Kindergarten Mathematics, Quarter 3, Unit 3.1

# Counting to 100 and Representing Quantities to 20

## Overview

**Number of Instructional Days:** 15 (1 day = 45–60 minutes)

### Content to be Learned

- Count to 100 by ones.
- Count forward from any given number.
- Write numerals 0–20.
- Represent number of objects with a numeral.
- Understand that the last number named in a count represents the total quantity of objects counted.
- Count to answer "How many?" questions about 20 objects in an arranged configuration or as many as 10 objects in a scattered configuration; given a number 1–20, count that many objects.
- Compare two numbers, 1–10, presented as numerals.

### Mathematical Practices to Be Integrated

Make sense of problems and persevere in solving them.

- Use concrete objects or pictures to help conceptualize and solve a problem.

Attend to precision.

- Communicate precisely to others.
- Calculate accurately and efficiently.
- Give carefully formulated explanations.

### Essential Questions

- Starting at one, count as high as you can.
- Starting at ____, count forward. I will tell you when to stop.
- How do you write the numeral ____? Show me. (0–20)
- What numeral do you write for this quantity of objects?_____
- Count these objects. (up to 20 arranged; up to 10 scattered) How many do you have? _____
- I have _____. You have _____. Which numeral is greater/less? Are they equal? How do you know?

# Written Curriculum

## Common Core State Standards for Mathematical Content

### Counting and Cardinality K.CC

**Know number names and the count sequence.**

K.CC.1 Count to 100 by ones ~~and by tens~~.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

**Count to tell the number of objects.**

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

**Compare numbers.**

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

## Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

According to *The Young Child and Mathematics* (Copley, 2001), "The teacher's role is to provide a bridge between the child's informal knowledge of mathematics and the more formal 'school' mathematics. Young children enter school with many intuitive mathematical understandings." (p. 47)

Students at this level may have had some experience counting objects and combining groups. They may have played a variety of games that include counting and one-to-one correspondence.

*Current Learning*

Early in the year, kindergarteners develop their counting and cardinality skills as they rote count to 50 by ones. They write numerals 0–10 and represent a number of objects with the numeral. Students count to answer "how many?" for up to 10 arranged or 10 scattered objects.

In this unit, kindergarteners rote count to 100 by ones. They count forward from any given number. Students write numerals 0–20 and represent a number of objects with the numeral. They work toward understanding relationships between numbers and quantities and connect counting to cardinality. Students count to answer "how many?" up to as many as 20 arranged or 10 scattered objects. In Unit 2.1, they identify the number of objects as *greater than, less than,* or *equal to* another quantity of objects. In this unit, students apply this understanding to compare two numerals between 1 and 10. This is an area of critical focus for kindergarten.

Later in the year, students rote count to 100 by tens. They also count to answer how many for up to 20 arranged and 20 scattered objects.

*Future Learning*

In grade 1, students will extend the counting sequence to 120. They will understand the connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). Students will use increasingly sophisticated strategies based on properties of numbers to understand combinations to 10.

## Additional Findings

According to *Principles and Standards for School Mathematics*, representing numbers with various physical materials should be a major part of mathematics instruction in the elementary grades. Understanding number and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics instruction for the elementary grades. (pp. 32 and 33)

Kindergarten Mathematics, Quarter 3, Unit 3.2

# Understanding Addition as Putting Together and Adding To and Subtraction as Taking Apart and Taking From

## Overview

**Number of Instructional Days:** 15 (1 day = 45–60 minutes)

### Content to be Learned

- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, and expressions.
- Solve addition and subtraction word problems, and add and subtract within 10 using objects and pictures to represent the problem.
- Decompose numbers less than or equal to 10 into pairs in more than one way using objects and drawings.
- Record decompositions with drawings.
- For any number from 1 to 9, find the number that makes 10 when added to the given number using objects and drawings. Record answers with drawings.

### Mathematical Practices to Be Integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and look for entry points to its solution.
- Plan a solution pathway.
- Monitor and evaluate progress and change course if necessary.
- Explain correspondences between verbal descriptions, drawings, and concrete representations.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Look for and express regularity in repeated reasoning.

- Continually evaluate the reasonableness of intermediate results.

### Essential Questions

- I know ____ +/– ____ = ____. How can you model or show me this?
- How many are there altogether? Show me how you know.
- I have ____. How many more do you need to make 10? Show me your thinking.
- Show me two numbers that go together to make 10. What is another pair you can show me?
- What is a way to show how to solve this problem?
- How does your drawing show your answer?
- How does your drawing show that your two numbers equal 10?

# Written Curriculum

## Common Core State Standards for Mathematical Content

### Operations and Algebraic Thinking K.OA

**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions~~, or equations~~.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing ~~or equation~~ (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing ~~or equation~~.

## Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way

terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Clarifying the Standards

*Prior Learning*

According to *The Young Child and Mathematics* (Copley, 2001), "The teacher's role is to provide a bridge between the child's informal knowledge of mathematics and the more formal 'school' mathematics. Young children enter school with many intuitive mathematical understandings." (p. 47)

Students at this level may have had some experience counting objects and combining groups. They may also have taken apart groups of objects through play.

*Current Learning*

Early in the year, students understand that addition is putting together and adding to. They represent addition with objects, fingers, mental images, acting out, sounds, and verbal explanations. Students also solve addition word problems and add within 10, using objects or drawings to represent the problem. They decompose numbers less than or equal to 10 into pairs by using objects or drawings.

In this unit, kindergarteners use objects, fingers, mental images, and drawings to represent addition *and* subtraction. They solve addition *and* subtraction word problems that are presented orally to them. Students add *and* subtract within 10, using objects or drawings to represent the problem. They also decompose numbers less than or equal to 10 into pairs in *more than one way*, using objects or drawings. Students record the decompositions with a drawing. Kindergarteners also find the number that makes 10 when added to a given number (1–9). They record this information with objects and drawings. For example, *You have 4 kittens. How many more do you need to make a group of 10?* Students also act out and model their ability to compose and decompose structures to 10.

Later in the year, students are introduced to writing equations as a method of representing a mathematical situation. This provides an additional way to record a mathematical situation involving addition and subtraction.

*Future Learning*

In grade 1, students will use addition and subtraction within 20 to solve word problems with unknowns in all positions. They will also solve word problems requiring the addition of three whole numbers, with a sum less than or equal to 20, using objects, drawings, and equations.

## Additional Findings

According to *K–5 OA Progressions* (p. 8), "Students act out adding and subtracting situations by representing quantities in the situation with objects, their fingers, and math drawings (MP5). To do this, students must mathematize a real-world situation (MP4), focusing on the quantities and their relationships rather than non-mathematical aspects of the situation."

Kindergarten Mathematics, Quarter 3, Unit 3.3

# Classifying and Counting Objects

## Overview

**Number of Instructional Days:** 5 (1 day = 45–60 minutes)

### Content to be Learned

- Classify objects into given categories.
- Count the number of objects in each category.
- Sort the categories by count.

### Mathematical Practices to Be Integrated

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate them to others, and respond to the arguments of others.
- Construct arguments using concrete objects, drawings, diagrams, and actions.

Attend to precision.

- Use clear definitions in discussion with others and in their own reasoning.
- Express numerical answers with a degree of precision appropriate for the problem context.

### Essential Questions

- Given a set of objects, how could you group them? (shape, color, size, length, etc.)
- How many _____ do you have in each group? How do you know?
- Look at your groups. Which group has the most/least? How do you know?
- How could you arrange your groups by the amount of items in each group?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Measurement and Data | K.MD |
|---|---|

**Classify objects and count the number of objects in each category.**

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

[3] Limit category counts to be less than or equal to 10.

## Common Core Standards for Mathematical Practice

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

"Young children (3–5 year olds) spend time comparing, sorting and classifying everyday objects. Curcio and Folkson (1996) suggested that preschool and kindergarten age children experience many exploratory data-collecting activities, using their own way of organizing information." (*Early Childhood Mathematics*, p. 98; Smith, 2001)

*Current Learning*

Earlier in the year, students count objects arranged in a line. Now they learn to count objects in more scattered arrangements. In this unit, students classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Counts should be limited to groups less than or equal to 10.

This is a critical area of instruction, as students begin to connect counting to cardinality. They are beginning to understand the relationships between numbers and quantities.

Later in the year, students compare numbers.

*Future Learning*

In grade 1, students will organize, represent, and interpret data with up to three categories. They will answer questions about the total number of data points, how many in each category, and how many more/less in one category than in another.

## Additional Findings

According to *K–3 Categorical Data Learning Progressions* (p. 5), "Students in kindergarten classify objects into categories initially specified by the teacher, and perhaps eventually elicited from students. Students can then count the number in each [group]. Students can use these category counts and their understanding of cardinality to say whether there are more than or less than."

Kindergarten Mathematics, Quarter 3, Unit 3.4

# Analyzing, Comparing, Creating, and Composing 2-D and 3-D Shapes

## Overview

**Number of Instructional Days:** 5 (1 day = 45–60 minutes)

### Content to be Learned

- Analyze and compare two- and three-dimensional shapes.
- Use informal language to describe similarities and differences (i.e., number of sides, corners) and other attributes.

### Mathematical Practices to Be Integrated

Construct viable arguments and critique the reasoning of others.

- Analyze situations.
- Justify conclusions and communicate them to others.
- Reason inductively about data.

Use appropriate tools strategically.

- Use concrete models, manipulatives, drawings, or computer-generated tools to compare two- and three-dimensional shapes.

### Essential Questions

- This is a _____. Which other shape has some of the same attributes?
- Do these two shapes have the same name? Why or why not? (e.g., two triangles with different orientations or different sizes) How are they the same? How are they different?
- How is a _____ like a _____? Explain your thinking. How are they different? (e.g., How is a circle like a sphere? How are a cone and cylinder similar and different?)
- How would you sort this group of shapes? (sort and describe by various attributes) What is another way you can show me how to sort these shapes? Why did you sort them that way?
- How is this shape (2-D) the same or different from this other shape (3-D)?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Geometry | K.G |
|---|---|

**Analyze, compare, create, and compose shapes.**

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

## Common Core Standards for Mathematical Practice

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

According to *Principles and Standards for School Mathematics,* young children are naturally inclined to observe and describe a variety of shapes and begin to notice their properties. (p. 41)

According to *The Young Child and Mathematics* (Copley, 2001), "The teacher's role is to provide a bridge between the child's informal knowledge of mathematics and the more formal 'school' mathematics. Young children enter school with many intuitive mathematical understandings about geometric shapes." (p. 47)

*Current Learning*

Earlier in the year, students are introduced to two- and three-dimensional shapes as they identify and describe the shapes by attributes. They also use shapes to form larger shapes.

In this unit, kindergarteners analyze and compare two- and three-dimensional shapes. They use informal language to distinguish between defining and nondefining attributes. Using manipulatives, students build and draw shapes that possess defining attributes. Kindergarteners are expected to name and work with squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

These geometric ideas are listed as a critical focus for kindergarten in the CCSS (p. 9). Students learn to describe their physical world using geometric ideas and vocabulary.

*Future Learning*

In grade 1, students will reason with shapes and their attributes as they distinguish between defining and nondefining attributes. They will use two- or three-dimensional shapes to create a composite shape and will compose new shapes from the composite shape. Students will recognize shapes from different perspectives and orientations. They will develop the background for measurement concepts.

## Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics,* "Overall, research indicates that all types of geometric ideas appear to develop over time, becoming increasingly integrated and synthesized. These ideas are originally intuitions grounded in action (building, drawing and perceiving)." (p. 152) In addition, "Children's ideas about shapes do not come from passive looking. Instead, they come as children's bodies, hands, eyes and minds engage in active construction." (p. 152)