## History of One Defeat:

# Reform of the Julian Calendar as Envisioned by Isaac Newton 

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#### Abstract

Here we discuss a proposal for the reform of the Julian and Ecclesiastical calendars, found among Newton's unpublished manuscripts, known among scholars as Yahuda MS 24. His calendar, if implemented, would have become for England a viable alternative to the Gregorian. We propose a reason for Newton's failure to implement it. We also suggest tentative dating for several manuscripts of Yahuda MS 24.


## Introduction

Circa 1700, England was one of many Protestant countries that did not join the calendar reform promulgated by Pope Gregory XIII in 1582. The resulting 10-day difference in dates caused problems in trade with the Continent. In 1700, the time disparity was expected to increase by a day due to the application of a Gregorian rule: that in years divisible by 100 , but not by 400 , a $29^{\text {th }}$ day should not be appended to the month of February.

It is no surprise that Isaac Newton, then a Master of the Mint, c. 1699 foresaw the necessity for change and took it upon himself to propose a calendar that would be astronomically more sound than the Gregorian. He had every chance for success; astronomy had made great strides forward in the 118 years since implementation of the

[^0]Gregorian reform in the Catholic countries. Besides, he was Newton, after all. Not yet "Sir Isaac," but Newton, the author of Principia Mathematica Philosophie Naturalis.

The manuscripts, known as Yahuda MS 24 after Sotheby's 1936 auction, were never published, though a draft of his letter to the Bishop of Worcester proves that Newton made an effort to convince the politically preeminent of the advantages of his calendar system. We first give a brief account of Newton's system, subdivided into a historical and a purely calendrical part; the latter in turn is subdivided into the solar, seasonal, and lunar parts of his system; and, finally, his proposed ecclesiastical reform. We present some general discussion and pose some open questions. A transcript of Newton's own manuscript is in the Appendix. The first brief description of the manuscripts was made by David Castilejo, a Newtonian scholar, at the request of the Hebrew University in Jerusalem, and we preserved his cataloguing numbers - A, B, C, D, E, F, G. - throughout the text, introducing additional pagination on the top of the letters.

## Historical Part

In his letter to the Bishop of Worcester (draft F) Newton gives a brief sketch of historical calendars. The original calendar, handed down by the Biblical patriarch Noah, was lunisolar. Egyptians worked only with a 365 -day solar year. Julius Caesar improved the Egyptian calendar by taking into account the remaining quarter of a day. Chaldeans used a lunisolar calendar and Jews obtained it from them while exiled in Babylonia, as proved by their identical names for the lunar months. In the desert, Moses could not observe new and full moons and appointed the first days of the months arbitrarily; but King David appointed 12 guards over Eretz Israel from the 12 tribes, according to the celestial 12-month division.

## Solar Part

In three consecutive drafts A1-A3 of the proposed reform, Newton quotes three different values for the solar year, assuming that it is shorter than the Julian year (of 365 d 6 h ) by ' 11 $1 / 4 \mathrm{~m}$ ' (A1 [p. 1]); ' $111 / 5 \mathrm{~m}$ ' (A2 [p. 5]); first ' $111 / 15 \mathrm{~m}$ ' and, finally, ' 11 m and 3 or 4s' (A3 [p. 8]). The first two values better fit the proposed Newtonian reform:
(1) to drop February 29 in years divisible by 100 but not in those divisible by 500 ;
(2) to add February 30 in years divisible by 5,000 .

The first rule makes Newton's calendar year shorter than the Julian by only 11m 31s; the second adds back about 17 s . The final value is about 365 d 5 h 48 m 46 s , just in between the first two values (with 45s and 48s respectively).

Fortuitously, the final value ( 365 d 5 h 48 m 46 s ) almost precisely matches the modern value of the tropical year, 365.2422 d . Such a calendar would last in historical time faultlessly. ${ }^{1}$ However, in A3, Newton crosses out the second value and puts the value 365 d 5 h 48 m 56 s in one place and $365 \mathrm{~d} 5 \mathrm{~h} 48 \mathrm{~m} 56 / 7 \mathrm{~s}$ in another. In D1 [p.21], discussing the advantages of his calendar, however, he claims that his calendar would err only a day in 10,000 years, while the Gregorian would err a day in 5,000 years. Since the Gregorian year is equal to 365d 5h 49m 12s, Newton made comparison of errors for the year of 365d 5h 48m 55s.

Newton saw also another advantage, beyond precision, in the arrangement of the solar part of his system. His calendar year (46s) is shorter than the true year (55s) and therefore its state gradually "approaches the state it had in the age of Christ" so that in 30,000 years, the vernal equinox would fall on March 24, and in 110,000 years, on April 1.

## Lunar Part

Newton suggested creating a Great Lunar Cycle of 49 months, subdivided into three smaller cycles of 17,15 , and 17 months respectively. In each of the three cycles, the odd months would have 30 days and the even, 29 days. Then 49 months would consist of 1447 days, and thus Newton's mean month would be equal to 29 d 12 h 44 m 4.9 s .

The synodic month, Newton believed to be true, can be found from his computations in C2 [p. 18], intended to find the time of mean Full Moons for many years into the future. There Newton used the following parameters:

Epoch: 1701.0 Julian Calendar $=$ Dec 31, noon, 1700 Julian Calendar (Jan 11, noon, 1701 Gregorian Calendar).

At Epoch [Moon Sun] difference in longitude: $24^{\circ} 33^{\prime} 57^{\prime \prime}$.
The increment (modulo full circles) for 60 years: $1 \mathrm{~s} 10^{\circ} 14^{\prime} 12^{\prime \prime}$.

[^1]From the last parameter we calculated that Newton assumed a daily increase in elongation to be $12^{\circ} 11^{\prime} 26.7^{\prime \prime}$ and therefore his synodic month was $29 \mathrm{~d} 12 \mathrm{~h} 44 \mathrm{~m} 3.16 \mathrm{~s} .^{2}$ This is 1.74 s smaller than the value coming from his calendar. The 1.74 s accumulate to 3 hours in 500 years, in complete agreement with his memorandum on the advantages of his calendar (drafts D1 [p. 22]) and D2 [p. 24]).

To organize the lunar calendar in a comparatively short, 4,000-year cycle, Newton proposed two different methods. The "deductive" would drop four months in those Great Cycles that embrace the end of $500,1000,1500$, etc., years. ${ }^{3}$ The goal of this procedure is not clear and will be discussed further in the Discussion section. The other, an "additive" device, was suggested in A2 [p. 7]: add two months every 250 years, which is the same as adding 32 months every 4,000 years. In A3 [p.10], Newton suggested adding one more day (to the last month of 29 days) in 4,000 years. To clarify his intentions, let us make several computations. In every 500 calendar years there are

$$
325.25 * 500-4=182,621 \mathrm{~d} .
$$

Divided by 1447, the last number gives 126 (Great Cycles) and remainder 299 (days). In the 4 months that have to be added every 500 years, there are 118 days. Subtracting 118 from 299 we get 181 days. To find the total remainder for the 4,000-year period, we should multiply it by 8 :

$$
181 \mathrm{~d} * 8=1448 \mathrm{~d}=1447 \mathrm{~d}+1 \mathrm{~d},
$$

or a Great Cycle and a day. Overall, there are

$$
126 * 8+1=1,009 \text { Great Cycles }
$$

while $8 * 4=32$ months remain outside. Therefore Newton's "master equation" is

$$
\begin{equation*}
1,460,968 \mathrm{~d}=4,000 \mathrm{y}=1,009 \text { Great cycles }+32 \text { months }+1 \mathrm{~d}=49,473 \text { months }+1 \mathrm{~d} . \tag{1}
\end{equation*}
$$

[^2]According to A3, the lone ' 1 day' had to be absorbed into the last month of 29 days. This makes the value of mean month equal to

$$
\begin{equation*}
\text { m_month }=1,460,968 \mathrm{~d} / 49,473 \text { months }=29.53061266 \mathrm{~d}=29 \mathrm{~d} 12 \mathrm{~h} 44 \mathrm{~m} 6.33 \mathrm{~s}, \tag{*}
\end{equation*}
$$

which is 3.5 seconds greater than the modern value of the mean month. This means that the price for having a comparatively short, 4,000-year cycle, will be a two-day delay of the calendar moon against its true position.

In A1-A3, Newton left two blank spaces, intended to fix January 1, 1701, inside the Great Cycle. In C2 [p. 18] he considered inter alia two possibilities:
either January 1, 1701, becomes the 2nd day of the 1st month of the 2nd Great Cycle, or January 1, 1701, becomes the 2nd day of the 8th month of the 3rd Great Cycle.

## Division of the Year and the Ecclesiastical Calendar

In drafts A1 and A2, Newton complains that the Julian calendar was maimed by the Roman Senate reform, made in 8 BC to honor Augustus by adding a 31st day to the month named after him and depriving February of a day. He believed that Pope Gregory could score a great accomplishment if he were to reverse that transfer and rearrange the months. However, in draft C1 [p. 13], Newton put forward quite a different idea - that of close imitation of the contemporary seasonal divisions of the year - a temptation that later was tried repeatedly by revolutionary nations. Newton suggested assigning 30 days to all winter months and 31 days to all summer months save the last one in the non-leap years. ${ }^{4}$

Newton's other proposed amendments are:

1) From the year 1701 on, the year should begin on January 1 and not on March 25.
2) Easter will be kept on the next Sunday after the 14th of the lunar month, which shall begin after March 7.
3) All the moveable feasts (Pentecost, etc.) that are dependent on Easter have to be set after the date of Easter is calculated.
[^3]According to A1-A3, all the fixed feasts had to stay in place after the first 11 days of December 1700 were omitted. However, in the tables of C2 [pp. 15,16], Newton suggested two quite different ways to deal with the fixed feasts (Christmas, Lady Day, etc.). He arranged the year in two parallel columns, shifted by some 22-23 days against one another, marking along the way all the fixed feasts. The goal was to return all the feasts to their original positions, as set in the 1st century (though from a brief historical discourse on the introduction of the feasts from Emperor Trajan's tenure on, it is clear that only three major feasts were established in the 1st century).

The right column (in case the arrangement of length of the months remains unchangeable and none of the days were omitted) suggested moving the fixed feasts to the contemporary cardinal points: Christmas would be moved to the winter solstice (December 11), Lady Day to the vernal equinox (March 11), etc. ${ }^{5}$ In September thirteen days are missing - from the 16 th to the 28 th.

The left column was arranged according to the new division of the year. September 31 had to be an intercalary day. Christmas was set on January 3, Lady Day on April 3. For that, he was ready to omit 22-23 days from the calendar, which would move all the feasts 8-9 days earlier, as he promised in draft $\mathrm{C} 1[\mathrm{p} .13]$.

## Discussion and Open Questions

## 1. Solar Part

In the two drafts A , he described the value of the year as 'Julian without c. $111 / 4$ minutes' (in A1) and 'Julian without c. $111 / 5$ minutes' (in A2). However, in the fair copy A3, Newton replaced the last fraction with ' $111 / 15 \mathrm{~m}$ ' and later even with ' $111 / 20 \mathrm{~m}$ '. The last value, equal '365d 5h 48m 57s' also appeared on the margin of D1 [p. 23], and a practically identical value was used in Newton's Theory of Moon's Motion [2], published by David Gregory in 1702.

Though the explicit " 118 years since Gregorian reform" in all three drafts A suggests only one possible dating - year 1699-1700 - it is difficult to believe that during one year Newton three times changed his opinion on the length of a solar year, the most important parameter

[^4]of lunisolar theory. We hypothesize that first two drafts A1 and A2 were composed much earlier and only waited for the proper time to be published.

The value Newton stayed with in writing drafts D1 and D2 was '365d 5h 48 m 55 s .' Only this value satisfactorily explains why he believed that the Gregorian calendar errs a day in 10,000 years, while his own errs a day in 5,000 years. However, that value leads to the continued fraction

$$
365.25-365.2423=0.0077=[0,1,3,2,1,7] / 100
$$

where the last approximant, $77 / 10,000$, points to the cycle of 10,000 years, twice the greater cycle than finally chosen by Newton. This means that in the actual calendar, Newton did not use this value. A question arises of how Newton actually obtained the values finishing with 55 s and 57 s .

A table, Annales de L'Observatoire de Bruxelles, taken from the NASA Astrophysics Data System ${ }^{6}$ provides the following data:

48 m 45.5 s for 1602 Tycho Brahe (Astron. Instaur progym),
48 m 57.6 s for 1627 Johannes Kepler (Tabulae Rudolphine),
49m 4.5s for 1645 Boulliaud (Astronomia philolaica),
48m 40s for 1651 Riccioli (Alm. Nov.),
48 m 8s for 1665 Street (Carolician tables),
48 m 57.5 s for 1687 Flamsteed (Newton, Principia?) ${ }^{7}$, and
48m 34.5s for 1719 Edmond Halley (Tabulae Astronom).

The most remarkable feature of this table is that Flamsteed adopted a value almost identical to that of Kepler while disregarding what was found later by his close contemporaries. The bridge between the two astronomers was Jeremiah Horrox. Though it is an open question why Newton stayed for a while with Tycho's value, we can tell why he later switched to Kepler's. An immediate answer is that Flamsteed picked up the Horroxian lunar theory together with its major parameters, like mean solar and lunar motions, while Horrox largely adopted Kepler's values. Collaborating in the late 1690s

[^5]with John Flamsteed, Astronomer Royal, in constructing a new Theory of the Moon, Newton picked up Flamsteed's parameters for mean solar and lunar motions. ${ }^{8}$

Actually, there is little factual evidence of Tycho's influence on Newton's views about the Sun. ${ }^{9}$ More likely, he used the equinoctial observations by Hipparchus, made about 1850 years before Newton's time. The 15 -second discrepancy in the value of a year would accumulate, counting backward to the mid-2nd century BC , to the full 8 -hour difference. Extensive computations in D1 [p. 21] show that Newton computed backward some of the 162-128 BC equinoxes, trying to find the best correlation with Hipparchus' recorded observations in the same years. ${ }^{10}$ There is no sign that he arrived at a definite conclusion.

The question remains of how Newton decided on the length of the cycle, 5,000 years. The same page C2 [p.18] shows that Newton first had chosen 5,000 as his solar cycle and looked only for the number of days that had to be deleted from it. The procedure can be described as follows. The difference between the Julian year and the modern value for a tropical year, 365.2422 ( 365 d 5 h 48 m 46 s ), converted into a continued fraction via

$$
365.25-365.2422=0.0078=[0,1,3,1,1,5] / 100
$$

yields the last rational approximant $39 / 5,000$, indicating that in 5,000 years, precisely 39 days have to be omitted.

Newton found this number for the Gregorian year to be 37.5 d , while for his basic procedure of omitting 4 days out of 500-40d. Two intermediate numbers 38 d and $39 \mathrm{~d}-$ led to years of 365 d 5 h 49 m 4 s and 365 d 5 h 48 m 46 s , respectively. It seems that Newton chose the latter version by some external (theological), rather than astronomical considerations: the value he believed to be the true year (with 55 s) lies exactly in between. The major consideration here was his wish that the calendar year be shorter than the true one. In that case, the equinoxes would drift in the calendar to the positions they occupied in the time of Jesus.

[^6]
## 2. Lunar part

In a series of papers (see [3]), Heiner Lichtenberg clarified several important issues related to the Gregorian reform proper that are contained in the Explicatio written c. 1605 by Clavius. The father of the reform seemed to be fully aware of the possibility of future changes, but insisted on a so-called 'Secular Principle' that allows further adjustments only in years divisible by 100 . Though it is unclear whether Newton carefully read Clavius' work, we see that he did respect this principle. ${ }^{11}$

Further, Newton remarked that the lunar part of the Gregorian calendar designed for the determination of Easter is too hard to implement because it rests on 3 or 4 tables. It is true that changes in the Golden Numbers and Epacts at the centennial boundaries do not have clear rules in the Gregorian calendar. It is known, however [3,7], that the Gregorian calendar precipitates more than 43 changes in Epact during the next 100 centuries. Moreover, Heiner Lichtenberg [3] claimed to have discovered a fundamental equation governing the Gregorian calendar:

$$
5,700,000 \mathrm{y}=70,499,183 \text { months }=2,081,882,250 \mathrm{~d}
$$

Because the number of days is divisible by 7, this equation also represents the Grand Cycle of the Gregorian calendar.

Newton did not leave us his Grand Cycle; however, it can be computed easily. If the lone ' 1 day' in equation $\left({ }^{*}\right)$ is not incorporated into one of the 29-day months, but is kept separate, then after 30 cycles it would grow to 1 month. Noticing that 120,000 years call for 24 additional days (February 30), we come to the equation

$$
120,000 \mathrm{y}=1,484,191 \text { months }=43,829,064 \mathrm{~d} .
$$

This improves the value of the mean month back to

$$
\text { m_month }=43,829,064 \mathrm{~d} / 1,484,191 \text { months }=29.53060893 \mathrm{~d}=29 \mathrm{~d} 12 \mathrm{~h} 44 \mathrm{~m} 4.9 \mathrm{~s} .
$$

[^7]Because the number of days in (\$) is not divisible by 7, the Newtonian Grand Cycle is 840,000 years. This cycle, however, is purely imaginary. The introduction of an extra month was never approved by Newton. ${ }^{12}$ The more subtle "deductive" device could be designed to absorb 1 extra day "naturally," inside a simple algorithm. How it was done is still an open question.

In all three drafts A, the two blank spaces reserved for the beginning of the Great lunar cycle remained empty. Two dates appear in C2 [p. 18], which is filled with extensive computations, within the lunisolar parameters described above, in Newton's Lunar Theory. The source for these parameters appears to be Flamsteed's 1681 Horroxian Lunar Theory. Computations, we made using a computer program [1], show only a 3 arc-seconds difference with the Horroxian daily elongation and lead to a synodic month identical with Horroxian synodic month of $29 \mathrm{~d} 1244 \mathrm{~m} 3.16 \mathrm{~s} .{ }^{13}$

On the other hand, the parameters in C2[p. 18] are neither consistent with Newton's 1702 Theory of the Moon's Motion [2] nor with the mean positions of the Sun and Moon quoted in the 1713 and 1726 editions of the Principia, Book 3, Proposition XXXV. They differ from both by more than 2 arc-minutes. It is an accepted fact that Newton began work on his own lunar theory in 1694-5. ${ }^{14}$ The implication is that the computations in C 2 [p. 18] should be dated sometime between 1681 and 1694.

The two dates written in C2 [p. 18] suggest even more precise dating. Naming January 1, 1701 as the 2 nd day of the 1 st month of the 2 nd Great Cycle points to an epoch of January 14 , 1697, while naming January 1,1701 , as the 2 nd day of the 8 th month of the 3 rd Great Cycle points to the epoch of June 1, 1692. As we know, the latter date (1692-3) is a time when Newton was actively involved in a theological exchange with John Locke and also was afflicted with a mysterious 'illness. ${ }^{15}$ The draft C3 on ancient chronology could belong to this period, as well.

[^8]Accordingly, December 31, 1700, was simply a convenient choice to start a lunar cycle. Except for being near January 1, it implicitly swallows, as can be seen from the "master equation" (1) above, one day remaining from the complete number of months. Because the true elongation at noon was more than $25^{\circ}$, it was easy to prove that the moon could be seen on that night at any location on Earth. To show that it could not be seen a night earlier (with true elongation at noon c. $15^{\circ}$ ) was a tricky problem, dependent on the particular lunar visibility theory; this will be reserved for another paper. ${ }^{16}$

## Epilogue

The final question to be addressed is why Newton failed to publish his calendar. The standard answer is that he could have been afraid of public controversy. Still, he made sure that his highly controversial Chronology of Ancient Kingdoms Amended would be published after his death, and it was (in 1728), while his calendar was buried in a pile of various other historical drafts.

One answer is that in Newton's eyes, his calendar - its major solar part - was imperfect. As we saw, in every consecutive draft, Newton increased his estimate for the true solar year from an initial 365d 5h 48m 45s to a final 365d 5h 48m 57s. Remarkably, in the Chronology [6, p. 81], he increased this value further, rounding it up to $365 \mathrm{~d} 5 \mathrm{~h} 49 \mathrm{~m} .{ }^{17}$ With that, the Gregorian year was only 12 s greater than the "true" one, while the Newtonian was already c. 14s shorter.

To conclude: drafts A1-A2-A3 were likely written in 1699, though the lengths of the solar year adopted in A1 and A2 are puzzling. Drafts C2-C3 and D1-D2 could be composed earlier, in 1692-3, after Newton's acquaintance with the Horroxian lunar theory and before his own work on the lunar theory. Draft F was composed after 1705 because the title "Sir" preceded Newton's name. Either the imperfection of his lunar system or his growing

[^9]disappointment with the solar part caused Newton to abandon his grand project. We are unaware whether the letter intended for the Bishop of Worcester was ever sent to him, though draft G, discussing a calendar proposal by Dr. Prideaux, indicates that the first, albeit timid, step was probably taken.

## Acknowledgments and Remarks

Newton's original text was transcribed by the first author (A.B.) with the cooperation of Ayval Leshem (Bar-Ilan University). We acknowledge use of Nick Kollerstrom's program [1,2] for comparing Newton's parameters in Yahuda MS 24 with the Flamsteed-Horroxian and Newton's 1702 lunar theories. Discussions with Kelly Ross and Heiner Lichtenberg about the Gregorian calendar are also acknowledged. Joan Griffith made helpful remarks on style and pointed to several errors in the Appendix.

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## Appendix. Newton / Yahuda MS 24*

[^10]
## (A1 [pp 1-3], A2 [pp 5-7], A3 [pp 8-10])

## Considerations about rectifying the Julian Calendar

Times were at first reckoned by returns of day and night, new and full moon, summer and winter. Whence the oldest years consisted of lunar months and when twelve months were found too short a thirteenth was added to make up the year. These months began not at the conjunction of Luminairies but at the first appearance of the new moon which used to be between 18 and 42 hours after the conjunction if the sky is clear. ${ }^{1}$ And because the new moon appeared at sunset the days of the lunar month begun in the evening.

The just length of the summer and winter is the return of the Sun to the same equinox, that is 365 days and 6 hours wanting about 11 minutes and 3 or 4 seconds ${ }^{2}$ [11 1/4 in A1 and $111 / 5$ in A2]. And there being something more than 12 moons in summer and winter and something more than 29 days and half in a Moon, the first ages look at next round numbers of 30 days to a Month and 12 months to a year and so made the civil year to consist of 360 days, whence came the division of a circle into 360 degrees.

But this year being too short by five days and almost six hours the Egyptians added five days to the end of it and so made the year to consist of 12 lunar months and five days. And this year was in use in Egypt at least from the days of Amenophes the grandson of Sesostris and seems to have being received in the Assyrian and Persian Monarchies.

The Greeks used lunar months first of 30 days and then of 29 and 30 alternately, and contrived several ways to adapt those months to the year, the principal of which was in every 19 years to intercale 7 months, ${ }^{3}$ whence came the golden number.

At length Julius Caesar ${ }^{4}$ in lieu of the six hours added a day once in four years to the year of 365 days and by adapting this measure to the old Roman year made a new year of 12 months of various length without any good order or uniformity or agreement of the months with the stay of the sun in the twelve signs. And the Senate in honour of Augustus took a day from February and added it to August. ${ }^{5}$ And so Caesar and the Senate together left us a year more irregular and intricate than the Egyptian, but better on this account that the same months keep better to the same seasons of the year. In the Kalendar of this year the Lunar years were supplied by setting the golden numbers to the days of the new Moons for 19 years together.

And because the Julian solar year proved too long by about 11 ' $1 / 15$ [ $1 / 4$ in A1 and $1 / 5$ in A2], that is by a day in 130 years $^{6}$ [128 years in A1, 128 or 129 years in A2], Pope Gregory XIII about 118 years $\mathrm{ago}^{7}$ ordained that three days be taken from it in four hundred years by omitting the 29th day of February in the end of every 100 years excepting at the end of every 400 . And to bring the Vernal [new page] Equinox to the 21st of March on which it fell in the time of the

Council of Nicea ${ }^{8}$ he took 10 days from this year: whence arose the difference of 10 days between the old and new stiles [styles] in the century which is now expiring. And because the rule for finding the new moons by the Golden number erred about an hour and an half in 19 years and a day in 312 years he corrected that rule every 300 years or thereabouts by the alteration of the day.

Had Julius Caesar divided the year into four equal quarters according to the four cardinal periods of soltices of mean equinoxes and then divided every quarter into three months as nearly equal as he could make them which he might have done by making the month of 30 and 31 days alternately and the last month of 31 days in leap years and 30 days in ordinary years so that in the leap year all the odd months should have 30 days and all the eaven 31 , he would have made the Roman year of a regular and convenient form and well adapted to the motion of the sun and periods of summer and winter. And the Pope's correction would have made it lasting.

But without the consent of a good part of Europe I do not think it advisable to alter the number of the days in the months. The question is now whether the old stile [style] should be retained in conformity with antiquity or the new received in conformity with the nations abroad. I press neither opinion but whenever the latter shall be resolved on I believe the best way may be to receive the new stile without the Gregorian calendar by an Act of Parliment to some such purpose as that which follows.

For avoiding the difference of recconing by the old and new stiles which is troublesome in commerce between this and other nations, it may be enacted that in the year of our Lord 1700 the first 11 days of December shall be omitted, rejected and abolished out of that year and the twelveth day of the sad month shall immediately succeed the month of November without any alteration in the days of the week or in the form of Julian calendar, excepting that the Golden number and epact may be omitted. And this accompl[ishment] or stile shall thence forward in all his Majesty's dominions be received used and understood in all Dates and recconings of time for keeping of set festivals fairs Birthdays and all other anniversary days and for performance of all covenants duties and services and payments of interest rent salary pension or wages and all other debts and dues whatsoever with an abatement of interest rent salary pension or wages for an proportional unto eleven days in the first payment of interest rent salary pension or wages which shall by virtue of any covenant grant act or deed had made or done before the day of become due on or after the 12th day of December abovementioned, that is to say with an abatement of the hundredth part of three years interest rent salary pension and wages. ${ }^{9}$

Provided nevertheless that all debts which ought to be paid and all things which ought to be done on any of the said eleven days of December which are hereby abolished, shall be payed and
done on the same [new page] day or days on which they should have been done if this Act had never been made.

And for avoiding the double recconing by the civil and ecclesiasticall years between the last day of December and the 25th day of March the ecclesiastical year shall in all his Majesty's dominions from (the month of December) and after the year of our Lord 1700 begin on the first day of January forever and be no longer dated from the 25th of March.

And that the year may be of a just length and the month remain constant to the seasons of the summer and winter, it may be further enacted that the 29th day of February shall be omitted in the last year of every century escaping the last year of every fifth century and that in the last year of every fiftieth century a day shall be added to the end of February, that is to say, the month of February in the years 1800, 1900, 2100 etc shall have 28 days and in the years 2000, 2500, 3000 etc each shall have 29 days and in the years 5000 and 10,000 etc (if the calendar should extend so far) each shall have 30 days.

And because the movable festivals and law-days depend upon the course of the Moon and the vulgar rule for determining that course needs frequent correction and is now grown very faulty, it may be further enacted that the lunar month shall be recconed to consist of 30 and 29 days alternately in three periods or cycles of months perpetually to succeed one another, each of which periods shall consist of an odd number of months, the two first of 17 and the third of 15 and the first and last month of each period shall contain 30 days so that all three periods summed up together shall make a larger period of 49 lunar months containing 1447 days or 4 solar years wanting a fortnight. [In A1: And the period of 15 months once in every 1000 years that is to say next ensuing the years of the Lord 2000, 3000, 4000, 5000 etc shall have eight months deduced from it, and shall consist of the seven remaining months and no more.] And the first day of January which shall be in the year of our Lord 1701 shall be the day of the month of the larger period of 49 months. And from thence forward the festival of Easter shall be kept on the Lord's day next after the 14th of that lunar month which shall begin next after the seventh day of March. And at the end of every four thousand years a day shall be added to the last lunar month of nine and twenty days.
[In A2: This rule for determining the course of the moon is much more simple and exact than that of the Golden number used by Pope Gregory for that rule errs an hour and an half in $191 / 2$ years and a day in 312 years and so needs frequent correction, this errs only a day in 4000 years. And if in the end of every 250 years the cicle of 15 months have two months of 29 and 30 days added to it so that all the three cycles do once consist of 17 months the rule will be much exacter.]
(B [pp 11-14])

## The use of the Kalendar for finding the Lord's day and the Moveable Feasts

Divide the year of our Lord by 28. Seek the remainder in the following table and you will find under it the Sunday Letter for that year. And in the third column of the Kalendar where you see that Sunday Letter the days are Sundays. In Leap year there are two Sunday Letters: the one obtaining with February 24? and the other for the rest of the year.

| 0123 | 4567 | 891011 | 12131415 | 16171819 | 20212223 | 24252627 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DBAG | FDC B | AFED | C A G F | E C B A | G E D C | B G F |
| C | E | G | B | D | F | A |

Divide the year of our Lord by 19 and the remainder increased by an unit shall be the Golden Number \{or Prime\} for that year. And in the first column of the Kalendar when you find that number the days are new moons ......according to the calendar of the 14th day of moon is the Full Moon. \{several lines crossed out\}....Easter day is always the first Lord's day after the Full moon which happens upon or next after the one and twentieth day of March.

| Sunday | is | weeks before Easter |  |
| :--- | :--- | :---: | :---: |
| Septuagesima | Nine | Rogation Sunday | 5 weeks |
| Sexagesima | Eight | Assention Day | 40 days |
| Quinquagesima | Seven | Whitsunday | 7 weeks |
| Qudrogesima | Six | Trinity Sunday | 8 weeks |

Advent Sunday is always the nearest Sunday to the Feast of St Andrew whether before or after.
[page full of computations, in the middle twice: The Lord Chief Justice Greby]
(C1 [p 13-14])
The Julian year now in use is very irregular. February has but 28 days and the other months 30 and 31 days without any regular order or reason for that irregularity.

The best form of the solar year is to divide it by 4 cardinal periods of the Equinoxes and Solstices into 4 quarters, so that the quarters of that year may begin with the Equinoxes and the solstices as they ought to do, and then to divide every quarter into 3 equal months which will be done by making the six winter months to consist of 30 days each and the six summer months of 31 days each excepting one of them suppose the last which in the leap year shall have 31 days in the other years only 30 days. Although the end of every hundred years omit the intercalary day in that
leap year excepting at the end of every five hundred years. For this rule is exacter than the Gregorian of omiting it at the end of every hundred years excepting at the end of every 400 years. And this recconing by five hundreds and thousands of years is rounder than the other by four, eight and twelve hundreds. And this I take to be the simplest and in all respects the best form of the civil year that can be thought of. And this is all the reformation of the year which need be made at first.

As for the Ecclesiastical year if the fixed feasts shall be placed on the same of the months of this new year as in the Julian year, they will come nearer to the truth than they do at present. For they are now become about 14 days later than they were in the first century in respect of summer and winter whereas in this new year they will be only eight or 9 sooner. So that the Calendar will be amended almost half (in best?) by this new year without translating the fixed feasts to other days of the months.

But if it may be allowed to translate them to other days of the months so as to bring them nearer to the places where they were in the first century in respect of Summer and Winter the Calendar made be still amended as follows.

1. Let Lady day [the first day of Ecclesiastical year] be removed from the 25th of March to the first of April and the Ecclesiastical year will begin at the Equinox and on the first day of the month as it ought to do, whereas in the present Julian year it begins neither at the Equinox nor on the first day of the month but on the 25th of March and 16 days after the Equinox.
2. Let Michaelmus be removed from the 29 of September to the 1st of October and their two principal days of payment? Will fall on the Equinoxes and on the first days of the month which begin the spring and autumn quarters of the year which is very proper and ready for recconing, and also more just for contracts. For the summer half year is 11 days longer?? than the winter half year in the vu[l]gar Calender but in this new one the difference will be but 5 days.
3. In like manner to regulate the days of quarterly payments let St John Baptist's day be removed from the 24th of June to the 4th of July and Christmas of 25th of December to the 1st of January, or perhaps to the 2nd that it will be distinguished from the New Years Day.

Thus will the year become fitter for civil uses and the festivals be reduced within a day or two to the places where they were in the first century in respect of summer and winter; whereas they now err 14 days from those places. And the like corrections maybe made of all the other moveavle festivals by setting them 7 or 8 days later.

Easter is determined by making it the first Sunday after the first full moon after the first of April and the rest of the moveable feasts are determined by their distance from Easter as in the Vulgar Calendar.

The old Rule for finding Easter by the Prime of Dominical Letter is to be corrected at the end of every hundred or two hundreds years by ecclesiastical authority and so is the Rule of finding the new Moon by the Epact in the margin of the Calendar. And with such correction both Rules maybe retained for ever.

## (C2 [pp. 15-18]) Ecclesiastical Calendar

P.15: calendar for July-September;
p. 16: calendar for January-June;
p. 17: calendar for January-April and computations of Dominical letters on the bottom
p .18: computations of the mean full moons for several hundered years ahead.

## (C3 [pp 19]) Notes about Ancient Chronology

The only feasts in the beginning till the reign of Trajan were the Lord day, Easter \& Whitsunday. See Origen b 8 cont. Cels. Christmas began to be celebrated diverse places about the year 190 (Throphilus Casarintis in epist. paschal.)

The Martyrs began to be commemorated on their passion days about the year 170 and these days at length were celebrated as feasts by the institution of Constantin the great (Euseb. in vit. Const. b.4) who also instituted the observation of Friday. Euseb. ib[id] The heathens feasts turned into Christia[n] Theodoret b 8 de martyribus and Greg. M. b 9 Cap 71 citante Hospin. De Origen. Christ. Fest. p 15.

The Greeks celebrated the Epiphany or Baptism of Christ on the same day with his birth, the Christmas on January 6 Hospin ad Jan 6.

Timothy martyred on Jan 24. Pauli Conversio Jan 25. The burning of light on Candlemas Day Feb 2 taken from the sacra of Ceres seeking her daughter Proserphina with light and torches. Feb 1. The Bacchinal rights turned into Christian carnivals in the first days of the Quinquagesima or week before Lent. Matthias Feb 24. Festus annunciationes Maria March 25. St Mark martyred Apr 25. The Greeks celebrated to all the apostles the feast of Peter and Paul Jan 29. The Latins that of Philyp and James May 1. Although length? the day is left Phil and James alone. Quadratus May 26. Nativity of John Baptist June 24. Peter and Paul on June 29 on which day the Heathens celebrated the feast of Hercules and the Muses. July 25 St James. Aug 24 St Bartholomew Sep 21 St Matthew. Sept 25 Cleopas. Sept 25 St Michael and all angels. Octob 18 St Luke. Octob 28 Simon and Jude. Novemb. 28 Adventus Domini. Nov 30 St Andrew.

## (D1 [p 21]) COMPUTATIONS

First four lines - the length of the year with $39,40,38,37 \frac{1}{2}$ days deleted out of 5,000 years OBSERVATIONES HYPPARCHI \{for years 162, 159, 158, 147, 146, 146, 143, 135, 128 BC\}

## ( [pp 21-22) Memorandum on the advantage of this Kalendar

And in the end of every 500 years the larger period of lunar months which should be then running shall contain 45 lunar months and the three lesser periods of which that larger period consists shall each of them contain only 15 lunar months, the two last months of the two periods containing 17 months being omitted.

The advantage of this Calendar above the Gregorian in respect of the solar year is that the solar year in the Gregorian errs a day in 5000 years and by that error recedes from the state it had in the age of Chirst, in this it errs a day in 10,000 years and by that error approaches the state it had in the age of Christ so that in 30,000 years the equinox will fall on the 24th of March as it did in the age of Christ and in 110,000 years the beginning of January will fall on the winter solstice as it ought to do. Also the recconing by $500,1000,1500$ etc runs in rounder and fewer numbers than by $400,800,1200,1600$ etc. And thou the Calendars differ yet they will agree in stile for 700 hundred years to come. ${ }^{10}$

The advantage in respect of the Lunar year is much greater for in the Gregorian Kalendar the full Moon on which Easter depends is not to be found without the help of three or four Tables, and when you have the full moon there is no rule in that Calendar for finding the other full moons and the new moons throught the year. But in this Kalendar all the new and full moons are found perpetually without any Tables or any other recconing then the continual addition of the 30 or 29 days [alternatively] which is so very easy a work that any Novice can perform it and besides this rule is much exacter than the Gregorian for that errs three hours in 39 years ${ }^{11}$ this errs but three hours in 500 years ${ }^{12}$, and may be corrected every 500 years to keep it exact.

## [p 23] COMPUTATIONS

On the right margin: Annus equinoxiatis 365 . $5 \mathrm{~h} 48^{\prime} 57^{\prime \prime}$ and below $365^{1 / 4}-11^{\prime} 1 / 20$
(D2 [pp 24])
The advantage of this Kalendar...
(F1 [pp 30-31])

## A draft of the letter to the Bishop of Worchester on Calendar Reform

Sir Isaac Newton represents that he did formerly discourse with our Lord-p about the ancient year of 360 days and represented to your Lord-p that it was the Calendar of the ancient Lunisolar year (of the Egyptians Syrians Chaldeans Asiatics Greeks etc, that it was) composed of the nearest round number of days in a lunar month and lunar months in solar year; that the ancients corrected this Kalendar monthly by the new moons and early by the returns of the four seasons, dropping a day or two when they found the calendar too long for the course of the Moon and adding a month to the end of the year when they found the calendar year too short for the return of the seasons; [ that for avoyding the trouble of so frequent corrections, several cycles of months and years were invented, as a cycle of 12 months consisting of 30 and 29 days alternately, the Diateris consisting of two years of 12 and 13 months alternately, the Octaeris consisting of four Diateris wanting a month, the cycles Decimenovalis, Dadecarteris of the Chaldeans, etc; that the Egyptians by adding 5 days to the end of their calendar year formed a solar year of 365 days which the Romans corrected by adding a day to the end of every four years; that the Arabians by omitting the intercalary months have formed their year of 12 lunar months; that the Luni-solar, the solar and the lunar years and their Calendars are all the sorts of years which he marks? with in antiquity;] that Moses in describing the flood uses the Kalendar months without correcting them by the course of the Moon, the cloudy rainy weather not suffering her? to appear; that when [the Athenians erected 360 statues to the Demetrius Phalarius according to the number of days in the year, or ] Herodotus reccons 30 days to the month 12 months to the year he understood the Kalendar year without correcting it by the courses of the Sun and Moon; that when Herodotus reccons by years of 12 and 13 months alternately for 70 years together he needs the Diatris continued for 70 years together without correcting it by the Lunar part? That when we meet with a week of years or a month of years or a year of years, we are to understand a Kalendar week of natural years, a Kalendar month of natural years and the Kalendar year of natural years, that is, 7 or 30 or 360 natural years, taking any number of natural years for so many revolutions of winter and summer; [that the Jews in returning from captivity called their own months by the names of the Chaldean, which argues that they were the same;] and that he meets with nothing in your Lord-p paper which in his opinion makes against what he then represented to your Lord-p.

He saith also that within the compass? of the four monarchies he marks? with no other year to this day than the Lunisolar propagated by Noah to his posterity, the solar of 365 days [corrected by the Romans by the addition of a day in 4 years,] and the Lunar used by the Mahometans, and their? Kalendars of three years?? ; lessening the trouble of correcting the primitiv[e] lunisolar
calendar every month by the moon in every year by the sun, various cycles were invented, as the annual cycle of 30 and 29 days in the month alternately, the Dieteris consisting of 12 and 13 months in the year alternately, Octaetris consisting of 4 Dieterises wanting a month, the Dodecaeteris of the Chaldees mentioned by Censorinus, and containing, as he thinks, 4 intercalary months, and the Sarus of Chaldees mentioned by Suidas (in $\Sigma \alpha \beta \circ \theta$ ) and consisting of 18 years of 12 lunar months each, besides six months each which he takes to be intercalary, a month being added every third year for 18 years together.

Sir Isaac saith further that in his opinion the original year of all nations was Lunisolar, the same with its calendar being propagated down from Noah to all his posterity. That for keeping to the courses of the Sun and the Moon and yearly by the sun and return of the seasons and fruits of the earth various cycles and amendations of the primitive calendar have being invented as the cycle of months consisting 30 and 29 days alternately during the whole year. The intercalation of a month every other year which made the Trieteries of the Anceients more properly called?? the Dieteris, the Octaeteris composed of 4 Dieterises wanting a month. The or Dodecaeteris of the Chalde[an]s........The Sarus of Chalde[an]s composed of 18 years and 6 intercalary months. The Egyptian cycle of 365 days. Julian and Gregorian correction of that year. The Arabian cycle of 12 lunar months perpetually without any intercalary months.
\{A note on the bottom of that page:
And tho the intervals should have been a year or two \{more or less than\} 70 yet I had rather allow that the Prophet might use the nearest round number of seventy than run into greater greater difficulties. For that Zerubbavel -- sight?

As for Iddo $\}$

## (G [ pp 36]). Kalendar by Dr. Prideaux

I have perused the paper which his Lord[shi]p the Bishop of Worcester sent to Dr. Pridaeux and found it full of excellent observations concerning the ancient year: but do not percieve that they amount to any thing more than a proof that the Kalendar of the ancient Lunisolar consists of 12 lunar month and each Kalendar month of 30 days. ....The first nations before use artificial cycles kept recconing of time by the courses of the sun and moon Gen. 1: 14. Courses ...
${ }^{1}$ This is related to Maimonides' lunar visibility theory, see [4].
${ }^{2}$ Inserted instead of the crossed out ' 12 ' before 'seconds.'
${ }^{3}$ The so-called Metonic cycle, ascribed to Meton in 431 BC.
${ }^{4}$ In 46 BC .
${ }^{5}$ In 8 BC .
${ }^{6} 11 \mathrm{~m} 4.6 \mathrm{~s}$. Crossed out: ‘ 129 ' days.
${ }^{7}$ In October 1582.
${ }^{8}$ In 325 AD .
${ }^{9} 3 * 365.25 d / 100=10.95 \sim 11 d$.
${ }^{10}$ The first difference between the Newtonian and Gregorian calendars has to appear in the year 2400.
${ }^{11}$ Or 1 day in 312 years, or $22.39 s$ in a month, which is the difference between the Julian 29 d 12 h 44 m 25.53 s and adopted by Newton Horroxian 29d 12 h 44 m 3.16 s months. It seems that Newton did not check the final value of the lunar Gregorian month, which, after elaborate system of epacts, is, according to H. Lichtenberg [3], only $0.5 s$ less than the modern mean month.
${ }^{12}$ About $1.7 s$ in a month.


[^0]:    - belenka@mail.biu.ac.il
    + edovila@vtr.net

[^1]:    ${ }^{1}$ The error will be less than 1 day in 100,000 years assuming that the Earth stops slowing down.

[^2]:    ${ }^{2}$ In Principia, Newton assumed a slightly greater value (though he never quotes the value of the synodic month explicitly, it can be recomputed from the value of the sidereal year and sidereal month). The 1713 Edition, Book 3, Proposition 37, Problem 18, Corollary 7 gives $27 \mathrm{~d} 7 \mathrm{~h} 431 / 5 \mathrm{~m}$ for a sidereal month. The value of a sidereal year ( 365.2565 d ) was cited in the 1726 edition of Principia, Book 3, Phenomenon 4. This brings the synodic month to the Ptolemaic value of 29d 12h 44m 3.3s. It is interesting that the 1687 edition gives only the approximate value, 27 d 7 h 43 m , for a sidereal month, while the 1726 edition gives $434 / 9 \mathrm{~m}$, where - N.B.! - the last fraction is an obvious typo. Henry Pemberton was no match to Roger Cotes!
    ${ }^{3}$ Either 8 months out of a 15 -month small cycle every 1,000 years (in the draft A1), or the two last months in both 17 -month small cycles every 500 years (in D1).

[^3]:    ${ }^{4}$ This arrangement had to correspond to the fact that in his time spring+summer+ fall+winter $=93 \mathrm{~d}+93.5 \mathrm{~d}+89.6 \mathrm{~d}+89.1 \mathrm{~d}$. The solar apogee c .1700 was at ca. $97^{\circ}$, roughly dividing summer-spring and fall-winter semicircles in half.

[^4]:    ${ }^{5}$ The rest were treated accordingly: St. Matthew to Feb 10, St. Mark to Apr 11, Sts. Philip and James to Apr 17.

[^5]:    ${ }^{6}$ Web site at <http://adsbit.harvard.edu/cgi-bin/nph-iarticle_query?journal=AnOBN\&year= n/a\&volume=1\&page_ind=238\&letter=.\&type=SCREEN_GIF >.
    ${ }^{7}$ We did not find in Principia any support for the value ascribed to Flamsteed. However this is exactly the value we deduced from Newton's 1702 Theory of the Moon's Motion [2]. The value we deduced from Horroxian tables [1] is 365 d 5 h 48 m 59.5 s .

[^6]:    ${ }^{8}$ We venture to hypothesize that the Horroxian tables, published by Flamsteed in 1681, mark the inception of Newton's interest in Keplerian planetary theory, which soon would play a prominent role in Principia.
    ${ }^{9}$ Westfall [8, p. 830] brings to light only one instance when Tycho's manuscript (on comets) received Newton's attention. Newton presented Tycho's manuscript to the Royal Society, and as president, ordered it printed. It is unclear whether it was his own manuscript, and if so, when he acquired it.
    ${ }^{10} \mathrm{~A}$ kind of a precursor of the future "least squares method."

[^7]:    ${ }^{11}$ It is interesting that the 20th century witnessed several proposals to amend the Gregorian calendar that were greatly inferior to Newton's - see a short review in [3].

[^8]:    ${ }^{12}$ Though in the end of draft C1 [p. 14] he allowed ecclesiastical authorities to intervene to correct Dominical letters.
    ${ }^{13}$ The value we found in the Rudolphine Tables is almost identical: 29d 12h 44m 3.18s.
    ${ }^{14}$ See Westfall, [8, pp. 540-48].
    ${ }^{15}$ See Ibid [8, pp. 530-40].

[^9]:    ${ }^{16}$ In draft E, discussed in a subsequent paper, Newton grossly relied on the lunar visibility theory described by Maimonides. The large southern latitude of the moon (ca. $4^{\circ}$ ) can be a reason for declaring non-visibility. See details in Neugebauer [4].
    ${ }^{17}$ This value, even greater by 3 s , can be computed from the speed of the equinoxes ( 50 " per year $=0.0141 \mathrm{~d}$ ) found in all three editions of Principia, and the value of a sidereal year (365.2565d), in the 3rd edition of the Principia, Book 3, Phenomenon 4. The fact that the value of a sidereal year was given with $1 / 10,000 \mathrm{~d}$ precision allows for the value 365 d 5 h 49m.

[^10]:    *Transcribed by Ayval Leshem and Ari Belenkiy from the originals in the National Library of the Hebrew University, Jerusalem. We preserved D. Castilejo's cataloguing numbers A, B, C, D, E, F, G-throughout the text, introducing additional pagination.

