APS DISTRICT HIGH SCHOOL MATHEMATICS CURRICULUM FRAMEWORK

Course Title: Math Strategies I **Course Number:** 31001

Department: Mathematics **NM STARS Number:** 20114913

Prerequisites: None

Length of Course: One Year **Credit/PRI Area:** .50 per Semester/Elective **Grade Level(s):** 9, 10

Important Notes: Math Strategies I is a Tier II math Response to Intervention(RTI) course that focuses on the development of Algebra I skills and strategies for the struggling student. Student placement in this course is based on standardized test results, teacher recommendation, and grades. A teacher can recommend placement to meet a student's IEP modifications and requirements. The success of the program depends on partnerships among administrators, teachers, parents, and students as well as sustained professional development and resources that are not only current, but enriching. It is recommended that PTR for this course should not exceed 18:1. The duration of this course is one year. The student takes Math Strategies I concurrently with his/her Algebra I course. The goal is to have the student meet all algebra standards by providing support in the areas not yet mastered by the student.

A school may choose to offer this course with a pass/fail option.

The intent is NOT to cover every standard listed in the framework, but to select the standards that address student needs.

COURSE DESCRIPTION: Math Strategies I is for the student who needs support with Algebra I skills and concepts (e.g., operations with integers, solving equations, graphing). The student receives personalized instruction and progress monitoring assessment in his/her areas of need and learns through a variety of strategies, techniques (e.g., graphic organizers, *Teaching and Learning with Text*), and technology (e.g., graphing calculators and computers). These strategies and techniques improve the student's abilities to interpret the text, problem solve, and make connections to real-world situations through a variety of opportunities to learn, practice, and internalize critical-thinking skills.

References in parentheses following each performance standard align with the National Council of Teachers of Mathematics Standards (NCTM), the State of New Mexico Mathematics Standards (NM), the Albuquerque Public Schools Mathematics Standards (APS – MA), the Albuquerque Public Schools Language Arts Standards (APS - LA), and New Mexico Career Readiness Standards (CR).

Note: Power standards are in italics.

STRATEGIES:
The "Illustrations" column in the *Program of Studies* provides exemplars of the performance standards, strategies, and best practices suggested by Mathematics teachers in the Albuquerque Public Schools (APS).

ASSESSMENTS:
Assessments may include: authentic and performance-based assessment, cooperative learning, teacher observations, checklists, tests and exams, formal and informal writing, small group and full class discussions, oral and multimedia presentations, projects, demonstrations, and portfolios. Assessments are based on appropriate rubrics.

SUGGESTED TEXTBOOKS AND INSTRUCTIONAL MATERIALS:

- Current state adopted mathematics textbooks
- Graphing calculators
- Geometer's Sketchpad
- *Discovering Algebra* – Key Curriculum Press
- *Teaching and Learning with Text* – APS
- *Real-Life Math* – Walter Sherwood – Walch – 1998
- *Making Math Count* – Educational Leadership – November, 2007
- *Literacy Strategies for Improving Mathematics Instruction* – Joan M. Kenney – ASCD – 2005
- *Algebra Rescue*
- *Carnegie Learning*

SUGGESTED TITLES/AUTHORS WEB SITES:

- Rubistar4teachers.com
- Nctm.org
- McDougal littell.com
- Hotmath.com
- http://www2.edc.org/Math Problems

Approved by HSCA: April 3, 2008

STRAND I: GLOBAL MATHEMATICS PROCESSES
CONTENT STANDARD: The student understands and uses mathematical processes.

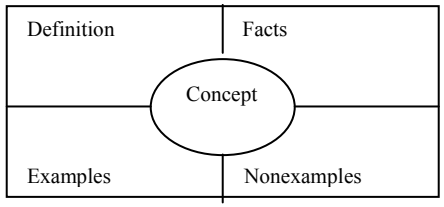
BENCHMARK: The student uses problem solving, reasoning and proof, communication, connections, and representations as appropriate in all mathematical experiences.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
		Note: Research shows that the key factor for student success in any class is effective teaching; teaching that employs quality instruction through the use of a variety of teaching strategies and practices, student engagement, continual use of formative and summative assessments, and appropriate materials. This alone does not guarantee student success or raise achievement levels. The authors of this course contend that the student's regular attendance, positive attitude, and organizational skills are also essential and contribute to his/her success. The intent of Math Strategies I is to support the student's development of vocabulary, language, and process skills (e.g., reasoning and logic) while learning the skills specific to Algebra I. Because of scheduling issues, it is possible that the Math Strategies I student's teacher is different from his/her Algebra I teacher. Therefore, it is critical that collaboration take place among the teachers of these courses. **NOTE: Illustrations include suggested activities for attaining each performance standard. Differentiated instruction through the use of a combination of games, manipulatives, group work, dialogue, graphic organizers, and various assessments provides for lesson enhancement and increased mathematical understanding and learning. A check for (√) refers to a key feature to look for while assessing student performance.**
	1. *Applies the "rule of four" (i.e., represents mathematics graphically, symbolically, verbally, numerically) (APS – all of Strand I).* 2. Prepares mathematically for future careers (APS – I. 14). 3. Uses graphing technology throughout the curriculum (APS – I.9; NM – IC.2). 4. *Uses reasoning and problem-solving strategies to solve new problems [APS – I.3; NM – IIA (5-7)].*	1 – 7. Suggested practices/activities: • The student maintains a notebook with daily entries to include tab dividers for vocabulary, notes, quizzes, sample problems, projects, activities, and grades or other categories depending on teacher preferences. • Test-taking strategies: The student takes practice tests similar to what he/she has to take to learn how to read and understand directions, to look at all the choices before deciding, to get the feel for open-ended questions, and to review rubrics that are used to assess them. • Study skills – Besides learning how to organize his/her notebook,

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	5. Makes connections among mathematical concepts (APS – I.12; NM – IA.6). 6. *Recognizes when to use previously learned strategies to solve new problems (APS – I.2; NM – IC.1, IID.2).* 7. Identifies how seemingly different mathematical situations may be essentially the same (e.g., The intersection of two lines is the same as the solution to a system of linear equations.) (APS – I.13; NM – IA.7, IA.10). **Note: The global processes are not taught in isolation but should be integrated in all strands throughout the curriculum.**	the student develops good study skills (e.g., time management, prioritizing assignments, task completion) though teacher modeling, monitoring, and practice. • The student should receive no homework in this class as he/she is more than likely getting homework in his/her Algebra I class. • The student looks for relationships and connections through every learning experience. He/She expresses these ideas verbally or in written format. This is especially valuable for students who are developing English language skills. • The student observes multiple modeling experiences of new concepts and practices problems that support the instruction. • The student explores concepts and their relationships through the use of technology (e.g., graphing calculators, computers). **Sample Illustration:** Working in small groups, the student selects a unit (e.g., metric, English) to measure a length of rope slightly longer than one meter. He/She records findings on a T-chart. He/She then ties one knot into the rope, re-measures, and records on the T-chart, continuing the process up to seven or eight knots. (Note: The knots need to be consistent in size and tightness. Number of Knots Length of Rope 0 38” 1 36” Using data from the T-chart, the student plots points on an x, y coordinate plane. He/She uses a manipulative (e.g., string, paper) to draw a line of best fit. With a pencil, the student marks on both sides of one of the knots, unties the knot, and measures the distance between the pencil marks. On the graph, the student chooses two points on the line of best fit and determines the rate of change (i.e., slope) between those two points. After using a graphing calculator to enter and graph the data, he/she presents findings to the class. ✓ task completion ✓ connections (e.g., fractions, slope) ✓ accuracy ✓ use of technology ✓ correct labels (e.g., units of measure) ✓ equation for the line of best fit ✓ effective presentation ✓ teamwork/collaboration Extension: The student responds to questions centered around applications of linear functions and browses through newspapers looking for graphs that represent trends.

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: The student understands algebraic concepts and applications.

BENCHMARKS: A. The student represents and analyzes mathematical situations and structures using algebraic symbols.
B. The student understands patterns, relations, functions, and graphs.
C. The student uses mathematical models to represent and understand quantitative relationships.
D. The student analyzes changes in various contexts.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark A: The student represents and analyzes mathematical situations and structures using algebraic symbols. 1. Classifies numbers and members of the following sets (NM – IA.1): • natural, • whole, • integers, • rationals, and • irrationals. *2. Simplifies numerical expressions using the order of operations, including exponents (NM – IA.2).* 3. Evaluates the numerical value of expressions of one or more variables that are (NM – IA.3): • polynomial, • rational, and • radical. 4. Simplifies algebraic monomial expressions raised to a power [e.g., $(5xy^2)^3$] and algebraic binomial [e.g., $(5x^2 + y)^2$] expressions raised to a power (NM – IA.4). 5. Compares and orders polynomial expressions by degree (NM – IA.5).	Note: The standards in this strand are expressed in the mathematical language that frequently creates difficulties for a student in gaining the understanding and skills necessary for success. Since language development is a focus of this course, the illustrations represented emphasize understanding the text and learning the vocabulary and language necessary to acquire the content skills. 1 – 5, 7 – 10, 12, 17, 34. The student participates in a "chalk talk" activity to reinforce vocabulary and vocabulary awareness. He/She responds to questions related to an algebraic concept (e.g., What is a _____ and how does it apply to _____?). The student engages in a mathematical dialogue with other students in the class while a student reporter scripts on butcher paper the responses. Through further questioning, clarification, modeling, and practice, the student gains understanding of the new concept. ✓ active participation ✓ language acquisition ✓ clarity in expression ✓ understanding of new concept ✓ thoughtful response to questions ✓ multiple representations Suggested strategies for learning new concepts: a) Frayer Model  b) mnemonic –PEMDAS for learning order of operations c) KWL charts – K (What you know); W (Want to know); L (What did you learn?)

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
		d) use of flow charts, Venn diagrams, and other graphic organizers to represent knowledge of specific concepts or demonstrate understanding e) compare and contrast (e.g., What is the difference between simplifying an expression and evaluating an expression?) f) Backward solving – The student sees the solution to a problem, starts with the answer, and works backward to the original problem while providing rationale for each step. g) The student applies skills on a regular basis to problems that reinforce understanding of earlier learned concepts. This can be achieved through questioning, asking the student to clarify or distinguish, or through writing. h) Journals – The student writes an explanation to a new concept, asks questions, or writes comments about what was learned. The student can use writing for creative purposes (e.g., poem, create a new problem similar to one being solved). i) Cognitive Academic Language Learning Approach (CALLA) – This approach provides support for ESL learners in both content and learning strategies as well as all students. It can be used as a problem solving procedure where the student works with a partner to identify key vocabulary words, discusses or writes the process to solve the problem, solves the problem, and explains his/her solution. The student can also use this approach to self evaluate. He/She identifies what was learned, what was easy, what was difficult, with what he/she needs more help, and how he/she feels about the new concept. j) word walls 1, 7, 8, 13 - 15, 16, 22, The student uses manipulatives (e.g., Algebra tiles) to learn integers, how to factor, and how to distinguish between X + X and X•X. After effective modeling, the student practices and applies the learned techniques to new problems. ✓ understanding of new concepts ✓ ability to represent concepts in multiple ways ✓ accuracy ✓ number sense and operations
	6. Represents and analyzes relationships using written and verbal expressions, tables, equations, and graphs, and describes the connections among those representations to (NM – IA.6): • *translates from verbal expression to algebraic formulae (e.g., "Set up the equations that represent the data in the following equation: John's father is 23 years older than John.*	6, 18 – 21,24, 26, 27, 30, 31. The student demonstrates in a variety of ways his/her understanding of locating points and naming locations. He/She participates in the game "Battleship", uses city and state maps to find addresses and towns, and uses computers and graphing calculators as tools to increase student understanding of locating points (e.g., Green Globs). Extension: Using a set of points or equations, the student graphs the lines to

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	John is 4 years older than his sister Jane. John's mother is 3 years younger than John's father. John's mother is 9 times as old as Jane. How old are John, Jane, John's mother, and John's father?"), • *given data in a table, constructs a function that represents these data (linear only), and* • *given a graph, constructs a function that represents the graph (linear only).* *7. Knows, explains, and uses equivalent representations for the same real number including (NM – IA.7):* • *integers,* • *decimals,* • *percents,* • *ratios,* • *scientific notation,* • *numbers with integer exponents,* • *inverses (reciprocal), and* • *prime factoring.* 8. Simplifies algebraic expressions using the distributive property (NM – IA.8). 9. Explains and uses the concept of absolute value (NM – IA.9). 10. Knows, explains, and uses equivalent representations for algebraic expressions (NM – IA.10). 11. Simplifies square roots and cube roots with monomial radicands that are perfect squares or perfect cubes (e.g., $9a^2x^4$) (NM – IA.11). *12. Solves (NM – IA.13):* • *formulas for specified variables, and* • *radical equations involving one radical.* 13. Factors polynomials, difference of squares and perfect square trinomials, and the sum and difference of cubes (NM – IA.14). 14. Simplifies fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms (NM – IA.15).	create a picture. ✓ location of points and/or lines on a coordinate plane ✓ accuracy ✓ understanding of key components of an ordered pair or a set of ordered pairs ✓ technology skills 11. The student learns that numbers having square roots that are integers are perfect squares and that a perfect cube is a number whose cube root is an integer. The student starts to explore the concept of a perfect square by using dot paper to draw some perfect squares (e.g., 1, 4, 16). Using a calculator the student then finds the square roots of those numbers. In small groups, the student discusses the relationship between the square and its square root. He/She creates a chart for the first 15 squares and their square roots and writes it in his/her notebook for future reference. (Repeat for cubes and cube roots.) Number Square Square Root Cube Cube Root 1 1 1 1 1 2 4 2 8 2 3 9 3 27 3 Extension: Once the student grasps the concepts of squares, cubes, square

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	15. Manipulates simple expressions with + and – exponents (NM – IA.16). 16. Uses the four basic operations (+, -, x, ÷) with (NM – IA.17): • linear expressions, • polynomial expressions, and • rational expressions. **Benchmark B: The student understands patterns, relations, functions, and graphs.** 17. Distinguishes between the concept of a relation and a function (NM – IB.1). 18. Describes the concept of a graph of a function (NM – IB.3). *19. Translates among tabular, symbolic, and graphical representations of functions (NM – IB.4).* 20. Explains and uses function notation (NM – IB.5). 21. Determines the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression (NM – IB.6). 22. Uses the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points (NM – IB.12). 23. Applies quadratic equations to physical phenomena (e.g., the motion of an object under the force of gravity) (NM – IB.13). **Benchmark C: The student uses mathematical models to represent and understand quantitative relationships.** *24. Models real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; apply quadratic equations to model throwing a baseball in the air) (NM – IC.1).*	roots, and cube roots with integers, the student examines finding the square root and cube root of variables and uses the rules of exponents to make conjectures. ✓ application of concepts ✓ teamwork/collaboration ✓ effective communication ✓ understanding of concepts ✓ technology skills ✓ connections 17, 20, 21, 23, 37, 38. The student uses real-life situations (e.g., rate of speed vs. distance traveled, cooking examples, age and height) to distinguish between a relation and a function. When looking at a variety of graphs, he/she uses the vertical line test to determine if the graph is a function and explains why or why not a graph is or is not a function. After extensive practice, the student, working alone or with a partner, applies concepts to create a fictional business and gives examples that represent ordered pairs (e.g., time considerations) and functions (e.g., equation to represent profit). ✓ application of concepts ✓ accuracy ✓ connections ✓ creativity ✓ relevant examples ✓ effective communication 24. Probably the single most difficult concept for a student to master is that of becoming a good mathematical problem solver. Inherent in this is making meaning of the text, understanding the vocabulary, making connections, and being able to apply what the student knows. It is important that the student learn early in the year some strategies that simplify the problem-solving process and help him/her comprehend what he/she reads and deepens his/her conceptual understanding. Through teacher modeling, guided practice, and independent practice of a variety of problems, the student reads the problem,

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9, 10</th><th>PERFORMANCE STANDARDS</th><th>ILLUSTRATIONS</th></tr>
<tr><td></td><td></td><td>identifies key words and their meaning, determines what was given, determines what is not known, considers what process should be used to solve the problem (e.g., draw a picture, a table), organizes the information, selects a variable to represent the unknown, translates the information into a mathematical sentence, solves the problem, and checks it for reasonableness.
✓ reading comprehension
✓ synthesis of information
✓ problem-solving strategies
✓ application of concepts
✓ connections
✓ accuracy
✓ logical solution
See also Teaching and Learning with Text – p. 9.

24 – 26, 28, 31. The student assumes the role of a sales rep for video games. He/She prepares and submits a report recommending a replacement plan for cellular phone service for the 12 sales reps in this company. Estimating that each sales rep spends between 40 and 60 minutes per weekday on his/her phone, the student uses the information from the given table to select the most cost-efficient plan for the company.
<table>
<tr><th>Plan</th><th>Monthly Access Charges</th><th>Minutes per month</th><th>Additional Airtime Charge Minute</th></tr>
<tr><td>Plan 1</td><td>$69.99</td><td>600</td><td>.26</td></tr>
<tr><td>Plan 2</td><td>$99.99</td><td>1000</td><td>.20</td></tr>
<tr><td>Plan 3</td><td>$149.99</td><td>1600</td><td>.18</td></tr>
</table>
In the report the student includes an estimate of a low, medium, and high range of number of minutes per month of expected use, an equation for each of the three plans in the table, a calculation of the cost of each plan based on the different monthly usage amounts, a graph that shows where the rates for the different phone plans overlap, the pros and cons of each plan, and a recommendation that includes a detailed analysis of the advantages of the selected plan.
✓ all required components
✓ analysis of information
✓ rationale for each plan and recommendation
✓ effective communication
✓ accuracy
✓ problem-solving strategies
✓ multiple representations of the information</td></tr>
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GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	25. *Uses a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools) (NM – IC.2).*	25. The student uses a variety of methods from calculating mentally, estimating, pencil and paper, and using calculators and computers in every problem throughout this document. Sometimes one method for solving the problem may dominate, but all of the mentioned methods are used consistently throughout the course. ✓ multiple forms of calculation
	26. Expresses the relationship between two variables using a table with a finite set of values and graphs the relationship (NM – IC.3). 27. Expresses the relationship between two variables using an equation and a graph to (NM – IC.4): • graphs a linear equation and linear inequality in two variables, • solves linear inequalities and equations in one variable, • solves systems of linear equations in two variables and graphs the solutions, and • uses the graph of a system of equations in two variables to help determine the solution. 28. Solves applications involving systems of equations (NM – IC.5).	26, 27, 30, 32, 33, 35, 37. The student uses rubber bands as a bungie cord to determine how far an object falls, collects data, plots the data,, writes a linear equation in y = mx + b form, and predicts the number of rubber bands needed for a test condition. ✓ multiple representations of information ✓ accuracy ✓ relationships and connections
	29. Evaluates numerical and algebraic absolute value expressions (NM – IC.6). 30. Creates a linear equation from a table of values containing co-linear data (NM – IC.7). 31. Determines the solution to a system of equations in two variables from a given graph (NM – IC.8). 32. Generates an algebraic sentence to model real-life situations (NM – IC.9). 33. Writes an equation of the line that passes through two given points (NM – IC.10). 34. Understands and uses (NM – IC.11): • such operations as taking the inverse, finding the reciprocal, taking a root, and raising to a fractional power, and • the rules of exponents.	29. The student uses a number line to count distance from a given point in either direction. He/She can also model this concept from the teacher's desk to a student's desk in either corner of the room. (Teacher's desk = 0, left corner = negative values, right corner = positive values). In a class discussion, the student verbalizes his/her understanding of the concept. Extension: Once the student has a solid grasp and understanding of absolute value, he/she applies that knowledge along with equation-solving skills to evaluate and solve algebraic absolute value expressions. ✓ individual participation ✓ comprehension ✓ clarity and conciseness of communication ✓ application of multiple concepts and skills

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	35. Verifies that a point lies on a line, given an equation of the line, and be able to derive linear equations by using the point-slope formula (NM – IC.12). **Benchmark D: The student analyzes changes in various contexts.** 36. Calculates the percentage of increase and decrease of a quantity (NM – ID.3). 37. Estimates the rate of change of a function or equation by finding the slope between two points on the graph (NM – ID.5). 38. *Evaluates the estimated rate of change in the context of the problem (NM – ID.6).*	36. The student participates in a class discussion and provides various situations where percents are used (e.g., discounts at stores, newspaper survey results, test grades). Working in small groups and using a variety of materials (e.g., newspapers, magazines) each group selects a project where the object is to show various representations of percents in everyday life. Along with the examples, the group must calculate the percent of increase and/or decrease of quantities. Each group presents its project in the form of a visual (e.g., collage, poster, game) to the rest of the class. ✓ individual participation ✓ teamwork/collaboration ✓ clarity in communication ✓ relevant examples ✓ creativity ✓ effective presentation

STRAND III: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: The student understands geometric concepts and applications.

BENCHMARKS: A. The student analyzes characterisitcs and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships.
B. The student specifies locations and describes spatial relationships using coordinate geometry and other representational systems.
D. The student uses visualization, spatial reasoning, and geometric modeling to solve problems.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark A: The student analyzes characterisitcs and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships. 1. Demonstrates an understanding of inductive and deductive reasoning, explains the difference between inductive and deductive reasoning, and identifies and provides examples of each (NM – IIA.6): • for inductive reasoning, demonstrates understanding that showing a statement is true for a finite number of examples does not show it is true for all cases unless the cases verified are all cases, and • for deductive reasoning, proves simple theorems. **Benchmark B: The student specifies locations and describes spatial relationships using coordinate geometry and other representational systems.** 2. Demonstrates understanding of the construction of the coordinate plane, knows the names of the origin, coordinate axes and four quadrants, draws and labels them correctly, finds the coordinates of an indicated point, and plots a point with given coordinates (NM – IIB.1). 3. Given two linear equations, determines whether the lines are parallel, perpendicular, or coincide (NM – IIB.3).	1. The student uses experiences from his/her own life to determine whether the example is inductive or deductive and justifies his/her response (e.g., If a fire truck goes by, there must be a fire, If $4x + 2 = 10$, then $x = 2$, following a pattern from a table to develop a function rule). The student looks at several solutions to equations and analyzes the solutions step-by-step citing a property, definition, or theorem to justify his/her response. ✓ application of concepts ✓ logical reasoning ✓ clarity of expression ✓ accuracy ✓ understanding of the difference between deductive and inductive reasoning 2. This standard is met in Strand II. Fundamental to an Algebra I course is understanding the coordinate plane, being able to locate points, and graph linear equations. 3. The student reviews the meaning of the words parallel, perpendicular, and coincide and models examples using objects found in the classroom (e.g., pencils, rulers). Once the student understands the concepts, he/she extends the learning to include how angle measures and slopes can help him/her determine if two lines are parallel, perpendicular, or coincide. The next step is to take a system of equations and solve them algebraically. The student practices after extensive modeling, guided, and independent practice. See also Strand II, the illustrations for performance standards #27 and #28. ✓ concrete examples ✓ application of content skills ✓ equation solving prowess

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
		✓ understanding of new concepts ✓ accuracy
	4. Uses basic geometric ideas (e.g., the Pythagorean Theorem, area, and perimeter of objects) in the context of the Euclidean Plane and calculates the perimeter of a rectangle with integer coordinates and sides parallel to the coordinate axes and with sides not parallel (NM – IIB.4). **Benchmark D: The student uses visualization, spatial reasoning, and geometric modeling to solve problems.** 5. Solves problems using the Pythagorean theorem (e.g., "Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder.") (NM – IID.4).	4, 5. Through a variety of activities, the student initiates his/her understanding of the Pythagorean Theorem and then advances to problem solving. He/She: • working with a partner and using inductive reasoning, responds to the question, "How are the lengths of the sides of a right triangle related?' The student explores this by using grid paper to measure the sides of a variety of right triangles. He/She records the measurements in a table and makes a conjecture. • draws a right triangle. Using each side of the triangle, he/she then draws a square and then conjectures as to the relationship of the three squares. • uses technology (e.g., Geometer's Sketchpad) to explore further relationships of right triangles. • applies the Pythagorean Theorem to determine the length of the missing sides of a right triangle. • integrates work from other classes (e.g., Woods, CAD) to create a project. ✓ thoughtful response to questions ✓ individual participation ✓ logical reasoning ✓ application of concepts ✓ teamwork/collaboration ✓ technology skills ✓ accuracy ✓ effective communication ✓ understanding of the Pythagorean Theorem

STRAND IV: DATA ANALYSIS AND PROBABILITY
CONTENT STANDARD: The student understands how to formulate questions, analyze data, and determine probabilities.

BENCHMARKS: B. The student selects and uses appropriate statistical methods to analyze data.
C. The student develops and evaluates inferences and predictions that are based on data.
D. The student understands and applies basic concepts of probability.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark B: The student selects and uses appropriate statistical methods to analyze data. 1. Understands the meaning of measurement data and categorical data, and of the term "variable" (NM – IIIB.1). 2. Understands the meaning of "univariate" (i.e., one variable) and "bivariate" (i.e., two variable) data (NM – IIIB.2). 3. For univariate data, displays the distribution and describes its shape using appropriate summary statistics, and understands the distinction between a statistic and a parameter (NM – IIIB.3): • constructs and interprets frequency tables, histograms, stem and leaf plots, and box and whisker plots, • calculates and applies measures of central tendency (mean, median, and mode) and measures of variability (range, quartiles, standard deviation), and • compares distributions of univariate data using back-to-back stem and leaf plots and parallel box and whisker plots.	1 – 5. The student participates in a variety of activities to gain understanding of data. He/She: • working alone or with a partner, measures his/her hand spread, collects the data from each student in the class, arranges the data, finds the median, upper and lower quartiles, and makes a box-and-whisker plot. • determines what day of the year he/she was born (e.g., 1st, 5th, 23rd), lines him/herself from smallest to the greatest, counts him/herself to the middle to get the median, determines that the mode is the most frequent occurrence, and determines the lower and upper quartiles (e.g., half way between the minimum and the median is the lower quartile – halfway between the median and the maximum is the upper quartile). • as part of a small group, considers a survey question that can be answered with a number (e.g., How many children are there in your family?, How many hours a day do you watch TV?). After selection of the question, the group decides upon a population for the survey and the minimum number of people to survey. The student collects the data, organizes the data in a frequency table, finds the percent of responses for each answer, and draws a bar graph or pictograph to display the data. He/She then finds the mean, median, mode, range, and all quartiles of the data. Each group presents his/her idea to the class. The group may or may not use technology. ✓ individual participation ✓ teamwork/collaboration ✓ completion of all required components ✓ organization of data ✓ clarity of expression ✓ effective presentation ✓ technology skills, if used

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Benchmark C: The student develops and evaluates inferences and predictions that are based on data.

4. Compares and draws conclusions between two or more sets of univariate data using basic data analysis techniques and summary statistics (NM – IIIC.1).

5. *Draws conclusions concerning the relationships among bivariate data (NM – IIIC.2):*
   - *makes predictions from a linear pattern in data,*
   - *determines the strength of the relationship between two sets of data by examining the correlation, and*
   - *understands that correlation does not imply a cause-and-effect relationship.*

Benchmark D: The student understands and applies basic concepts of probability.

6. Explains the concept of a random variable (NM – IIID.1).

7. Understands the concept of probability as relative frequency (NM – IIID.2).

8. Uses simulations to compute the expected value and probabilities of random variables in simple cases (NM – IIID.3).

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4, 5. The student simulates the techniques observed in popular TV programs (e.g., CSI, detective stories) to solve a variety of contrived "mysteries." Sample problem: The student arrives at class and discovers that the candy jar has been taken. He/She observes that the culprit has left behind a series of footprints. Using what the student knows about ratios and proportions, he/she sets out to solve the mystery and determine "Who took the candy?" The student presents his/her solution to the class and provides rationale for his/her solution.

- ✓ logical reasoning
- ✓ accuracy of prediction
- ✓ critical-thinking skills
- ✓ connections/relationships
- ✓ clarity of expression
- ✓ effective presentation

6 – 8. The student participates in a class discussion to gain understanding of key vocabulary words (e.g., random event, likely outcomes, theoretical probability). The student enters the new vocabulary in his/her notebook. The student then works in small groups and uses real objects or manipulatives (e.g., dice, cards, spinners, coins) to carry out a variety of activities.

Sample activities:

- A software program selects a name at random from a database that contains the names and phone numbers of every person in the United States. The student uses a given table to estimate the probability that the person called is a female.
- The student rolls a die and finds the probability of rolling a two or a five.
- The student tosses a coin 100 times and completes the following table.

Tosses	Heads	Tails
10	?	?
20	?	?
. . ;		
100	?	?

The student states in writing what conclusion he/she can draw about the chance of tossing a head and explains his/her reasoning.

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GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
		• The student finds a graph in a newspaper or magazine and writes two probability problems that can be answered by data in the graph. ✓ individual participation in all activities ✓ reasoning skills ✓ vocabulary development ✓ elements of effective writing ✓ conceptual understanding of probability

STRAND V: LITERACY
CONTENT STANDARD: The student communicates mathematical principles through reading, writing, and speaking opportunities.

BENCHMARK: The student demonstrates through a variety of writing and speaking requirements proficiency in reading comprehension, specialized vocabulary, and reasoning.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	The following performance standards are aligned with 9th grade APS Language Arts Standards.	Note: The very nature of mathematics courses require the student to read the textbook (e.g., word problems); to learn the vocabulary of mathematics; to communicate symbolically, orally, and in written formats; and to think critically through problem solving. Through consistent integration of the mathematical processes, the student works collaboratively with other students, requiring whole or small group discussions; listens to other's viewpoints whether it be via print, technology, or guest speaker; displays data in an organized fashion; and makes connections. Consequently, literacy strategies are integrated and reflected in every strand. The following citations illustrate specific examples of these strategies although numerous opportunities are presented throughout the year and throughout the curriculum.
	1. Develops and demonstrates proficiency with the following strategies to approach reading for information across content areas: (APS – LA I.1): • scans reading selection to determine whether a text contains relevant information, • uses the headings and subheadings of the material to make predictions and to validate comprehension of text, • reads and rereads to decode meaning, and • reviews and summarizes essential elements of text for overview. 2. Identifies and uses roots, prefixes, and suffixes to determine meaning of words (APS – LA I.4). 3. Uses textual evidence to develop and support an interpretation of a scientific process or concept (APS – LA II.2).	1 – 3. See Strand I – the illustration on test-taking strategies; Strand II – the illustrations that incorporate the use of KWL charts, other graphic organizers, the problem-solving process, and the last illustration on the percent activity; Strand IV – the last illustration; and Strand VI – the 1st illustration.
	4. Develops increased competence in using the writing process to create a final product (APS – LA III.1).	4 – 7. See Strand I – notebook maintenance; Strand II – the illustrations that require journal entries, CALLA activities, vocabulary development, and the cell phone illustration; Strand III – 3rd illustration and the illustration for performance standards #4 and #5 that requires technology to explore the

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	5. Develops increased competence in using elements of effective writing (APS – LA III.2). 6. Supports an informed opinion: (APS – LA III.6): • uses appropriate language, reasoning, and organizational structure for the audience and purpose, • provides relevant and convincing reasons, uses various types of evidence, and • demonstrates an awareness of possible questions, concerns, or counterarguments. 7. Responds to a variety of written, electronic, and other media (APS – LA III.7). 8. Develops increased competence with speaking and language conventions (APS – LA IV.3). 9. Listens to and analyzes a presentation or discussion (APS – LA V.1). 10. Conducts research and collects data from in-depth field studies (APS – LA VI.1). 11. Uses a variety of technology (APS – LA VI.5). 12. Synthesizes information from multiple research studies to draw conclusions and inferences that go beyond those found in any of the individual studies (APS – LA VI.9). 13. Synthesizes and organizes information from a variety of sources to inform and persuade an audience (APS – LA VI.9).	Pythagorean Theorem; Strand IV – the last illustration; and VI – both illustrations. 8, 9. See Strand I illustration; Strand II – the chalk talk and bungee cord illustrations; Strand III – the 1st illustration; Strand IV – all three illustrations; and Strand VI – both illustrations. 10 – 13. See Strand I illustration; Strand II – the cell phone illustration and the last illustration involving the percent activity; Strand IV – the 1st illustration; and Strand VI – the 1st illustration.

STRAND VI: CAREER APPLICATIONS
CONTENT STANDARD: The student explores mathematics career pathways and recognizes skills necessary to seek employment in the field.

BENCHMARKS: A. The student identifies education, training, and skills required for employment in the math-related careers.
B. The student examines employment opportunities, workplace environments, and career growth potential for mathematics fields.

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	1. Identifies personal interests and aptitudes for proper course selection and career choices (CR – 1A). 2. Explores a career path consistent with career interests, aptitudes, and abilities to meet career goals and objectives that includes further study beyond high school (CR – 1C). 3. Exhibits ethical conduct, positive behaviors and personal responsibility within the school, workplace, and community (CR – 4A).	1 – 6. The student engages in a variety of activities that expand his/her awareness of career opportunities, options, and requirements that increase success. He/She: • researches careers and/or personal hobbies • explores the mathematics within various careers/hobbies • develops awareness of high school graduation requirements • follows class and school rules • mentors lower level students. • models and demonstrates techniques to less proficient students. • reviews and revises his/her Next Step Plan. • looks at higher level high school courses which include honors and AP. For selected activities in which the student participates, he/she writes a reflection paper and/or compiles a portfolio on what he/she heard, learned, observed, or did as part of that experience. ✓ exhibition of work ✓ positive behaviors ✓ examination of ways to improve skills and knowledge ✓ reflection/analysis ✓ effective communication ✓ cooperation/teamwork ✓ evidence of proficiency and growth in algebra
	4. Works cooperatively with persons from diverse backgrounds to accomplish a common goal (CR - 4B, 5C). 5. Uses the critical-thinking skills gained to solve problems in daily life (NM – IC.1; CR – 5E). 6. Engages in continuous self-assessment and goals modification for	4. Early in the school year, the student works collaboratively with the other students to determine the class mission and vision statement. Upon agreement, the class breaks into smaller groups to prepare and present in writing the class mission and vision statement. The student uses technology to develop a visual that represents his/her assignment. ✓ teamwork/collaboration ✓ positive behaviors (e.g., adherence to rules)

GRADE 9, 10	PERFORMANCE STANDARDS	ILLUSTRATIONS
	personal and professional growth (CR – 2A, 5F).	✓ task accomplishment ✓ technology skills ✓ quality product ✓ creativity ✓ effective presentation of visual