

## Topic A

# Multiplicative Patterns on the Place Value Chart

## 5.NBT.1, 5.NBT.2, 5.MD.1

<b>Focus Standard:</b>	5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.
	5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
	5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
<b>Instructional Days:</b>	4	
<b>Coherence -Links from:</b>	G4–M1	Place Value, Rounding, and Algorithms for Addition and Subtraction
<b>-Links to:</b>	G6–M2	Arithmetic Operations Including Dividing by a Fraction

Topic A begins with a conceptual exploration of the multiplicative patterns of the base ten system. This exploration extends the place value work done with multi-digit whole numbers in Grade 4 to larger multi-digit whole numbers and decimals. Students use place value disks and a place value chart to build the place value chart from millions to thousandths. Students compose and decompose units crossing the decimal with a view toward extending students' knowledge of the *10 times as large* and *1/10 as large* relationships among whole number places to that of adjacent decimal places. This concrete experience is linked to the effects on the product when multiplying any number by a power of ten. For example, students notice that multiplying 0.4 by 1000 shifts the position of the digits to the left 3 places, changing the digits' relationships to the decimal point and producing a product with a value that is  $10 \times 10 \times 10$  as large (400.0) (**5.NBT.2**). Students explain these changes in value and shifts in position in terms of place value. Additionally, students learn a new and more efficient way to represent place value units using exponents, e. g., 1 thousand =  $1000 = 10^3$  (**5.NBT.2**). Conversions among metric units such as kilometers, meters, and centimeters give an opportunity to apply these extended place value relationships and exponents in a meaningful context by exploring word problems in the last lesson of Topic A (**5.MD.1**).

**A Teaching Sequence Towards Mastery of Multiplicative Patterns on the Place Value Chart**

**Objective 1:** Reason concretely and pictorially using place value understanding to relate adjacent base ten units from millions to thousandths.  
(Lesson 1)

**Objective 2:** Reason abstractly using place value understanding to relate adjacent base ten units from millions to thousandths.  
(Lesson 2)

**Objective 3:** Use exponents to name place value units and explain patterns in the placement of the decimal point.  
(Lesson 3)

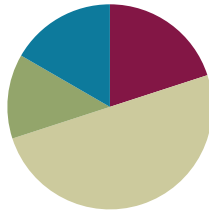
**Objective 4:** Use exponents to denote powers of 10 with application to metric conversions.  
(Lesson 4)

## Lesson 1

**Objective:** Reason concretely and pictorially using place value understanding to relate adjacent base ten units from millions to thousandths.

### Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problems	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
<b>Total Time</b>	<b>(60 minutes)</b>



#### NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Throughout *A Story of Units*, place value language is key. In earlier grades, teachers use units to refer to numbers such as 245 as two *hundred* forty five. Likewise, in Grades 4 and 5, decimals should be read emphasizing their unit form. For example, 0.2 would be read *2 tenths* rather than *zero point two*. This emphasis on unit language not only strengthens student place value understanding, but also builds important parallel between whole number and decimal fraction understanding.

### Fluency Practice (12 minutes)

- Multiply by 10 **4.NBT.1** (8 minutes)
- Rename the Units **2.NBT.1** (2 minutes)
- Decimal Place Value **4.NF.5–6** (2 minutes)

### Sprint: Multiply by 10 (8 minutes)

Materials: (S) Multiply by 10 Sprint

Note: Reviewing this fluency will acclimate students to the Sprint routine, a vital component of the fluency program.

Please see *Sprints* in the Appendix for directions on administering.

### Rename the Units—Choral Response (2 minutes)

Notes: This fluency will review foundations that will lead into today's lesson.

T: (Write 10 ones = \_\_\_\_ ten.) Say the number sentence.

S: 10 ones = 1 ten.

T: (Write 20 ones = \_\_\_\_ tens.) Say the number sentence.

S: 20 ones = 2 tens.

T: 30 ones.

S: 3 tens.

Repeat the process for 80 ones, 90 ones, 100 ones, 110 ones, 120 ones, 170, 270, 670, 640, and 830.

**Decimal Place Value (2 minutes)**

Materials: (S) Personal white boards

Note: Reviewing this Grade 4 topic will help lay a foundation for students to better understand place value to bigger and smaller units.

T: (Project place value chart from millions to hundredths. Write 3 ten disks in the tens column.) How many tens do you see?

S: 3 tens.

T: (Write 3 underneath the disks.) There are 3 tens and how many ones?

S: Zero ones.

T: (Write 0 in the ones column. Below it, write 3 tens = \_\_\_\_.) Fill in the blank.

S: 3 tens = 30.

Repeat the process for 3 tenths = 0.3.

T: (Write 4 tenths = \_\_\_\_.) Show the answer in your place value chart.

S: (Students write four 1 tenth disks. Below it, they write 0.4.)

Repeat the process for 3 hundredths, 43 hundredths, 5 hundredths, 35 hundredths, 7 ones 35 hundredths, 9 ones 24 hundredths, and 6 tens 2 ones 4 hundredths.

**Application Problem (8 minutes)**

Farmer Jim keeps 12 hens in every coop. If Farmer Jim has 20 coops, how many hens does he have in all? If every hen lays 9 eggs on Monday, how many eggs will Farmer Jim collect on Monday? Explain your reasoning using words, numbers, or pictures.

Note: This problem is intended to activate prior knowledge from Grade 4 and offer a successful start to Grade 5. Some students may use area models to solve while others may choose to use the standard algorithm. Still others may draw tape diagrams to show their thinking. Allow students to share work and compare approaches.

$$\begin{array}{|c|c|} \hline 10 & 2 \\ \hline 20 & 200 & 40 \\ \hline \end{array}$$

$$200 + 40 = 240 \text{ hens}$$

$$\begin{array}{|c|c|} \hline 200 & 40 \\ \hline 9 & 1800 & 360 \\ \hline \end{array}$$

$$\begin{array}{r} 1800 \\ + 360 \\ \hline 2,160 \text{ eggs} \end{array}$$

**Concept Development (30 minutes)**

Materials: (S) Personal place value mats, disks, and markers

The place value chart and its  $\times 10$  and relationships are familiar territory for students. New learning in Grade 5 focuses on understanding a new fractional unit of *thousandths* as well as the decomposition of larger units to those that are  $1/10$  as large. Building the place value chart from right (tenths) to left (millions) before

beginning the following problem sequence may be advisable. Encourage students to multiply then bundle to form next largest place (e.g.,  $10 \times 1$  hundred = 10 hundreds, which can be bundled to form 1 thousand).

### Problem 1

Divide single units by 10 to build the place value chart to introduce **thousandths**.

- T: Show 1 million using disks on your chart. How can we show 1 million using hundred thousands? Work with your partner to show this on your mat.
- S: 1 million is the same as 10 hundred thousands.
- T: What is the result if I divide 10 hundred thousands by 10? Talk with your partner and use your mat to find the quotient.
- T: (Circulate.) I saw that David put 10 disks in the hundred thousands place, then put them in 10 equal groups. How many are in each group?
- S: When I divide 10 hundred thousands by 10, I get 1 hundred thousand in each group.
- T: Let me record what I hear you saying. (Record on class board.)

$$10 \text{ hundred thousands} \div 10 = 1 \text{ hundred thousand}$$

$$1 \text{ million} \div 10 = 1 \text{ hundred thousand}$$

$$1 \text{ hundred thousand is } 1/10 \text{ as large as 1 million}$$

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
							●		
							●		

Diagram description: A place value chart with 10 columns. The first column is labeled 'Millions'. The second column is labeled 'Hundred Thousands'. The third column is labeled 'Ten Thousands'. The fourth column is labeled 'Thousands'. The fifth column is labeled 'Hundreds'. The sixth column is labeled 'Tens'. The seventh column is labeled 'Ones'. The eighth column is labeled 'Tenths'. The ninth column is labeled 'Hundredths'. The tenth column is labeled 'Thousandths'. A vertical line is drawn between the 'Ones' and 'Tenths' columns. A dot is placed in the 'Tenths' column. Another dot is placed in the 'Hundredths' column. In the 'Millions' column, there is a '1' with a division symbol and '10' next to it. An arrow points from this '1' to the '1' in the 'Hundred Thousands' column.

- T: Put 1 hundred thousand disk on your chart. What is the result if we divide 1 hundred thousand by 10? Show this on your mat and write a division sentence.

Continue this sequence until the hundredths place is reached emphasizing the unbundling for 10 of the smaller unit and then the division. Record the place values and **equations** (using unit form) on the board being careful to point out the *1/10 as large* relationship:

MP.7

$$1 \text{ million} \div 10 = 1 \text{ hundred thousand}$$

$$1 \text{ hundred thousand} \div 10 = 1 \text{ ten thousand}$$

$$1 \text{ ten thousand} \div 10 = 1 \text{ thousand}$$

$$1 \text{ thousand} \div 10 = 1 \text{ hundred}$$

$$\text{(and so on, through } 1 \text{ tenth} \div 10 = 1 \text{ hundredth)}$$



#### NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students who have limited experience with decimal fractions may be supported by a return to Grade 4's Module 6 to review decimal place value and symmetry with respect to the ones place.

Conversely, student understanding of decimal fraction place value units may be extended by asking for predictions of units  $1/10$  as large as the thousandths place, and those beyond.

MP.8

- T: What patterns do you notice in the way the units are named in our place value system?
- S: The ones place is the middle. There are tens on the left and tenths on the right, hundreds on the left and hundredths on the right.
- T: (Point to the chart.) Using this pattern, can you predict what the name of the unit that is to the right of the hundredths place ( $1/10$  as large as hundredths) might be? (Students share. Label the thousandths place.)
- T: Thinking about the pattern that we've seen with other adjacent places, talk with your partner and predict how we might show 1 hundredth using thousandths disks and show this on your chart.
- S: Just like all the other places, it takes 10 of the smaller unit to make 1 of the larger so it will take 10 thousandths to make 1 hundredth.
- T: Use your chart to show the result if we divide 1 hundredth by 10 and write the division sentence. (Students share. Add this equation to the others on the board.)



### NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Proportional materials such as base ten blocks can help children with language differences distinguish between place value labels like *hundredth* and *thousandth* more easily by offering clues to their relative sizes.

These students can be encouraged to name these units in their native language and then compare them to their English counterparts. Often the roots of these number words are very similar. These parallels enrich the experience and understanding of all students.

### Problem 2

Multiply copies of one unit by 10, 100, and 1000.

$$0.4 \times 10$$

$$0.04 \times 10$$

$$0.004 \times 10$$

- T: Draw number disks to represent 4 tenths at the top on your place value chart.
- S: (Students write.)
- T: Work with your partner to find the value of 10 times 0.4. Show your result at the bottom of your place value chart.
- S:  $4 \text{ tenths} \times 10 = 40 \text{ tenths}$ , which is the same as 4 wholes.  $\rightarrow 4 \text{ ones}$  is 10 times as large as 4 tenths.
- T: On your place value chart, use arrows to show how the value of the digits has changed. (On place value chart, draw an arrow to indicate the shift of the digit to the left, write  $\times 10$  above arrow.)
- T: Why does the digit move one place to the left?
- S: Because it is 10 times as large, it has to be bundled for the next larger unit.

100	10	1	.	1	1
Hundreds	Tens	Ones	.	Tenths	Hundredths
				4	
		4			

An arrow points from the 4 in the Tenths column to the 4 in the Ones column, with the text  $\times 10$  written above the arrow.

Repeat with  $0.03 \times 10$  and  $0.003 \times 1000$ . Use unit form to state each problem and encourage students to articulate how the value of the digit changes and why it changes position in the chart.

### Problem 3

Divide copies of one unit by 10, 100, and 1000.

$$6 \div 10$$

$$6 \div 100$$

$$6 \div 1000$$

Follow similar sequence to guide students in articulating changes in value and shifts in position while showing on the place value chart.

Repeat with  $0.7 \div 10$ ;  $0.7 \div 10$ ;  $0.05 \div 10$ ; and  $0.05 \div 100$ .

### Problem 4

Multiply mixed units by 10, 100, and 1000.

$$2.43 \times 10$$

$$2.43 \times 100$$

$$2.43 \times 1000$$

MP.4

T: Write the digits two and forty-three hundredths on your place value chart and multiply by 10, then 100, and then 1000. Compare these products with your partner.

Lead students to discuss how the digits shift as a result in their change in value by isolating one digit, such as the 3, and comparing its value in each product.

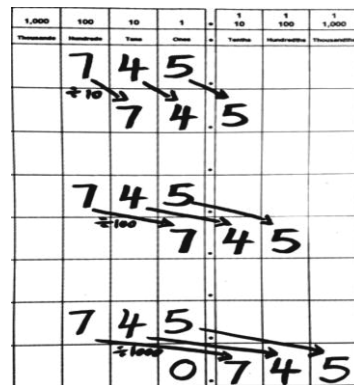
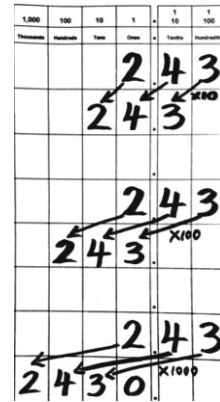
### Problem 5

$$745 \div 10$$

$$745 \div 100$$

$$745 \div 1000$$

Engage in a similar discussion regarding the shift and change in value for a digit in these division problems. See discussion above.



## Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the careful sequencing of the problem set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Assign incomplete problems for homework or at another time during the day.

## Student Debrief (10 minutes)

**Lesson Objective:** Reason concretely and pictorially using place value understanding to relate adjacent base ten units from millions to thousandths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- Compare the solutions you found when multiplying by 10 and dividing by 10 ( $32 \times 10$  and  $32 \div 10$ ). How do the solutions in these two **equations** relate to the value of the original quantity? How do they relate to each other?
- What do you notice about the number of zeros in your products when multiplying by 10, 100, and 1000 relative to the number of places the digits shift on the place value chart? What patterns do you notice?

Name Judea Date \_\_\_\_\_

1. Record the digits of the first factor on the top row of the place value chart. Draw arrows to show how the value of each digit changes when you multiply. Record the product on the second row of the place value chart. The first one has been done for you.

a.  $3,452 \times 10 = \underline{34,520}$

b.  $3,452 \times 100 = \underline{345,200}$

c.  $3,452 \times 1,000 = \underline{3,452,000}$

d. Explain how and why the value of the "5" changed in (a), (b) and (c).  
The value of the 5 in 3,452 is 5 hundredths.  
In (a), the 5 becomes 5 tenths. In (b), the 5 becomes 5 ones. In (c), the value of the 5 changes to 5 tens.  
The value keeps changing because I multiplied and made the 5 ten times, then 100 times, and finally 1,000 times greater.

COMMON CORE Lesson 1: Reason Concretely and Pictorially Using Place Value Understanding to Relate Adjacent Base Ten Units from Millions to Thousandths  
Date: 5/2/13 engage<sup>ny</sup> X.X.1

NY COMMON CORE MATHEMATICS CURRICULUM 5•1

2. Record the digits of the dividend on the top row of the place value chart. Draw arrows to show how the value of each digit changes when you divide. Record the quotient on the second row of the place value chart. The first one has been done for you.

a.  $345 \div 10 = \underline{34.5}$

b.  $345 \div 100 = \underline{3.45}$

c.  $345 \div 1,000 = \underline{0.345}$

d. Explain how and why the value of the "4" changed in the quotients in (a), (b) and (c).  
In all of the problems the "4" got smaller + smaller.  
It started out every time as 4 tens. In (a) it became 4 tenths because I divided by 10. In (b) it moved 2 places smaller because I divided by 100 - which is like dividing by 10 twice.  
In (c) it got the smallest. It moved 3 places because I divided by 1000 - which is like dividing by 10 3 times.

COMMON CORE Lesson 1: Reason Concretely and Pictorially Using Place Value Understanding to Relate Adjacent Base Ten Units from Millions to Thousandths  
Date: 4/6/13 engage<sup>ny</sup> 1.A.11



- What is the same and what is different about the products for Problems 1(a), 1(b), and 1(c)? (Encourage students to notice that digits are exactly the same, only the values have changed.)
- When solving Problem 2(c), many of you noticed the use of our new place value. (Lead brief class discussion to reinforce what value this place represents. Reiterate the symmetry of the places on either side of the ones place and the size of **thousandths** relative to other place values like tenths and ones.)

### Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NYS COMMON CORE MATHEMATICS CURRICULUM

3. The manufacturer has made 7,234 boxes of coffee stirrers. Each box contains 1000 stirrers. How many stirrers did they make? Explain your thinking and include a statement of the solution.

*Handwritten solution:*

1000	100	10	1		
7	2	3	4	0	0

They made 7,234,000 stirrers.

*Handwritten note:* It would be 7234 thousands. That's the same as 7 million 234 thousand.

4. A student used his place value chart to show a number. After the teacher instructed him to multiply his number by 10, the chart showed 3200.4. Draw a picture of what the place value chart looked like at first.

	3	2	0	.	0	4			

a. Explain how you decided what to draw on your place value chart. Be sure to include your reasoning about how the value of the digits was affected by the multiplication. Use words, pictures, or numbers.

*Handwritten explanation:* If he multiplied by 10, then the number he started with moved to the left one place value. I just moved them all back one place to the right. Also, if he ended up with 3 thousands then he started with 3 hundreds  $300 \times 10 = 3000$ .

5. A microscope has a setting that magnifies an object so that it appears 100 times as large when viewed through the eyepiece. If a tiny insect is 0.095 cm long, how long will the insect appear in centimeters through the microscope? Explain how you know.

*Handwritten solution:*

100	10	1		
0	0	9	5	0

The insect will appear to be 9.5 cm in the microscope. Because 9 hundredths  $\times 100$  is 900 hundredths. That's the same as 9 ones.

COMMON CORE Lesson 1: Reason concretely and pictorially using place value understanding to relate adjacent base ten units from millions to thousandths. engage<sup>ny</sup> 1.A.12  
Date: 6/28/13

**A**

# Correct \_\_\_\_\_

Multiply.

1	$12 \times 10 =$		23	$34 \times 10 =$	
2	$14 \times 10 =$		24	$134 \times 10 =$	
3	$15 \times 10 =$		25	$234 \times 10 =$	
4	$17 \times 10 =$		26	$334 \times 10 =$	
5	$81 \times 10 =$		27	$834 \times 10 =$	
6	$10 \times 81 =$		28	$10 \times 834 =$	
7	$21 \times 10 =$		29	$45 \times 10 =$	
8	$22 \times 10 =$		30	$145 \times 10 =$	
9	$23 \times 10 =$		31	$245 \times 10 =$	
10	$29 \times 10 =$		32	$345 \times 10 =$	
11	$92 \times 10 =$		33	$945 \times 10 =$	
12	$10 \times 92 =$		34	$56 \times 10 =$	
13	$18 \times 10 =$		35	$456 \times 10 =$	
14	$19 \times 10 =$		36	$556 \times 10 =$	
15	$20 \times 10 =$		37	$950 \times 10 =$	
16	$30 \times 10 =$		38	$10 \times 950 =$	
17	$40 \times 10 =$		39	$16 \times 10 =$	
18	$80 \times 10 =$		40	$10 \times 60 =$	
19	$10 \times 80 =$		41	$493 \times 10 =$	
20	$10 \times 50 =$		42	$10 \times 84 =$	
21	$10 \times 90 =$		43	$96 \times 10 =$	
22	$10 \times 70 =$		44	$10 \times 580 =$	

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**B**

Improvement \_\_\_\_\_ # Correct \_\_\_\_\_

Multiply.

1	$13 \times 10 =$		23	$43 \times 10 =$	
2	$14 \times 10 =$		24	$143 \times 10 =$	
3	$15 \times 10 =$		25	$243 \times 10 =$	
4	$19 \times 10 =$		26	$343 \times 10 =$	
5	$91 \times 10 =$		27	$743 \times 10 =$	
6	$10 \times 91 =$		28	$10 \times 743 =$	
7	$31 \times 10 =$		29	$54 \times 10 =$	
8	$32 \times 10 =$		30	$154 \times 10 =$	
9	$33 \times 10 =$		31	$254 \times 10 =$	
10	$38 \times 10 =$		32	$354 \times 10 =$	
11	$83 \times 10 =$		33	$854 \times 10 =$	
12	$10 \times 83 =$		34	$65 \times 10 =$	
13	$28 \times 10 =$		35	$465 \times 10 =$	
14	$29 \times 10 =$		36	$565 \times 10 =$	
15	$30 \times 10 =$		37	$960 \times 10 =$	
16	$40 \times 10 =$		38	$10 \times 960 =$	
17	$50 \times 10 =$		39	$17 \times 10 =$	
18	$90 \times 10 =$		40	$10 \times 70 =$	
19	$10 \times 90 =$		41	$582 \times 10 =$	
20	$10 \times 20 =$		42	$10 \times 73 =$	
21	$10 \times 60 =$		43	$98 \times 10 =$	
22	$10 \times 80 =$		44	$10 \times 470 =$	

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Name \_\_\_\_\_

Date \_\_\_\_\_

1. Record the digits of the first factor on the top row of the place value chart. Draw arrows to show how the value of each digit changes when you multiply. Record the product on the second row of the place value chart. The first one has been done for you.

a.  $3.452 \times 10 = \underline{34.52}$

			3	.	4	5	2
			3		4	5	2

b.  $3.452 \times 100 = \underline{\hspace{2cm}}$

			3	.			

c.  $3.452 \times 1000 = \underline{\hspace{2cm}}$

				.			

- d. Explain how and why the value of the 5 changed in (a), (b), and (c).

2. Record the digits of the dividend on the top row of the place value chart. Draw arrows to show how the value of each digit changes when you divide. Record the quotient on the second row of the place value chart. The first one has been done for you.

a.  $345 \div 10 = \underline{\quad 34.5 \quad}$

				•			
	3	4	5				
		3	4				

*(Note: In the original image, blue arrows point from the 3 in the top row to the 3 in the bottom row, from the 4 in the top row to the 4 in the bottom row, and from the 5 in the top row to the 5 in the bottom row.)*

b.  $345 \div 100 = \underline{\hspace{2cm}}$

				•			

c.  $345 \div 1000 = \underline{\hspace{2cm}}$

				•			

- d. Explain how and why the value of the 4 changed in the quotients in (a), (b), and (c).

3. A manufacturer made 7,234 boxes of coffee stirrers. Each box contains 1000 stirrers. How many stirrers did they make? Explain your thinking and include a statement of the solution.
4. A student used his place value chart to show a number. After the teacher instructed him to multiply his number by 10, the chart showed 3200.4. Draw a picture of what the place value chart looked like at first.

				●			

- a. Explain how you decided what to draw on your place value chart. Be sure to include your reasoning about how the value of the digits was affected by the multiplication. Use words, pictures, or numbers.
5. A microscope has a setting that magnifies an object so that it appears 100 times as large when viewed through the eyepiece. If a tiny insect is 0.095 cm long, how long will the insect appear in centimeters through the microscope? Explain how you know.

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Write the first factor above the dashed line on the place value chart and the product or quotient under the dashed line, using arrows to show how the value of the digits changed. Then write your answer in the blank.

a.  $6.671 \times 100 =$  \_\_\_\_\_

				.			
-----							

b.  $684 \div 1000 =$  \_\_\_\_\_

				.			
-----							