RECURSIVE ROUTINE APPLICATIONS

LESSON 2.3

Represent recursive routine applications with graphs, tables and words.

Recursive routines are useful when dealing with a variety of real-world situations. Recursive routines can be illustrated with graphs, tables and by words. Using multiple ways of showing a recursive routine helps to reach a variety of audiences. It is important to think about what type of graphic (table, graph, words, etc.) best illustrates each situation.

EXPLORE! SAVING AND SPENDING

William and his sister, Jennifer, each worked summer jobs. William mowed lawns in his neighborhood. Jennifer baby-sat for two different families. By the end of the summer, William had put $410 in a savings account. Jennifer put $275 in her account. After school started, Jennifer continued baby-sitting and earned $20 per week. She put all of her earnings in her savings account. William stopped working and withdrew $15 per week from his savings account for spending money.

Step 1: Write a recursive routine (start value and operation) for the amount in William's savings account each week after school begins.

Step 2: Write a recursive routine for the amount in Jennifer's savings account each week after school begins.

Step 3: Copy the input-output tables shown below and fill in each for the first 10 weeks after school starts.

Weeks After School Starts	William's Total Savings
0	
1	
2	

Weeks After School Starts	Jennifer's Total Savings
0	
1	
2	

Step 4: On the SAME first-quadrant coordinate plane, graph William and Jennifer's total savings for the first ten weeks. Use ● to designate Jennifer's amounts and ▲ to represent William's amounts. Put weeks on the x-axis and total savings in dollars on the y-axis.

Step 5: After what week does Jennifer have more money than her brother? Which illustration (table, graph or recursive routine) best shows this?

EXAMPLE 1

Matt pays a fee of $25 per month for his cell phone plan. He is charged $0.15 per text message he sends or receives.

a. Write a recursive routine that describes Matt's monthly cell phone bill based on the number of text messages he sent or received.

b. Create an input-output table for the first ten text messages.

c. Create a linear plot that shows his total monthly bill for up to ten text messages.

d. Determine Matt's total bill for the month of January if he sent or received 16 text messages.

Solutions

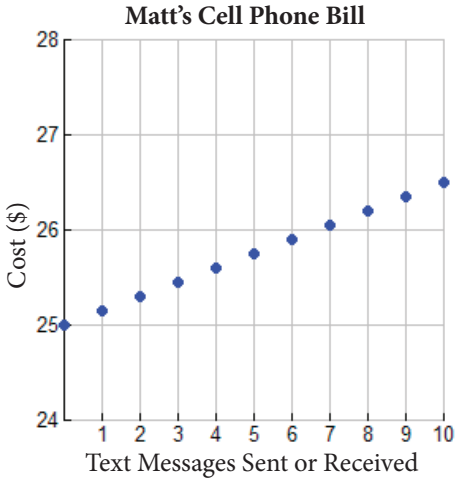
a. Start Value = $25
Operation = Add $0.15 (or + 0.15)

b.

Text Messages Sent or Received	Total Bill
0	$25
1	$25.15
2	$25.30
3	$25.45
4	$25.60
5	$25.75
6	$25.90
7	$26.05
8	$26.20
9	$26.35
10	$26.50

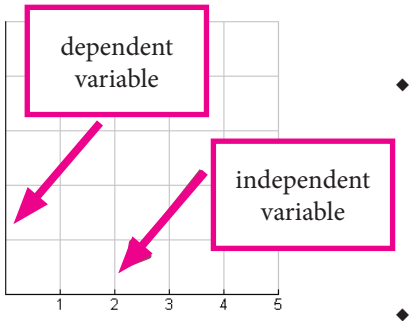
Add $0.15 for each text message received.

c.



d. When using a calculator, enter the start value, 25, and press ENTER or =. Then enter your operation, + 0.15, and press ENTER or = sixteen times. You should arrive at the answer of $27.40.

A few things to remember when creating tables and graphing:



- Always put the **independent variable** in the first column of a table and on the x-axis of a graph. The independent variable is the variable the y-value is dependent on. The y-axis corresponds to the **dependent variable**.
- Most real-world situations take place in the first quadrant. Think about your situation before graphing and decide if negative numbers would ever make sense. For example, you will not have a negative amount for the cost of a cell phone bill, so you will only need to use the first quadrant.
- Choose a range (lowest to the highest number) for the y-axis that allows the viewer of your graph to see all points easily. Also, make sure your increments on the y-axis are reasonable.

EXERCISES

Determine an appropriate range for the y-axis. State what increments you would use on the graph.

1.

Minutes	Distance Traveled
0	8
1	20
2	32
3	44
4	56
5	68

2.

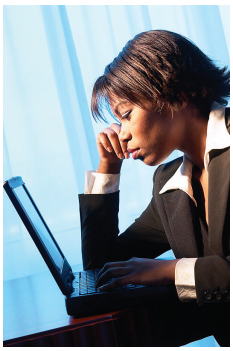
Sales Made	Salary
0	$120
1	$150
2	$180
3	$210
4	$240
5	$270

3.

Years	Car's Worth
0	$12,000
1	$10,500
2	$9,000
3	$7,500
4	$6,000
5	$4,500

4. Jackson got his driving license one year ago. When Jackson got his driver's license, his car insurance cost $82 per month. Each time he gets a speeding ticket, his insurance goes up $26 per month.

a. Write a recursive routine that describes Jackson's monthly car insurance bill based on the number of tickets he has received.
b. Create an input-output table for 0 to 5 speeding tickets.
c. Create a linear plot that shows his total monthly bill through the first five tickets.
d. Jackson has received 12 speeding tickets. How much is his monthly bill? Show all work necessary to justify your answer.



5. Maggie bought a laptop computer for $799. Each year, the value of her laptop decreases by $70.

a. Write a recursive routine that describes the value of Maggie's laptop based on the number of years she has owned it.
b. Create an input-output table for the value of the laptop for 0 to 5 years.
c. Create a linear plot that shows the value of the laptop through the first five years.
d. How many years will it take before the laptop is not worth anything? Use words and/or numbers to show how you determined your answer.

6. Frank borrowed $200 from his parents to buy a mountain bike. Each week, he uses $14 of his allowance to pay back his parents.

a. Write a recursive routine that describes the total amount Frank owes his parents based on the number of weeks that have passed since he borrowed the money.

b. Create an input-output table that shows the amount he still owes for 0 to 5 weeks.

c. Create a linear plot that shows the amount Frank still owes his parents through the first five weeks.

d. How much will Frank's last payment be? Show all work necessary to justify your answer.

7. Quincy hiked up a slope in Desert Shores, California (one of the few places below sea level in the United States). He began at an elevation 61 feet below sea level. Each minute that he hiked, he rose 7 feet in elevation.

a. Write a recursive routine that describes Quincy's elevation based on the number of minutes he hiked.

b. Create an input-output table to find his elevation for 0 to 10 minutes of hiking.

c. Create a linear plot that shows Quincy's change in elevation through the first 10 minutes.

d. How many minutes did it take for Quincy to get above sea level? Support your answer with mathematics.

8. Victor and Mike had a pizza-eating contest. Victor had already eaten three pieces when the competition started. Mike had only eaten one piece. Once the competition started, Victor was able to eat $\frac{1}{2}$ of a piece every minute. Mike was able to eat a little faster. He ate $\frac{3}{4}$ of a piece every minute.

a. Write two recursive routines, one that describes Victor's pizza-eating and the other describing Mike's pizza-eating. Label them accordingly.

b. The pizza-eating competition lasted for 8 minutes. Create two input-output tables that show the number of pieces each boy had eaten for each of the first 8 minutes.

c. Who won the competition at the end of 8 minutes?

9. When Kathy was born, her grandparents started an account for her college education with $1,000 in it. Each year, on her birthday, they add $250.

a. Write a recursive routine that gives the amount of money in Kathy's account based on her age, not including interest.

b. Determine the total amount her grandparents will have contributed after her 18th birthday.

c. Overall, the entire account earned 28% interest. Determine the total amount the account was worth when she withdrew it after her 18th birthday. Show all work necessary to justify your answer.

10. Marin and Jimmy's father asked them to figure out a problem about an antique desk he owned. The antique desk was currently worth $200. Each year that passed, it was worth $45 more. He wanted to know how many years it would take before the desk would be worth over $500. They created a recursive sequence shown below:

200, 245, 290, 335, 380, 425, 470, 515

Jimmy says it will take 8 years but Marin says it will only take 7 years. Who do you agree with? Explain your reasoning.

11. Diana's resting metabolic rate is 1,550 calories per day. She participates in a 30-minute aerobics class in the morning and burns calories at a rate of 7 calories per minute. After school she rides her horse. If she burns 4 calories per minute while riding, how long will she need to ride her horse to burn a total of 2,000 calories? Show all work necessary to justify your answer.

REVIEW

Find the missing values in each sequence. Identify the start value and the operation that must be performed to arrive at the next term.

12. −14, −11, _____, _____, −2, _____

13. 5.8, 4.6, _____, 2.2, _____, _____

14. 9, _____, 21, _____, 33, _____

15. $\frac{1}{3}$, 1, _____, $2\frac{1}{3}$, _____, _____

Solve each equation. Show all work necessary to justify your answer.

16. $x + 28 = 102$

17. $\frac{x}{6} = -7$

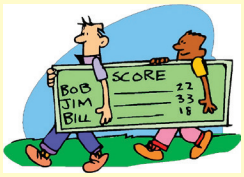
18. $-3x + 5 = 38$

19. $5x + 7 = 7x - 9$

20. $3 = \frac{x}{2} - 1.5$

21. $2x + 7 = 4$

Tic-Tac-Toe ~ Card Game



Use a regular deck of playing cards for this activity. Take out all the face cards (Jacks, Queens and Kings). Create a card game that can be played with two people. The card game must make the players use recursive routines to earn "points". Be creative with your rules.

Some ideas to think about:

- How many cards does each person start with?
- Do some colors and/or suits represent negative integers or subtraction?
- How will the cards be used to determine a start value of a recursive routine?
- How will the cards be used to determine an operation of a recursive routine?
- How are points scored or how does the person progress towards a finish line?
- Do players ever have to find a specific term in the sequence based on a number they draw from the deck?

Once you have designed your game, ask two different pairs of people to try it out. Ask each player to write a short review of your game once they have played it. Read the reviews and write a one-page paper summarizing the feedback. Also include in your paper any changes you would make in the rules before the game was played again. Turn in your paper along with the original set of rules for your game.