IE 361 Exam 2
Spring 2011

I have neither given nor received unauthorized assistance on this exam.

KEY

Name   Date

Below are 25 True-False Questions, worth 2 points each. Write one of "T" or "F" in front of each.

F 1. The percent impurity in one fluid ounce of a liquid product is tested and plotted once per hour in a production facility. The appropriate control chart limits are p chart limits.

T 2. A mean number of non-conformities per unit plotted on Shewhart control chart can exceed 1.0.

T 3. The upper standards given control limit for ranges increases as the sample size increases. $D_2\sigma$ look at the table of control chart constants

F 4. The upper standards given control limit for standard deviations increases as the sample size increases. $B_6\sigma$

F 5. Upper and lower standards given control limits for fractions non-conforming get further apart as the sample size increases. They are $6\sqrt{\frac{p(1-p)}{n}}$ apart!

F 6. A point plotting outside of control limits on a Shewhart chart always suggests process degradation. Consider a point below LCLs

F 7. A standard "σ" used in setting $\bar{x}$ chart control limits represents only "process" variation. It must represent both process and measurement variation

F 8. Engineering specifications on dimension A are $1.00' \pm .02'$, while specifications on dimension B are $1.00' \pm .01'$. Then samples of size $n = 5$ must produce $\bar{x}$ chart control limits that are tighter for dimension B than for dimension A.

T 9. Engineering feedback control can play a part in establishing industrial process stability that is then monitored using statistical process control.

T 10. "Special cause variation" is another name for "process change" that Shewhart control charting is meant to detect.

F 11. Retrospective control limits are meant to help answer the question "Are process parameters at their standard values?"

F 12. Statistical tolerance limits are intended to indicate the requirements on a measurement in order for a corresponding item to be functional.

F 13. Statistical prediction limits are intended to locate most of the future output of a stable process based on a sample from that process.

T 14. C_{pk} is a measure of present process performance (rather than process potential).

F 15. If a 95% confidence interval for C_{pk} is $(1.5, 1.7)$ then it is reasonably clear that a "6 sigma" process performance goal has been achieved.

T 16. The ARL concept is a tool for aiding the choice/design of a process monitoring scheme, in that it is a quantification of monitoring scheme performance under a particular model of process behavior.

T 17. The "Western Electric Alarm Rules" are meant to provide the ability to quickly detect non-random patterns on a Shewhart chart.

T 18. Tool wear in a turning process that would naturally make consecutively machined cylinders increase in size can potentially be compensated for by the use of engineering feedback control.

F 19. A physically stable process will of necessity produce acceptable product.

F 20. Trends on an individuals chart tend to make the corresponding value of $\overline{MR}/1.128$ "too small" as an estimate of "σ".

T 21. Two different machining centers produce supposedly identical cylinders. A consistent difference between those machines (in terms of diameters produced) if ignored would produce "sample" averages of two diameters from each machine that would tend to look "too/unbelievably stable."

T 22. Samples (or rational subgroups) of size $n=1$ make completely reliable estimation of σ impossible.

T 23. If a process has a number of known "knobs" that can be used to change an output variable, y, establishing a level of "baseline" variation for y might be done by holding those fixed and control charting process output.

F 24. Lack of physical stability means that basic changes to process configuration or operation are necessary in order to reduce observed variation.

F 25. Normal plotting and confirmation of a normal distributional shape are necessary before it is possible to make any form of statistical prediction limits.

The next 5 pages each have a 10 point "work out" problem on them (numbered W1, W2,W3,W4, and W5.). Answer all 5.

W1. Below are some means and standard deviations for samples of size $n = 3$ surface roughness measurements (units are μ-inches).

Sample	1	2	3	4	5	6	7	8	9	10	
$\bar{x}$	19.4	19.2	21.1	19.8	19.9	19.6	20.3	19.7	18.7	20.1	$\sum \bar{x} = 197.8$
s	.3	1.5	2.0	1.6	1.8	1.3	.9	1.2	.7	.9	$\sum s = 12.2$

Suppose that process standards are $\mu = 20$ and $\sigma = 1$. Is there evidence of change from these standard values in these data? Show appropriate calculations and explicitly say whether there is any evidence of change from the process standard values.

Standards given control limits are

For $\bar{x}$:

$$UCL_{\bar{x}} = \mu + 3\frac{\sigma}{\sqrt{n}} = 20 + 3\frac{(1)}{\sqrt{3}} = 21.73$$

$$LCL_{\bar{x}} = \mu - 3\frac{\sigma}{\sqrt{n}} = 20 - 3\frac{(1)}{\sqrt{3}} = 18.27$$

For s:

$$UCL_s = B_6\sigma = 2.276(1) = 2.276$$

$$LCL_s = B_5\sigma = NA$$

No $\bar{x}$'s and no s's are outside control limits. There is no evidence of change from standard process parameter values.

Evidence of change from standard values? (Circle the correct response.)

In means? yes/no

In standard deviations? yes/no

W2. Below are numbers of radiators inspected and total leaks found in those inspections over a number of 1 hour periods.

	Period	1	2	3	4	5	6	7	8	9	10	
X	Leaks	2	0	1	1	2	1	0	6	0	3	$\sum$ Leaks = 16
k	Number Inspected	2	1	2	1	3	3	2	1	2	3	*Total* = 20
$\hat{u}=\frac{X}{k}$		1	0	.5	1	.67	.33	0	6.0	0	1	

Determine whether there is any evidence of process instability in these data. Show appropriate calculations and say clearly where (if at all) there is evidence of instability.

$$\hat{\lambda}_{\text{pooled}} = \frac{\#\text{ leaks}}{\#\text{ inspected}} = \frac{16}{20} = .8$$

So we can either convert all counts to $\hat{u} = \frac{\text{count}}{\#\text{ inspected}}$ and use (retrospective) control limits

$$.8 \pm 3\sqrt{\frac{.8}{k}}$$

or use limits $.8k \pm 3\sqrt{.8k}$ for the raw counts.

Considering the latter one gets

k	UCL_X	LCL_X
1	3.48	NA
2	5.39	NA
3	7.05	NA

(control limits for $\hat{u}$ are those for X divided by k

k	$UCL_{\hat{u}}$	$LCL_{\hat{u}}$
1	3.48	NA
2	2.70	NA
3	2.35	NA

)

Period 8 has X (and $\hat{u}$) outside control limits and thus no evidence of process instability/change.

W3. Below is a `JMP` report for $n = 25$ measured weld strengths (units are psi).

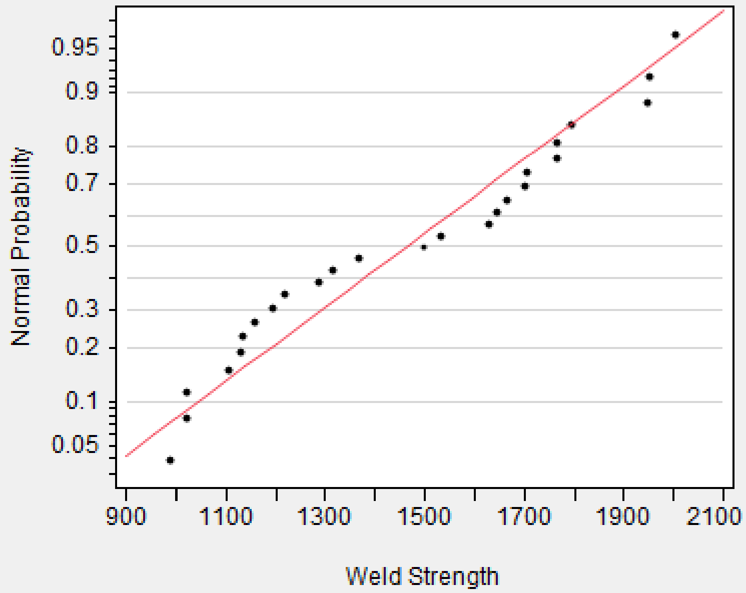
Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	1461.72	1327.7567	1595.6833
Dispersion	σ	324.53948	253.40964	451.48381

-2log(Likelihood) = 359.067286916631

Diagnostic Plot



a) Why is consideration of the kind of plot shown here wise before considering process capability measures? There are at least 2 reasons:

1) Capability indices (involving as they do 6σ or 3σ) are implicitly built on "normal dsn" thinking. Without a normal process dsn, it's not so clear what they even mean or how they should be interpreted.

2) Many potentially appropriate/helpful statistical methods for quantifying process performance (like some prediction and tolerance interval methods) rely for their validity on a "normal process" model assumption.

b) Specifications on such strengths are 1100 psi to 1800 psi. Give 95% confidence limits for C_{pk}.

$\hat{C}_{pk} = \min\left(\frac{1800-1462}{325}, \frac{1462-1100}{325}\right) = \min(1.04, 1.11)$ Then 95% confidence limits for C_{pk} are

$\hat{C}_{pk} \pm z\sqrt{\frac{1}{9n} + \frac{(\hat{C}_{pk})^2}{2n-2}}$ i.e. $1.04 \pm 1.96\sqrt{\frac{1}{9(25)} + \frac{(1.04)^2}{48}}$

W4. Suppose that a pelletizing process is physically stable, producing constant fraction non-conforming p. Samples of size $n = 100$ are used to do Shewhart control charting with standard value $p = .2$.

a) What is the ARL if p is at its standard value?

Consider an np chart. $UCL_X = np + 3\sqrt{np(1-p)} = 20 + 3\sqrt{100(.2)(.8)} = 20 + 12 = 32$

$LCL_X = np - 3\sqrt{np(1-p)} = 20 - 3\sqrt{100(.2)(.8)} = 20 - 12 = 8$

So the $p = .2$ ARL is $1/q$ for

$q = \underbrace{P[X < 8]}_{.0003} + \underbrace{P[X > 32]}_{.0016}$ for $X \sim Bi(100, .2)$

i.e. $ARL \approx \frac{1}{.0019} \approx 526$

b) What is the ARL if p is twice its standard value?

The $p = .4$ ARL is $1/q$ for

$q = \underbrace{P[X < 8]}_{\approx 0} + \underbrace{P[X > 32]}_{.9385}$ for $X \sim Bi(100, .4)$

So $ARL \approx \frac{1}{.9385} = 1.07$

W5. Below is an artificial series of observations (samples of size $n=1$) collected from consecutive items (suppose the units are inches).

2,4,6,8,10,12,14,16,18,20

a) How does the best available estimate of "σ" here (based on control charting ideas) compare to the sample standard deviation? Provide some rationale why the larger of these is larger.

$\overline{MR} = 2$ so $\overline{MR}/1.128 = 1.77$ in On the other hand, $s = 6.06$ in. obviously the bigger

Appearances here are that we're looking at a pure deterministic/non-random trend (there's little that "looks" random" here). That trend, if treated as part of random variation inflates one's view of "σ." In some sense, s takes the whole trend into account and ought to be bigger that $\overline{MR}/1.128$ that operates locally and will be inflated by only as much of the trend that is evident in consecutive values.

b) Set up control limits for future monitoring of individual measurements of this type using a process mean of 10 and an appropriate estimate of process standard deviation.

We want to set limits (for "$\bar{x}$ based on $n=1$"!) for individuals as

$$UCL_x = \mu + 3\sigma = 10 + 3\sigma$$
$$LCL_x = \mu - 3\sigma = 10 - 3\sigma$$

The only question is what to set for σ. There is no good answer, but the least bad answer I know is to use $\hat{\sigma} = \overline{MR}/1.128 = 1.77$. This gives

$$UCL_x = 10 + 3(1.77) = 15.31$$
$$LCL_x = 10 - 3(1.77) = 4.69$$