MATH 20 FOUNDATIONS UNIT #1 REVIEW ~ INDUCTIVE AND DEDUCTIVE REASONING TEXT MS STAMM IF YOU HAVE QUESTIONS 306 631 4417

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. (1 point) Justin gathered the following evidence.

    $17(22) = \mathbf{374}$ $14(22) = \mathbf{308}$ $36(22) = \mathbf{742}$ $18(22) = \mathbf{396}$

    Which conjecture, if any, is Justin most likely to make from this evidence?

    a.   When you multiply a two-digit number by 22, the last and first digits of the product are the digits of the original number.
    b.   When you multiply a two-digit number by 22, the first and last digits of the product are the digits of the original number.
    c.   When you multiply a two-digit number by 22, the first and last digits of the product form a number that is twice the original number.
    d.   None of the above conjectures can be made from this evidence.

2. (1 point) Sasha made the following conjecture:

    All polygons with six equal sides are regular hexagons (all sides and interior angles are equal)

    Which figure, if either, is a counterexample to this conjecture? Explain.
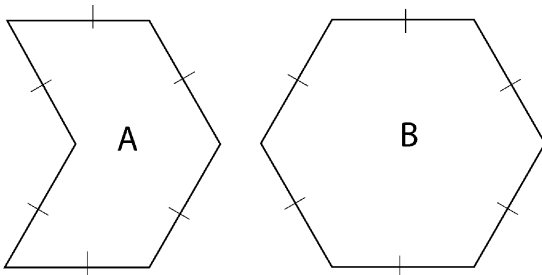
    a.   Figure A is a counterexample, because all six sides are equal and it is a regular hexagon.
    b.   Figure B is a counterexample, because all six sides are equal and it is a regular hexagon.
    c.   Figure B is a counterexample, because all six sides are equal and it is not a regular hexagon.
    d.   Figure A is a counterexample, because all six sides are equal and it is not a regular hexagon.

3. (1 point) Loretta made the following conjecture:

   When you add a multiple of 6 and a multiple of 9, the sum will be a multiple of 9.

   Is the following equation a counterexample to this conjecture? Explain.

   $12 + 27 = 39$

   a. Yes, it is a counterexample, because 39 is not a multiple of 9.
   b. No, it is not a counterexample, because 39 is a multiple of 3.
   c. Yes, it is a counterexample, because 39 is a multiple of 9.
   d. No, it is not a counterexample, because 39 is not a multiple of 9.

4. (1 point) Which of the following choices, if any, uses **<u>deductive reasoning</u>** to show that the sum of two odd integers is even?

   a. $3 + 5 = 8$ and $7 + 5 = 12$
   b. $(2x + 1) + (2y + 1) = 2(x + y + 1)$
   c. $2x + 2y + 1 = 2(x + y) + 1$
   d. None of the above choices

5. (1 point) What type of error, if any, occurs in the following deduction?

   Saturday is not a school day for most students.
   Therefore, students should not wear red clothing on Saturdays.

   a. a false assumption or generalization
   b. an error in reasoning
   c. an error in calculation
   d. There is no error in the deduction.

6. (1 point) Which type of reasoning does the following statement demonstrate?

   All birds have feathers.
   Robins are birds.
   Therefore, robins have feathers.

   a. inductive reasoning
   b. neither inductive nor deductive reasoning
   c. deductive reasoning
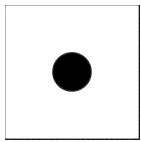
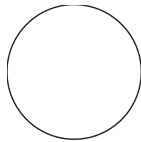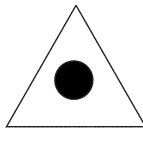7. (1 point) Choose the next figure in this sequence.
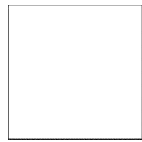

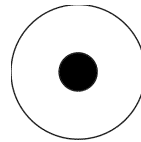Figure 1   Figure 2   Figure 3   Figure 4   Figure 5   Figure 6
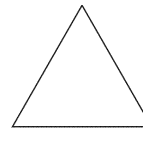
a.

b.

c.

d.

8. (1 point) Mary and Victor are playing darts. Mary has a score of 45. To win, she must reduce her score to zero and have her last counting dart be a double. Which of the following scores on the dart board, in order, would **not** give her the win?

dart in this ring scores triple points

dart in this ring scores double points

dart in outer bull scores 25 points

dart in inner bull scores 50 points (the inner bull also counts as a double)
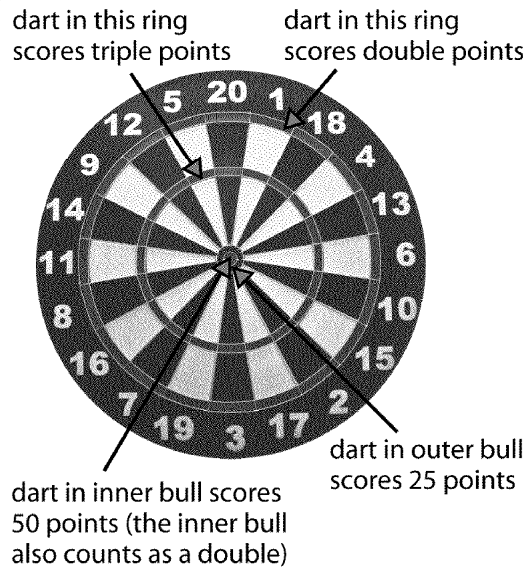
   a.   1, 4, double 20
   b.   15, 20, double 5
   c.   20, 5, double 10
   d.   double 15, 5, 10

9. (1 point) Debbie gathered the following evidence.

   4(33) = **132**   5(33) = **165**   6(33) = **198**

   Which conjecture, if any, is Debbie most likely to make from this evidence?

   a.   When you multiply a two-digit number by 33, the first and last digits of the product form
        a number that is twice the original number.
   b.   When you multiply a one-digit number by 33, the first and last digits of the product form
        a number that is three times the original number.
   c.   When you multiply a one-digit number by 33, the sum of the digits in the product is
        equal to the original number.
   d.   None of the above conjectures can be made from this evidence.

10. (1 point) Jessica noticed a pattern when dividing these numbers by 4: 5^3, 9^3, 13^3.

    Determine the pattern and make a conjecture.

    a.   When the cube of an odd number that is 1 more than a multiple of 4 is divided by 4, the
         decimal part of the result will be .75.
    b.   When the cube of an odd number that is 1 less than a multiple of 4 is divided by 4, the
         decimal part of the result will be .75.
    c.   When the cube of an odd number that is 1 more than a multiple of 4 is divided by 4, the
         decimal part of the result will be .25.
    d.   When the cube of an odd number that is 1 less than a multiple of 4 is divided by 4, the
         decimal part of the result will be .25.

11. (1 point) All alligators are reptiles. All reptiles are covered with scales.
    Tashi is a cat. **What can be deduced about Tashi?**

    1. Tashi has scales.
    2. Tashi is a reptile.

    a.   Choice 1 and Choice 2
    b.   Choice 1 only
    c.   Choice 2 only
    d.   Neither Choice nor Choice 2

12. (1 point) Which of the following choices, if any, uses **inductive reasoning** to show
    that the sum of two odd integers is even?

    a. $(2x + 1) + (2y + 1) = 2(x + y + 1)$
    b. $2x + 2y + 1 = 2(x + y) + 1$
    c.   None of the above choices
    d. $3 + 5 = 8$ and $7 + 5 = 12$

13. (1 point) What type of error, if any, occurs in the following proof?

$$
\begin{aligned}
2 &= 2 \\
4(2) &= 4(1 + 1) \\
4(2) + 3 &= 4(1 + 1) + 3 \\
8 + 3 &= 6 + 3 \\
11 &= 9
\end{aligned}
$$

    a.   a false assumption or generalization
    b.   an error in reasoning
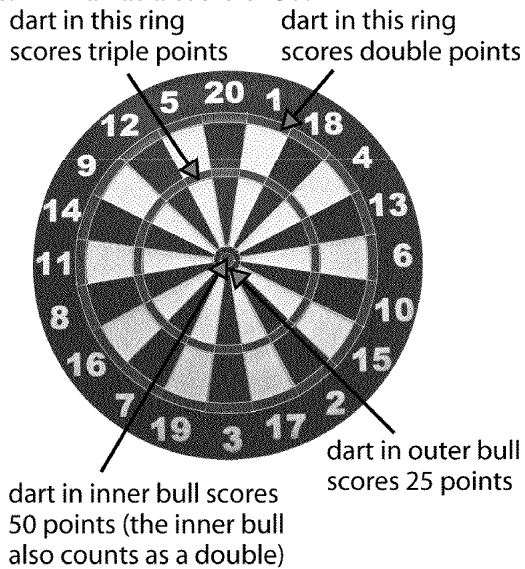   c.   an error in calculation
    d.   There is no error in the proof.

14. (1 point) Emma and Alexander are playing darts. Emma has a score of 37. To win, she must reduce her score to zero and have her last counting dart be a double. Which of the following scores on the dart board, in order, would give her the win?

dart in this ring scores triple points

dart in this ring scores double points

dart in outer bull scores 25 points

dart in inner bull scores 50 points (the inner bull also counts as a double)

    a.   15, 16, 6
    b.   8, 7, double 10
    c.   double 11, 6, 9
    d.   9, 6, double 11

Short Answer

15. (2 points) Guilia created the following table to show a pattern.

Multiples of 9	18	27	36	45	54
Sum of the Digits	9	9	9	9	9

Based on this evidence, Guilia made the following conjecture.

The sum of the digits of a multiple of 9 is equal to 9.

Try more examples. Is this conjecture reasonable? Briefly justify your decision.

16. (2 points) What can you deduce about the sum of two even numbers and an odd number? Justify your answer using deductive reasoning.

17. (2 points) What type of error occurs in the following deduction? Briefly justify your answer.

All videos have large groups of dancers. The western band is recording a new video, so it must have a large group of dancers.

18. (2 points) What type of error occurs in the following proof? Briefly justify your answer.

$$
\begin{aligned}
2 &= 2 \\
4(2) &= 4(1 + 1) \\
4(2) + 3 &= 4(1 + 1) + 3 \\
8 + 3 &= 6 + 3 \\
11 &= 9
\end{aligned}
$$

19. (2 points) Does the following statement demonstrate inductive reasoning or deductive reasoning?

    Every high school student in western Canada has to take math.
    You are a high school student in western Canada, therefore you have to take math.

20. (2 points) What number should appear in the centre of Figure 4?
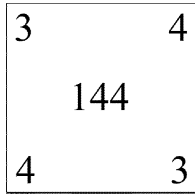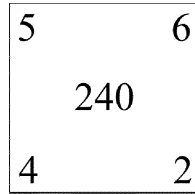
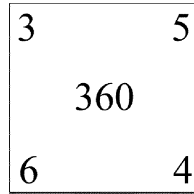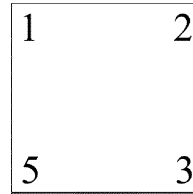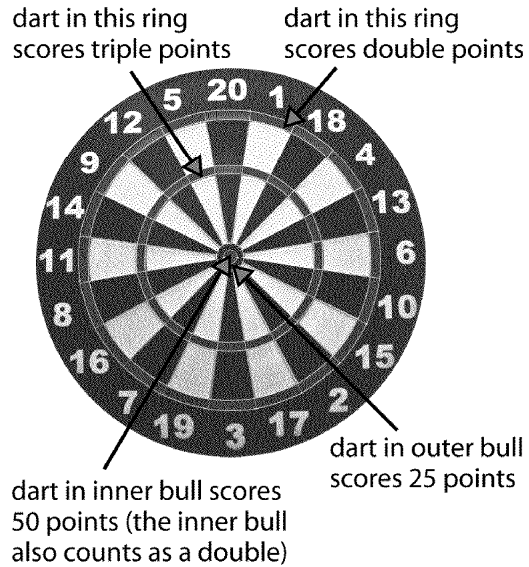| 3 4	5 6	3 5	1 2
144	240	360	
4 3	4 2	6 4	5 3
Figure 1	Figure 2	Figure 3	Figure 4

21. (2 points) Bob and Anne are playing darts. Bob has a score of 36. To win, he must reduce his score to zero and have his last counting dart be a double. Give a strategy that Bob might use to win.

dart in this ring scores triple points

dart in this ring scores double points

dart in outer bull scores 25 points

dart in inner bull scores 50 points (the inner bull also counts as a double)

22. (2 points) Jason created the following table to show a pattern.

Multiples of 27	54	81	108	135	162
Sum of the Digits	9	9	9	9	9

Based on this evidence, Jason made the following conjecture:

The sum of the digits of a multiple of 27 is divisible by 9.

Try more examples. Is this conjecture reasonable? Briefly justify your decision.

23. (2 points) Bradley made the following conjecture:

All professional hockey players can skate well.

Do you agree or disagree? Briefly justify your decision with a counterexample if possible.

24. (2 points) Cheyenne made the following conjecture:

As you travel farther south, toward the equator, the climate gets hotter.

Do you agree or disagree? Briefly justify your decision with a counterexample if possible.

25. (2 points) Place the numbers 2 to 8 in a V-shape, as shown, so the two arms of the V have the same total.
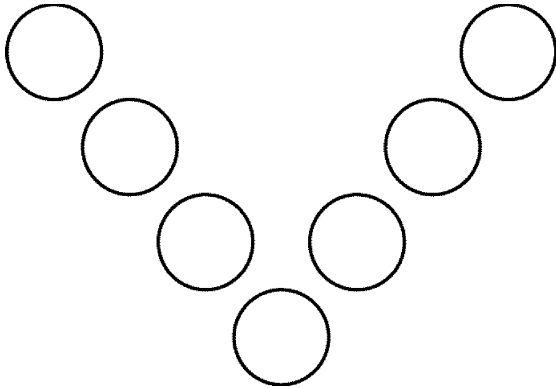
Problem

26. (5 points) Alexandra created a number trick in which she always ended with the number 3. When Alexandra tried to prove her trick, however, it did not work. In which step does the calculation error occur? What is the error?

n	Use n to represent any number.
$n + 2$	Add 2.
$4n + 8$	Multiply by 4.
$4n + 12$	Add 4.
$n + 12$	Divide by 4.
12	Subtract the original number.

27. (5 points) A set of 10 cards, each showing one of the digits from 0 to 9, is divided between five envelopes so that there are two cards in each envelope. The sum of the cards inside each envelope is written on the envelope:
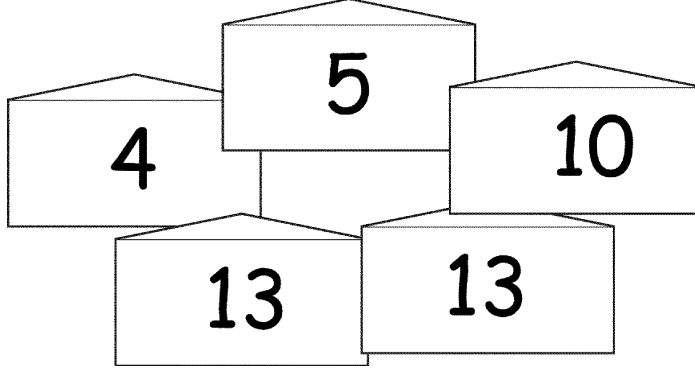
What pair of cards is definitely in an envelope marked 13? Explain.

28. (5 points) In a magic square, the columns, rows, and diagonals all add up to the same total. Use the natural numbers from 1 to 25 to complete this magic square. Use each number only once.

17		1		15
	5		14	
		13		22
	12	19		
11		25		9

29. (5 points) Solve this KenKen puzzle using only the numbers 1 to 4. Do not repeat a number in any row or column. The darkly outlined sets of squares are cages. The numbers in each cage must combine in any order to produce the target number, using the operation shown. A number may be repeated in a cage as long as it is not in the same row or column.

24 ×			2 −
6 ×	5 +		
	2 ÷	7 +	
		1 −	

MATH 20 FOUNDATIONS UNIT #1 REVIEW ~ INDUCTIVE AND DEDUCTIVE REASONING TEXT MS STAMM IF YOU HAVE QUESTIONS 306 631 4417
Answer Section

MULTIPLE CHOICE

   1. ANS: D   REF: Lesson 1.1   TOP: conjectures and Inductive Reasoning
   2. ANS: D   REF: Lesson 1.3   TOP: Disproving conjectures: Counterexamples
   3. ANS: A   REF: Lesson 1.3   TOP: Disproving conjectures: Counterexamples
   4. ANS: B   REF: Lesson 1.4   TOP: Proving conjectures| deductive reasoning
   5. ANS: B   REF: Lesson 1.5   TOP: invalid proofs| deductive reasoning
   6. ANS: C   REF: Lesson 1.6   TOP: reasoning to solve problems
   7. ANS: D   REF: Lesson 1.6   TOP: reasoning to solve problems
   8. ANS: D   REF: Lesson 1.7   TOP: analyzing puzzles and games
   9. ANS: B   REF: Lesson 1.1   TOP: conjectures and Inductive Reasoning
  10. ANS: C   REF: Lesson 1.1   TOP: conjectures and Inductive Reasoning
  11. ANS: D   REF: Lesson 1.4   TOP: Proving conjectures| deductive reasoning
  12. ANS: D   REF: Lesson 1.4   TOP: Proving conjectures| deductive reasoning
  13. ANS: C   REF: Lesson 1.5   TOP: invalid proofs| deductive reasoning
  14. ANS: D   REF: Lesson 1.7   TOP: analyzing puzzles and games

SHORT ANSWER

  15. ANS:
       No, the conjecture is not reasonable, because $9(11) = 99$ is a multiple of 9, and the sum of its digits is 18, not 9.

       REF: Lesson 1.1   TOP: conjectures and Inductive Reasoning
  16. ANS:
       For example, the sum will be odd.
       $2x + 2y + (2z + 1) = 2(x + y + z) + 1$

       REF: Lesson 1.4   TOP: Proving conjectures| deductive reasoning
  17. ANS:
       There is a faulty assumption: not all videos have dancers in them.

       REF: Lesson 1.5   TOP: Invalid proofs| deductive reasoning
  18. ANS:
       There is an error in calculation in the 4th step: $4(1 + 1) = 8$, not 6.

       REF: Lesson 1.5   TOP: invalid proofs| deductive reasoning
  19. ANS:
       deductive reasoning

       REF: Lesson 1.6   TOP: reasoning to solve problems

20. ANS:
    30

    REF: Lesson 1.6   TOP: reasoning to solve problems
21. ANS:
    For example, score double 18 on first throw and not throw any more darts.

    REF: Lesson 1.7   TOP: analyzing puzzles and games
22. ANS:
    Yes, this conjecture is reasonable, because
    27(11) = 297 and 2 + 9 + 7 = 18, which is divisible by 9.
    Also, 27(12) = 324 and 3 + 2 + 4 = 9, which is divisible by 9.

    REF: Lesson 1.1   TOP: conjectures and Inductive Reasoning
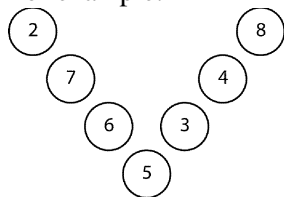23. ANS:
    For example, agree: I couldn't think of a counterexample. A person would have to be able to skate very well to play hockey professionally.

    REF: Lesson 1.3   TOP: Disproving conjectures: Counterexamples
24. ANS:
    For example, agree: the closer you get to the equator, the warmer the climate gets.

    REF: Lesson 1.3   TOP: Disproving conjectures: Counterexamples
25. ANS:
    For example:

    REF: Lesson 1.7   TOP: analyzing puzzles and games

PROBLEM

26. ANS:
    In the second last step, the "12" in "$n + 12$" was not divided by 2.
    In the last step, the number on the left should be 3.

    REF: Lesson 1.5   TOP: invalid proofs| deductive reasoning

27. ANS:
    Make a list of possible pairs in each envelope:
    4: (0, 4), (1, 3)
    5: (0, 5), (1, 4), (2, 3)
    10: (1, 9), (2, 8), (3, 7), (4, 6)
    13: (4, 9), (5, 8), (6, 7)
    Suppose the two envelopes marked 13 contain the pairs
    (4, 9) and (5, 8). Since the 4, 8, and 9 cards are used, the only possible
    pair in the envelope marked 10 is (3, 7). But that leaves no possible pairs
    for the other envelopes because 3, 4, and 5 are all used. That means
    (6, 7) must be in one of the envelopes marked 13.
    If it is, there are two possible solutions:
    (0, 4), (2, 3), (1, 9), (5, 8), (6, 7) and (1, 3), (0, 5), (2, 8), (4, 9), (6, 7).

    REF: Lesson 1.6   TOP: reasoning to solve problems

28. ANS:

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

    REF: Lesson 1.7   TOP: analyzing puzzles and games

29. ANS:

24 × 4	3	2	2 − 1
6 × 2	5 + 1	4	3
1	2 ÷ 2	7 + 3	4
3	4	1 − 1	2

    REF: Lesson 1.7   TOP: analyzing puzzles and games