

Prince Sultan University Orientation Mathematics Program

MATH 223 Class Major Test I Semester II, Term 142 Tuesday, March 17th, 2015

Time Allowed: 90 minutes

Student Name: ______

Student ID #: _____

Important Instructions:

- 1. Once you start the exam, there are no bathroom breaks.
- 2. You may use a scientific calculator that does not have programming or graphing capabilities.
- 3. You may NOT borrow a calculator from anyone.
- 4. You may NOT use notes or any textbook.
- 5. There should be NO talking during the examination.
- 6. No usage of phone during exams. Turn it off before starting the exam.
- 7. Your exam will be taken immediately if your mobile phone is seen or heard.
- 8. Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1	14	
2	08	
3	17	
4	10	
5	15	
6	11	
Total	75	

1. Consider the matrices

$$A = \begin{bmatrix} 2 & 4 & x \\ 1 & 0 & 5x \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3x \\ -2 & 5 \\ 0 & -1 \end{bmatrix}.$$

- (a) Compute the determinant of C = AB
- (b) Compute the trace: tr(C)
- (c) Is the matrix $BA + 5I_3$ a square matrix.? Why or why not?
- (d) Compute the inverse C^{-1}
- 2. Assume that a system in 5 variables v, w, x, y, z has been reduced to a row reduced augmented form:
 - $\begin{bmatrix} 1 & 0 & 0 & 4 & 4 & 7 \end{bmatrix}$
 - 0 1 1 1 0 6
 - (a) What are the free variables?
 - (b) Solve the system?
- 3. Solve the system of equations for *x*, *y*, and *z* using an augmented matrix and **row reduction**. Also solve the system for only z using **Cramer's rule**.

x + 3z = 2

y + 4z = 1

- x + 2y + 7z = 0
- 4. Let plane D have equation 3x 4y + z = 1 and plane E have equation 6x 8y + 2z = 3.
 - (a) Show that plane D is parallel to plane E.
 - (b) Obtain any point P that is on D.
 - (c) Use (b) to compute the distance between plane D and plane E.
- 5. Let vectors u = (6,1) and a = (3,-9).
 - (a) Why is *u* not orthogonal to *a*?
 - (b) What is the angle between *u* and *a* in radians to 2 decimal places?
 - (c) Compute $\| \operatorname{proj}_a u \|$.
 - (d) Get the vector component of u orthogonal to a.
- 6. Let vectors u = (4, 2, 0) and v = (3, -1, 1).
 - (a) Compute ||u|| 2||v|| and ||u|| + ||-2v||.
 - (b) Compute vector w defined by $w = (2u) \times (-3v)$.
 - (c) Compute the volume of the parallelepiped determined by *u*,*v* and *w*. (It is not necessarily a box!!)

BONUS Short answer questions [6 points]

- (a) What elementary matrix of size 4×4 is obtained when 5 times row 2 is added to row 3?
- (b) Let u, v be vectors in \mathbb{R}^3 . Circle the expressions that make sense: $|u \cdot v|, |u \times v|, ||u \cdot v||, ||u \times v||$
- (c) Let u, v, and w be vectors in \mathbb{R}^3 . Is $(u \cdot v) + w \cdot (u \times w)$ is meaningful? Why or why not?

