CNN optimizations for embedded systems and FFT

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Abstract

CNNs have proven to be a very successful yet computationally expensive technique which made them slow to be adopted in mobile and embedded systems.

There is a number of possible optimizations: minimizing the memory footprint, using lower precision and approximate computation, reducing computation cost of convolutions with FFTs. These have been explored recently and were shown to work.

This project take ideas of using FFTs further and develops an alternative way to computing CNN – purely in frequency domain. As a side result it develops intuition about nonlinear elements: why do they work and how new types can be created.

1. Introduction

Today mobile devices such as smartphones and tables are the most often used compute platform and digital camera as such they are the best target for various computer vision applications. These devices are battery powered which limits available resources and poses a significant challenge to advanced algorithms such as CNN.

Traditionally CNNs have been developed for clusters of desktop grade CPUs and GPUs which consume about 80 and 200W respectively, while mobile platforms are limited at 5-10W. This gets even worse for wearable and "always on" devices which only have 0.1-1W.

A common approach to alleviate performance and power problem (to some degree) is to move computation to "the cloud", however this raises privacy concerns about doesn't solve the power problem if we want to apply CNNs in real time. Sending even small 256x256 images at 25fps over the data network will quickly drain the battery, not to mention huge amounts of latency.

The only way to solve this problem is by building specialized hardware such as NeuFlow[15] or DaDianNao[17]. Alternatively, one can try to adjust an algorithm to better fit existing hardware and it's limitations. The best results could be archived by applying both approaches at the same time and co designing algorithms and hardware at the same time.

2. Problem statement

In specialized hardware, most of the gains are archived by select "the right" algorithm – the one which can be implemented efficiently.

For embedded platform the measure of efficiency is power which is equal to number of operations times energy per operations. To optimize power we can either do less operations or spend less energy doing it.

2.1. Energy table

First of all it's important to understand how the energy is spent on various steps in computer program.

Operation	Energy, pJ	Relative cost
16b Int ADD	0.06	1
16b Int MULT	0.8	13
16b FP ADD	0.45	8
16b FP MULT	1.1	18
32b FP ADD	1.0	17
32b FP MULT	4.5	80
Register File, 1kB	0.6	10
L1 Cache, 32kB	3.5	58
L2 Cache, 256kB	30.2	500
on-chip DRAM	160	2667
DRAM	640	10667
Wireless transfer	60000	1000000

Table 1: Energy cost of common operations.

Numbers in Table 1 heavily depend on technology parameters, such as manufacturing node (feature size), operating voltage, frequency, etc. but the general trend will be same:

- communication is extremely expensive
- computation is cheaper than memory access
- memory access depends on it's capacity (register file vs cache vs DRAM)
- integer arithmetic is cheaper than floating point

- everything depends on data precision(16b vs 32b)

This clearly shows the importance of data locality and memory footprint optimization to energy efficiency.

2.2. Choosing CNN implementation

For a baseline architecture in embedded system we'd want to use one of the top performing submitions in ImageNet challenge, that has the minimal working set. In other words, the CNN with smallest number of parameters.

CNN	Year	Parameters
AlexNet[3]	2012	60M
Clarify[7]	2013	65M
OverFeat[5]	2013	70M
VGG [8]	2014	135M
GoogLeNet[6]	2014	7M

Table 2: Number of parameters in state-of-the-art CNNs.

As the summary table 2 shows, in general, the trend has been towards increasing the number of weights. This is cause by increasing complexity of CNN and adding more layers.

An exception to this trend is 2014 winner – GoogLeNet. By comparing it with VGG, we can notice that there are more convolution layers – 59 stages of varying sizes in 21 layers vs 16 stages of 3x3 convolutions over 16 layers in VGG[8]. At the same time GoogLeNet[6] has about 20x less parameters because there is a one instead three Fully Connected layers.

Even though such a reduction is great we are still need 7m*4Byte=28MB of storage this is still too high from SRAM and would require DRAM access. Additionally we are doing about 3x more computations (59 vs 21 convolution stages).

GoogLeNet use a lot of convolutions, in fact it spends majority of computations doing them. For this reason it's critical to have a very efficient implementation of convolution. One of the options is by using Fast Fourier Transformation. This will turn convolutions into point-wise multiplications and reduce complexity from $O(n^2*k^2)$ to $O(n^2)$ where n is the input width/height and k is the filter size. Section 4 describes how this idea can be developed further.

3. Overview of previously explored ideas

The remaining problem with the number of parameters can be addressed by the following optimizations:

- Setting some weights to 0
- Quantizing weights to fewer bits
- Weights Deduplication

And the computation increase can be offset by:

- Interger/Fixed point arithmetic instead of floats
- Aproximate/Imprecise computation
- Use less precision / fewer bits
- Do convolutions in frequency domain

All these ideas have merit and there are very recent papers (from 2015) that explored them, however those papers used old CNN designs as a baseline link AlexNet[3], OverFeat[5] or Maxout[4], no paper have considered GoogLeNet[6], but in general the same methods will likely work.

3.1. Setting some weights to 0

The main assumption is that some weights are less important than others and can be set to 0 and that this will have small to moderated affect on the accuracy.

M. D. Zeiler and R. Fergus[9] showed that the convolotional layers gradually increase the accuracy of CNN which supports my assumptions. Furthermore techniques like DropConnect [10] suggest that remove some parameters is not only acceptable, but could also be beneficial and certain regularizations types are also known to encourage sparse parameters as shown in Elastic Net [11].

3.2. Quantizing weights

L2 is most commonly used regularization type it penalizes large values in weights and encourages that all parameters are used a little. This suggest that we can expect relatively small dynamic range in weights thus use smaller number of bits to represent them.

3.3. Weights deduplication

Data deduptication is a well know technique to reduce memory requirement. It doesn't change the data and thus will not affect the accuracy and it can be efficiently integrated into computer architecture as was shown by HICAMP[14].

The simplest way to simulate the benefits of deduplication is by using compression, like Zip.

3.4. Lower precision arithmetic

O. Temam [16] investigated neural network hardware accelerator geared towards defect tolerance and energy efficiency. This is a very desirable feature of CNN which allows cheaper and more efficient hardware implementations. The fault tolerance can be investigated in software by artificially injecting random errors during computation. It seem that the fault tolerance could the consequence of using more precision than required by the algorithm.

Courbariaux and David investigated the use of low precision arithmetic for deep learning in [12]. Their result shows that Maxout[4] architecture can use only 10bits for computation and 12bits storage without significantly affecting the accuracy.

S. Gupta et al. [13] also successfully used lower precision arithmetic both for CNN and fully connected architecture. Their paper used very simple CNN with 2 layers and small data set like CFAIR10, but they showed that it's possible to use none standard 12bit float instead of 32bit single precision with small effect on the accuracy. They report archiving energy efficiency of 37 GOps/second/W with low precision implementation in fpga vs 1-5 GOps/second/W achievable on CPU/GPUs (Intel i7-3720QM, NVIDIA GT650m and the GTX780)

3.5. FFTs

A well know property of FFT is that it turns convolution into element wise multiplication. Not only this requires significantly less operations to compute, but it also eliminates reeducation step in convolution (summation) and thus exposes extra level of parallelism. This is a very desirable characteristic in every parallel system, but especially in GPUs which are optimized for fully independent threads. Not surprisingly FFTs were used for GPU optimization, fist by Mathieu et al.[1] who reported up to 3 times faster performance and in resent for of N. Vasilache et al[2] which archived 1.4 to 14.5 time better performance than cuDNN by custom implementation of FFTs tuned to small kernel sizes.

4. CNNs with FFTs

4.1. Problem statement and related work

Works of Mathieu et al.[1] and Vasilache et al[2] are the most related papers to my project. Both papers used traditional CNN structure and interpretation of weights, which means there have to do FFT, element wise multiplication and inverse FFT on *every* convolution layer. Both FFT and iFFT are $O(n^2 log\ n)$ operations (vs $O(n^2)$ for element wise multiplication). This greatly reduces the benefits especially for small filters (see [1] for more details).

My idea is to comute FFT only once on the input image and do iFFT after the last convolution layer (or at the very end). Modern CNNs like VGG[8] and GoogLeNet[6] have up to 59 convolution stages in 21 layer and the savings would be big, but it would also require doing both NonLinearity and Pooling layers in frequency domains.

The next section develops mathematic framework to approach this problem and reports experimental results for Pooling layer and Nonlinearity.

4.2. Linear system analysis

A common way to analyze and work with linear systems is through the use of a transfer function. A complicated system can be broken into simple stages, each stage is modifying a spectrum according to the transfer function and feeds the output to the next stage.

Such method can't be directly applied to CNN because of none linearity. Switching between time and frequency domain (as in [1] and [2]) is one way to address it. The advantage of this approach is that it can deal with all types on none linear functions, but at the expense of doing iFFT before none linearity and FFT after.

Alternatively we can use the properties of a particular none linearity function and stay in the frequency domain. Section 4. 5 will develop mathematic justification for the case of ReLu() function which allows to treat it almost like a linear system.

4.3. Convolution layer

As was mentioned before, convolution in frequency domain becomes an element wise multiplication of the Fourier components.

To perform element wise multiplication, the two arrays must be of equal size, so it might seem that we have increase the number of parameters from k^2 to n^2. But in fact all n^2 Fourier coefficients can be expressed through k^2 original parameters because we can view the coefficient as a weighted sum of 2D delta functions:

$$\sum_{i,j} \delta(x - x_i, y - y_j) * w(i,j)$$

Delta function is defined over entire n^2 domain, and it's spectrum is known.

4.4. Pooling layer

Pooling layer can be done ether with MAX operation or AVERAGE. This is the same as image decimation – a standard practice image processing, it is well described by Bouman in [25].

Decimation is equivalent to image blur, which removes high frequencies for an image followed by size reduction. This can be done in frequency domain by doing convolution and discarding extra frequency components.

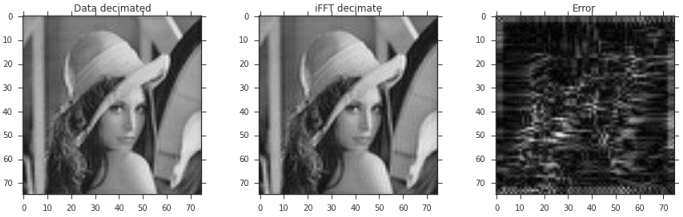


Figure 1: Pooling Layer in Frequency domain

MAX and Average are special kinds of blur kernels used for their computation efficeincy (in space domain). The later one is also known as box filter. Both are actually considered suboptimal compared to 2D *sinc* function [25] which better suppresses aliacing.

The comparison beween space and frequency domain implementation is shown on Figure 1. It should be noted that in frequency domain using box filter intead of *sinc* doesn't give any benefits because both are just element wise multiplications.

4.5. Nonlinearity in frequency domain

Any stage in the system can be viewed as applying a certain function g(y) to the input function f(x),so analysis in the frequency domain comes to finding the Fourier transform F(g(f(x)) with respect to F(f(x)).

In general problem doesn't have an analytical solution, but we can find one in case g(y)=ReLu(y)

The most common way none linearity is ReLu, which acts as data clipping in time domain. It creates sharp corners in the signal, so in the frequency domain this would add higher frequency harmonics to the spectrum.

Mathematically we can express ReLu(f(x)) function through f(x) as a multiplication with the sign(f(x)) : which is equal to 1 if f(x)>0 and 0 otherwise :

$$ReLu\big(f(x)\big) = \max\{f(x), 0\} = sign\big(f(x)\big) * f(x)$$

Because we are working with limited intervals (number of samples) of function f(x), we can express ReLu through the multiplication with sum of delta functions:

$$sign\big(f(x)\big) * f(x) = f(x) * \sum_i \delta(x - x_i)\,, f(x_i) > 0$$

The Fourier transform of a delta function is given by:

$$F\big(\delta(x - x_0)\big)(k) = e^{2\pi j k\, x_0}$$

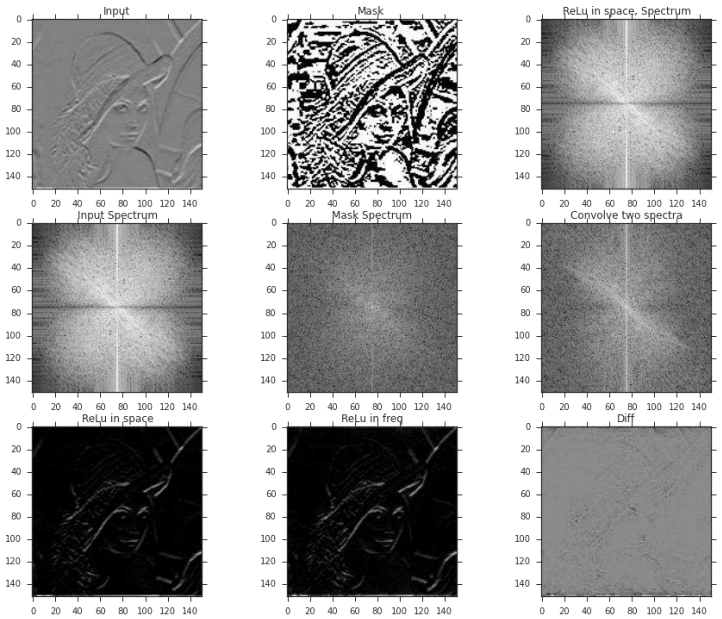
Using linearity of FFTs and convolution theorem we can express the Fourier transform of ReLu(f(x)) through the Fourier transform of f(x):

$$F(ReLu(f(x)))(k) = \left(\sum_i e^{2\pi j k\, x_i}\right) \otimes F\big(f(x)\big)$$

This shows that in frequency domain, ReLu() acts as a convolution with the function of known form. However, this function depends on the input and we need to find positions in space domain $x_i : f(x_i) > 0$. To do that we need to take inverse transforms of the input and solve the inequality.

The key factor is that once we have found x_i, we know the transfer function of the ReLu for this input and don't need to calculate FFT.

Figure 2 illustrates the method of computing ReLu in frequency domain. We can see that the result is very close, but not identical. The power spectrum plots indicate that



the difference comes from the edges (high frequency) and is likely caused by limited image dimensions.

4.6. Fully connected layer

Fully connected layer is a special case of convolution layer where the result is computed for a single point. As such, it turns into element wise multiplication in frequency domain.

Typically, the input to this layer is very deep, but narrow and short. But batching multiple image we can make it wider and taller to increase the computation efficiency.

4.7. Softmax

Softmax takes a vector as an input, but in frequency domain every element of this vector is spread among all frequencies. So we would need to use a matrix as an input to softmax, or convert back to special domain with iFFT.

4.8. Computational complexity

Figure 2: ReLu in Frequency domain

Since ReLu is equivalent to convolution in frequency domain it might seem that we haven't gained anything by using FFTs.

This is not the case because pooling in frequency reduces the data by discarding elements after a certain index. As a result, the convolution doesn't need to compute that data and has to generate only ¼ of the points.

Overall the complexity of the algorithm seems to be:

$O(n^2 \log n)$ vs $O(n^2 k^2)$ in space domain. However, the constant factor and lower order terms seems to be smaller. Additionally, authors of [2] showed that the benefit of independent operation in element by element multiplication. It maps well on modern SIMD architecture and runs faster, however a careful code tuning is required to take full advantage of this property

Because asymptotically $O(n^2 \log n)$ is worse than $O(n^2 k^2)$, we can expect that there is a point where two methods run in the same time.

4.9. Putting it all together

For the final experiment, I have implement all layers python, such that they interface with each other by passing *frequency* activations instead of special ones.

This required soling a number of issues like: in dimensions are even the location of 0 fequency in the array is not obvious (different form MatLab), all frequencies and spectrum have to account the number of samples, etc.

However the one issue worth noting is that in frequency domain, just like in space domain, we have to deal with the boundary conditions by using more elements than in the incoming activations. In special domain this was a simple padding with 0, and in frequency domain this requires changing all elements of array (because in numpy implementation frequency depends on the number of samples).

In the end my implantation ran much slower compared to optimized computations we used in homework's and I was getting poor prediction quality because the weights were trained on a traditional network in space domain and then transferred over to frequency CNNs, which doesn't not compute identical result due to boundary conditions and other implementation details hinted earlier.

Figure 3 illustrate this on a single block of three consecutive operations: Convolution, ReLu, Pool. An input was 151 by 151'Lenna' image and Sobel filter operator: [[-1,-2,-1],[0,0,0],[1,2,1]].

We can see that even though the error on each layer are small (as shown in Fig.1 and 2), they accumulate and they are not concentrated in a small or not important area of the image.

This issue will likely be corrected by using the same frequency computations during training, instead of doing them only during testing. Also we can back propagate purely in the frequency domain – we are doing same kinds of operations – multiplications, additions and convolutions.

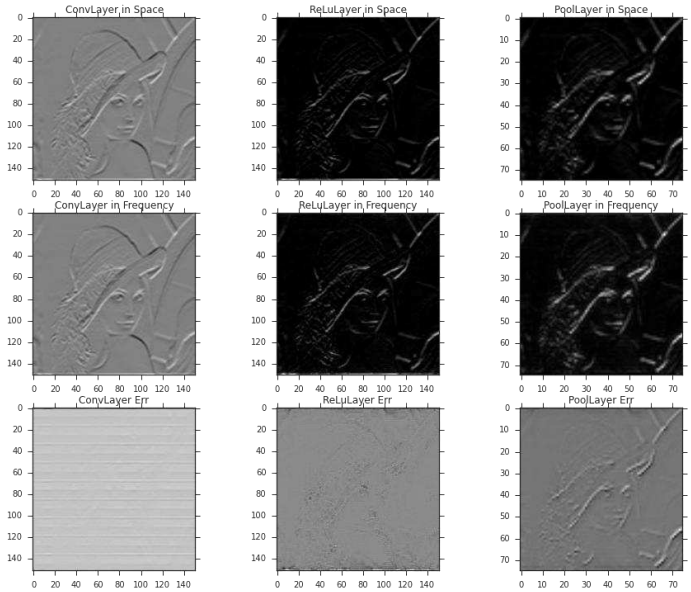


Figure 3: Conv-ReLu-Pool – error accumulation frequency domain

5. Intuition about nonlinear elements

Nonlinear elements are often described as "magic" that makes CNN works. They are given as the reason why CNNs can express "interesting" functions that separate object classes, but very little is given as explanation of their mechanics other that the fact that ReLus "seem to perform better than others".

We have seen that ReLu acts as convolution in Frequency domain. Let's try to develop an intuition about other common types: *sigmoid* and *tanh* by looking at their modification to a spectrum of function *sin(50x)*.

In case of a linear system, we'd simply derived a transfer function by applying a step function as an input. But since the system is not linear, we can only develop intuition without mathematical backing.

As we can see from Figure 4, all three act similarly by "spreading" information from a single frequency band at 50Hz to other harmonics: 2x and 4x in case of ReLu and 3x in case of sigmoid and tanh. Also it's clear that ReLu redistributes information better – the peak at 2x is 20% of the oginal, while sigmoid only 20% at the original location and a tiny portion at 3x, putting most of the energy in DC.

High emphasis of nonlinearity on DC component is bad because all the frequency would be smashed into the same bin and become indistinguishable.

We also know that ReLu is followed by Pooling which discards some frequency components, so a more even redistribution has a better chance of preserving some of the informational content from the frequency that will be discarded in other components

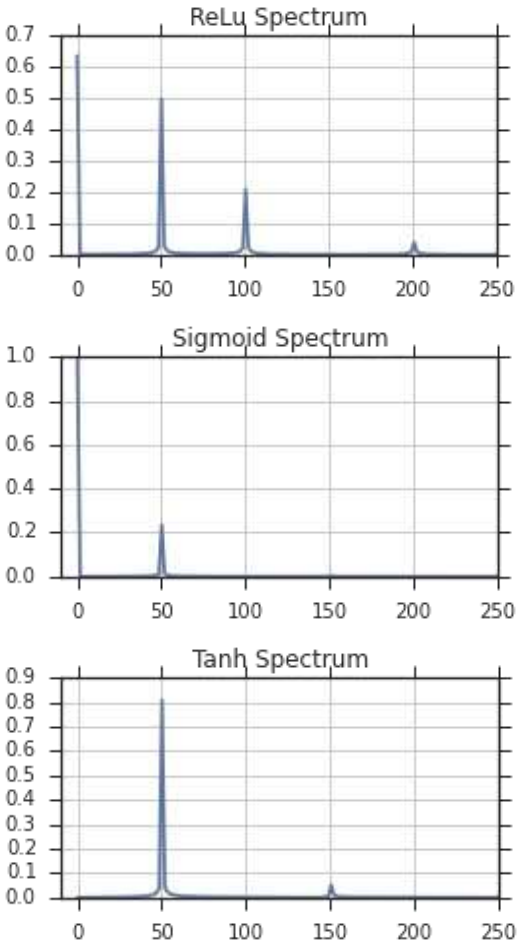


Figure 4: Effect of none linearity

5.1. Generating new nonlinearities

As we saw, *ReLu*, *sigmoid* and *tanh* act similarly, by redistributing information for original image, we can design new kind of functions to do the same thing.

A possible strategy is to use convolutions in frequency domain (just like ReLu does), but employ different kenrels than ReLu. And important feature is that the *kernel applied to each activation has to depend on that activation*, in other words it's not constant. ReLu bases the kernel on mask which translates to a large kernel in frequency domain, but we can use smaller kernel and even one based on frequencies only instead of information from special domain. We can also create one that doesn't emphases DC component.

Of cause, we would have to train the network differently and do a proper back propagation.

6. Conclusion and future works

In this project I did a review of possible methods to optimize CNN for embedded platform and developed an new way to perform all the computation in frequency domain. This method exposed an intriguing "duality" of CNN: a convolution operation is require both in special domain and in frequency domain, however they have different purpose and meaning. Convolution in space captures special locality in the data, while convolution in frequency redistributes information to different components in order to mitigate information loss in the Pooling layer.

The goal of doing computation in frequency domain was to eliminate convolutions, while it's impossible to do without constantly switching between space and frequency (which is expensive), computation entirely in frequency domain has to calculate ¼ of convolution results compared to special domain.

The project has demonstrated that this is a viable approach but computation has to be done the same way during training and testing. We also saw that it's possible to match the exact behavior of ReLu, but it's computationally expensive. However this is probably not required for the successful operation and other nonlinearities are possible.

A good follow up would be to explore different kinds of nonlinearities by performing convolutions in space, a more optimized implementation similar to work in [2] and performing training in frequency domain.

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