Math 131 Fall 2010 Abrams / Rossing **Exam 1** Name ______________________

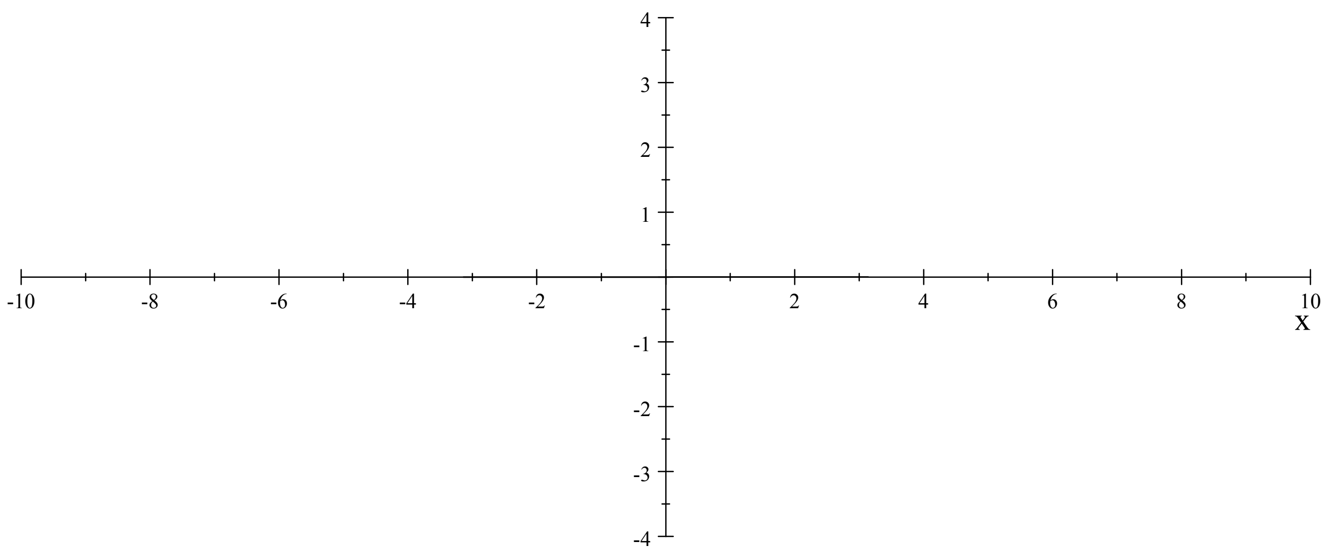
Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Only a non-graphing calculator with no QWERTY keyboard is allowed. All answers involving angles must be given in radians (degree measurement is not allowed here). Use the back of the sheet if you need more space. **Circle answers when appropriate**. Good luck !!

1. (2 pt) Find the equation of the line having slope -2 which passes through the point $(3,5)$. Give your answer in $y = mx + b$ form.

2. (3 pt total) (a) Solve for x: $\sin(x) = \frac{\sqrt{3}}{2}$

(b) $\sin^{-1}(\frac{\sqrt{3}}{2}) =$

3. (3 pt) Carefully draw the function $y = 2\sin x$ on these axes. Label each of the x-intercepts with exact values involving π.



4. (1 pt each) (a) Simplify to an expression of the form e^a : $\frac{e^{-2}e^{7}}{e^{5}e^{-3}}$

(b) $\log_2(16) =$

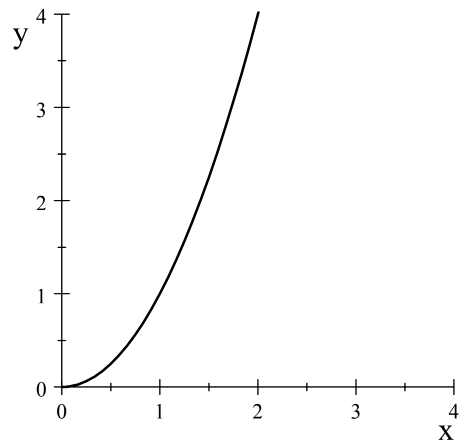
5. (2 pt) Find the inverse function for the function $f(x) = x^3 + 7$.

6. (2 pt) If $f(x) = 3x + 1$ and $g(x) = x^2 + 4x$, find and simplify $g \circ f(x)$.

7. (4 pt total) The graph of the function f is shown here. (The domain of f is $\{x \mid x \geq 0\}$.)

(a) What property of f ensures that f has an inverse function?

(b) On the same axes, sketch a graph of $f^{-1}(x)$.



8. (1 pt each) Short answer

(a) For any angle α, what is the fundamental summation equation which relates $\sin(\alpha)$ and $\cos(\alpha)$?

(b) Compute the exact value of $\tan(\frac{\pi}{3})$.

(c) Find the exact value of $\ln(e^2)$.

(d) Find the exact value of $\cos(\cos^{-1}(\frac{1}{3}))$

9. (2 pt) Give PRECISELY the meaning of the statement: $\lim_{x \to a} f(x) = L$.

10. (1 pt each) (a) (Circle the correct answer): The statement $\lim_{x \to a} f(x) = L$ intuitively means:

i. as the values of x get closer and closer to a, then eventually there is an input value whose output value equals L.

ii. All of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

iii. the values of $f(x)$ never equal L as the values of x get closer and closer to a.

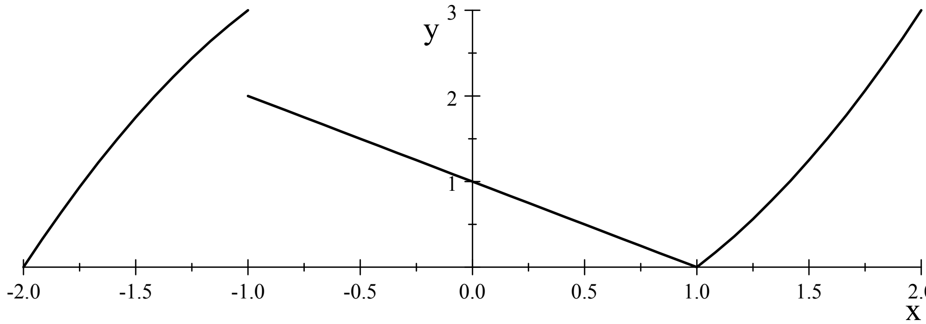
iv. Some of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

(b) **T** **F** For every function f, if $f(a) = L$ then $\lim_{x \to a} f(x) = L$.

(c) **T** **F** For every function f, if $\lim_{x \to a} f(x) = L$ then a is in the domain of f.

(d) The idea / concept on which all of calculus is based is the notion of ______________.

11. (1/2 pt each) Using the graph of the function f given here, find the indicated limits.

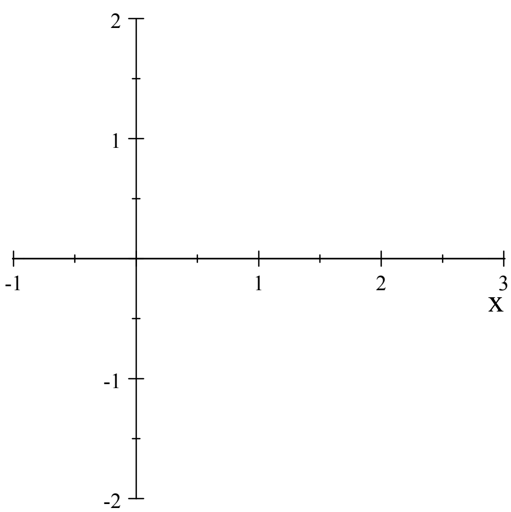


(a) $\lim_{x\to -1^-} f(x) =$ (b) $\lim_{x\to -1^+} f(x) =$ (c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 0^+} f(x) =$

(e) $\lim_{x\to 1^+} f(x) =$ (f) $\lim_{x\to 1^-} f(x) =$ (g) $\lim_{x\to 1} f(x) =$ (h) $f(0) =$

12. (4 pt) On the left hand axes below for this problem, sketch CLEARLY a graph of a function f which has: domain $-1 \le x \le 3$; $\lim_{x\to 0^-} f(x) = 1$; $\lim_{x\to 0^+} f(x) = -1$; $\lim_{x\to 2^-} f(x) = 0$; $\lim_{x\to 2^+} f(x) = 1$; $f(2) = 0$; and $f(0) = 1$. (Use open circles as appropriate.)

13. (2 pt) On the right hand axes below for this problem, sketch CLEARLY a graph of a function f which has: domain $-1 \le x \le 3$; $\lim_{x\to 1} f(x) = 2$; and for which 1 is not in the domain of f. (Use open circles as appropriate.)



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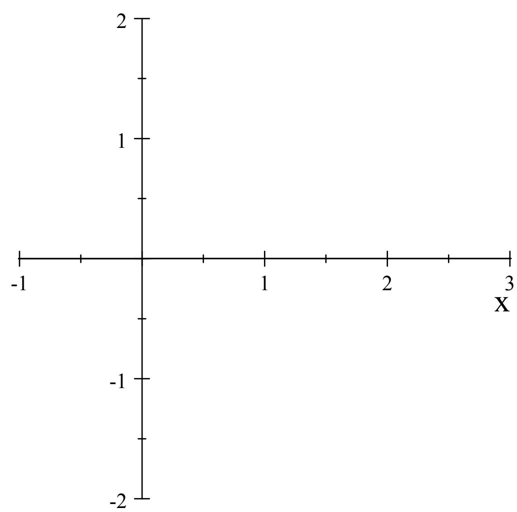

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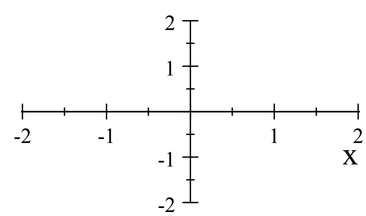
14. (2 pt) Sketch clearly the graph of the function $f(x) = \frac{|x|}{x}$.



(Use open circles as appropriate.)

15. (6 pt total) Find the indicated limit. **Use the most informative of the phrases 'does not exist', ∞, or $-\infty$ whenever appropriate**. Remember to circle your answer.

(a) $\lim_{x\to -1} \dfrac{x^2+5x+4}{x+1} =$

(b) $\lim_{x\to -1} \dfrac{x^2-3x+4}{x+3} =$

(c) $\lim_{x\to\infty} \dfrac{4-x-x^2}{9+3x^2} =$

.

(d) $\lim_{x\to 0} \dfrac{\sin(x)}{2x} =$

.

(e) $\lim_{x\to 4} \dfrac{1}{(x-4)^2} =$

.

16. (2 pt total). (a) Suppose $f, g,$ and k are functions which have $f(x) \leq g(x) \leq k(x)$ for all values of x. If $\lim_{x\to a} f(x) = L = \lim_{x\to a} k(x)$, then

$$\lim_{x\to a} g(x) = \_\_\_\_\_\_$$

(b) What is the name of the result you used to get your answer to part (a)? (Hint: there are two possible correct answers.)

17. (1 pt) **T** **F** If f is any function with domain $[a,b]$, and $f(a) < 0$, and $f(b) > 0$, then $f(c) = 0$ for some value c in $[a,b]$.

18. (4 pt each) Compute the indicated limit. **Show all your work**, **justify each step**, **and make sure your notation is clear and precise**. Remember to circle your answer.

(a) $\lim_{h\to 0} \dfrac{(-4+h)^2+7(-3+h)+5}{h} =$

(Problem 18, continued) (b) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} =$

19. (6 pt total) **Make sure to include the correct units in all parts of your answers to this question**. The position function of an object is given by $s(t) = 110t - 16t^2$, where $s(t)$ is in feet, and t is in seconds (In fact, $s(t)$ gives the distance above the ground, after t seconds, of an object which is thrown directly upwards at an initial speed of $110 ft/\mathrm{sec.}$)

(a) Find the average velocity of the object for the time period beginning when $a = 2$ and lasting

(i) 0.1 second

(ii) 0.01 second

(b) Compute the limit $\lim_{h \to 0} \frac{s(2+h) - s(2)}{h}$.

(c) Interpret your answer to part (b) in real-world terms.