Annotated Bibliography

Applebee, Arthur N. (1984). Writing and Reasoning. Review of Educational Research, 54(4), 577-596. *This is a synthesis of research on writing as a means of learning. Some results that are noted are that the amount of manipulation increases long-term memory, the topic written about it remembered more deeply (sometimes to the exclusion of other things) and students excel at different types of assignments depending on how they mentally organize the material.*

Azzolino, Aggie (1990). Writing as a Tool for Teaching Mathematics: The Silent Revolution. In the 1990 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics. *This chapter notes a variety of types of ways writing can be used, and gives a number of examples of sentence length assignments.*

Beidleman, James, Jones, Doug, and Wells, Pamela (1995). Increasing Students' Conceptual Understanding of First Semester Calculus Through Writing. Primus, V(4), 297-316. *This article gives some examples of writing examples in calculus and student samples. Also included is the guide the authors give to their students on how to write in calculus.*

Buerk, Dorothy, ed. (1994). Empowering Students by Promoting Active Learning in Mathematics. Reston, VA: National Council of Teachers of Mathematics. *This book contains a number of examples of how teachers used writing in their classroom, including Helen Gibson's description of the Math Metaphors assignment.*

Borasi, Raffaella and Rose, Barbara J. (1989). Journal Writing and Mathematics Instruction. Educational Studies in Mathematics, 20, 347-365. *This article relates the use of journals in a college Algebra course as a means to have students reflect on the content of the course and share their feelings.*

Clarke, David J., Waywood, Andrew, and Stephens, Max (1993). Probing the Structure of Mathematicsal Writing. Educational Studies in Mathematics, 25, 235-250. *This research deals with the long-term reflective journal use of secondary students and finds a stronger correlation between depth of reasoning and how long students have used a journal than between depth of reasoning and student age.*

Connolly, Paul and Vilardi, Teresa, eds. (1989). Writing to Learn Mathematics and Science. New York: Teachers College Press. *This volume contains a number of examples of writing use, mostly at the college level, and some general expositions on the philosophy of writing to learn.*

Countryman, Joan (1992). Writing to Learn Mathematics. Portsmouth, NH: Heinemann Educational Books, Inc. *This book is intended for K-12 mathematics and addresses some commons types of writing and issues of evaluation.*

Ehrich, Patricia (1991). Cognitive process writing in mathematics : a middle school study. MA Thesis, Cornell University. *In this study, the author concludes that writing even after a solution is reached confirms and deepens student understanding.*

Emig, Janet (1977). Writing as a Mode of Learning. College Composition and Communication, 28, 122-127. *Emig is one of the originators of the writing to learn movement.*

Gopen, George D. and Smith, David A. (1989). What's an Assignment Like You Doing in a Course Like This?: Writing to Learn Mathematics. *Gopen and Smith highlight a few questions that help students to write better and clarifying their reasoning as they write. Most of these questions center on thinking about how a reader will understand what is written.*

Hufferd-Ackles, K., Fuson, K.C., and Gamoran, Sherin M.(2004), Describing Levels and Components of a Math-Talk Learning Community, *Journal for Research in Mathematics Education*, 35(2), p. 81-116. *This article describes a categorization of different levels of classroom discourse. While mainly aimed at spoken discourse, some of the ideas are transferable to written discourse.*

Meier, John and Rishel, Thomas (1998). Writing in the Teaching and Learning of Mathematics.: The Mathematical Association of America. *This book contains ideas for using writing, most of which were used in an introductory college math course for students who do not like math.*

Miller, L. Diane (1991). Writing to Learn Mathematics. Mathematics Teacher, 84(7), 516-521. *This article gives a variety of ideas for how writing can be used in mathematics, as well as thinking about some of the practicalities involved.*

Price, J.J. (1989). Learning Mathematics Through Writing: Some Guidelines. The College Mathematics Journal, 20(5), 393-401. *This includes some guidelines used for writing by a college professor.*

Shepard, Richard G. (1993). Writing for Conceptual Development in Mathematics. Journal of Mathematical Behavior, 12, 287-293. *This article contains some ideas about how different types of assignments will be correlated with different types of assignments.*

Shield, Mal and Galbraith, Peter (1998). The Analysis of Student Expository Writing in Mathematics. Educational Studies in Mathematics, 36, 29-52. *This article discusses one method to analyze student writing for 8th grade students as a prelude to further research on writing in mathematics.*

Sterrett, Andrew, ed. (1990). Using Writing to Teach Mathematics.: The Mathematical Association of America. *This volume contains many wonderful examples of using writing in mathematics classrooms.*

Whitin, Phyllis and Whitin, David J. (2000). Math is Language Too: Talking and Writing in the Mathematics Classroom. USA: National Council of Teachers of English. *This book, although focused on writing mathematics in elementary classrooms, highlights many important mathematical concepts that writing supports, such as naming concepts and providing multiple solutions.*

Wolfe, Patricia. (2001) Brain Matters: Translating Research into Classroom Practice. Alexandria, VA: Association for Supervision and Curriculum Development. *This book gives a very accessible look at different factors that may influence students memory, and uses writing as one example.*

Zinsser, William (1988). Writing to learn. New York: Harper & Row, Publishers, Inc. *This book is the standard source for the ideas of writing to learn. It contains examples of writing assignments in many different subject areas. It is interesting that there are very few writing assignments in the sciences and almost none in math that are listed.*

A Classification of Secondary Mathematics Writing Tasks

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Presentation at the Joint Mathematics Meetings in the MAA Session on Getting Students to Discuss and to Write about Mathematics, January 13, 2006

This presentation is based on my Masters thesis in Education, the whole text of which may be found at http://www.math.cornell.edu/~kacam/thesis.htm

List of Categories

1. Teacher Intentions and Goals	
1	**Intended Area of Change/Purpose**
2. Assignment	
Practical Product Specifications	
2.1.1	**Writer**
2.1.2	**Collaborators**
2.1.3	**Audience**
2.1.4	**Reader**
2.1.5	**Sources**
2.2.1.1	**Units of Writing**
2.2.1.2	**Length Restrictions**
2.2.2	**Grammar and Spelling Requirements**
2.2.3	**Breadth of Topic**
2.2.4	**Use of Visuals and Graphics**
2.2.5	**Multiplicity of Answers and Methods**
2.3	**Level of Student Manipulation**
2.4	**Student Choice**
Theoretical Product Specifications	
2.5	**Function of Writing**
2.6	**Bloom's Taxonomy**
2.7.1	**Level of Language**
2.7.2	**Aspects of Mathematics**
2.8	**Connections**

Process and Context	
2.9.1	**Groupings**
2.9.2	**Sequencing of Activities**
2.10.1	**Modifying Writing Within the Same Structure**
2.10.2	**Adapting Writing into a New Structure**
2.11.1	**Task Frequency**
2.11.2	**Importance/Centrality of Assignment Type**
2.11.3	**Overlap of Material with Other Assignments**
2.11.4	**Placement in Learning Sequence**
2.11.5	**Relationship Between Words and Symbols**
2.12.1	**Mode of Assignment**
2.12.2	**Modeling of Assignment**
2.13.1	**Place of Completion**
2.13.2	**Time to Complete Assignment**
2.13.3	**Amount of Classroom Time**
2.13.4	**Expected Time at Home**
2.13.5	**Timing Within Class Period**
2.14	**Relevance of Task Context**
3. Response & Assessment	
3.1.1	**Teacher Comments**
3.1.2	**Feedback Frequency**
3.1.3	**Peer Response**
3.1.4	**Collection of Writing**
3.1.5	**Availability to Students**
3.2	**Use of Completed Assignments**
3.3.1	**Standards for Grading**
3.3.1c.1	**Content vs. Expression**
3.3.1c.2	**Standards for Justification**
3.3.2	**Grading Method**
3.3.3	**Grader**
3.3.4	**Percentage of Grade**
3.3.5	**Student Awareness of Assessment Parameters**

Samples of Availability to Students (3.1.5): excerpts from Assignment Summaries

Discarded by teacher:*Class Summary (Rosenberg)*
Students are occasionally asked at the end of class to write down one thing they learned that day in order to bring closure. This is not corrected or graded, and is used to help the teacher assess student learning and identify student misconceptions. This assignment is used to help focus students at the end of class when time is available.

Kept at discretion of student: *Bingo Sentences (Vertuno)*
Sixth grade students are given two 3x3 grids, one with mathematics terms and the other with numbers. Students choose one term and one number and then write a sentence that includes both and makes a true mathematical statement. For example, if they choose the number 6 and the word "multiples" their sentence might be "Six multiples of 3 are 3,6,9,12,15, and 18." The goal of this assignment is to familiarize students with the vocabulary while having some fun. The teacher does not grade these sentences, but assesses what students know by their choice of words and the how they are used.

Compiled (by student): *Notebooks (Adams)*
Students are required to keep a notebook with four sections: Vocabulary, Problem of the Day (POD), Classwork and Summaries. In the vocabulary section, the teacher assigns words that the students need to look up in the glossary and copy into their notebooks…

Compiled with reflection (by student): *Portfolios (Cabral)*
At the end of each unit, high school students choose 3-5 items of work from the unit that represent each of the following: (a) the main ideas from the unit and (b) their best work from the unit…These items can be homework, quizzes, notes, classwork or tests. They also need to write a cover letter for the unit which (c) highlights the main ideas and skills from the unit, (d) justifies their choice for each assignment chosen for (a), and (e) gives their personal reaction to the unit: likes/dislikes, speed, ease/difficulty. Credit is lost in the respective categories if not all the important concepts are covered, if they do not describe each item included separately, or if they do not answer the question completely.

Compiled with reflection (by teacher): *Problems of the Week (Stofferan)*
Seventh grade students were given a POW on Monday and expected to turn in a response to the problem on Friday. The intention of this task was to give students a chance to write about their problem solving process, thereby allowing metacognition, and to provide students with an opportunity to find multiple methods for solving the problem. Students were asked to include four parts in their responses to the POWs: a problem statement, a plan, work, and their answer. This method of response is adapted from Tsuruda (1994). After two weeks, Stofferan shared examples of student work, emphasizing "clear and descriptive explanations" (Stofferan, 2005, p.36) and multiple problem solving methods. Each student's POWs were collected in a folder, and this whole folder was commented on and returned to students.

Questionnaire on Availability to Students (3.1.5)
56. Are completed written products available to students at a later date? Choose the ONE best answer.

3.1.5/

No; products are discarded…….... a
Possibly; returned to students, who may discard……..……. b
Products are saved in some form of compilation (see below). c
Products are compiled and students are required to reflect on them……(see below)……... d

IF COMPILED:
Who compiles the written products? Choose ONE answer.
Student…….... a
Teacher……. b

Why do you want to use writing?

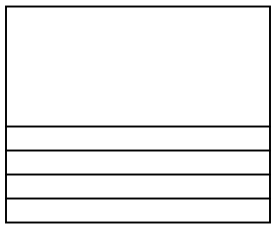
Below find some reasons to use writing that teachers gave and correlated descriptions of sample assignments. (Fuller descriptions of many of these assignments can be found at www.math.cornell.edu/~kacam/thesis.html in Appendix A)

1. Intended Area of Change/Purpose		
1.1	Teacher assessment	
1.1.1	Assess instructional program	
	Student Evaluations	16.4; 20
	Outcome of content area tasks	44
1.1.2	Assess individual student learning	4
1.1.3	Assess student attitudes and beliefs	
	Math Biographies	6; 46.1
	Math Metaphors	22
	Questions about certain topics	16.4,5; 23
1.2	Student attitudes	
1.2.1	Student-teacher relationships	
	Provide open forum for discussion	16
	See 1.1.3	
1.2.2	Attitude toward mathematics	
	Mathematician/Statistician Biographies	11
	Variety of assignments	
1.2.3	Beliefs about mathematics	
	Ask students consider beliefs	22; 46.3
	Emphasize multiple solutions	44
	Importance of definitions	28; 32
	Connect to careers	35
1.3	Student self-evaluation	
1.3.1	Study habits	16.3; 20
1.3.2	Goal setting	16.3; 20; 46
1.3.3	Metacognition	
	Test corrections	40
	Reflection on thought processes	26; 44

1.4	Mathematical content	
	LEARNING PRINCIPLES	
1.4.A	Multiple Intelligences	
	Linguistic	ALL
	Kinetic	ALL
	Visual	43
	Intrapersonal	44
	Interpersonal	25
1.4.B	Increasing Memory	
	Consolidation: summaries	21; 30
	Increasing meaning/relevance	22; 35
1.4.C	Unique qualities of writing	
	Permanence - Compilation	32; 36; 44
	Increased need for precision	30; 44
1.4.D	Externalization of thought	
	Freewrites/prep for discussion	36
	Revision	25.1; 40
	AREAS	
1.4.1	Vocabulary	
	Flipbooks	1
	Sorting & explaining activities	2; 28
	Bingo sentences	4
	Explain ___ in 1-2 sentences	
	Write out definitions	32
	Give examples of vocabulary	
	Make up a crossword with clues	
1.4.2	Procedural understanding	
	Explain a process	47.1
	Work an example with explanation	19; 25.1
	Make up examples and work them	41

	1.4.3	Conceptual understanding	16; 47.2; 52
		Explain the meaning of something	28;
		Booklets/scrapbooks	14; 15; 41
		Compare/contrast	
		Projects	8; 10; 13; 42
	1.4.4	Metacognition	44
		See 1.3.3	
	1.4.5	Logical argument or proof	
		Generalizing patterns/conjecturing	7
		Explain reasons for a result	36; 44
		Read other work & critique	12; 45
1.5	Connections		
	1.5.1	To experience	
		Student-created problems	40
		Applications of concepts	47.3
		Careers	35
	1.5.2	To other mathematical content	
		Concept maps	18
		Review/organization of material	23
		Booklets/scrapbooks	41
		To previous knowledge	44; 47.4
1.6	Communication ability		
	1.6.1	"People-talk": normal speech	
		Letters to a non-class person	
		Personal understandings of content	47; 52
	1.6.2	"Math-talk": disciplinary specific standards	
		Writing for class peers	45
		Publish in student contests	
		Precision & clarity	6; 10; 12; 13
	1.6.3	Standardized test-preparation	1.2; 5
	1.6.4	Writing Across the Curriculum	3; 25; 28

1.Flip-books (Vertuno)



Students make a flip-book which serves as a reference for related vocabulary terms. A number of pages of the same width but different lengths are stacked so that each extends an inch beneath the previous. These exposed sections each have a term written on them and you can flip it open to draw a picture that goes with the term or to include further explanation. The last page can be used for student reflections on the terms or what the student has learned.

1.1: *Prefixes/Polygons*:

After 6th grade students have (a) sorted many terms with number prefixes (tri-, quad-, oct-, and hex-) and (b) cut out convex and non-convex polygons and named them with the appropriate terms (triangle, quadrilateral, hexagon, octagon), they make a flip book using the same prefixes to make words with –ped or –cycle and draw corresponding pictures for each one of them.

1.2: *Units of measure*:

To help prepare 8th grade students for the Minnesota Basic Skills Test (MBST) by giving them benchmarks for different measures, students make a flipbook with the terms millimeter, centimeter, decimeter, and inch and give examples of the objects with these measurements drawn in their actual size.

2. Triangle Sorting (Vertuno)

Sixth grade students are given a colored sheet of paper with different triangles on it. Students are asked to cut them out and sort them into three categories and glue them on another piece of paper in these categories. Then students write a sentence to describe the qualities that each group of triangles shared. It is expected that this is a first try to get students to recognize the difference between equilateral, isosceles and scalene triangles and describe those categories; the vocabulary is supplied after the activity.

3. Equilateral Triangle Factory (Vertuno)

After exploring properties of equilateral triangles, 6th grade students were given 10 minutes to respond to the following prompt: "Pretend you are the owner of an equilateral triangle factory. Write a letter to an inspector telling him what an equilateral triangle looks like so he can make sure the triangles made in the factory are equilateral. Write about the sides, angles and symmetries of equilateral triangles." They were given the starter: "My name is _____. I own the _____ Equilateral Triangle factory. All the equilateral triangles made in our factory…" They were expected to look back over their work to make sure it was correct, making minor revisions as necessary. The goal of this assignment was to familiarize students with properties of equilateral triangles.

4. Bingo Sentences (Vertuno)

Sixth grade students are given two 3x3 grids, one with mathematics terms and the other with numbers. In one example, the terms were multiple, factor, perfect (as in perfect numbers), proper factor, common factor, divisible by, common multiple, prime, & composite. Students choose one term and one number and then write a sentence that includes both and makes a true mathematical statement. For example, if they choose the number 6 and the word "multiples" their sentence might be "Six multiples of 3 are 3,6,9,12,15, and 18." The goal of this assignment is to familiarize students with the vocabulary while having some fun. The teacher does not grade these sentences, but assesses what students know by their choice of words and the how they are used.

5. Test Preparation (Vertuno)

Eighth grade students are given a worksheet that has a number of word problems (similar to those they will find on the Minnesota Basic Skills Test) as well as a set of answers to choose from (16 answers, 9 problems). They are also given a second worksheet to help them think about the problems. The goal of this activity is to help them learn to think about what the problems mean, and choose appropriate answers. They also do some of the problems. Partly, this is teaching test taking skills and partly learning how to think about word problems.

First of all, students are asked to finish sentences about the words in the problems to make sure they understand what the problem means. For example, in a problem where students want to find average attendance at a county fair, they are asked to complete the sentence: "In this problem, attendance means_____." Then, for each problem students are asked to pick two answers from the answer bank that are reasonable answers before they solve the problem. The answer bank has answers that include numbers without units, dollar amounts, and degrees. This helps them think about the type and magnitude of answer each problem expects. Finally they are to solve 5 of the problems that they think are easier and find the answers.

They are given half credit for showing all their work in these solutions and full credit for correct solutions with all work shown.

6.Autobiography (Favata)

This is the first assignment in an AP Statistics class. In one page (double spaced) each student is to write four paragraphs which (a) tell something about his- or herself, (b) share a memory from a previous mathematics class or about a mathematics teacher, (c) tell about their plans for college and (d) explain why they are taking AP Statistics. This assignment is used to "get to know a little about my students: their writing abilities, creativity, as well as their lives outside the classroom" (personal communication, January 31, 2005). Limiting students to one page emphasizes clarity and concision, goals in students' later writing for the class.

7. Toothpicks I & II (Favata)

These assignments are used in the first week of a Geometry class. The tasks are intended to emphasize problem solving and the possibility of multiple solutions to a mathematics problem. The structure of the assignment has students work in groups so that they talk about mathematics to each other and then individually write up and explain their work.

In Toothpicks I, students need recognize a pattern of how many toothpicks are necessary to make n squares in a row. They are given an example, and then fill in a chart with the number of toothpicks needed to make 1,2,3,4,5, and 6 squares. Next they are asked to find how many toothpicks would be needed to make a row of 10 squares, and finally to determine the number of toothpicks needed for 100 squares. Students are to explain their answers to the last two problems, and prompted by "I want sentences and your math."

In Toothpicks II, students solve a problem where they start with three piles of toothpicks, containing 11, 7 and 6 toothpicks respectively, and want to even them out with exactly three moves that must follow certain rules. This is intended to encourage students to read carefully, since one of the course goals is to work on interpreting problems. They figure out how to do this in groups, and are responsible for making sure that everyone in their groups understands how to do it. Then they must each write a paragraph for homework explaining how to solve the problem and draw pictures for each move. These assignments have been used together in the same 39 minute meeting, with the students writing the responses for homework.

8. Designing a Food Container (Favata)

This project is from Holt, Rinehart & Winston's *Geometry*, p. 493. Students were given the choice between this and two other projects as an extra credit option at the end of the quarter for a Geometry class. The project is intended to help students think about minimizing surface area of solids with given volume. They imagine a product they want to sell and choose 3 of 4 solids (prism, pyramid, cylinder, or cone) as possible packaging shapes. They are to create multiple nets for each shape (with tabs and known dimensions) so that the containers can be assembled and will each have volume of 100 cubic inches. Then they are to compare the surface area of the different containers and draw conclusions about which shapes take the least material to make. Finally, they choose an optimal container, considering a variety of factors including the amount of material required. This is graded with the majority of the credit going to choosing the container and finding correct surface areas. The students who did this project wrote it up step by step with sketches and explanations.

10. Olympic Games: analysis of long-jump information (Favata)

This assignment is from Bock, Velleman & DeVeaux's *Stats: Modeling the World* (2004) and is handed out to students on a sheet of paper. AP Statistics students are given data on the gold medal long-jump distances in all the Olympic Games. They are asked to individually (a) do numerical and graphical analyses, (b) discuss the trend in long jumps based on a linear model, (c) explain the decisions made in creating the model with historical analysis of gaps in the data and departures from the trend, and (d) predict the distance that will win in the 2004 Games, including their faith in that prediction. This assignment was done after these topics had already been covered in lecture and in group activities. The rubric specifies what is desired, defining graphical analysis as a scatterplot, and numerical analysis as finding a linear model, defining variables, calculating r and r^2, and a description of the direction, form, scatter and meaning of r^2. The latter description is 5 of 12 points in this section. In terms of (c), they were expected to note gaps for the World Wars, and big jumps between the first two Olympics and in 1968, as well as their analysis of linearity. Finally, there were points for the prediction. The analysis was more than half the points. Some bonus points were available for work beyond what the project asked for; for example, some students searched for the athlete who was the outlier. The assignment was graded out of 30 points.

This assignment is part of a series of assignments, so students have an understanding of the requirements for the write-up. Students do a similar project at the end of most chapters, for a total of 7-10 assignments of this

type throughout the course. Projects like this make up 25% of the course grade. Students are expected to be clear and concise and answer all the questions, as well as including all the graphs and data and supporting all their conclusions. They have seen what is acceptable from previous assignments and have the opportunity to ask any questions during the week they are doing the project or after the project is graded. They also have a sample grading rubric they can use as a checklist. The writing affects the grading only as a tool for communicating. If the statistics is done accurately and in context and communicated clearly, this is all that matters. Favata reports that in this assignment, students have already had enough experience in the class so that "they are handing in projects that are well written and edited" (personal communication, July 13, 2005). Students are expected to recognize that communication is a major part of the work of statistics.

Students are given freedom regarding the form in which they write the projects; for example, they can add extra graphs or write it in the form of a newspaper article. The length is determined by the information that they need to communicate.

Each paper is commented on extensively in accordance with the rubric in the areas of content and clarity. The grading is intended to model an AP grader's rubric. The comments are intended to help students correct their mistakes and misconceptions.

11. Statistician Biographies (Favata)

This assignment is used in an AP Statistics class after the AP test as one of the final projects, together with a group project [13] and an article critique [12]. Students write a biography of a statistician, either by interviewing a statistician (they are near a GE Research & Development Center) or researching a famous statistician. They choose the statistician and must OK it with the teacher, who ensures that each statistician is researched by at most one person. The report is to include personal and professional biographical information, and then focus on the statistical work. They are to describe the theory of the work, explain it with examples, show applications of it, and describe its implications. Some points (25%) are given for the quality and clarity of explanations and for citing sources (40 points total). Length is not specified.

12. Statistics Article Critique (Favata)

This assignment is used in an AP Statistics class after the AP test as one of the final projects, together with a group project [13] and the biography of a statistician [11]. Students are to choose a statistical article from a professional journal. They are to summarize the study and critique the methods used. Specifically, they are asked to define the sample and variables, note the date and location of the study, define the objective of the study, describe the statistical methods used, discuss the quality of the statistical methods and note any limitations to the study. 20% of grade is due to choosing an appropriate article and turning it in, 60% of the grade is reporting on the article, and 20% is the discussion of quality and limitations, for a total of 50 points. Length is not specified.

13. Statistics Final Group Project (Favata)

This assignment is used in an AP Statistics class after the AP test as one of the final projects, together with an article critique [12] and the biography of a statistician [11]. Students work in groups of two to design, conduct and analyze an experiment. The teacher must OK their hypothesis test question, and then is consulted throughout the project. They make both oral and written reports. The oral reports are 5-7 minutes each and the written reports vary in length. They are written in the form of a statistics article with an abstract, null hypothesis and alternative hypothesis, data, graphs, levels of confidence and significance, etc. It is intended to be a capstone project and is given 60 points.

Sample projects:

- whether there is a significant difference between the level of political knowledge in students and teachers
- whether there is a difference in tasting Dr. Pepper vs. Diet Dr.Pepper
- whether one weighted side of a quarter could affect the probability or outcome of heads or tails when the quarters are spun
- whether the distribution of colors in bags of Skittles are as the company claims (expected percentage was 20% per color)
- whether there is a relationship between time of swimming pool use and the age of the patron
- determine whether music and gender are independent

14. Distributions Scrapbook (Favata)

This is an ongoing project in the first quarter of AP Statistics. Students collect distributions from various publications, and then choose 10 of them to present in scrapbook format or on a piece of posterboard (distributions are mounted and discussions are word-processed in both cases). They choose a large variety of distributions, include a statistical discussion of each (shape, center, spread, numerical summary data), and discuss the graphical excellence of the distribution according to given standards. The work is assessed based on the number and variety of distributions (10 points), descriptions of the statistical properties of each distribution (20 points, 2 points each), the discussions of graphical excellence (10 points, 1 point each), and adherence to the format and discussion of flaws of (at most 4) bad graphs (10 points). As a class they have also discussed clarity, concision, and the context of the work, as well as graphical excellence.

15. Constructions Scrapbook (Favata)

Geometry students create a booklet of 10 constructions, one per page, after having studied constructions for a week. They choose which 10 constructions they want in their booklet. For each construction, they are required to do the construction, label it and then carefully write out the steps to do the construction. This can be either hand-written or word-processed. Then they are required to make a cover for their project. Five points are given for each construction, 1 for the name, 2 for the construction and 2 for an explanation, so the whole assignment counts for 50 points. Accuracy is the main concern in all these cases.

16. Journals (Danforth)

As part of a community college Calculus class (which has four lectures and one computer lab each week), students are asked to write weekly journals. The journals are handed out Monday and students turn them in Friday. It is similarly used in Math Skills (arithmetic and basic algebra) and Intermediate Algebra classes. Each journal is worth 20 points: 70% content and 30% presentation (neat, understandable by classmates, complete sentences). Expectations differ slightly by the course. Students are allowed to work together, but the finished product must be individual. The journals usually incorporate a problem, a journal topic and an opportunity to write comments. Explanations of problems are expected to be easily understandable to other people in the class. The journal topics frequently focus on their work habits and feelings about the class. Overall, the journals are used to push students to think, assess their understanding, and promote student-teacher relationships.

16.4: *Calculus #12*: Students are asked to find the volume of a solid of revolution formed by rotating an area enclosed by two functions around the x-axis. This is a standard question so the students can focus on the journal topic, which requests students to evaluate the course and share one aspect of the course they would change and how. The instructor summarizes the student comments in class to convey the importance he places on these comments, and occasionally adjusts the class accordingly.

16.5: *Math Skills #1*: Students are given a problem that can be solved by algebra or by trial and error. The problem has a personal touch in that it talks about a hypothetical situation involving the professor. As the first journal of the year, this problem is to help assess whether students can find a solution and express their reasoning. The instructor can gauge student problem solving ability and begin to develop the ability to communicate their reasoning. There is also a journal topic where students are asked to describe their feelings about how they do in mathematics. This is an important opportunity for these students, usually unsuccessful in mathematics, to share their fears and experiences. The instructor responds, sometimes at length, in order to show care and understanding to the students, and provide trust to start working on their mathematics.

16.6: *Math Skills #6*: Students are given a word problem relating a few different quantities and asked to write algebraic expressions for different aspects of this problem. This builds to writing an equation that students must solve to determine the answer to the problem. In this task, students are asked to confront their fear of word problems. They are given some structure to begin the process of translating words into algebra and communicating their solutions.

16.7: *Math Skills #14*: This is the last entry of the semester for all courses. Students are asked to share the most important mathematical and non-mathematical things they learned during the semester. This is considered a central formative activity.

18. Concept Map (Danforth)

Students are to create a concept map of the main ideas of calculus. This is intended to help students organize their knowledge and help them recall details, as well as communicate how the students connect the different concepts in the class. The concept maps are graded based on the variety of

details, the number and complexity of links, and the integration of the various pieces with each other and with previously known material. Also noted are obvious omissions of concepts or connections.

19. Unit 4 Investigation (Rosenberg)

This assignment was intended to help students learn about modeling football kicks and springboard dives with mathematical functions. It is based on the Unit 4 investigation in *Contemporary Mathematics in Context: Course 2* (2003). Students work in expert groups for about 20 minutes to understand one of the parts of the investigation, working together to answer given questions. Then they work in regular groups at least 20 minutes to explain these answers to each other. Finally, students are given two days to write individual reports. These reports focus on two football kicks and two springboard dives; for each situation, students (a) draw and label graphs, (b) write equations explaining the source of specific numbers, and (c) give a table of values with 8 equally spaced x-values. For one of the situations, they are also to find what the maximum height is and when the ball/diver returns to the surface, explaining their answers completely. All explanations are expected to be in complete sentences. Tables and equations are each given one point, and graphs are given one point each for correctness and one point overall for labeling and completeness. One point is given for each of the maximum height and time of return to the surface, and two points for a complete explanation. Finally, one point is available for visual appeal. Students have this rubric as they complete the assignment. The writing assignment is used as a follow-up to the investigation, where the learning is intended to take place, and is supposed to (1) help them focus on the important mathematics concepts, (2) keep them accountable for the investigation, and (3) help the teacher assess their understanding of the investigation.

20. Student Self-evaluation (Rosenberg)

At the transition between marking periods, students do a self-evaluation. After noting what their quarter grade, exam grade and last unit grade were, each student grades themselves on effort, homework, tests and quizzes, behavior, participation, attitude and overall performance. Next they are asked to write three things they think they did well, and two areas they can improve, as well as a plan to help improve in these areas. Then they are asked to evaluate the class by naming three things they enjoyed and two things that could be improved and how they could be improved. This does not have any bearing on their grade.

21. Class Summary (Rosenberg)

Students are occasionally asked at the end of class to write down one thing they learned that day in order to bring closure. This is not corrected or graded, and is used to help the teacher assess student learning, and possibly identify student misconceptions. This assignment is used to help focus students at the end of class when time is available.

22. Metaphors (Gibson)

(Buerk, 1994, p.7-12)

This assignment is used to get insight into students' conceptions of and attitudes toward mathematics. First, students list words and phrases they would use to describe mathematics to a friend. Then they imagine themselves doing mathematics and list feelings that come to mind. Thirdly, students list objects that they think mathematics is like. Finally, students are prompted to read over their lists and choose a word from the third list that best describes what mathematics is like for them. They are asked to finish the sentence: "For me, math is most like a(n) _____." and then to write a paragraph explaining their choice. Students were given 5-7 minutes to create each list, and 15 minutes for the paragraph. Students could be given more support through prompts after they have written the three lists. These prompts ask them to think what mathematics would be like if it fell in a certain category, such as food, weather, color, etc. (Buerk, 1994, p.12) and add to their lists. Then students would read over their lists and write the paragraph as above. These papers could be submitted with or without names.

There are a number of variations on this assignment, depending on the teacher's goals. If the teacher wants to gauge the change in students' attitudes and perceptions, they should do this at the beginning and end of the term. Teachers could also lead a discussion based on this assignment, or respond to students individually based on their answers. Teachers can also decide whether they want to comment on the writing, and whether they return the writing. It is important that students are comfortable with writing what they actually think, so they should be alerted that this will be kept in confidence. Similarly, the word "metaphor" is confusing to students and was avoided.

23. Portfolios (Cabral)

At the end of each unit, students choose 3-5 items of work from the unit that represent each of the following: (a) the main ideas from the unit (10 pts) and (b) their best work from the unit (10 pts). These points are earned as long as

the correct number of assignments from the unit is selected and the work is complete and legible. These items can be homework, quizzes, notes, classwork or tests. They also need to write a cover letter for the unit which (c) highlights the main ideas and skills from the unit (10 pts), (d) justifies their choice for each assignment chosen for (a) (10 pts), and (e) gives their personal reaction to the unit: likes/dislikes, speed, ease/difficulty (10 pts). Credit is lost in the respective categories if not all the important concepts are covered, if they do not describe each item included separately, or if they do not answer the question completely. No credit is lost for poor spelling or grammar, although it may be corrected. This is a total of 50 points, or half of a test grade.

25. TIPS (Spriggs, Adams)

In the Think-Ink-Pair-Share (TIPS) process, students are asked to think about a problem, write about it, share it with a partner and then share with the whole group. This is a pattern that fits into the school emphasis on writing.

25.1: (Adams) *Problem Of the Day (POD)*

Students are regularly given a problem of the day that is done using the TIPS process. For example, students were asked to solve a problem based on the previous day's lesson: "Solve: $7N + 4 = 32$. Using words, explain how you would solve this equation." They are reminded that they need to write in complete sentences for their explanations - using words, not numbers - and should write 3-4 sentences. Students are asked to write as if they were writing an E-mail or letter to a friend explaining how to do the problem. Students are given about 30 seconds to think, 2 minutes to write, 1-2 minutes to share with a partner, and then each table of four students was asked to have one person read exactly what they wrote to the class. Students were asked to write in such a way that it would be a good explanation for someone else in the class. After some students read to the class, all were given a short period of time to Re-Ink, that is, revise what they had written. Adams goes over the POD after students have written and shared about it, asking questions until they reach the answer. This assignment was recorded in their class notebooks. At this point in the course, students have already done a number of hands on activities related to linear equations, as well lecture and class discussion and homework exercises. This type of task is usually used to check understanding on the most recent lesson or to introduce a new lesson. It could be used for individual assessment, in which case Adams would draw a line after what students had written individually and then allow them to write below the line after interaction with peers and the class.

26. What We Were Doing Today…(Spriggs)

When students were confused by a warm-up, the teacher inserted a writing assignment: "In at least five lines (every other), what were you supposed to work on in your group today? Be specific." An example was given with the weather to explain what it meant to be specific. Students wrote for two minutes. Then students were asked to share some of what they had been asked to do, and as they shared, others in the class were to put x'es over anything they also had, and add anything they had missed. This was used as a chance to refocus after an activity didn't go as planned and give them another chance at the activity.

28. Regular vs. Irregular Polygons (Spriggs)

Students respond to the prompt: "Explain the difference between a regular and irregular polygon." This assignment was used in two classes with slight variations that are described below.

28.1: After asking students to look at pictures of regular and irregular polygons, the class makes a list noting what is the same between all of these polygons and what is different between the two groups. Students are told to remember this list and it is erased. Students are then asked, within the "Today's Question" format (see [27]), to write five lines in complete sentences responding to the prompt and given two minutes. This assignment was on the same sheet of paper as What We Were Doing Today…[26] and Angles in Triangles [27] assignments and was turned in at the end of class.

28.2: Students have separated shapes of triangles and then quadrilaterals into regular and irregular polygons. After checking the sorting and a little discussion, they have two minutes to answer the question above. A few people read their answers until there was a complete answer and there was some discussion. This knowledge was used as part of the next activity: picking examples that are regular and irregular. This assignment was not turned in.

30. Class Summary (Spriggs)

Students are regularly asked to summarize the class. They were asked "If you were summarizing today's math lesson for a friend who was absent, what would you tell your friend?" This can be done in a variety of ways by including peer interaction, counting it as a quiz or doing it entirely verbally.

32. Notebook (Adams)

Students are required to keep a notebook with four sections: Vocabulary, Problem of the Day (POD), Classwork and Summaries. In the vocabulary section, the teacher assigns words that the students need to look up in the glossary and copy into their notebooks. The problem of the day section holds work from the PODs which are now being done with TIPS [25]. The activities and the follow-up questions from the *Connected Mathematics Project* curriculum are in the classwork section. The students keep daily summaries in the Summary section [31]. The course is graded as 50% tests and 50% homework and classwork. (Homework is collected daily.) Points are given for labeling and dating the work in the notebook, and for neatness (5 points each). One point is given for each vocabulary word, 5 points for each activity and follow-up, and 5 points each for all the summaries and PODs.

35. Who Uses Math? (Patterson)

At the beginning of the year, students are given an item and need to think of a career where it is used. Students think of three different ways that mathematics is used in that career. Then students are expected to interview an adult and write a paragraph telling about that person's job and 3 ways that mathematics is used in that job. Students are encouraged to use examples as much as possible. This activity is intended to help students realize that mathematics is a part of everyday life and open lines of communication with parents or other adults about mathematics. This assignment is graded, but with more leniency as it is the first of the year. If information about who was interviewed, what their job is, and three ways that mathematics is used in that job are included, it will usually get an A or A-. The inclusion of examples or pictures raises the grade.

36. Journals (Patterson)

Each student has a spiral-bound notebook to keep a journal. The journal is used most days for problem solving. Students are expected to show how they are thinking using diagrams, calculations, drawings and increasingly words. They are frequently encouraged to summarize their thinking in words. This writing helps students think before discussing possible solutions and is partially in preparation for standardized assessments in the spring. These journals are not formally assessed as a whole. A few times a month these problems will be assessed. In this case, they will either rip a page out of their journal or solve the problem on a worksheet. These solutions should contain a labeled diagram, labeled calculation or a paragraph explaining where the solution comes from. A correct answer with no work earns a C, but you can earn an A- with an incorrect answer if the work is logical and clearly explained but contains a small mistake.

Problem examples:

36.1: Mary Meant loves the number eight. How many 8's will she write altogether if she writes the numbers 1 through 100?

36.2: Mrs. Patterson used chips to make a design. In the first design, she had one row with 2 chips in it. The second design had two rows with three chips in each row. The 3rd design had 3 rows with 4 chips in each row. How many chips will she need for the 10^{th} design? The 100th design? The xth design? (This is actually explained using diagrams rather than descriptions.)

40. Test Corrections (Cabral)

After each exam, students have an opportunity to earn back half of the points they missed if they do "rewrites". This means that they must (a) explain in words what they did incorrectly, (b) make up a similar problem, and (c) correctly solve this problem. Students have opportunities to correct mistakes they make on the rewrites with no penalty, so there may be considerable back and forth between teacher and student. This is intended to provide remediation for material that students didn't understand and motivate them to get the extra help they need.

41. Fraction Booklet (Patterson)

Accelerated 6^{th} grade students create a booklet that shows they understand the addition, subtraction, multiplication and division of fractions. They choose two specific fractions and create a situation or situations where different operations on the fractions occur and find the answers.

42. Correlation of Body Parts (Patterson)

Students pick two "school-acceptable" body parts and study the correlation between their lengths. They collect data, make a scatterplot, make charts and then write a conclusion. For example, students may compare height to circumference of the head.

43. Interactive Notebooks (King)

Middle school students keep a spiral notebook that is intended to help them process, organize and interact with the information that the teacher presents in class. Teacher input is placed on the right page, and student processing of this content is done on the left page. Teacher input can include class notes, discussion notes, reading notes, and handouts or prompts that are glued into the notebook. Students are expected to use the left page to systematically reorganize the material that is on the right page, and express opinions, thoughts and feelings about this material. They are expected to use color, highlighters, and graphics or illustrations to make their work visually attractive. It is hoped that the reprocessing of information and the chance to express themselves will help students learn the information in their own learning styles, especially those with strong visual and linguistic intelligences. As the teacher gives information for the right page, specific ways that they can use color or organize their work are modeled. Assignments are also given for the students to complete on the left page; this can give students a chance to preview new ideas, review topics, or hypothesize and make predictions about classroom activities. The notebook holds most of the classwork and homework for the class. The notebooks are graded approximately every three weeks. Students keep a table of contents for their notebook, which also serves as a grading and evaluation sheet. Notebooks are graded on "thoroughness, quality, organization, and visual appeal." Prior to formal grading, the teacher informally does checks on the notebooks by glancing at them during class or looking while they are working on another assignment to make sure they are on track. Models of outstanding work are distributed and students work in pairs to evaluate their work in light of these examples. Excellent work is encouraged by allowing students to use their notebooks on quizzes. Before formal grading, students do a self-evaluation of their notebook before the teacher grades it. Not every entry is necessarily graded. The notebooks serve as a personal portfolio of students' mathematical learning. This assignment is adapted from a model developed by History Alive!

44. Problem of the Week (POW) (Stofferan)

(Stofferan, 2005)

Seventh grade students were given a POW on Monday and expected to turn in a response to the problem on Friday. The intention of this task was to give students a chance to write about their problem solving process, thereby allowing metacognition, and to provide students with an opportunity to find multiple methods for solving the problem. Students were asked to include four parts in their responses to the POWs: a problem statement, a plan, work, and their answer. This method of response is adapted from Tsuruda (1994). For the problem statement, students were supposed to restate the problem in their own words, clearly enough that anyone having only their paper and not the original problem statement would understand what they were supposed to do. As the plan, students are asked to tell what they did in preparation for solving the problem; they are asked to think about similar problems they may have solved, and what strategies they might be able to use, and make a guess as to the answer. In the work, students share the details of how they solved the problem, using graphics as appropriate. They share what they tried, what worked, what didn't work and how they got past places where they were stuck. Next, students state the answer they got for the problem. Last, they reflect on their work, considering whether the answer makes sense, whether there could be other correct answers, how it compares to their original guess, and what they learned from solving the problem that they could use to solve other problems. After two weeks, Stofferan shared examples of student work, emphasizing "clear and descriptive explanations" (Stofferan, 2005, p.36) and multiple problem solving methods. Over time, the directions were adjusted to give students more detail about what was expected. These responses were not graded since they were a part of action research. However, some responses were assessed according to a rubric that took into account students' process, presentation or explanation, and answer. Each student's POWs were collected in a folder, and this whole folder was commented on and returned to students.

44.1: List 10 possible combinations you could get with 4 darts The numbers on the target are 7, 5, 3, 1. Try to establish some logical method of doing this.

44.2: Ron and Darrell mow lawns to earn money in the summer. Ron has a riding mower. He charges $18 a yard and pays his little brother $7 to help. He is able to do 9 yards in a day. Darrell has a walking mower. He charges $20 a yard and does 8 yards a day. Who do you think does better? Explain your answer.

44.3: Compare different pay scales. Decide if it is better to receive $300 a week or to be paid hourly at a rate of $7.50 per hour. What factors could affect your decision?

44.4: Rainforests cover only a small part of the Earth, but they are home to more than half the world's plants and animals It is estimated that Rainforests are cut down at the rate of 100 trees in a minute. At this rate, how many are cut down every 24 hours? Every week? Explain your answer.

44.5: Look around one room in your home and find as many different shapes as you can. Make a tally to help you determine which shapes are used most often. Why do you think this is so?

44.6: Choose 10 students from your class. Tally the number of vowels in each person's name. Use this to predict the results for the rest of the class.

44.7: In the old days there were elevator operators to transport passengers. Don Downs always started his day in the basement. He went up 20 floors to take his boss some coffee. Then we went down 8 floors to take a donut to his friend. He went up 7 floors to check things out. This was the halfway point in the building. How many floors are in this building?

44.8: Sam made the following scores on unit tests for the quarter: 92, 98, 15, 92, 87, and 92. Sam's teacher said that his grade would be based on the mean of his grades. Find the mean and the median of Sam's grades. Which do you think best reflects Sam's work for the quarter?

44.9: Here are the average temperatures in Hawaii for 7 weeks in 1999 and the same 7 weeks in 2000. Determine which type of graph would be the most appropriate and make it. Don't forget labels, title, and the correct scale. Justify why you chose that type of graph.

1999:	87	92	98	99	98	101	90
2000:	91	92	99	102	100	88	89

44.10: Sylvester measured his pulse and found that his heart beat at a rate of 80 beats a minute at rest. At this rate, how many days will it take his heart to beat 1,000,000 times?

45. Peer-Editing of Problem Solutions (Stofferan)

(Stofferan, 2005)

In addition to expanding the directions for the POWs, Stofferan created an activity to help students understand the criteria better. Each student was given a different story problem from the book *Problem of the Day* (Evan-Moor, 2001) and asked to write out their answer to the problem in the same format required for POWs [44]. No names were included on these assignments, but they were numbered so that they could be returned to the authors. Stofferan collected and redistributed the work to other students who filled out a peer evaluation form (Stofferan, 2005, Appendix F), which was returned with the original assignment to the author. The teacher wanted students to learn from this feedback so that they could recognize their strengths and weaknesses and improve their responses. The peer evaluation form asks students to evaluate each step required by the format, to share what they did or did not understand about the student's work, and conclude whether it was solved correctly. Finally, they gave a score of 1-3 to the exposition of the problem based on a teacher-supplied rubric. The teacher looked at these evaluations before returning them to students in order to assess that the students understood the POW expectations.

46. Affective & Cognitive Writing Prompts (Stofferan)

(Stofferan, 2005)

Twice a month 7th grade students were asked to respond to a writing prompt. At first these prompts were more affective in nature in order to help students become comfortable with writing about mathematics, and then cognitive prompts were used that asked students to explain concepts they were learning in class. Students worked on these prompts individually and without help from the teacher. These prompts were not graded, but were used by the teacher to help decide if the concepts needed to be retaught. The teacher did not respond to the prompts.

The prompts were as follows:

Affective Writing Prompts:

46.1: Dear Mrs. Stofferan,…: Go back as far as you can remember – what did you like about learning math? What did you not like?

46.2: What is your goal for this quarter in math?

46.3: Should students be able to use calculators in mathematics class?

Cognitive Prompts:

46.4: You are going to buy 10 CDs that cost $12.95 each. Explain how you would figure out the total cost (before tax) without the use of a calculator.

46.5: Explain how you would change a number into scientific notation. Give an example of a number and an explanation as to why you might see a number written in this form rather than as a decimal.

47. Writing Activities (Stofferan)

(Stofferan, 2005)

A variety of writing activities were used once a month. These activities included group collaboration and communication about mathematics as well as individual and occasionally group writing. The teacher used these

activities so that students could communicate about mathematics in both speech and writing, hoping that students would think and share about mathematics concepts and make a personal connection with the material. The teacher used the writing to assess students' understanding, and could differentiate between students who understood the content and those who only wrote down what the group said. The activities below were adapted from DeBolt (1998):

47.1: *List the Steps* (based on DeBolt, 1998, p.22):

> Students were asked to explain the steps required to find the cost of an item when given the original cost and a percentage discount. All students had their own item. In pairs, each student explained to the other the steps they would use to determine the sale price and wrote down the other student's steps. Students checked their partner's description and discussed any inaccuracies. Then the students took their classmate's steps that they had written down and wrote up their own explanation of how to determine the sale price of their item.

47.2: *Define It* (based on DeBolt, 1998, p.28):

> Individually, students were asked to come up with their own definitions for four given vocabulary words from their current unit of study. Then the students shared their definitions in small groups and the groups used the individual definitions to determine a group definition that they thought most clearly defined the word. These group definitions were then shared with the whole class using the blackboard.

47.3: *Real Life Applications* (based on DeBolt, 1998, p.68):

> Students were asked to work in groups to think of different situations where measuring was necessary. Each student had to write about three of these applications. This was intended to help students make connections between what they were learning and everyday life.

47.4: *Pre-assessment of Measuring* (based on DeBolt, 1998, p.24):

> Prior to a unit on measurement, students worked in groups to brainstorm as many ideas verbally as they could about measurement. Then each student was required to explain in their own words what they knew about measuring, using the ideas from the small groups however they wished. This was intended to help students make connections between what they already know and what they would be learning in the unit. It was also intended to arouse students' curiosity about measurement and help them see its importance.

52. Algebra Project: Tripline (Moses, Dubinsky & Quinn)

(Moses, Dubinsky & Quinn, 2005)

The Tripline is a significant unit in the Algebra Project curriculum, taking more than half a year. It is intended to help students conceptualize the integers so that properties and operations make sense. Students begin by going on a trip, noticing interesting sites on the trip, and marking these on a line. This line is then abstracted to a number line that students use to think about properties of integers.

For each new concept, this curriculum moves students through a number of stages of communication about this trip: from "people-talk" or normal speech to "math-talk." In the process, they recognize and internalize the mathematical properties that are the goal of the curriculum. This process starts with "people-talk," asks students to pay attention to specific features in the writing, and then moves them into iconic representations using personal symbols, and finally into the use of standard mathematical symbols and language, or "math-talk". For example, students are first asked to write observation sentences about a trip they had taken and represented on a line. In making this line, if students first saw the Mermaid Statue and then the Bus Station, the Mermaid Statue would be placed further left on the line. Then the students can state the observation sentence given in the text: "The Mermaid Statue is to the left of the Bus Station" (p. 54). Students then notice features – people or objects, actions, and relationships – in this sentence and rewrite it to highlight these features: "The location of the Mermaid Statue is to the left of the Bus Station" (p. 60). In this sentence, the feature of location is the subject rather than the objects themselves, adding a level of abstraction. Also the relationship "to the left of" is highlighted as a feature. Next, students make icons to represent the Mermaid Statue, Bus Station and the relationship "to the left of." These are personal icons and could be anything, resulting in a sentence like " ←  " where represents "Mermaid Statue", ← represents "to the left of" and represents "Bus Station." Finally, the sentence is put into the mathematical symbols "x < y", with x now representing "Mermaid Statue", < representing "to the left of", and y representing "Bus Station." The intention of this process is to help students understand integers, as represented by locations on a line, and to therefore make sense of the properties of integers. Specific observation sentences are then used to go through the same process to justify many properties of integers.

This process is combined with a variety of levels of student work at each stage. To make each abstraction in language, students usually work individually, in groups and as a full class. Frequently their work is posted around the class.