NOTES 12: SEQUENCES AND SERIES	Name: Key Date:________ Period:______ Mrs. Nguyen's Initial:________

LESSON 12.1 – SEQUENCES AND SUMMATION NOTATION

Definition of a Sequence	A sequence is a set of numbers written in a specific order. Each number in the sequence is called a term and is denoted as $a_1, a_2, a_3, \ldots, a_n$.	**Example:** $a_1 \quad a_2 \quad a_3 \quad a_4 \quad \ldots \quad a_n$ $3 \quad 6 \quad 12 \quad 24$

Practice Problems: Find the first four terms and the 100th term of the following sequences.

1. $a_n = 2n+3$ Arithmetic	2. $a_n = (-1)^{n+1}\dfrac{n}{n+1}$	3. $a_n = n^2 - 2n$
$a_1 = 5$ $a_2 = 7$ $a_3 = 9$ $a_4 = 11$ $a_{100} = 203$ x | y: 1 | 5; 2 | 7; 3 | 9; 4 | 11; 100 | 203	$a_1 = \frac{1}{2}$ $a_2 = -\frac{2}{3}$ $a_3 = \frac{3}{4}$ $a_4 = \frac{-4}{5}$ $a_{100} = \frac{-100}{101}$	$a_1 = -1$ $a_2 = 0$ $a_3 = 3$ $a_4 = 8$ $a_{100} = 9800$

rule $a_n = ?$ start $n = 1$

Practice Problems: Find the nth term of a sequence whose first several terms are given.

4. $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots$	5. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$	6. $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, \ldots$
mult by $-\frac{1}{3}$ ∴ geometric $a_n = a_1 r^{n-1}$ $= -\frac{1}{3}\left(\frac{-1}{3}\right)^{n-1}$ $\boxed{a_n = \left(\frac{-1}{3}\right)^n}$	$\boxed{a_n = \dfrac{n+2}{n+3}}$	$1^1, 2^{-1}, 3^1, 4^{-1}, 5^1, 6^{-1} \ldots$ $\boxed{a_n = n^{(-1)^{(n+1)}}}$

Practice Problems: Find the first five terms of the given recursively defined sequence.

7. $a_n = \frac{a_{n-1}}{2}$ and $a_1 = -8$	8. $a_n = \frac{1}{1+a_{n-1}}$ and $a_1 = 1$	9. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ and $a_1 = a_2 = a_3 = 1$
$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-8}{2} = -4$ $a_3 = \frac{a_{3-1}}{2} = \frac{a_2}{2} = \frac{-4}{2} = -2$ $a_4 = \frac{a_3}{2} = \frac{-2}{2} = -1$ $a_5 = \frac{-1}{2}$	$a_2 = \frac{1}{1+a_1} = \frac{1}{2}$ $a_3 = \frac{1}{1+a_2} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$ $a_4 = \frac{1}{1+\frac{2}{3}} = \frac{3}{5}$ $a_5 = \frac{1}{1+3/5} = 5/8$	$a_4 = a_3 + a_2 + a_1$ $= 1 + 1 + 1 = 3$ $a_5 = a_4 + a_3 + a_2 = 3+1+1 = 5$ $a_6 = 5 + 3 + 1 = 9$

Partial Sum of a Sequence	The nth partial sum of a sequence is found by adding up the first n terms of the sequence.	**Example:** Sum of the first 5 terms of example # 9 $S_5 = 1 + 1 + 1 + 3 + 5 = 11$

Practice Problems: Find the first four partial sums and the nth partial sum of the sequence a_n.

10. $a_n = \frac{3}{2^n}$	11. $a_n = \frac{4}{n+3} - \frac{4}{n+2}$
$a_1 = \frac{3}{2}$ $a_2 = \frac{3}{4}$ $a_3 = \frac{3}{8}$ $a_4 = \frac{3}{16}$ $S_1 = \frac{3}{2}$ $S_2 = \frac{3}{2} + \frac{3}{4} = \frac{9}{4}$ $S_3 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} = \frac{21}{8}$ $S_4 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$ $= \frac{45}{16}$	$a_1 = \frac{-1}{3}$ $a_2 = \frac{-1}{5}$ $a_3 = \frac{-2}{15}$ $a_4 = \frac{-2}{21}$ $S_1 = \frac{-1}{3}$ $S_2 = \frac{-1}{3} - \frac{1}{5} = \frac{-8}{15}$ $S_3 = \frac{-8}{15} - \frac{2}{15} = \frac{-2}{3}$ $S_4 = \frac{-2}{3} - \frac{2}{21} = \frac{-16}{21}$

Sigma/Summation Notation	$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$	**Example:** $a_2 + a_3 + a_4$ $\sum_{i=2}^{4} i^2 = 4 + 9 + 16$ $= 29$

Practice Problems: Find each sum.

12. $\sum_{k=1}^{4} k^2$	13. $\sum_{i=1}^{3} i2^i$	14. $\sum_{j=1}^{5} \frac{1}{j}$
$a_1 + a_2 + a_3 + a_4$ $1 + 4 + 9 + 16$ $= \boxed{30}$	$a_1 + a_2 + a_3$ $2 + 8 + 24$ $= \boxed{34}$	$a_1 + a_2 + a_3 + a_4 + a_5$ $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ $= \boxed{\frac{137}{60}}$

LESSON 12.2 – ARITHMETIC SEQUENCES

Definition of an Arithmetic Sequence	An arithmetic sequence is a sequence in which the difference between consecutive terms is constant. That difference is called the common difference.	**Example:** $a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_n$ $2, \ 4, \ 6, \ 8, \ 10 \ldots$ $d = 2 \quad 2 \quad 2 \quad 2$
Arithmetic Sequence Formula	The **nth term** of an arithmetic sequence is $a_n = a_1 + (n-1)d$ ← common diff ↑ first term	**Example:** $a_n = 2 + (n-1)2$ $= 2 + 2n - 2$ $a_n = 2n$

Practice Problems: Find the nth term of the arithmetic sequence with the given first term *a* and common difference *d*.

$a_n = ?$

1. $a = -6$, $d = 3$ $a_n = -6 + (n-1)3$ $= -6 + 3n - 3$ $\boxed{a_n = 3n - 9}$	2. $a = 7$, $d = -2$ $a_n = 7 + (n-1)(-2)$ $= 7 - 2n + 2$ $\boxed{a_n = -2n + 9}$

Practice Problems: Find the specified term.

3. 18th term of 3, 7, 11, … (4, 4) $a_1 = 3 \quad d = 4$ $n = 18$ $a_{18} = 3 + (18-1)(4)$ $= 3 + (17)(4)$ $\boxed{a_{18} = 71}$	4. 9th term of 3, -2, -7,… (−5, −5) $a_1 = 3 \quad d = -2 - 3 = -5$ $n = 9$ $a_9 = 3 + (9-1)(-5)$ $= 3 + (-40)$ $\boxed{a_9 = -37}$	5. 15th term of 20, 23, 26, 29, … $a_1 = 20 \quad d = 3$ $n = 15$ $a_{15} = 20 + (15-1)(3)$ $= 20 + 42$ $\boxed{a_{15} = 62}$

Practice Problems: Determine the common difference, the fifth term, the nth term, and the 100th term of the arithmetic sequence.

6. 1, 5, 9, 13, … $a_1 = 1 \quad d = 4$ $a_n = 1 + (n-1)4$ $= 1 + 4n - 4$ $\boxed{a_n = 4n - 3}$ $a_5 = 4(5) - 3 = \boxed{17}$ $a_{100} = 4(100) - 3 = \boxed{397}$	7. $\frac{7}{6}, \frac{5}{3}, \frac{13}{6}, \frac{8}{3}, \ldots$ $a_1 = \frac{7}{6} \quad d = \frac{5}{3} - \frac{7}{6} = \frac{3}{6} = \frac{1}{2}$ $a_n = \frac{7}{6} + (n-1)\frac{1}{2}$ $= \frac{7}{6} + \frac{n}{2} - \frac{1}{2} = \frac{1}{2}n + \frac{2}{3}$ $a_5 = \frac{1}{2}(5) + \frac{2}{3} = \boxed{\frac{19}{6}}$ $\boxed{a_n = \frac{1}{2}n + \frac{2}{3}}$ $a_{100} = \frac{1}{2}(100) + \frac{2}{3} = \boxed{50\frac{2}{3}}$	8. $-t, -t+3, -t+6, -t+9, \ldots$ $a_1 = -t \quad d = -t + 3 - -t = 3$ $a_n = -t + (n-1)3$ $\boxed{a_n = -t + 3n - 3}$ $\boxed{a_5 = -t + 12}$ $\boxed{a_{100} = -t + 297}$

Practice Problems: Find the first term of each sequence described. Then find a_{100}.

9. $a_8 = 12,\ a_{15} = 61$

$$a_n = a_1 + (n-1)d$$
$$a_{15} = a_8 + (15-8)d$$
$$61 = 12 + (7)d$$
$$49 = 7d \rightarrow d = 7$$
$$a_8 = a_1 + (8-1)7$$
$$12 = a_1 + 49 \rightarrow \boxed{a_1 = -37}$$
$$a_n = -37 + (n-1)7$$
$$= -37 + 7n - 7 = 7n - 44$$
$$a_{100} = 7(100) - 44 = \boxed{656}$$

10. $a_5 = 10,\ a_{12} = 32$

$$a_{12} = a_5 + (12-5)d$$
$$32 = 10 + 7d \rightarrow d = \frac{22}{7}$$
$$a_5 = a_1 + (5-1)\frac{22}{7}$$
$$10 = a_1 + \frac{88}{7} \rightarrow \boxed{a_1 = \frac{-18}{7}}$$
$$a_{100} = \frac{-18}{7} + (99)\left(\frac{22}{7}\right)$$
$$= \boxed{\frac{2160}{7}}$$

11. $a_6 = 10, a_{21} = 55$

$$a_{21} = a_6 + (21-6)d$$
$$55 = 10 + 15d \rightarrow d = 3$$
$$a_6 = a_1 + (6-1)3$$
$$10 = a_1 + 15 \rightarrow \boxed{a_1 = -5}$$
$$a_{100} = -5 + (99)(3)$$
$$= -5 + 297$$
$$= \boxed{292}$$

Definition of Arithmetic Series

The arithmetic series is the sum of all the terms in an arithmetic sequence.

$1 + 2 + 3$

$\cancel{98} + 99 + 100$

$1 + 2 + 3 + 4 \cdots + 100$

Arithmetic Series Formula:

The sum of the first n terms of an arithmetic series is: $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

$$S_{100} = 100\left(\frac{1 + 100}{2}\right) = \frac{100 \cdot 101}{2} = (50)(101)$$

Practice Problems: Find the partial sum S_n of the arithmetic sequence that satisfies the given condition.

12. $a = 3,\ d = 2,\ n = 12$

$$S_{12} = 12\left(\frac{3 + \boxed{a_{12}}}{2}\right)$$
$$a_n = 3 + (n-1)(2)$$
$$= 3 + 2n - 2$$
$$a_n = 2n + 1$$
$$a_{12} = 2(12) + 1 = 25$$
$$S_{12} = 12\left(\frac{3 + 25}{2}\right)$$
$$= 6(28)$$
$$= \boxed{168}$$

13. $a = 100,\ d = -5,\ n = 8$

$$a_8 = 100 + (8-1)(-5)$$
$$= 100 - 35 = 65$$
$$S_8 = 8\left(\frac{100 + 65}{2}\right)$$
$$= 4(165)$$
$$= \boxed{660}$$

14. $a_2 = 8,\ a_5 = 9.5,\ n = 15$

$$a_5 = a_2 + (5-2)d$$
$$9.5 = 8 + 3d$$
$$1.5 = 3d \rightarrow d = \frac{1}{2}$$
$$a_2 = a_1 + (2-1)\frac{1}{2}$$
$$8 = a_1 + \frac{1}{2} \rightarrow a_1 = \frac{15}{2}$$
$$a_{15} = \frac{15}{2} + (15-1)\frac{1}{2}$$
$$= \frac{15}{2} + 7 = \frac{29}{2}$$
$$S_{15} = 15\left(\frac{\frac{15}{2} + \frac{29}{2}}{2}\right)$$
$$= 15(11) = \boxed{165}$$

Practice Problems: A partial sum of an arithmetic series is given. Find the sum.

15. $2+6+10+\ldots+118$ $a_1=2 \quad a_n=118 \quad d=4$ $118=2+(n-1)4$ $\frac{116}{4}=n-1 \rightarrow n-1=29 \rightarrow n=30$ $S_{30}=30\left(\frac{2+118}{2}\right)=30(60)=\boxed{1800}$	16. $6+3+0-3-6-\ldots-57$ $a_1=6 \quad d=-3 \quad a_n=-57$ $-57=6+(n-1)(-3)$ $\frac{-63}{-3}=n-1 \rightarrow n-1=21 \rightarrow n=22$ $S_{22}=22\left(\frac{6-57}{2}\right)=11(-51)=\boxed{-561}$
17. $\sum_{i=1}^{12}(-2i-7)=S_{12}$ ($n=12$) $a_1 \quad a_2 \quad a_3 \ \ldots \ a_{12}$ $-9 \quad -11 \quad -13 \quad\quad -31$ $S_{12}=12\left(\frac{-9-31}{2}\right)=12(-20)=\boxed{-240}$	18. $\sum_{k=1}^{40}(6k+1)$ $n=40 \quad a_1=7 \quad a_{40}=241$ $S_{40}=40\left(\frac{7+241}{2}\right)=20(248)$ $=\boxed{4960}$

Practice Problems: Find n for the given sum S_n

$S_n=n\left(\frac{a_1+a_n}{2}\right) \rightarrow a_n=a_1+(n-1)d$

19. $4+7+10+13+16+19+\ldots;\ S_n=175$ $a_1=4 \quad d=3$ $a_n=4+(n-1)3=4+3n-3$ $=3n+1$ $S_n=175=n\left(\frac{a_1+a_n}{2}\right)=n\left(\frac{4+3n+1}{2}\right)$ $175=\frac{3n^2+5n}{2} \rightarrow 3n^2+5n-350=0$ $(3n+35)(n-10)=0 \rightarrow \boxed{n=10}$	20. $100+110+120+130+\ldots;\ S_n=600$ $a_1=100$, $d=10$ $\}$ $a_n=100+(n-1)10$ $=100+10n-10=10n+90$ $600=n\left(\frac{100+10n+90}{2}\right)$ $600=n(95+5n)$ $0=5n^2+95n-600$ $0=n^2+19n-120$ $0=(n-5)(n+24) \rightarrow \boxed{n=5},\ \cancel{-24}$
21. $9+13+17+21+25+\ldots;\ S_n=2272$ $a_1=9$, $d=4$ $\}$ $a_n=9+(n-1)4$ $=9+4n-4=4n+5$ $2272=n\left(\frac{9+4n+5}{2}\right)$ $2272=n(7+2n)$ $0=2n^2+7n-2272$ $=\frac{-7\pm\sqrt{49+18176}}{4}=\frac{-7\pm135}{4}$ $=\frac{128}{4}=\boxed{32}$	22. $-7+(-4)+(-1)+2+5+\ldots;\ S_n=210$ $a_1=-7$, $d=-4-{}^-7=3$ $\}$ $a_n=-7+(n-1)3$ $=-7+3n-3=3n-10$ $210=n\left(\frac{-7+3n-10}{2}\right)$ $420=-17n+3n^2$ $0=3n^2-17n-420$ $0=\frac{17\pm\sqrt{289+5040}}{6}$ $=\frac{17\pm73}{6} \rightarrow \frac{90}{6}=\boxed{15}$

LESSON 12.3 – GEOMETRIC SEQUENCES

Definition of a Geometric Sequence	A <u>geometric sequence</u> is a sequence in which the ratio between consecutive terms is constant. That ratio is called the <u>common ratio</u>.	**Example:** $2, 4, 8, 16, \dots, a_n$ $r = 2$
Geometric Sequence Formula	The **nth term** of a geometric sequence is… $a_n = a_1 \cdot r^{n-1}$	**Example:** $a_n = 2(2)^{n-1} = \cancel{4^{n-1}}$

<u>Practice Problems</u>: Find the nth term of the geometric sequence with the given first term *a* and common ratio *r*.

1. a = -6 and r = 3 $a_n = -6(3)^{n-1}$	2. a =12 and r = -2 $a_n = 12(-2)^{n-1}$

<u>Practice Problems</u>: Determine the common ratio, the fifth term, and the nth term of the geometric sequence.

3. $7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \dots$ $r = \frac{\frac{14}{3}}{7} = \frac{14}{3} \cdot \frac{1}{7} = \boxed{\frac{2}{3}}$ $\boxed{a_n = 7\left(\frac{2}{3}\right)^{n-1}}$ $a_5 = 7\left(\frac{2}{3}\right)^4 = 7\left(\frac{16}{81}\right) = \boxed{\frac{112}{81}}$	4. $-8, 2, -\frac{1}{2}, \frac{1}{8}, \dots$ $r = \frac{2}{-8} = -\frac{1}{4} \to \boxed{a_n = -8\left(\frac{-1}{4}\right)^{n-1}}$ $a_5 = -8\left(-\frac{1}{4}\right)^4 = \frac{-8}{256} = \boxed{\frac{-1}{32}}$	5. $5, 5^{c+1}, 5^{2c+1}, 5^{3c+1}, \dots$ $r = \frac{5^{c+1}}{5^1} = 5^{c+1-1} = \boxed{5^c}$ $a_n = 5(5^c)^{n-1} = \boxed{5(5^{cn-c})}$ $a_5 = 5(5^{4c}) = \boxed{5^{4c+1}}$

<u>Practice Problems</u>: Find the first term of each sequence described. Then find a_{10}.

6. $a_2 = 5, a_4 = \frac{1}{5}$ $a_n = a_1 r^{n-1}$ $a_4 = a_2 (r)^2$ $\frac{1}{5} = 5(r)^2$ $\frac{1}{25} = r^2 \to r = \pm\frac{1}{5}$ $5 = a_1\left(\frac{1}{5}\right)^1 \to \boxed{a_1 = 25}$	7. $a_2 = 28, a_5 = -1792$ $a_5 = a_2 r^{5-2}$ $-1792 = 28 r^3$ $-64 = r^3 \to \boxed{r = -4}$ $a_2 = a_1(-4)^1$ $28 = a_1(-4) \to \boxed{a_1 = -7}$ $a_{10} = a_1 r^{10-1} \to -7(-4)^9$ $\boxed{1{,}835{,}008}$	8. $a_3 = 8, a_6 = -64$ $-64 = 8r^3 \to r^3 = -8$ $\boxed{r = -2}$ $8 = a_1(-2)^2 \to \boxed{a_1 = 2}$ $a_{10} = 2(-2)^9$ $= 2(-512)$ $= \boxed{-1024}$

$a_n = 25\left(\frac{1}{5}\right)^{n-1} \to a_{10} = 25\left(\frac{1}{5}\right)^9 = \frac{25}{5^9} = \frac{1}{5^7} = \boxed{\frac{1}{78125}}$

Geometric Series Formula	The sum S_n of the first n terms of a geometric series with common ratio $r \neq 1$ is: $$S_n = \frac{a_1(1-r^n)}{1-r}$$

Practice Problems: Find the partial sum S_n of the geometric sequence that satisfies the given conditions.

9. $a = \frac{2}{3}$, $r = \frac{1}{3}$, $n = 4$ $S_4 = \frac{\frac{2}{3}\left(1-\left(\frac{1}{3}\right)^4\right)}{1-\frac{1}{3}}$ $= \frac{\frac{2}{3}\left(1-\frac{1}{81}\right)}{\frac{2}{3}} = 1-\frac{1}{81}$ $= \boxed{\frac{80}{81}}$	10. a = -4, r = 4, n = 10 $S_{10} = \frac{-4(1-4^{10})}{1-4} = \frac{-4(1-4^{10})}{-3}$ $= -1398100$	11. $a_3 = -64, a_7 = -\frac{1}{4}$, n = 12 $a_7 = a_3 r^4 \rightarrow -\frac{1}{4} = -64 r^4$ $r^4 = \frac{1}{256} \rightarrow r = \pm\frac{1}{4}$ Assume $r = \frac{1}{4}$ $-64 = a_1\left(\frac{1}{4}\right)^2 \rightarrow -64 = a_1\left(\frac{1}{16}\right)$ $a_1 = -1024$ $S_{12} = \frac{-1024\left(1-\left(\frac{1}{4}\right)^{12}\right)}{1-\frac{1}{4}} \approx \boxed{-1365.333}$

Practice Problems: Find the sum.

12. 2 – 4 + 8 + -16 + … - 65536 $r = \frac{-4}{2} = -2$ $a_n = 2(-2)^{n-1} = -65536$ $(-2)^{n-1} = -32768$ $(-2)^{n-1} = (-2)^{15} \rightarrow n-1 = 15$, $n = 16$ $S_{16} = \frac{2(1-(-2)^{16})}{1+2} = \frac{2(1-65536)}{3} = \boxed{-43690}$	13. $-60 + 20 - \frac{20}{3} + \ldots - \frac{20}{2187}$ $r = -\frac{1}{3}$ $\frac{-20}{2187} = -60\left(-\frac{1}{3}\right)^{n-1}$ $\frac{1}{6561} = \left(\frac{-1}{3}\right)^{n-1} \rightarrow \left(\frac{-1}{3}\right)^8 = \left(\frac{-1}{3}\right)^{n-1} \rightarrow n = 9$ $S_n = \frac{-60\left(1-\left(-\frac{1}{3}\right)^9\right)}{4/3} = -\frac{98420}{2187} \approx \boxed{-45.00}$
14. $\sum_{i=0}^{15} 4\left(\frac{1}{2}\right)^i$ $r = \frac{1}{2}$ $a_0 = 4$ $n = 16$ $a_0 + a_1 + a_2 + \cdots + a_{15}$ $S_{16} = \frac{4\left(1-\left(\frac{1}{2}\right)^{16}\right)}{1/2} = \boxed{7.99987793}$	15. $\sum_{i=1}^{10} \frac{1}{2}(6)^{i-1}$ $a_1 = \frac{1}{2}$ $r = 6$ $n = 10$ $S_{10} = \frac{\frac{1}{2}(1-(6)^{10})}{-5} = \boxed{6046617.5}$

Practice Problems: Find n for the given sum S_n of each geometric series.

16. $1 + (-4) + 16 + (-64) + \ldots,$ $S_n = -819$

$r = -4 \quad a_1 = 1$

$-819 = \dfrac{1(1-(-4)^n)}{1--4}$

$-4095 = 1 - (-4)^n$

$-4096 = -(-4)^n$

$4096 = (-4)^n$

$(-4)^{\text{even}} = (-4)^n$

$(-4)^6 = (-4)^n \rightarrow n = 6$

17. $2 - 4 + 8 - 16 + \ldots,$ $S_n = 86$

$r = -2 \quad a_1 = 2$

$86 = \dfrac{2(1-(-2)^n)}{3}$

$129 = 1 - (-2)^n$

$128 = -(-2)^n$

$-128 = (-2)^n$

$(-2)^{\text{odd}} = (-2)^n$

$(-2)^7 = (-2)^n \rightarrow \boxed{n = 7}$

18. $-8 - 32 - 128, -512, \ldots,$ $S_n = -44739240$

$r = 4 \quad a_1 = -8$

$-44739240 = \dfrac{-8(1-4^n)}{-3}$

$-16777215 = 1 - 4^n$

$16777216 = 4^n$

$4^{12} = 4^n$

$\boxed{n = 12}$

Infinite Geometric Series Formula	If $\lvert r\rvert < 1$, then the sum of an infinite geometric series is:	If $\lvert r\rvert \geq 1$, the series has no sum.
$\sum_{i=1}^{\infty} a_1 r^{i-1}$	$\boxed{S = \dfrac{a_1}{1-r}}$ $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ $r = \frac{1}{2}$	$2 + 4 + 8 + 16 + \cdots$ $r = 2$

Practice Problems: Find the sum of the infinite geometric series if it has one.

19. $\dfrac{2}{5} + \dfrac{4}{25} + \dfrac{8}{125} + \ldots$

$r = \dfrac{4}{25} \cdot \dfrac{5}{2} = \dfrac{2}{5} < 1$

$S = \dfrac{\frac{2}{5}}{1 - \frac{2}{5}} = \dfrac{\frac{2}{5}}{3/5} = \dfrac{2}{5} \cdot \dfrac{5}{3} = \boxed{\dfrac{2}{3}}$

20. $\dfrac{1}{\sqrt{2}} + \dfrac{1}{2} + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{4} + \ldots$

$r = \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2} < 1$

$S = \dfrac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}-1} = \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{\sqrt{2}+1}{1} = \boxed{\sqrt{2}+1}$

21. $\sum_{n=0}^{\infty} 3\left(\dfrac{1}{2}\right)^n$

$r = \dfrac{1}{2} < 1$

$S = \dfrac{3}{1 - \frac{1}{2}} = \dfrac{3}{\frac{1}{2}} = \boxed{6}$

22. $\sum_{k=0}^{\infty} 3\left(\dfrac{4}{3}\right)^k$

$r = \dfrac{4}{3} > 1$

No Sum

LESSON 12.6 – THE BINOMIAL THEOREM

Pascal's Triangle **Note:** The values in Pascal's Triangle are the coefficients in a binomial expansion.	$(a+b)^0 = 1$ $(a+b)^1 = a+b$ $(a+b)^2 = a^2+2ab+b^2$ $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ $n=0$: 1 $n=1$: 1 1 $n=2$: 1 2 1 $n=3$: 1 3 3 1 $n=4$: 1 4 6 4 1 $n=5$: 1 5 10 10 5 1 $n=6$: 1 6 15 20 15 6 1 $n=7$: 1 7 21 35 35 21 7 1
The Binomial Theorem $_nC_r = C(n,r) = \frac{n!}{(n-r)!r!}$	The binomial expansion of $(a+b)^n$ for any positive integer n is: $(a+b)^n = {}_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + {}_nC_3a^{n-3}b^3 + \ldots + {}_nC_na^0b^n$

Practice Problems: Use Pascal's Triangle to expand the expression.

1. $(x+2)^4$ Degree = 4 Terms = 5

$1x^4 2^0 + 4x^3 2^1 + 6x^2 2^2 + 4x^1 2^3 + 1x^0 2^4$

$\boxed{x^4 + 8x^3 + 24x^2 + 32x + 16}$

2. $(u+v^2)^5$ D = 5 Terms = 6

$1u^5(v^2)^0 + 5u^4(v^2)^1 + 10u^3(v^2)^2 + 10u^2(v^2)^3 + 5u^1(v^2)^4 + 1u^0(v^2)^5$

$u^5 + 5u^4v^2 + 10u^3v^4 + 10u^2v^6 + 5uv^8 + v^{10}$

3. $(a+2b^3)^4$

$1a^4 + 4a^3(2b^3)^1 + 6a^2(2b^3)^2 + 4a^1(2b^3)^3 + 1(2b^3)^4$

$a^4 + 8a^3b^3 + 24a^2b^6 + 32ab^9 + 16b^{12}$

4. $(2x-y^2)^3$	$D = 3$ Terms $= 4$

$1(2x)^3(-y^2)^0 + 3(2x)^2(-y^2)^1 + 3(2x)^1(-y^2)^2 + 1(2x)^0(-y^2)^3$

$\boxed{8x^3 - 12x^2y^2 + 6xy^4 - y^6}$

5. $\left(\sqrt{a}+\sqrt{b}\right)^6$	$D = 6$ Terms $= 7$

$1\sqrt{a}^6\sqrt{b}^0 + 6\sqrt{a}^5\sqrt{b}^1 + 15\sqrt{a}^4\sqrt{b}^2 + 20\sqrt{a}^3\sqrt{b}^3 + 15\sqrt{a}^2\sqrt{b}^4 + 6\sqrt{a}^1\sqrt{b}^5 + 1\sqrt{a}^0\sqrt{b}^6$

$a^3 + 6a^{5/2}b^{1/2} + 15a^2b + 20a^{3/2}b^{3/2} + 15ab^2 + 6a^{1/2}b^{5/2} + b^3$

Practice Problems: Find the given term in the expansion of the given binomial.

6. Binomial: $(x+3)^4$, Term: x^2

$_4C_0 x^4 \quad {_4C_1} x^3 \quad \boxed{6}\, x^2\, 3^2 = 54x^2$

$_4C_2$

$$_4C_2 = \frac{4!}{2!\,2!} = \frac{\cancel{4}\cdot 3\cdot 2\cdot 1}{\cancel{2}\cdot 1\cdot \cancel{2}\cdot 1} = 6$$

7. Binomial: $(x-2y)^6$, Term: x^3y^3

$_6C_3\, x^3(-2y)^3$

$$\frac{6!}{3!\,3!} = \frac{6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}\; 3\cdot 2\cdot 1} = 20$$

$20x^3(-8y^3) = \boxed{-160x^3y^3}$

8. Binomial: $\left(3x-2y^2\right)^{10}$, Term: y^6

$_{10}C_3\, (3x)^7(-2y^2)^3$

$$_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\; 3\cdot 2\cdot 1}$$

$= 10\cdot 3\cdot 4 = 120$

$120(2187x^7)(-8y^6) = \boxed{-2099520x^7y^6}$

9. Binomial: $(3x^2+y)^6$, Term: x^8

$_6C_2\, (3x^2)^4 y^2$

$$_6C_2 = \frac{6!}{4!\,2!} = \frac{6\cdot 5\cdot 4!}{4!\;2} = 15$$

$15(81x^8)y^2 = \boxed{1215x^8y^2}$