

PRACTICE TEST (Chapters 5 & 6)

Probability & Random Variables

MULTIPLE CHOICE (1 point each)

- 1) The probability that you will be ticketed for illegal parking on campus is about $\frac{1}{3}$. During the last nine days, you have illegally parked every day and have NOT been ticketed. Today, on the 10th day, you again decide to park illegally. Assuming the outcomes are independent from day to day, the probability that you will be caught is 1) _____

a) $\frac{1}{3}$ b) $\frac{1}{3} + \left(\frac{1}{3}\right)^9$ c) $\frac{1}{3} - \left(\frac{1}{3}\right)^9$ d) $\frac{1}{10}$ e) $\frac{9}{10}$

- 2) A friend has placed a large number of plastic disks in a hat and invited you to select one a random. He informs you that they have numbers on them, and that one of the following is the probability model for the number on the disk you have chosen. Which one is it? 2) _____

a)

Number of Disks	1	2	3	4	5
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b)

Number of Disks	1	2	3	4	5
Probability	1	2	3	4	5

c)

Number of Disks	1	2	3	4	5
Probability	0.1	0.2	0	0.3	0.4

d)

Number of Disks	1	2	3	4	5
Probability	0.10	0.11	0.25	0.05	0.26

e)

Number of Disks	1	2	3	4	5
Probability	1	0	-1	0	1

For 3 – 5, use the two-way table that gives information on juniors and seniors at a high school and the means they typically get to school.

	Car	Bus	Walk	Total
Juniors	146	106	48	300
Seniors	146	64	40	250
Total	292	170	88	550

- 3) You select one student from this group at random. What is the probability that this student typically takes a bus to school? 3) _____

a) 0.256 b) 0.309 c) 0.353 d) 0.455 e) 0.604

- 4) You select one student from this group at random. If the student says he is a junior, what is the probability that he walks to school? 4) _____

a) 0.073 b) 0.160 c) 0.455 d) 0.660 e) 0.833

- 5) You select one student from this group at random. Which of the following statements is true about the events "Typically walks to school" and "Junior?" 5) _____

- a) The events are mutually exclusive and independent.
 b) The events are not mutually exclusive but they are independent.
 c) The events are mutually exclusive, but they are not independent.
 d) The events are not mutually exclusive, nor are they independent.
 e) The events are independent, but we do not have enough information to determine if they are mutually exclusive.

- 6) People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population has O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor? 6) _____

a) 0 b) 0.280 c) 0.526 d) 0.720 e) 1

- 7) Of people who died in the United States in a recent year, 86% were white, 12% were black, and 2% were Asian. (We will ignore the small number of deaths among other races.) Diabetes caused 2.8% of deaths among whites, 4.4% among blacks, and 3.5% among Asians. The probability that a randomly chosen death was due to diabetes is about _____
- a) 0.96 b) 0.107 c) 0.042 d) 0.038 e) 0.030

- 8) In your top dresser drawer are 6 blue socks and 10 grey socks, unpaired and mixed up. One dark morning you pull two socks from the drawer (without replacement, of course!). What is the probability that the two socks match? _____
- a) 0.075 b) 0.250 c) 0.500 d) 0.531 e) 0.567

For 9 – 10, use the following information: In order to set premiums at profitable levels, insurance companies must estimate how much they will have to pay in claims on cars of each make and model, based on the value of the car and how much damage it sustains in accidents. Let C be a random variable that represents the cost of a randomly selected car of one model to the insurance company. The probability distribution of C is given below.

C	\$0	\$500	\$1000	\$2000
$P(C)$	0.60	0.05	0.13	0.22

- 9) The expected value of C is _____
- a) \$155 b) \$595 c) \$875 d) \$645 e) \$495

- 10) Which of the following is the best interpretation of expected value? (In the choices below, “ $\text{Exp}(C)$ ” represents expected value you found in question 9). _____
- a) If the company insures 10 cars of this model, they know they will incur $10 \times \text{Exp}(C)$ in costs.
 b) The maximum cost to the company for insuring this car model is $\text{Exp}(C)$ per car.
 c) The company must insure at least $\text{Exp}(C)$ of these cars to make a profit.
 d) If the company insures a large number of these cars, they can expect the cost per car to average approximately $\text{Exp}(C)$.
 e) If the company insures a large number of these cars, they can expect the variability in cost per car to average approximately $\text{Exp}(C)$.

- 11) A randomly chosen subject arrives for a study of exercise and fitness. Consider these statements. _____
- I. After 10 minutes on an exercise bicycle, you ask the subject to rate his or her effort on the Rate of Perceived Exertion (RPE) scale. RPE ranges in whole-number steps from 6 (no exertion at all) to 20 (maximum exertion).
 II. You measure VO_2 , the maximum volume of oxygen consumed per minute during exercise. VO_2 is generally between 2.5 liters per minute and 6 liters per minute.
 III. You measure the maximum heart rate (beats per minute).

The statements that describe a discrete random variable are

- a) I b) II c) I, III d) I, II, III e) None of the statements describes a discrete random variable.
- 12) If A = result of a single roll of a six-sided die and B = result of a single roll of an 8-sided die, then $\mu_A = 3.5$, $\sigma_A = 1.71$, $\mu_B = 4.5$, and $\sigma_B = 2.29$. If D = the difference $B - A$, then which of the following is true? _____
- a) $\mu_D = 1$, $\sigma_D = 4.000$
 b) $\mu_D = 1$, $\sigma_D = 0.058$
 c) $\mu_D = 1$, $\sigma_D = 2.858$
 d) $\mu_D = 1$, $\sigma_D = 1.523$
 e) $\mu_D = 1$, $\sigma_D = 2.000$

For 13 – 14, use the following information: It has been estimated that about 30% of frozen chickens are contaminated with enough salmonella bacteria to cause illness if improperly cooked. Chickens are delivered to grocery stores in crates of 24. Assume the chickens are independently selected for inclusion in the crate.

13) The probability that a certain crate has more than 4 contaminated chickens is 13) _____

- a) 0.0424 b) 0.0686 c) 0.8889 d) 0.9313 e) 0.9576

14) The mean and standard deviation of the number of contaminated chickens in a crate are 14) _____

- a) $\mu = 7; \sigma = 2.24$
b) $\mu = 7; \sigma = 2.68$
c) $\mu = 7; \sigma = 5.04$
d) $\mu = 7.2; \sigma = 2.24$
e) $\mu = 7.2; \sigma = 5.04$

15) Which of the following random variables is geometric? 15) _____

- a) The number of phone calls received in a one-hour period
b) In a shuffled deck of cards, turn over one card at a time from the top of the deck until you get an ace.
c) The number of digits I will read beginning at a randomly selected starting point in a table of random digits until I find a 7.
d) The number of 7s in a row of 40 random digits.
e) All four of the above are geometric random variables.

FREE RESPONSE (3 points each). Drawing pictures, showing organized work, including correct units, and writing your answers in context and complete sentences will be part of how each question is graded.

For 16 – 18, use the following scenario: Suppose your school is in the midst of a flu epidemic. The probability that a randomly-selected student has the flu is 0.35, and the probability that a student who has the flu also has a high fever is 0.90. But there are other illnesses making the rounds, and the probability that a student who doesn't have the flu does have a high fever is 0.12.

16) What is the probability that a randomly selected student at this school has a high fever? 16) _____
Draw a tree diagram to help.

17) Suppose a student walks into the nurse's office with a high fever. 17) _____
What is the probability that he or she has the flu?

18) At this school, are having the flu and having a high fever independent? 18) _____
Explain mathematically why you said independent or not independent.

For 19 – 20, use the following scenario: Meadowbrook School surveys the families of its students and determines the following: if a family is chosen at random, the probability that they own a dog is 0.38, the probability they own a cat is 0.23, and the probability they own both a dog and a cat is 0.12.

19) What is the probability that a randomly selected family owns a dog or a cat? 19) _____

20) Given that a randomly selected family owns a dog or a cat, what is the probability that they own a cat? 20) _____

For 21 – 24, use the following information: The Census Bureau reports that 27% of California residents were born outside the United States.

21) Suppose that you randomly choose 4 Californians. What is the probability that exactly 1 of the chosen Californians were born outside the U.S.? 21) _____

22) Suppose that you randomly choose 100 Californians. What is the probability that at least 25 of the chosen Californians was born outside the U.S.? 22) _____

23) Find and interpret the expected number of foreign-born people in a randomly selected sample of size 100. 23) _____

24) Find and interpret the standard deviation of the number of foreign-born people in a randomly selected sample of size 100. 24) _____

For 25 – 26, use the following scenario: The weight of adult men is approximately Normally distributed with a mean of 190 pounds and a standard deviation of 30 pounds.

25) If you randomly select three men, what are the mean and standard deviation of the sum of their weights? 25) _____

26) An elevator in a small apartment building has a maximum weight capacity of 600 pounds. If three randomly selected adult men get on the elevator, what is the probability that they exceed the maximum capacity? 26) _____

For 27 – 28, use the following scenario: As a special promotion for its 20-ounce bottle of soda, a soft drink company printed a message on the inside of each cap. Some of the caps said, "Please try again," while others said, "You're a winner!" The company advertised the promotion with the slogan "1 in 6 wins a prize." Suppose the company is telling the truth and that every 20-ounce bottle of soda it fills has a 1-in-6 chance of being a winner. Alan decides to keep buying one 20-ounce bottle of the soda at a time until he gets a winner.

27) Find the probability that he buys exactly 5 bottles. 27) _____

28) Find the probability that he buys no more than 8 bottles. 28) _____

MULTIPLE CHOICE (1 point each)

1) The probability that you will be ticketed for illegal parking on campus is about $\frac{1}{3}$. During the last nine days, you have illegally parked every day and have NOT been ticketed. Today, on the 10th day, you again decide to park illegally. Assuming the outcomes are independent from day to day, the probability that you will be caught is

1) A

- a) $\frac{1}{3}$ b) $\frac{1}{3} + \left(\frac{1}{3}\right)^9$ c) $\frac{1}{3} - \left(\frac{1}{3}\right)^9$ d) $\frac{1}{10}$ e) $\frac{9}{10}$

2) A friend has placed a large number of plastic disks in a hat and invited you to select one a random. He informs you that they have numbers on them, and that one of the following is the probability model for the number on the disk you have chosen. Which one is it?

2) C

a)

Number of Disks	1	2	3	4	5
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b)

Number of Disks	1	2	3	4	5
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3) B

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5) B

- a) The events are mutually exclusive and independent.
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6) People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population has O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

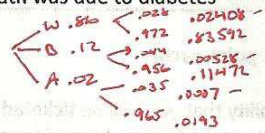
6) C

- a) 0 b) 0.280 c) 0.526 d) 0.720 e) 1

$P(X \geq 1)$
 $1 - P(X = 0)$
 $1 - (.928)^{10}$
 ≈ 0.526

7) Of people who died in the United States in a recent year, 86% were white, 12% were black, and 2% were Asian. (We will ignore the small number of deaths among other races.) Diabetes caused 2.8% of deaths among whites, 4.4% among blacks, and 3.5% among Asians. The probability that a randomly chosen death was due to diabetes is about 7) E

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$$\frac{6}{16} \cdot \frac{5}{15} + \frac{10}{16} \cdot \frac{9}{15}$$

For 9 – 10, use the following information: In order to set premiums at profitable levels, insurance companies must estimate how much they will have to pay in claims on cars of each make and model, based on the value of the car and how much damage it sustains in accidents. Let C be a random variable that represents the cost of a randomly selected car of one model to the insurance company. The probability distribution of C is given below.

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9) The expected value of C is 9) B

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$$\sum x \cdot p_i$$

10) Which of the following is the best interpretation of expected value? (In the choices below, "Exp(C)" represents expected value you found in question 9). 10) D

- a) If the company insures 10 cars of this model, they know they will incur $10 \times \text{Exp}(C)$ in costs.
- b) The maximum cost to the company for insuring this car model is $\text{Exp}(C)$ per car.
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- a) $\mu_D = 1$, $\sigma_D = 4.000$
 b) $\mu_D = 1$, $\sigma_D = 0.058$
 c) $\mu_D = 1$, $\sigma_D = 2.858$
 d) $\mu_D = 1$, $\sigma_D = 1.523$
 e) $\mu_D = 1$, $\sigma_D = 2.000$

$$\mu_D = 4.5 - 3.5 = 1$$

$$\sigma_D = \sqrt{2.29^2 + 1.72^2}$$

$$= 2.858$$

For 13 – 14, use the following information: It has been estimated that about 30% of frozen chickens are contaminated with enough salmonella bacteria to cause illness if improperly cooked. Chickens are delivered to grocery stores in crates of 24. Assume the chickens are independently selected for inclusion in the crate.

- 13) The probability that a certain crate has more than 4 contaminated chickens is $P(X > 4) = 1 - P(X \leq 4) = 1 - \binom{24}{4} (.3)^4 (.7)^{20} = 1 - .1111 = .8889$ 13) C
- a) 0.0424 b) 0.0686 c) 0.8889 d) 0.9313 e) 0.9576

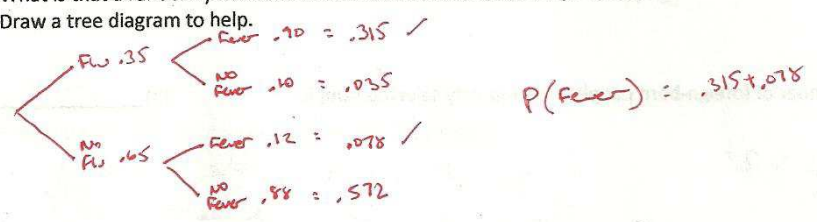
- 14) The mean and standard deviation of the number of contaminated chickens in a crate are 14) D
- a) $\mu = 7; \sigma = 2.24$
 b) $\mu = 7; \sigma = 2.68$
 c) $\mu = 7; \sigma = 5.04$
 d) $\mu = 7.2; \sigma = 2.24$
 e) $\mu = 7.2; \sigma = 5.04$
- $\mu = 24(.3) = 7.2$
 $\sigma = \sqrt{24(.3)(.7)} = 2.24$

- 15) Which of the following random variables is geometric? BITS 15) C
- a) The number of phone calls received in a one-hour period *not binary*
 b) *the probability of a card being an ace at the top of the deck will you get an ace = not independent*
 c) The number of digits I will read beginning at a randomly selected starting point in a table of random digits until I find a 7.
 d) The number of 7s in a row of 40 random digits. *not binary*
 e) All four of the above are geometric random variables.

FREE RESPONSE (3 points each). Drawing pictures, showing organized work, and writing your answers in context and complete sentences will be part of how each question is graded.

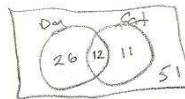
For 16 – 18, use the following scenario: Suppose your school is in the midst of a flu epidemic. The probability that a randomly-selected student has the flu is 0.35, and the probability that a student who has the flu also has a high fever is 0.90. But there are other illnesses making the rounds, and the probability that a student who doesn't have the flu does have a high fever is 0.12.

- 16) What is that a randomly selected student at this school has a high fever? the probability 16) .393
 Draw a tree diagram to help.



- 17) Suppose a student walks into the nurse's office with a high fever. What is the probability that he or she has the flu? 17) .8015
- $P(\text{Flu} | \text{Fever}) = \frac{.315}{.393}$

- 18) At this school, are having the flu and having a high fever independent? Explain clearly why you say independent or not independent. 18) not independent
- $P(\text{Flu} | \text{Fever}) \stackrel{?}{=} P(\text{Flu})$
 $.8015 \neq .35$
- $P(\text{Fever} | \text{Flu}) \stackrel{?}{=} P(\text{Fever})$
 $\frac{.315}{.35} = .903$
 $.9 \neq .393$



For 19 – 20, use the following scenario: Meadowbrook School surveys the families of its students and determines the following: if a family is chosen at random, the probability that they own a dog is 0.38, the probability they own a cat is 0.23, and the probability they own both a dog and a cat is 0.12.

19) What is the probability that a randomly selected family owns a dog or a cat?

19) 0.49

	cat	dog	
dog	.12	.26	.38
no dog	.11	.51	.62
	.23	.77	1

$$.38 + .23 - .12$$

20) Given that a randomly selected family owns a dog or a cat, what is the probability that they own a cat?

20) 0.4694

$$\frac{P(\text{cat and dog})}{P(\text{dog or cat})}$$

$$P(\text{cat} | \text{own dog or cat}) = \frac{.12}{.49}$$

For 21 – 24, use the following information: The Census Bureau reports that 27% of California residents were born outside the United States.

21) Suppose that you randomly choose 4 Californians. What is the probability that exactly 1 of the chosen Californians were born outside the U.S.?

21) 0.4201

$$P(X=1) = \binom{4}{1} (.27)^1 (.73)^3 = .4201$$

22) Suppose that you randomly choose 100 Californians. What is the probability that at least 25 of the chosen Californians was born outside the U.S.?

22) 0.7091

$$P(X \geq 25) = 1 - P(X \leq 24) = 1 - \left[\sum_{k=0}^{24} \binom{100}{k} (.27)^k (.73)^{100-k} \right]$$

$$= 1 - .2909$$

$$= .7091$$

23) Find and interpret the expected number of foreign-born people in a randomly selected sample of size 100.

23) 27

$$(100)(.27) = 27$$

In samples of 100, the average number of foreign born residents will be about 27.

24) Find and interpret the standard deviation of the number of foreign-born people in a randomly selected sample of size 100.

24) 4.44

$$\sqrt{(100)(.27)(.73)} = 4.44$$

In samples of 100, the number of foreign born residents will be about 4.44 away from the mean.

For 25 – 26, use the following scenario: The weight of adult men is approximately Normally distributed with a mean of 190 pounds and a standard deviation of 30 pounds.

- 25) If you randomly select three men, what are the mean and standard deviation of the sum of their weights?

$S = \text{sum of 3 men}$

$$\mu_S = 190 + 190 + 190 = 570$$

$$\sigma_S = \sqrt{30^2 + 30^2 + 30^2} = 51.9615$$

$$\mu_S = 570 \text{ pounds}$$

$$25) \sigma_S = 51.9615 \text{ pounds}$$

- 26) An elevator in a small apartment building has a maximum weight capacity of 600 pounds. If three randomly selected adult men get on the elevator, what is the probability that they exceed the maximum capacity?

$Y = \text{sum of 3 men}$

$$P(Y > 600) = P\left(Z > \frac{600 - 570}{51.9615}\right) = P(Z > .58) = 1 - .7190$$

$$26) \underline{.2810}$$

For 27 – 28, use the following scenario: As a special promotion for its 20-ounce bottle of soda, a soft drink company printed a message on the inside of each cap. Some of the caps said, "Please try again," while others said, "You're a winner!" The company advertised the promotion with the slogan "1 in 6 wins a prize." Suppose the company is telling the truth and that every 20-ounce bottle of soda it fills has a 1-in-6 chance of being a winner. Alan decides to keep buying one 20-ounce bottle of the soda at a time until he gets a winner.

- 27) Find the probability that he buys exactly 5 bottles.

$$P(Y=k) = (1-p)^{k-1} \cdot p$$

$$P(Y=5) = \left(1 - \frac{1}{6}\right)^{5-1} \cdot \left(\frac{1}{6}\right) = .0804$$

$$27) \underline{.0804}$$

- 28) Find the probability that he buys no more than 8 bottles.

$$P(Y \leq 8) = \left(1 - \frac{1}{6}\right)^{8-1} \cdot \left(\frac{1}{6}\right) = .7674$$

$$L_1 = 1, 2, 3, 4, 5, 6, 7, 8$$

$$28) \underline{.7674}$$