 1 \\
&\equiv \operatorname{cosec}^2 A - 1 \quad (\text{use } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta) \\
&\equiv 1 + \cot^2 A - 1 \\
&\equiv \cot^2 A \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(d) R.H.S.} &\equiv (\sec^2 \theta - 1) (1 - \sin^2 \theta) \\
&\equiv \tan^2 \theta \times \cos^2 \theta \quad (\text{use } 1 + \tan^2 \theta \equiv \sec^2 \theta \text{ and} \\
&\cos^2 \theta + \sin^2 \theta \equiv 1) \\
&\equiv \frac{\sin^2 \theta}{\cos^2 \theta} \times \cos^2 \theta \\
&\equiv \sin^2 \theta \\
&\equiv 1 - \cos^2 \theta \equiv \text{L.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(e) L.H.S.} &\equiv \frac{1 - \tan^2 A}{1 + \tan^2 A} \\
&\equiv \frac{1 - \tan^2 A}{\sec^2 A} \\
&\equiv \frac{1}{\sec^2 A} (1 - \tan^2 A) \\
&\equiv \cos^2 A \left(1 - \frac{\sin^2 A}{\cos^2 A} \right) \\
&\equiv \cos^2 A - \sin^2 A \\
&\equiv (1 - \sin^2 A) - \sin^2 A \\
&\equiv 1 - 2 \sin^2 A \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(f) R.H.S.} &\equiv \sec^2 \theta \operatorname{cosec}^2 \theta \\
&\equiv \sec^2 \theta (1 + \cot^2 \theta) \\
&\equiv \sec^2 \theta + \frac{1}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta} \\
&\equiv \sec^2 \theta + \frac{1}{\sin^2 \theta} \\
&\equiv \sec^2 \theta + \operatorname{cosec}^2 \theta \equiv \text{L.H.S.}
\end{aligned}$$

$$\text{(g) L.H.S.} \equiv \operatorname{cosec} A \sec^2 A$$

$$\begin{aligned}
&\equiv \operatorname{cosec} A (1 + \tan^2 A) \\
&\equiv \operatorname{cosec} A + \frac{1}{\sin A} \times \frac{\sin^2 A}{\cos^2 A} \\
&\equiv \operatorname{cosec} A + \frac{\sin A}{\cos^2 A} \\
&\equiv \operatorname{cosec} A + \frac{\sin A}{\cos A} \times \frac{1}{\cos A} \\
&\equiv \operatorname{cosec} A + \tan A \sec A \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(h) L.H.S.} &\equiv (\sec \theta - \sin \theta) (\sec \theta + \sin \theta) \\
&\equiv \sec^2 \theta - \sin^2 \theta \\
&\equiv (1 + \tan^2 \theta) - (1 - \cos^2 \theta) \\
&\equiv 1 + \tan^2 \theta - 1 + \cos^2 \theta \\
&\equiv \tan^2 \theta + \cos^2 \theta \equiv \text{R.H.S.}
\end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 7

Question:

Given that $3 \tan^2 \theta + 4 \sec^2 \theta = 5$, and that θ is obtuse, find the exact value of $\sin \theta$.

Solution:

$$\begin{aligned}3 \tan^2 \theta + 4 \sec^2 \theta &= 5 \\ \Rightarrow 3 \tan^2 \theta + 4(1 + \tan^2 \theta) &= 5 \\ \Rightarrow 3 \tan^2 \theta + 4 + 4 \tan^2 \theta &= 5 \\ \Rightarrow 7 \tan^2 \theta &= 1 \\ \Rightarrow \tan^2 \theta &= \frac{1}{7} \\ \Rightarrow \cot^2 \theta &= 7 \\ \Rightarrow \operatorname{cosec}^2 \theta - 1 &= 7 \\ \Rightarrow \operatorname{cosec}^2 \theta &= 8 \\ \Rightarrow \sin^2 \theta &= \frac{1}{8}\end{aligned}$$

As θ is obtuse (2nd quadrant), so $\sin \theta$ is +ve.

$$\text{So } \sin \theta = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Solutionbank

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Exercise D, Question 8

Question:

Giving answers to 3 significant figures where necessary, solve the following equations in the given intervals:

(a) $\sec^2 \theta = 3 \tan \theta, 0 \leq \theta \leq 360^\circ$

(b) $\tan^2 \theta - 2 \sec \theta + 1 = 0, -\pi \leq \theta \leq \pi$

(c) $\operatorname{cosec}^2 \theta + 1 = 3 \cot \theta, -180^\circ \leq \theta \leq 180^\circ$

(d) $\cot \theta = 1 - \operatorname{cosec}^2 \theta, 0 \leq \theta \leq 2\pi$

(e) $3 \sec \frac{1}{2}\theta = 2 \tan^2 \frac{1}{2}\theta, 0 \leq \theta \leq 360^\circ$

(f) $(\sec \theta - \cos \theta)^2 = \tan \theta - \sin^2 \theta, 0 \leq \theta \leq \pi$

(g) $\tan^2 2\theta = \sec 2\theta - 1, 0 \leq \theta \leq 180^\circ$

(h) $\sec^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 1, 0 \leq \theta \leq 2\pi$

Solution:

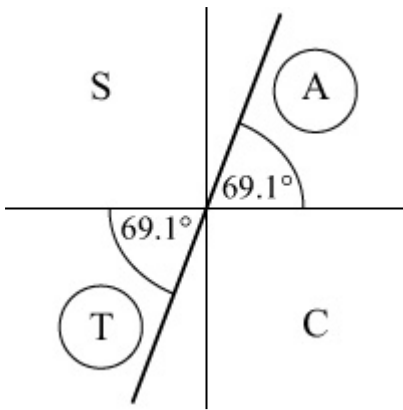
(a) $\sec^2 \theta = 3 \tan \theta \quad 0 \leq \theta \leq 360^\circ$

$$\Rightarrow 1 + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow \tan^2 \theta - 3 \tan \theta + 1 = 0$$

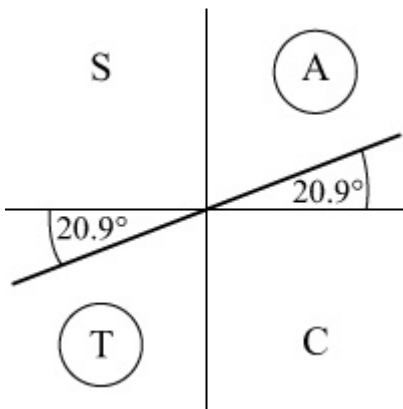
$$\tan \theta = \frac{3 \pm \sqrt{5}}{2} \quad (\text{equation does not factorise}).$$

For $\tan \theta = \frac{3 + \sqrt{5}}{2}$, calculator value is 69.1°



Solutions are 69.1° , 249°

For $\tan \theta = \frac{3 - \sqrt{5}}{2}$, calculator value is 20.9°



Solutions are 20.9° , 201°

Set of solutions: 20.9° , 69.1° , 201° , 249° (3 s.f.)

$$(b) \tan^2 \theta - 2 \sec \theta + 1 = 0 \quad -\pi \leq \theta \leq \pi$$

$$\Rightarrow (\sec^2 \theta - 1) - 2 \sec \theta + 1 = 0$$

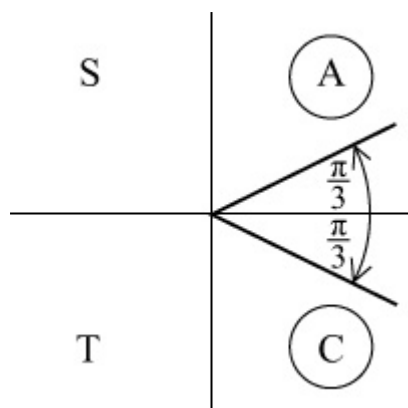
$$\Rightarrow \sec^2 \theta - 2 \sec \theta = 0$$

$$\Rightarrow \sec \theta (\sec \theta - 2) = 0$$

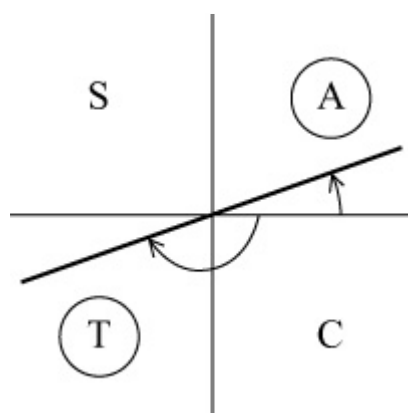
$$\Rightarrow \sec \theta = 2 \quad (\text{as } \sec \theta \text{ cannot be } 0)$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}$$



$$\begin{aligned}
 \text{(c) } \operatorname{cosec}^2 \theta + 1 &= 3 \cot \theta & -180^\circ \leq \theta \leq 180^\circ \\
 \Rightarrow (1 + \cot^2 \theta) + 1 &= 3 \cot \theta \\
 \Rightarrow \cot^2 \theta - 3 \cot \theta + 2 &= 0 \\
 \Rightarrow (\cot \theta - 1)(\cot \theta - 2) &= 0 \\
 \Rightarrow \cot \theta = 1 \text{ or } \cot \theta = 2 \\
 \Rightarrow \tan \theta = 1 \text{ or } \tan \theta = \frac{1}{2}
 \end{aligned}$$

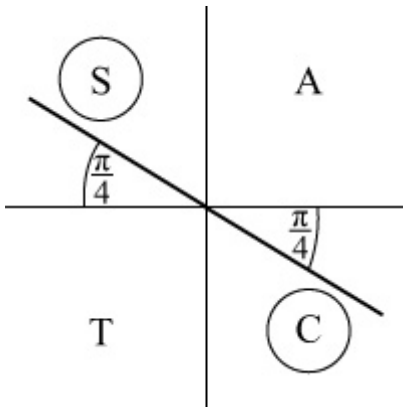


$$\begin{aligned}
 \tan \theta = 1 &\Rightarrow \theta = -135^\circ, 45^\circ \\
 \tan \theta = \frac{1}{2} &\Rightarrow \theta = -153^\circ, 26.6^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) } \cot \theta = 1 - \operatorname{cosec}^2 \theta & \quad 0 \leq \theta \leq 2\pi \\
 \Rightarrow \cot \theta = 1 - (1 + \cot^2 \theta) \\
 \Rightarrow \cot \theta = -\cot^2 \theta \\
 \Rightarrow \cot^2 \theta + \cot \theta = 0 \\
 \Rightarrow \cot \theta (\cot \theta + 1) = 0 \\
 \Rightarrow \cot \theta = 0 \text{ or } \cot \theta = -1
 \end{aligned}$$

For $\cot \theta = 0$ refer to graph: $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

For $\cot \theta = -1, \tan \theta = -1$



So $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

Set of solutions: $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

$$(e) 3 \sec \frac{1}{2}\theta = 2 \tan^2 \frac{1}{2}\theta \quad 0 \leq \theta \leq 360^\circ$$

$$\Rightarrow 3 \sec \frac{1}{2}\theta = 2 \left(\sec^2 \frac{1}{2}\theta - 1 \right) \quad (\text{use } 1 + \tan^2 A \equiv \sec^2 A \text{ with}$$

$$A = \frac{1}{2}\theta)$$

$$\Rightarrow 2 \sec^2 \frac{1}{2}\theta - 3 \sec \frac{1}{2}\theta - 2 = 0$$

$$\Rightarrow \left(2 \sec \frac{1}{2}\theta + 1 \right) \left(\sec \frac{1}{2}\theta - 2 \right) = 0$$

$$\Rightarrow \sec \frac{1}{2}\theta = -\frac{1}{2} \text{ or } \sec \frac{1}{2}\theta = 2$$

Only $\sec \frac{1}{2}\theta = 2$ applies as $\sec A \leq -1$ or $\sec A \geq 1$

$$\Rightarrow \cos \frac{1}{2}\theta = \frac{1}{2}$$

As $0 \leq \theta \leq 360^\circ$

so $0 \leq \frac{1}{2}\theta \leq 180^\circ$

Calculator value is 60°

This is the only value in the interval.

$$\text{So } \frac{1}{2}\theta = 60^\circ$$

$$\Rightarrow \theta = 120^\circ$$

$$\text{(f) } (\sec \theta - \cos \theta)^2 = \tan \theta - \sin^2 \theta \quad 0 \leq \theta \leq \pi$$

$$\Rightarrow \sec^2 \theta - 2 \sec \theta \cos \theta + \cos^2 \theta = \tan \theta - \sin^2 \theta$$

$$\Rightarrow \sec^2 \theta - 2 + \cos^2 \theta = \tan \theta - \sin^2 \theta \quad \left(\sec \theta \cos \theta = \right.$$

$$\left. \frac{1}{\cos \theta} \times \cos \theta = 1 \right)$$

$$\Rightarrow (1 + \tan^2 \theta) - 2 + (\cos^2 \theta + \sin^2 \theta) = \tan \theta$$

$$\Rightarrow 1 + \tan^2 \theta - 2 + 1 = \tan \theta$$

$$\Rightarrow \tan^2 \theta - \tan \theta = 0$$

$$\Rightarrow \tan \theta (\tan \theta - 1) = 0$$

$$\Rightarrow \tan \theta = 0 \text{ or } \tan \theta = 1$$

$$\tan \theta = 0 \Rightarrow \theta = 0, \pi$$

$$\tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\text{Set of solutions: } 0, \frac{\pi}{4}, \pi$$

$$\text{(g) } \tan^2 2\theta = \sec 2\theta - 1 \quad 0 \leq \theta \leq 180^\circ$$

$$\Rightarrow \sec^2 2\theta - 1 = \sec 2\theta - 1$$

$$\Rightarrow \sec^2 2\theta - \sec 2\theta = 0$$

$$\Rightarrow \sec 2\theta (\sec 2\theta - 1) = 0$$

$$\Rightarrow \sec 2\theta = 0 \text{ (not possible) or } \sec 2\theta = 1$$

$$\Rightarrow \cos 2\theta = 1 \quad 0 \leq 2\theta \leq 360^\circ$$

Refer to graph of $y = \cos \theta$

$$\Rightarrow 2\theta = 0^\circ, 360^\circ$$

$$\Rightarrow \theta = 0^\circ, 180^\circ$$

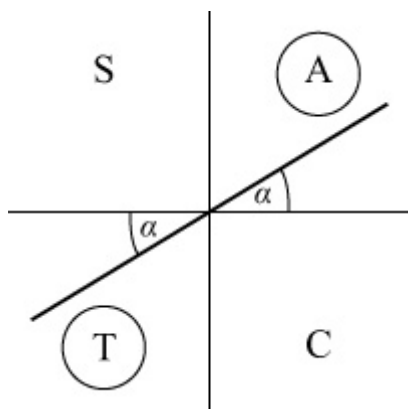
$$\text{(h) } \sec^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 1 \quad 0 \leq \theta \leq 2\pi$$

$$\Rightarrow (1 + \tan^2 \theta) - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 1$$

$$\Rightarrow \tan^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 0$$

$$\Rightarrow (\tan \theta - \sqrt{3}) (\tan \theta - 1) = 0$$

$$\Rightarrow \tan \theta = \sqrt{3} \text{ or } \tan \theta = 1$$



First answer (α) for $\tan \theta = \sqrt{3}$ is $\frac{\pi}{3}$

Second solution is $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

First answer for $\tan \theta = 1$ is $\frac{\pi}{4}$

Second solution is $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Set of solutions: $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 9

Question:

Given that $\tan^2 k = 2 \sec k$,

(a) find the value of $\sec k$.

(b) deduce that $\cos k = \sqrt{2} - 1$

(c) hence solve, in the interval $0 \leq k \leq 360^\circ$, $\tan^2 k = 2 \sec k$, giving your answers to 1 decimal place.

Solution:

(a) $\tan^2 k = 2 \sec k$

$$\Rightarrow (\sec^2 k - 1) = 2 \sec k$$

$$\Rightarrow \sec^2 k - 2 \sec k - 1 = 0$$

$$\Rightarrow \sec k = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

As $\sec k$ has no values between -1 and 1

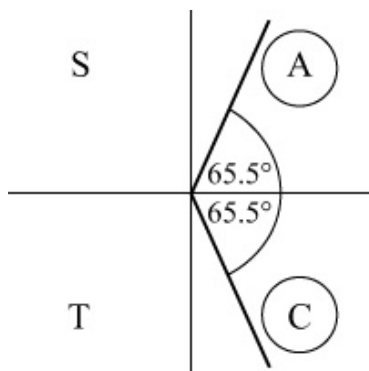
$$\sec k = 1 + \sqrt{2}$$

$$(b) \cos k = \frac{1}{1 + \sqrt{2}} = \frac{\sqrt{2} - 1}{(1 + \sqrt{2})(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

(c) Solutions of $\tan^2 k = 2 \sec k$, $0 \leq k \leq 360^\circ$
are solutions of $\cos k = \sqrt{2} - 1$

Calculator solution is 65.5°

$$\Rightarrow k = 65.5^\circ, 360^\circ - 65.5^\circ = 65.5^\circ, 294.5^\circ \text{ (1 d.p.)}$$



Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 10

Question:

Given that $a = 4 \sec x$, $b = \cos x$ and $c = \cot x$,

(a) express b in terms of a

(b) show that $c^2 = \frac{16}{a^2 - 16}$

Solution:

(a) As $a = 4 \sec x$

$$\Rightarrow \sec x = \frac{a}{4}$$

$$\Rightarrow \cos x = \frac{4}{a}$$

As $\cos x = b$

$$\Rightarrow b = \frac{4}{a}$$

(b) $c = \cot x$

$$\Rightarrow c^2 = \cot^2 x$$

$$\Rightarrow \frac{1}{c^2} = \tan^2 x$$

$$\Rightarrow \frac{1}{c^2} = \sec^2 x - 1 \quad (\text{use } 1 + \tan^2 x \equiv \sec^2 x)$$

$$\Rightarrow \frac{1}{c^2} = \frac{a^2}{16} - 1 \quad \left(\sec x = \frac{a}{4} \right)$$

$$\Rightarrow 16 = a^2 c^2 - 16c^2 \quad (\text{multiply by } 16c^2)$$

$$\Rightarrow c^2 (a^2 - 16) = 16$$

$$\Rightarrow c^2 = \frac{16}{a^2 - 16}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 11

Question:

Given that $x = \sec \theta + \tan \theta$,

(a) show that $\frac{1}{x} = \sec \theta - \tan \theta$.

(b) Hence express $x^2 + \frac{1}{x^2} + 2$ in terms of θ , in its simplest form.

Solution:

(a) $x = \sec \theta + \tan \theta$

$$\frac{1}{x} = \frac{1}{\sec \theta + \tan \theta}$$

$$= \frac{\sec \theta - \tan \theta}{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}$$

$$= \frac{\sec \theta - \tan \theta}{\sec^2 \theta - \tan^2 \theta}$$

$$= \sec \theta - \tan \theta \quad (\text{as } 1 + \tan^2 \theta \equiv \sec^2 \theta \Rightarrow \sec^2 \theta - \tan^2 \theta \equiv 1)$$

(b) $x + \frac{1}{x} = \sec \theta + \tan \theta + \sec \theta - \tan \theta = 2 \sec \theta$

$$\Rightarrow \left(x + \frac{1}{x} \right)^2 = 4 \sec^2 \theta$$

$$\Rightarrow x^2 + 2x \times \frac{1}{x} + \frac{1}{x^2} = 4 \sec^2 \theta$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4 \sec^2 \theta$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 12

Question:

Given that $2 \sec^2 \theta - \tan^2 \theta = p$ show that $\operatorname{cosec}^2 \theta = \frac{p-1}{p-2}, p \neq 2$.

Solution:

$$2 \sec^2 \theta - \tan^2 \theta = p$$

$$\Rightarrow 2(1 + \tan^2 \theta) - \tan^2 \theta = p$$

$$\Rightarrow 2 + 2 \tan^2 \theta - \tan^2 \theta = p$$

$$\Rightarrow \tan^2 \theta = p - 2$$

$$\Rightarrow \cot^2 \theta = \frac{1}{p-2} \quad \left(\cot \theta = \frac{1}{\tan \theta} \right)$$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + \frac{1}{p-2} = \frac{(p-2) + 1}{p-2} = \frac{p-1}{p-2}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

Question:

Without using a calculator, work out, giving your answer in terms of π , the value of:

(a) $\arccos 0$

(b) $\arcsin(1)$

(c) $\arctan (-1)$

(d) $\arcsin \left(-\frac{1}{2} \right)$

(e) $\arccos \left(-\frac{1}{\sqrt{2}} \right)$

(f) $\arctan \left(-\frac{1}{\sqrt{3}} \right)$

(g) $\arcsin \left(\sin \frac{\pi}{3} \right)$

(h) $\arcsin \left(\sin \frac{2\pi}{3} \right)$

Solution:

(a) $\arccos 0$ is the angle α in $0 \leq \alpha \leq \pi$ for which $\cos \alpha = 0$

Refer to graph of $y = \cos \theta \Rightarrow \alpha = \frac{\pi}{2}$

So $\arccos 0 = \frac{\pi}{2}$

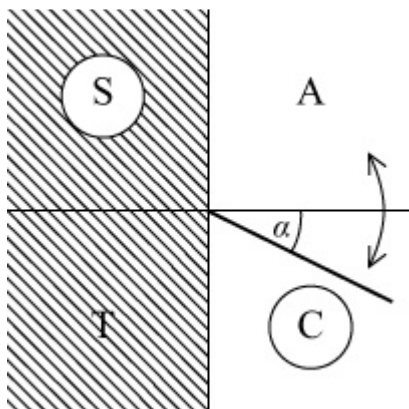
(b) $\arcsin 1$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which $\sin \alpha = 1$

Refer to graph of $y = \sin \theta \Rightarrow \alpha = \frac{\pi}{2}$

$$\text{So } \arcsin 1 = \frac{\pi}{2}$$

(c) $\arctan(-1)$ is the angle α in $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ for which $\tan \alpha = -1$

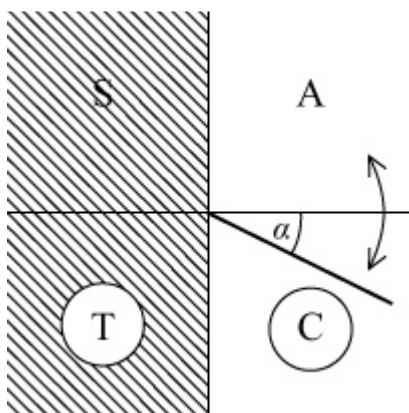
$$\text{So } \arctan(-1) = -\frac{\pi}{4}$$



(d) $\arcsin\left(-\frac{1}{2}\right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

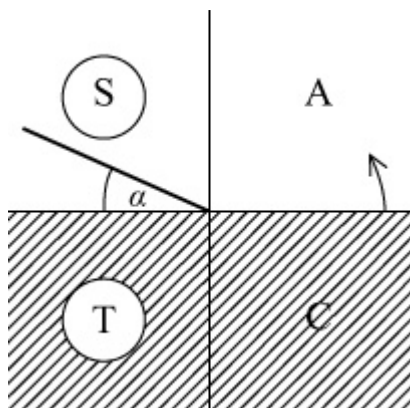
$$\sin \alpha = -\frac{1}{2}$$

$$\text{So } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$



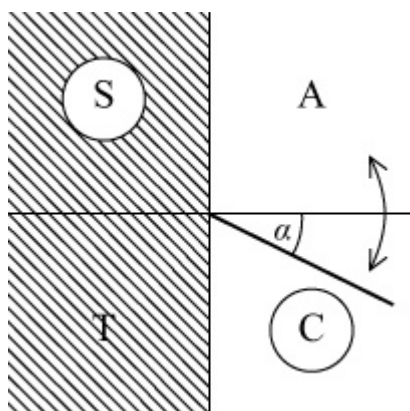
(e) $\arccos\left(-\frac{1}{\sqrt{2}}\right)$ is the angle α in $0 \leq \alpha \leq \pi$ for which $\cos \alpha = -\frac{1}{\sqrt{2}}$

$$\text{So } \arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$



(f) $\arctan \left(-\frac{1}{\sqrt{3}} \right)$ is the angle α in $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ for which $\tan \alpha = -\frac{1}{\sqrt{3}}$

So $\arctan \left(-\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6}$



(g) $\arcsin \left(\sin \frac{\pi}{3} \right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

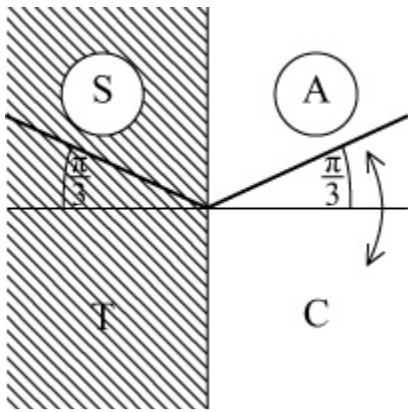
$$\sin \alpha = \sin \frac{\pi}{3}$$

So $\arcsin \left(\sin \frac{\pi}{3} \right) = \frac{\pi}{3}$

(h) $\arcsin \left(\sin \frac{2\pi}{3} \right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

$$\sin \alpha = \sin \frac{2\pi}{3}$$

So $\arcsin \left(\sin \frac{2\pi}{3} \right) = \frac{\pi}{3}$



Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 2

Question:

Find the value of:

(a) $\arcsin \left(\frac{1}{2} \right) + \arcsin \left(-\frac{1}{2} \right)$

(b) $\arccos \left(\frac{1}{2} \right) - \arccos \left(-\frac{1}{2} \right)$

(c) $\arctan (1) - \arctan (- 1)$

Solution:

(a) $\arcsin \left(\frac{1}{2} \right) + \arcsin \left(-\frac{1}{2} \right) = \frac{\pi}{6} + \left(-\frac{\pi}{6} \right) = 0$

(b) $\arccos \left(\frac{1}{2} \right) - \arccos \left(-\frac{1}{2} \right) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

(c) $\arctan (1) - \arctan (- 1) = \frac{\pi}{4} - \left(-\frac{\pi}{4} \right) = \frac{\pi}{2}$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 3

Question:

Without using a calculator, work out the values of:

(a) $\sin \left(\arcsin \frac{1}{2} \right)$

(b) $\sin \left[\arcsin \left(-\frac{1}{2} \right) \right]$

(c) $\tan [\arctan (-1)]$

(d) $\cos(\arccos 0)$

Solution:

(a) $\sin \left(\arcsin \frac{1}{2} \right)$

$$\arcsin \frac{1}{2} = \alpha \text{ where } \sin \alpha = \frac{1}{2}, -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$\text{So } \arcsin \frac{1}{2} = \frac{\pi}{6}$$

$$\Rightarrow \sin \left(\arcsin \frac{1}{2} \right) = \sin \frac{\pi}{6} = \frac{1}{2}$$

(b) $\sin \left[\arcsin \left(-\frac{1}{2} \right) \right]$

$$\arcsin \left(-\frac{1}{2} \right) = \alpha \text{ where } \sin \alpha = -\frac{1}{2}, -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$\text{So } \arcsin \left(-\frac{1}{2} \right) = -\frac{\pi}{6}$$

$$\Rightarrow \sin \left[\arcsin \left(-\frac{1}{2} \right) \right] = \sin \left(-\frac{\pi}{6} \right) = -\frac{1}{2}$$

(c) $\tan [\arctan (-1)]$

$$\arctan (-1) = \alpha \text{ where } \tan \alpha = -1, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

$$\text{So } \arctan(-1) = -\frac{\pi}{4}$$

$$\Rightarrow \tan[\arctan(-1)] = \tan\left(-\frac{\pi}{4}\right) = -1$$

(d) $\cos(\arccos 0)$

$$\arccos 0 = \alpha \text{ where } \cos \alpha = 0, 0 \leq \alpha \leq \pi$$

$$\text{So } \arccos 0 = \frac{\pi}{2}$$

$$\Rightarrow \cos(\arccos 0) = \cos \frac{\pi}{2} = 0$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

Question:

Without using a calculator, work out the exact values of:

$$(a) \sin \left[\arccos \left(\frac{1}{2} \right) \right]$$

$$(b) \cos \left[\arcsin \left(-\frac{1}{2} \right) \right]$$

$$(c) \tan \left[\arccos \left(-\frac{\sqrt{2}}{2} \right) \right]$$

$$(d) \sec \left[\arctan \left(\sqrt{3} \right) \right]$$

$$(e) \operatorname{cosec} \left[\arcsin \left(-1 \right) \right]$$

$$(f) \sin \left[2 \arcsin \left(\frac{\sqrt{2}}{2} \right) \right]$$

Solution:

$$(a) \sin \left(\arccos \frac{1}{2} \right)$$

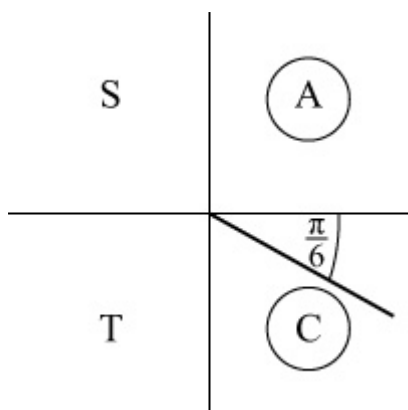
$$\arccos \frac{1}{2} = \frac{\pi}{3}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$(b) \cos \left[\arcsin \left(-\frac{1}{2} \right) \right]$$

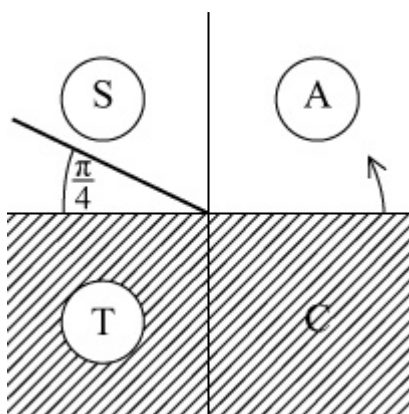
$$\arcsin \left(-\frac{1}{2} \right) = -\frac{\pi}{6}$$

$$\cos \left(-\frac{\pi}{6} \right) = +\frac{\sqrt{3}}{2}$$



$$(c) \tan \left[\arccos \left(-\frac{\sqrt{2}}{2} \right) \right]$$

$$\arccos \left(-\frac{\sqrt{2}}{2} \right) = \alpha \text{ where } \cos \alpha = -\frac{\sqrt{2}}{2}, 0 \leq \alpha \leq \pi$$



$$\text{So } \arccos \left(-\frac{\sqrt{2}}{2} \right) = \frac{3\pi}{4}$$

$$\tan \frac{3\pi}{4} = -1$$

$$(d) \sec (\arctan \sqrt{3})$$

$$\arctan \sqrt{3} = \frac{\pi}{3} \quad (\text{the angle whose tan is } \sqrt{3})$$

$$\sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$$

$$(e) \operatorname{cosec} [\arcsin (-1)]$$

$$\arcsin (-1) = \alpha \text{ where } \sin \alpha = -1, -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$\text{So } \arcsin(-1) = -\frac{\pi}{2}$$

$$\Rightarrow \operatorname{cosec}[\arcsin(-1)] = \frac{1}{\sin(-\frac{\pi}{2})} = \frac{1}{-1} = -1$$

$$(f) \sin \left[2 \arcsin \left(\frac{\sqrt{2}}{2} \right) \right]$$

$$\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

$$\text{So } \sin \left[2 \arcsin \left(\frac{\sqrt{2}}{2} \right) \right] = \sin \frac{\pi}{2} = 1$$

Solutionbank

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Exercise E, Question 5

Question:

Given that $\arcsin k = \alpha$, where $0 < k < 1$ and α is in radians, write down, in terms of α , the first two positive values of x satisfying the equation $\sin x = k$.

Solution:

As k is positive, the first two positive solutions of $\sin x = k$ are $\arcsin k$ and $\pi - \arcsin k$

i.e. α and $\pi - \alpha$

(Try a few examples, taking specific values for k).

Solutionbank

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Exercise E, Question 6

Question:

Given that x satisfies $\arcsin x = k$, where $0 < k < \frac{\pi}{2}$,

(a) state the range of possible values of x

(b) express, in terms of x ,

(i) $\cos k$ (ii) $\tan k$

Given, instead, that $-\frac{\pi}{2} < k < 0$,

(c) how, if at all, would it affect your answers to (b)?

Solution:

(a) $\arcsin x$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ such that $\sin \alpha = x$

In this case $x = \sin k$ where $0 < k < \frac{\pi}{2}$

As \sin is an increasing function

$$\sin 0 < x < \sin \frac{\pi}{2}$$

i.e. $0 < x < 1$

$$(b) (i) \cos k = \pm \sqrt{1 - \sin^2 k} = \pm \sqrt{1 - x^2}$$

k is in the 1st quadrant $\Rightarrow \cos k > 0$

$$\text{So } \cos k = \sqrt{1 - x^2}$$

$$(ii) \tan k = \frac{\sin k}{\cos k} = \frac{x}{\sqrt{1 - x^2}}$$

(c) k is now in the 4th quadrant, where $\cos k$ is positive.

So the value of $\cos k$ remains the same and there is no change to $\tan k$.

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Exercise E, Question 7

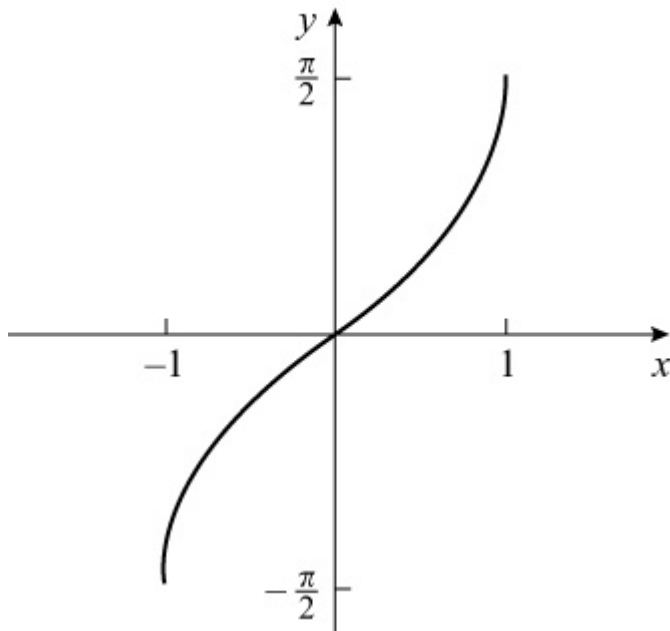
Question:

The function f is defined as $f : x \rightarrow \arcsin x$, $-1 \leq x \leq 1$, and the function g is such that $g(x) = f(2x)$.

- Sketch the graph of $y = f(x)$ and state the range of f .
- Sketch the graph of $y = g(x)$.
- Define g in the form $g : x \rightarrow \dots$ and give the domain of g .
- Define g^{-1} in the form $g^{-1} : x \rightarrow \dots$

Solution:

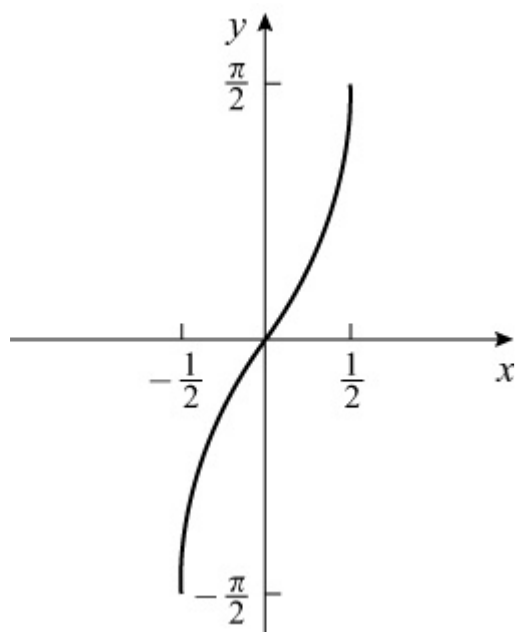
(a) $y = \arcsin x$



Range: $-\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$

- (b) Using the transformation work, the graph of $y = f(2x)$ is the graph of $y = f(x)$ stretched in the x direction by scale factor $\frac{1}{2}$.

$y = g(x)$



(c) $g : x \rightarrow \arcsin 2x, \quad -\frac{1}{2} \leq x \leq \frac{1}{2}$

(d) Let $y = \arcsin 2x$

$$\Rightarrow 2x = \sin y$$

$$\Rightarrow x = \frac{1}{2} \sin y$$

So $g^{-1} : x \rightarrow \frac{1}{2} \sin x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

Solutionbank

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Exercise E, Question 8

Question:

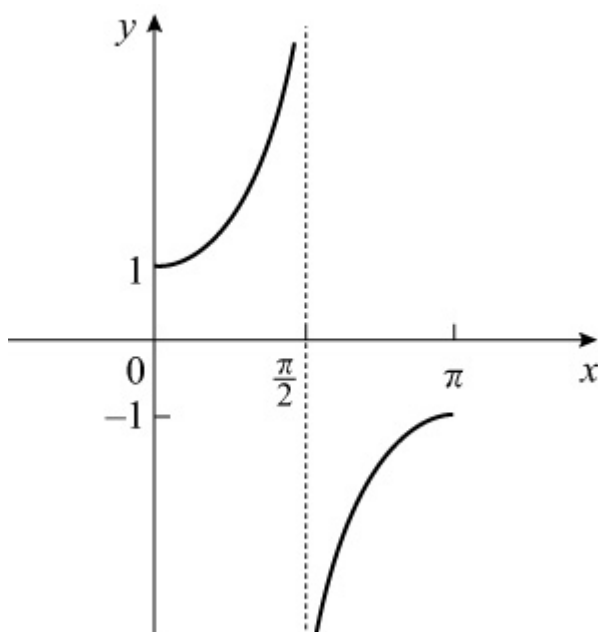
(a) Sketch the graph of $y = \sec x$, with the restricted domain

$$0 \leq x \leq \pi, \quad x \neq \frac{\pi}{2}.$$

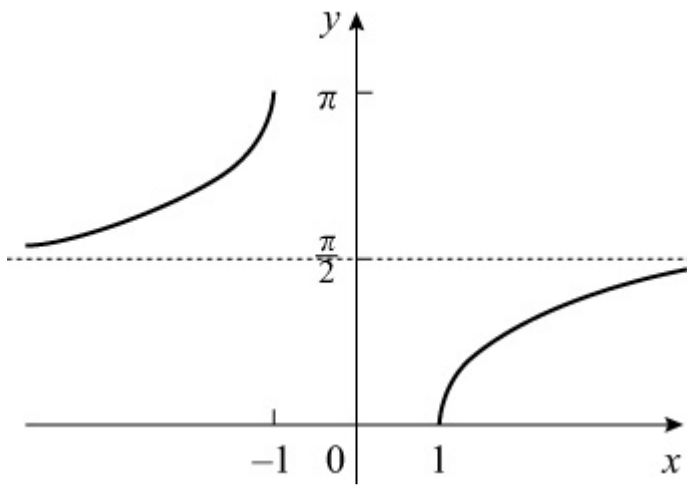
(b) Given that $\operatorname{arcsec} x$ is the inverse function of $\sec x$, $0 \leq x \leq \pi$, $x \neq \frac{\pi}{2}$, sketch the graph of $y = \operatorname{arcsec} x$ and state the range of $\operatorname{arcsec} x$.

Solution:

(a) $y = \sec x$



(b) Reflect the above graph in the line $y = x$
 $y = \operatorname{arcsec} x$, $x \leq -1$, $x \geq 1$



Range: $0 \leq \text{arcsec } x \leq \pi$, $\text{arcsec } x \neq \frac{\pi}{2}$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 1

Question:

Solve $\tan x = 2 \cot x$, in the interval $-180^\circ \leq x \leq 90^\circ$. Give any non-exact answers to 1 decimal place.

Solution:

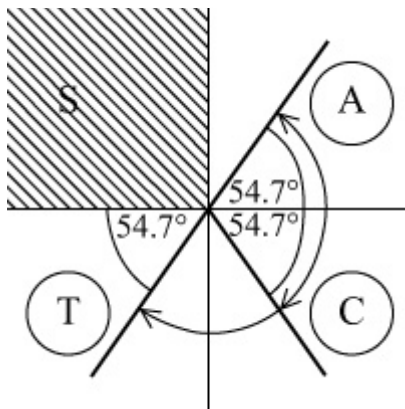
$$\tan x = 2 \cot x, \quad -180^\circ \leq x \leq 90^\circ$$

$$\Rightarrow \tan x = \frac{2}{\tan x}$$

$$\Rightarrow \tan^2 x = 2$$

$$\Rightarrow \tan x = \pm \sqrt{2}$$

Calculator value for $\tan x = +\sqrt{2}$ is 54.7°



Solutions are required in the 1st, 3rd and 4th quadrants.

Solution set: $-125.3^\circ, -54.7^\circ, +54.7^\circ$

Solutionbank

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Exercise F, Question 2

Question:

Given that $p = 2 \sec \theta$ and $q = 4 \cos \theta$, express p in terms of q .

Solution:

$$p = 2 \sec \theta \Rightarrow \sec \theta = \frac{p}{2}$$

$$q = 4 \cos \theta \Rightarrow \cos \theta = \frac{q}{4}$$

$$\sec \theta = \frac{1}{\cos \theta} \Rightarrow \frac{p}{2} = \frac{1}{\frac{q}{4}} \Rightarrow p = \frac{8}{q}$$

Solutionbank

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Exercise F, Question 3

Question:

Given that $p = \sin \theta$ and $q = 4 \cot \theta$, show that $p^2 q^2 = 16 (1 - p^2)$.

Solution:

$$p = \sin \theta \quad \Rightarrow \quad \frac{1}{p} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

$$q = 4 \cot \theta \quad \Rightarrow \quad \cot \theta = \frac{q}{4}$$

Using $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$

$$\Rightarrow 1 + \frac{q^2}{16} = \frac{1}{p^2} \quad (\text{multiply by } 16p^2)$$

$$\Rightarrow 16p^2 + p^2 q^2 = 16$$

$$\Rightarrow p^2 q^2 = 16 - 16p^2 = 16 (1 - p^2)$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 4

Question:

Give any non-exact answers to 1 decimal place.

(a) Solve, in the interval $0 < \theta < 180^\circ$,

(i) $\operatorname{cosec} \theta = 2 \cot \theta$

(ii) $2 \cot^2 \theta = 7 \operatorname{cosec} \theta - 8$

(b) Solve, in the interval $0 \leq \theta \leq 360^\circ$,

(i) $\sec(2\theta - 15^\circ) = \operatorname{cosec} 135^\circ$

(ii) $\sec^2 \theta + \tan \theta = 3$

(c) Solve, in the interval $0 \leq x \leq 2\pi$,

(i) $\operatorname{cosec} \left(x + \frac{\pi}{15} \right) = -\sqrt{2}$

(ii) $\sec^2 x = \frac{4}{3}$

Solution:

(a) (i) $\operatorname{cosec} \theta = 2 \cot \theta, 0 < \theta < 180^\circ$

$$\Rightarrow \frac{1}{\sin \theta} = \frac{2 \cos \theta}{\sin \theta}$$

$$\Rightarrow 2 \cos \theta = 1$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

(ii) $2 \cot^2 \theta = 7 \operatorname{cosec} \theta - 8, 0 < \theta < 180^\circ$

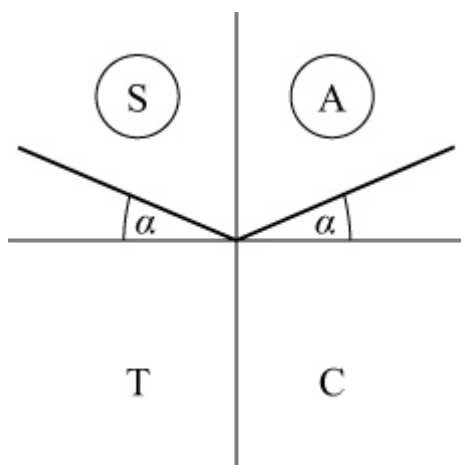
$$\Rightarrow 2(\operatorname{cosec}^2 \theta - 1) = 7 \operatorname{cosec} \theta - 8$$

$$\Rightarrow 2 \operatorname{cosec}^2 \theta - 7 \operatorname{cosec} \theta + 6 = 0$$

$$\Rightarrow (2 \operatorname{cosec} \theta - 3)(\operatorname{cosec} \theta - 2) = 0$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{3}{2} \text{ or } \operatorname{cosec} \theta = 2$$

$$\text{So } \sin \theta = \frac{2}{3} \text{ or } \sin \theta = \frac{1}{2}$$



Solutions are α° and $(180 - \alpha)^\circ$ where α is the calculator value.

Solutions set: $41.8^\circ, 138.2^\circ, 30^\circ, 150^\circ$

i.e. $30^\circ, 41.8^\circ, 138.2^\circ, 150^\circ$

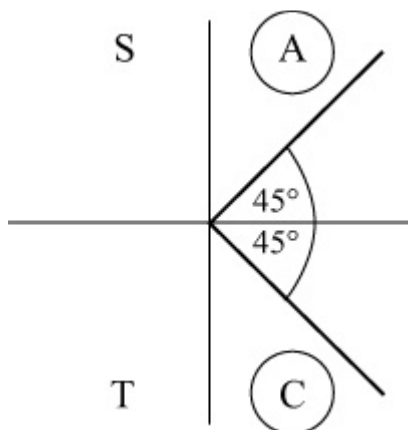
$$(b) (i) \sec(2\theta - 15^\circ) = \operatorname{cosec} 135^\circ, 0 \leq \theta \leq 360^\circ$$

$$\Rightarrow \cos(2\theta - 15^\circ) = \frac{1}{\operatorname{cosec} 135^\circ} = \sin 135^\circ = \frac{\sqrt{2}}{2}$$

$$\text{Solve } \cos(2\theta - 15^\circ) = \frac{\sqrt{2}}{2}, -15^\circ \leq 2\theta - 15^\circ \leq 705^\circ$$

$$\text{The calculator value is } \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

\cos is positive, so $(2\theta - 15^\circ)$ is in the 1st and 4th quadrants.



$$\text{So } (2\theta - 15^\circ) = 45^\circ, 315^\circ, 405^\circ, 675^\circ$$

$$\Rightarrow 2\theta = 60^\circ, 330^\circ, 420^\circ, 690^\circ$$

$$\Rightarrow \theta = 30^\circ, 165^\circ, 210^\circ, 345^\circ$$

$$(ii) \sec^2 \theta + \tan \theta = 3, 0 \leq \theta \leq 360^\circ$$

$$\Rightarrow 1 + \tan^2 \theta + \tan \theta = 3$$

$$\Rightarrow \tan^2 \theta + \tan \theta - 2 = 0$$

$$\Rightarrow (\tan \theta - 1)(\tan \theta + 2) = 0$$

$$\Rightarrow \tan \theta = 1 \text{ or } \tan \theta = -2$$

$$\tan \theta = 1 \Rightarrow \theta = 45^\circ, 180^\circ + 45^\circ, \text{ i.e. } 45^\circ, 225^\circ$$

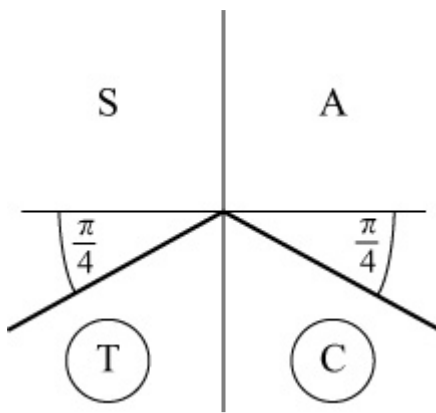
$$\tan \theta = -2 \Rightarrow \theta = 180^\circ + (-63.4)^\circ, 360^\circ + (-63.4)^\circ, \text{ i.e. } 116.6^\circ, 296.6^\circ$$

$$\text{(c) (i) } \operatorname{cosec} \left(x + \frac{\pi}{15} \right) = -\sqrt{2}, 0 \leq x \leq 2\pi$$

$$\Rightarrow \sin \left(x + \frac{\pi}{15} \right) = -\frac{1}{\sqrt{2}}$$

$$\text{Calculator value is } \sin^{-1} \left(-\frac{1}{\sqrt{2}} \right) = -\frac{\pi}{4}$$

$\sin \left(x + \frac{\pi}{15} \right)$ is negative, so $x + \frac{\pi}{15}$ is in 3rd and 4th quadrants.



$$\text{So } x + \frac{\pi}{15} = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\Rightarrow x = \frac{5\pi}{4} - \frac{\pi}{15}, \frac{7\pi}{4} - \frac{\pi}{15} = \frac{75\pi - 4\pi}{60}, \frac{105\pi - 4\pi}{60} = \frac{71\pi}{60}, \frac{101\pi}{60}$$

$$\text{(ii) } \sec^2 x = \frac{4}{3}, 0 \leq x \leq 2\pi$$

$$\Rightarrow \cos^2 x = \frac{3}{4}$$

$$\Rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$$

Calculator value for $\cos x = +\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$

As $\cos x$ is \pm , x is in all four quadrants.

Solutions set: $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 5

Question:

Given that $5 \sin x \cos y + 4 \cos x \sin y = 0$, and that $\cot x = 2$, find the value of $\cot y$.

Solution:

$$5 \sin x \cos y + 4 \cos x \sin y = 0$$

$$\Rightarrow \frac{5 \sin x \cos y}{\sin x \sin y} + \frac{4 \cos x \sin y}{\sin x \sin y} = 0 \quad (\text{divide by } \sin x \sin y)$$

$$\Rightarrow \frac{5 \cos y}{\sin y} + \frac{4 \cos x}{\sin x} = 0$$

$$\text{So } 5 \cot y + 4 \cot x = 0$$

$$\text{As } \cot x = 2$$

$$5 \cot y + 8 = 0$$

$$5 \cot y = -8$$

$$\cot y = -\frac{8}{5}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 6

Question:

Show that:

$$(a) (\tan \theta + \cot \theta) (\sin \theta + \cos \theta) \equiv \sec \theta + \operatorname{cosec} \theta$$

$$(b) \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$$

$$(c) (1 - \sin x) (1 + \operatorname{cosec} x) \equiv \cos x \cot x$$

$$(d) \frac{\cot x}{\operatorname{cosec} x - 1} - \frac{\cos x}{1 + \sin x} \equiv 2 \tan x$$

$$(e) \frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \equiv 2 \sec \theta \tan \theta$$

$$(f) \frac{(\sec \theta - \tan \theta) (\sec \theta + \tan \theta)}{1 + \tan^2 \theta} \equiv \cos^2 \theta$$

Solution:

$$\begin{aligned} (a) \text{ L.H.S.} &\equiv (\tan \theta + \cot \theta) (\sin \theta + \cos \theta) \\ &\equiv \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) (\sin \theta + \cos \theta) \\ &\equiv \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) (\sin \theta + \cos \theta) \\ &\equiv \left(\frac{1}{\cos \theta \sin \theta} \right) (\sin \theta + \cos \theta) \\ &\equiv \frac{\sin \theta}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} \\ &\equiv \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \\ &\equiv \sec \theta + \operatorname{cosec} \theta \equiv \text{R.H.S.} \end{aligned}$$

(b)

$$\begin{aligned}
\text{L.H.S.} &\equiv \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \\
&\equiv \frac{\frac{1}{\sin x}}{\frac{1}{\sin x} - \sin x} \\
&\equiv \frac{\frac{1}{\sin x}}{\frac{1 - \sin^2 x}{\sin x}} \\
&\equiv \frac{1}{\sin x} \times \frac{\sin x}{1 - \sin^2 x} \\
&\equiv \frac{1}{1 - \sin^2 x} \\
&\equiv \frac{1}{\cos^2 x} \quad (\text{using } \sin^2 x + \cos^2 x \equiv 1) \\
&\equiv \sec^2 x \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(c) L.H.S.} &\equiv (1 - \sin x)(1 + \operatorname{cosec} x) \\
&\equiv 1 - \sin x + \operatorname{cosec} x - \sin x \operatorname{cosec} x \\
&\equiv 1 - \sin x + \operatorname{cosec} x - 1 \quad \left(\text{as } \operatorname{cosec} x = \frac{1}{\sin x} \right) \\
&\equiv \operatorname{cosec} x - \sin x \\
&\equiv \frac{1}{\sin x} - \sin x \\
&\equiv \frac{1 - \sin^2 x}{\sin x} \\
&\equiv \frac{\cos^2 x}{\sin x} \\
&\equiv \frac{\cos x}{\sin x} \times \cos x \\
&\equiv \cos x \cot x \equiv \text{R.H.S.}
\end{aligned}$$

(d)

$$\begin{aligned}
\text{L.H.S.} &\equiv \frac{\cot x}{\operatorname{cosec} x - 1} - \frac{\cos x}{1 + \sin x} \\
&\equiv \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - 1} - \frac{\cos x}{1 + \sin x} \\
&\equiv \frac{\frac{\cos x}{\sin x}}{\frac{1 - \sin x}{\sin x}} - \frac{\cos x}{1 + \sin x} \\
&\equiv \frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} \\
&\equiv \frac{\cos x(1 + \sin x) - \cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} \\
&\equiv \frac{2 \cos x \sin x}{1 - \sin^2 x} \\
&\equiv \frac{2 \cos x \sin x}{\cos^2 x} \\
&\equiv 2 \frac{\sin x}{\cos x} \\
&\equiv 2 \tan x \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(e) L.H.S.} &\equiv \frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \\
&\equiv \frac{(\operatorname{cosec} \theta + 1) + (\operatorname{cosec} \theta - 1)}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \\
&\equiv \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} \\
&\equiv \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} \quad (1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta) \\
&\equiv \frac{2}{\sin \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} \\
&\equiv 2 \times \frac{1}{\cos \theta} \times \frac{\sin \theta}{\cos \theta} \\
&\equiv 2 \sec \theta \tan \theta \equiv \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(f) L.H.S.} &\equiv \frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}{1 + \tan^2 \theta} \\
&\equiv \frac{\sec^2 \theta - \tan^2 \theta}{\sec^2 \theta} \\
&\equiv \frac{(1 + \tan^2 \theta) - \tan^2 \theta}{\sec^2 \theta} \\
&\equiv \frac{1}{\sec^2 \theta} \\
&\equiv \cos^2 \theta \equiv \text{R.H.S.}
\end{aligned}$$

Solutionbank

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Exercise F, Question 7

Question:

(a) Show that $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} \equiv 2 \operatorname{cosec} x$.

(b) Hence solve, in the interval $-2\pi \leq x \leq 2\pi$, $\frac{\sin x}{1 + \cos x} +$

$$\frac{1 + \cos x}{\sin x} = -\frac{4}{\sqrt{3}}.$$

Solution:

(a) L.H.S. $\equiv \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}$

$$\equiv \frac{\sin^2 x + (1 + \cos x)^2}{(1 + \cos x) \sin x}$$

$$\equiv \frac{\sin^2 x + 1 + 2 \cos x + \cos^2 x}{(1 + \cos x) \sin x}$$

$$\equiv \frac{2 + 2 \cos x}{(1 + \cos x) \sin x} \quad \left(\sin^2 x + \cos^2 x \equiv 1 \right)$$

$$\equiv \frac{2(1 + \cos x)}{(1 + \cos x) \sin x}$$

$$\equiv \frac{2}{\sin x}$$

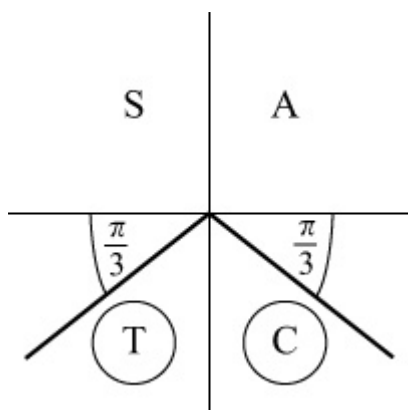
$$\equiv 2 \operatorname{cosec} x \equiv \text{R.H.S.}$$

(b) Solve $2 \operatorname{cosec} x = -\frac{4}{\sqrt{3}}$, $-2\pi \leq x \leq 2\pi$

$$\Rightarrow \operatorname{cosec} x = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin x = -\frac{\sqrt{3}}{2}$$

Calculator value is $-\frac{\pi}{3}$



Solutions in $-2\pi \leq x \leq 2\pi$ are

$$-\frac{\pi}{3}, \quad -\pi + \frac{\pi}{3}, \quad \pi + \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3},$$

i.e. $-\frac{\pi}{3}, \quad -\frac{2\pi}{3}, \quad \frac{4\pi}{3}, \quad \frac{5\pi}{3}$

Solutionbank

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Exercise F, Question 8

Question:

Prove that $\frac{1 + \cos \theta}{1 - \cos \theta} \equiv (\operatorname{cosec} \theta + \cot \theta)^2$.

Solution:

$$\begin{aligned} \text{R.H.S.} &\equiv (\operatorname{cosec} \theta + \cot \theta)^2 \\ &\equiv \left(\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right)^2 \\ &\equiv \frac{(1 + \cos \theta)^2}{\sin^2 \theta} \\ &\equiv \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta} \\ &\equiv \frac{(1 + \cos \theta)(1 + \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\ &\equiv \frac{1 + \cos \theta}{1 - \cos \theta} \equiv \text{L.H.S.} \end{aligned}$$

Solutionbank

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Exercise F, Question 9

Question:

Given that $\sec A = -3$, where $\frac{\pi}{2} < A < \pi$,

(a) calculate the exact value of $\tan A$.

(b) Show that $\operatorname{cosec} A = \frac{3\sqrt{2}}{4}$.

Solution:

(a) $\sec A = -3$, $\frac{\pi}{2} < A < \pi$, i.e. A is in 2nd quadrant.

$$\text{As } 1 + \tan^2 A = \sec^2 A$$

$$1 + \tan^2 A = 9$$

$$\tan^2 A = 8$$

$$\tan A = \pm \sqrt{8} = \pm 2\sqrt{2}$$

As A is in 2nd quadrant, $\tan A$ is -ve.

$$\text{So } \tan A = -2\sqrt{2}$$

(b) $\sec A = -3$, so $\cos A = -\frac{1}{3}$

$$\text{As } \tan A = \frac{\sin A}{\cos A}$$

$$\sin A = \cos A \times \tan A = -\frac{1}{3} \times -2\sqrt{2} = \frac{2\sqrt{2}}{3}$$

$$\text{So } \operatorname{cosec} A = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{2 \times 2} = \frac{3\sqrt{2}}{4}$$

Solutionbank

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Exercise F, Question 10

Question:

Given that $\sec \theta = k$, $|k| \geq 1$, and that θ is obtuse, express in terms of k :

(a) $\cos \theta$

(b) $\tan^2 \theta$

(c) $\cot \theta$

(d) $\operatorname{cosec} \theta$

Solution:

$$\sec \theta = k, |k| \geq 1$$

θ is in the 2nd quadrant $\Rightarrow k$ is negative

(a) $\cos \theta = \frac{1}{\sec \theta} = \frac{1}{k}$

(b) Using $1 + \tan^2 \theta = \sec^2 \theta$
 $\tan^2 \theta = k^2 - 1$

(c) $\tan \theta = \pm \sqrt{k^2 - 1}$

In the 2nd quadrant, $\tan \theta$ is -ve.

So $\tan \theta = -\sqrt{k^2 - 1}$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-\sqrt{k^2 - 1}} = -\frac{1}{\sqrt{k^2 - 1}}$$

(d) Using $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\operatorname{cosec}^2 \theta = 1 + \frac{1}{k^2 - 1} = \frac{k^2 - 1 + 1}{k^2 - 1} = \frac{k^2}{k^2 - 1}$$

So $\operatorname{cosec} \theta = \pm \frac{k}{\sqrt{k^2 - 1}}$

In the 2nd quadrant, $\operatorname{cosec} \theta$ is +ve.

As k is -ve, $\operatorname{cosec} \theta = \frac{-k}{\sqrt{k^2 - 1}}$

Solutionbank

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Exercise F, Question 11

Question:

Solve, in the interval $0 \leq x \leq 2\pi$, the equation $\sec \left(x + \frac{\pi}{4} \right) = 2$, giving your answers in terms of π .

Solution:

$$\sec \left(x + \frac{\pi}{4} \right) = 2, 0 \leq x \leq 2\pi$$

$$\Rightarrow \cos \left(x + \frac{\pi}{4} \right) = \frac{1}{2}, 0 \leq x \leq 2\pi$$

$$\Rightarrow x + \frac{\pi}{4} = \cos^{-1} \frac{1}{2}, 2\pi - \cos^{-1} \frac{1}{2} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$\text{So } x = \frac{\pi}{3} - \frac{\pi}{4}, \frac{5\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12}, \frac{20\pi - 3\pi}{12} = \frac{\pi}{12}, \frac{17\pi}{12}$$

Solutionbank

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Exercise F, Question 12

Question:

Find, in terms of π , the value of $\arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right)$.

Solution:

$\arcsin\left(\frac{1}{2}\right)$ is the angle in the interval $-\frac{\pi}{2} \leq \text{angle} \leq \frac{\pi}{2}$ whose sine is $\frac{1}{2}$.

$$\text{So } \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\text{Similarly, } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\text{So } \arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right) = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

Solutionbank

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Exercise F, Question 13

Question:

Solve, in the interval $0 \leq x \leq 2\pi$, the equation $\sec^2 x - \frac{2\sqrt{3}}{3}\tan x - 2 = 0$, giving your answers in terms of π .

Solution:

$$\sec^2 x - \frac{2\sqrt{3}}{3}\tan x - 2 = 0, 0 \leq x \leq 2\pi$$

$$\Rightarrow (1 + \tan^2 x) - \frac{2\sqrt{3}}{3}\tan x - 2 = 0$$

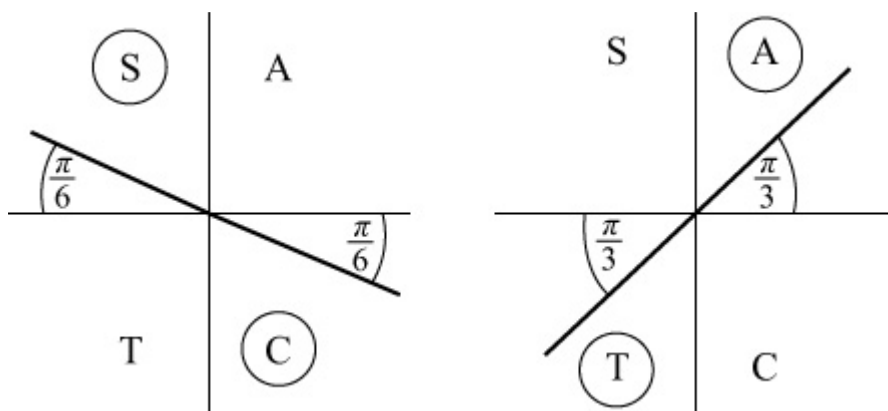
$$\tan^2 x - \frac{2\sqrt{3}}{3}\tan x - 1 = 0$$

(This does factorise but you may not have noticed!)

$$\left(\tan x + \frac{\sqrt{3}}{3}\right)(\tan x - \sqrt{3}) = 0$$

$$\Rightarrow \tan x = -\frac{\sqrt{3}}{3} \text{ or } \tan x = \sqrt{3}$$

Calculator values are $-\frac{\pi}{6}$ and $\frac{\pi}{3}$.



Solution set: $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$

Solutionbank

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Exercise F, Question 14

Question:

- (a) Factorise $\sec x \operatorname{cosec} x - 2 \sec x - \operatorname{cosec} x + 2$.
- (b) Hence solve $\sec x \operatorname{cosec} x - 2 \sec x - \operatorname{cosec} x + 2 = 0$, in the interval $0 \leq x \leq 360^\circ$.

Solution:

$$\begin{aligned} \text{(a) } \sec x \operatorname{cosec} x - 2 \sec x - \operatorname{cosec} x + 2 \\ &= \sec x (\operatorname{cosec} x - 2) - (\operatorname{cosec} x - 2) \\ &= (\operatorname{cosec} x - 2) (\sec x - 1) \end{aligned}$$

$$\begin{aligned} \text{(b) So } \sec x \operatorname{cosec} x - 2 \sec x - \operatorname{cosec} x + 2 &= 0 \\ \Rightarrow (\operatorname{cosec} x - 2) (\sec x - 1) &= 0 \\ \Rightarrow \operatorname{cosec} x = 2 \text{ or } \sec x = 1 \\ \Rightarrow \sin x = \frac{1}{2} \text{ or } \cos x = 1 \end{aligned}$$

$$\sin x = \frac{1}{2}, \quad 0 \leq x \leq 360^\circ$$

$$\Rightarrow x = 30^\circ, \quad (180 - 30)^\circ$$

$$\cos x = 1, \quad 0 \leq x \leq 360^\circ,$$

$$\Rightarrow x = 0^\circ, \quad 360^\circ \quad (\text{from the graph})$$

Full set of solutions: $0^\circ, 30^\circ, 150^\circ, 360^\circ$

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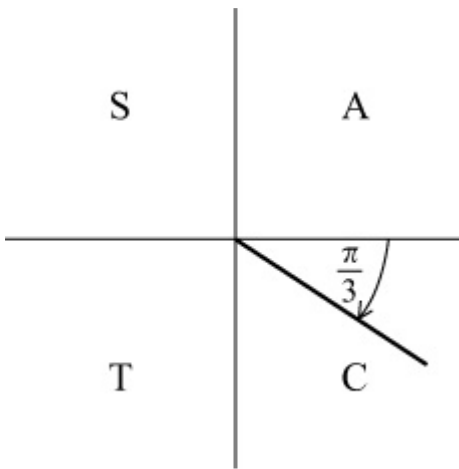
Exercise F, Question 15

Question:

Given that $\arctan (x - 2) = -\frac{\pi}{3}$, find the value of x .

Solution:

$$\arctan (x - 2) = -\frac{\pi}{3}$$



$$\Rightarrow x - 2 = \tan \left(-\frac{\pi}{3} \right)$$

$$\Rightarrow x - 2 = -\sqrt{3}$$

$$\Rightarrow x = 2 - \sqrt{3}$$

Solutionbank

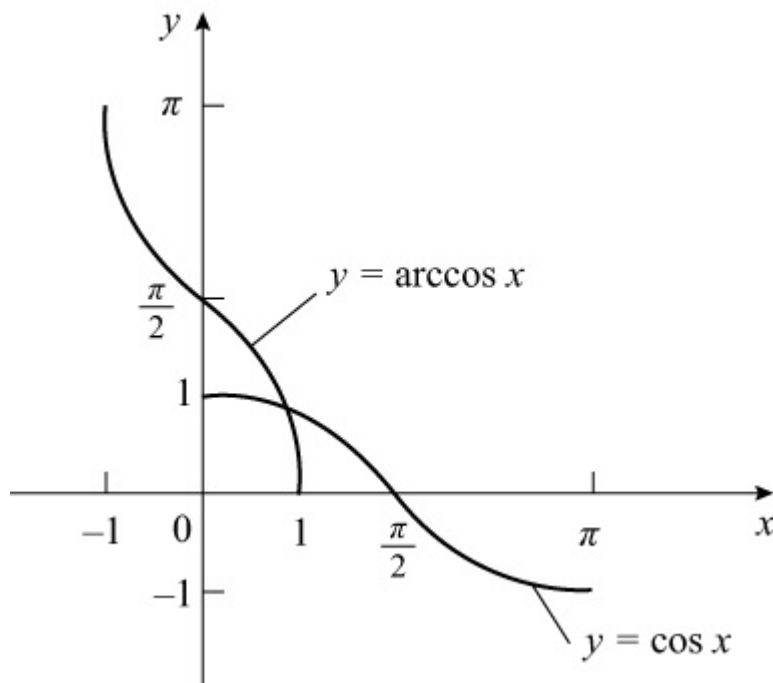
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Exercise F, Question 16

Question:

On the same set of axes sketch the graphs of $y = \cos x$, $0 \leq x \leq \pi$, and $y = \arccos x$, $-1 \leq x \leq 1$, showing the coordinates of points in which the curves meet the axes.

Solution:



...

Solutionbank

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Exercise F, Question 17

Question:

(a) Given that $\sec x + \tan x = -3$, use the identity $1 + \tan^2 x \equiv \sec^2 x$ to find the value of $\sec x - \tan x$.

(b) Deduce the value of

(i) $\sec x$

(ii) $\tan x$

(c) Hence solve, in the interval

$-180^\circ \leq x \leq 180^\circ$, $\sec x + \tan x = -3$. (Give answer to 1 decimal place).

Solution:

(a) As $1 + \tan^2 x \equiv \sec^2 x$

$$\sec^2 x - \tan^2 x \equiv 1$$

$$\Rightarrow (\sec x - \tan x)(\sec x + \tan x) \equiv 1 \quad (\text{difference of two squares})$$

As $\tan x + \sec x = -3$ is given,

$$\text{so } -3(\sec x - \tan x) = 1$$

$$\Rightarrow \sec x - \tan x = -\frac{1}{3}$$

(b) $\sec x + \tan x = -3$

$$\text{and } \sec x - \tan x = -\frac{1}{3}$$

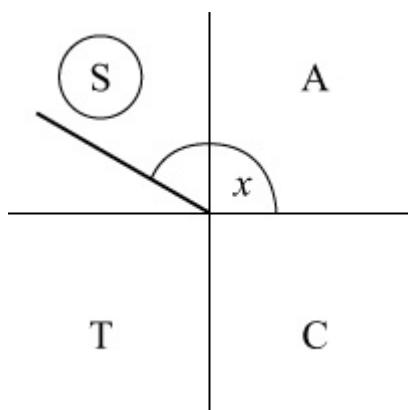
$$(i) \text{ Add the equations } \Rightarrow 2 \sec x = -\frac{10}{3} \Rightarrow \sec x = -\frac{5}{3}$$

$$(ii) \text{ Subtract the equation } \Rightarrow 2 \tan x = -3 + \frac{1}{3} = -\frac{8}{3} \Rightarrow \tan x = -$$

$$\frac{4}{3}$$

(c) As $\sec x$ and $\tan x$ are both $-ve$, $\cos x$ and $\tan x$ are both $-ve$.

So x must be in the 2nd quadrant.



Solving $\tan x = -\frac{4}{3}$, where x is in the 2nd quadrant, gives $180^\circ + \left(-53.1^\circ \right) = 126.9^\circ$

Solutionbank

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Exercise F, Question 18

Question:

Given that $p = \sec \theta - \tan \theta$ and $q = \sec \theta + \tan \theta$, show that $p = \frac{1}{q}$.

Solution:

$$p = \sec \theta - \tan \theta, q = \sec \theta + \tan \theta$$

Multiply together:

$$pq = (\sec \theta - \tan \theta) (\sec \theta + \tan \theta) = \sec^2 \theta - \tan^2 \theta = 1 \quad (\text{since } 1 + \tan^2 \theta = \sec^2 \theta)$$

$$\Rightarrow p = \frac{1}{q}$$

(There are several ways of solving this problem).

Solutionbank

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Exercise F, Question 19

Question:

(a) Prove that $\sec^4 \theta - \tan^4 \theta \equiv \sec^2 \theta + \tan^2 \theta$.

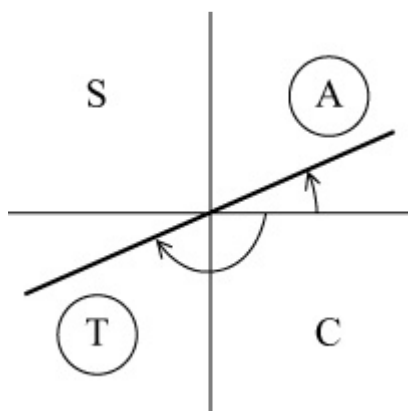
(b) Hence solve, in the interval

$-180^\circ \leq \theta \leq 180^\circ$, $\sec^4 \theta = \tan^4 \theta + 3 \tan \theta$. (Give answers to 1 decimal place).

Solution:

(a) L.H.S. $\equiv \sec^4 \theta - \tan^4 \theta$
 $\equiv (\sec^2 \theta + \tan^2 \theta) (\sec^2 \theta - \tan^2 \theta)$
 $\equiv (\sec^2 \theta + \tan^2 \theta) (1)$
 $\equiv \sec^2 \theta + \tan^2 \theta \equiv \text{R.H.S.}$

(b) $\sec^4 \theta = \tan^4 \theta + 3 \tan \theta$
 $\Rightarrow \sec^4 \theta - \tan^4 \theta = 3 \tan \theta$
 $\Rightarrow \sec^2 \theta + \tan^2 \theta = 3 \tan \theta$ [using part (a)]
 $\Rightarrow (1 + \tan^2 \theta) + \tan^2 \theta = 3 \tan \theta$
 $\Rightarrow 2 \tan^2 \theta - 3 \tan \theta + 1 = 0$
 $\Rightarrow (2 \tan \theta - 1) (\tan \theta - 1) = 0$
 $\Rightarrow \tan \theta = \frac{1}{2} \text{ or } \tan \theta = 1$



In the interval $-180^\circ \leq \theta \leq 180^\circ$

$$\tan \theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1} \frac{1}{2}, \quad -180^\circ + \tan^{-1} \frac{1}{2}$$

$$\frac{1}{2} = 26.6^\circ, \quad -153.4^\circ$$

$$\tan \theta = 1 \Rightarrow \theta = \tan^{-1} 1, \quad -180^\circ + \tan^{-1} 1 = 45^\circ, \quad -135^\circ$$

Set of solutions: $-153.4^\circ, -135^\circ, 26.6^\circ, 45^\circ$ (3 s.f.)

Solutionbank

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Exercise F, Question 20

Question:

(Although integration is not in the specification for C3, this question only requires you to know that the area under a curve can be represented by an integral.)

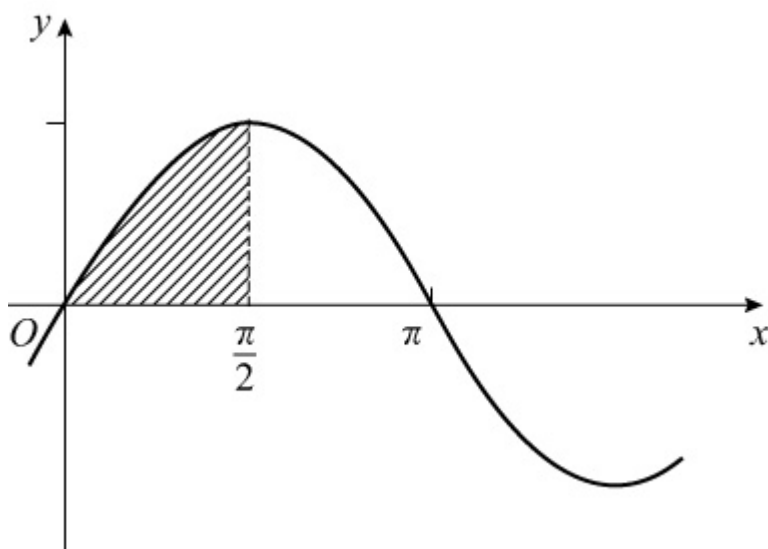
(a) Sketch the graph of $y = \sin x$ and shade in the area representing $\int_0^{\frac{\pi}{2}} \sin x \, dx$.

(b) Sketch the graph of $y = \arcsin x$ and shade in the area representing $\int_0^1 \arcsin x \, dx$.

(c) By considering the shaded areas explain why $\int_0^{\frac{\pi}{2}} \sin x \, dx + \int_0^1 \arcsin x \, dx = \frac{\pi}{2}$.

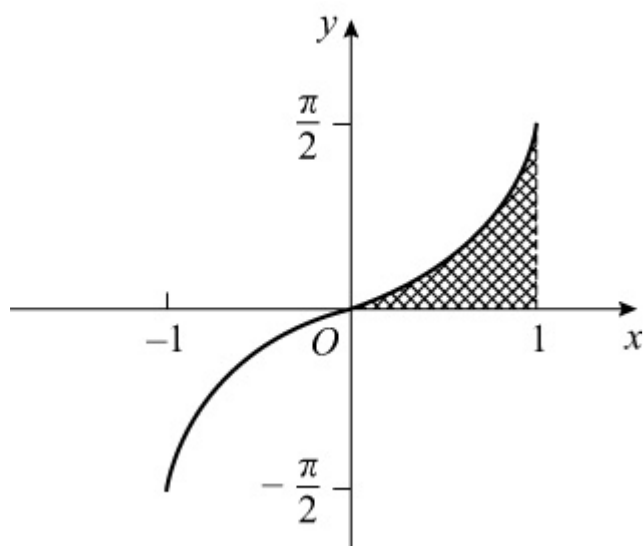
Solution:

(a) $y = \sin x$



$\int_0^{\frac{\pi}{2}} \sin x \, dx$ represents the area between $y = \sin x$, x -axis and $x = \frac{\pi}{2}$.

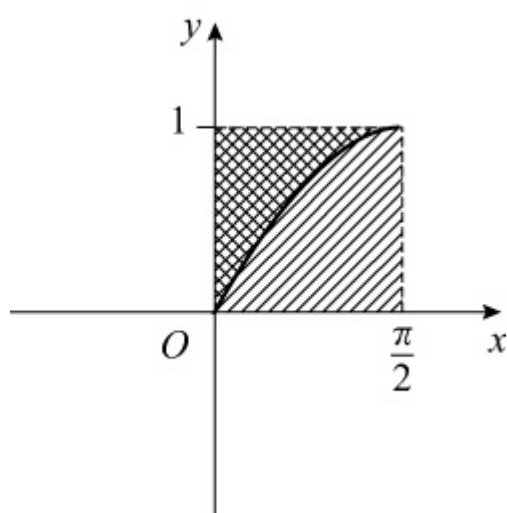
(b) $y = \arcsin x$, $-1 \leq x \leq 1$



$\int_0^1 \arcsin x \, dx$ represents the area between the curve, x -axis and $x = 1$.

(c) The curves are the same with the axes interchanged.

The shaded area in (b) could be added to the graph in (a) to form a rectangle with sides 1 and $\frac{\pi}{2}$, as in the diagram.



Area of rectangle = $\frac{\pi}{2}$

So $\int_0^{\frac{\pi}{2}} \sin x \, dx + \int_0^1 \arcsin x \, dx = \frac{\pi}{2}$