

RAMAPO-INDIAN HILLS SCHOOL DISTRICT

Dear Ramapo-Indian Hills Student:

Please find attached the summer packet for your upcoming math course. The purpose of the summer packet is to provide you with an opportunity to review prerequisite skills and concepts in preparation for your next year's mathematics course. While you may find some problems in this packet to be easy, you may also find others to be more difficult; therefore, you are not necessarily expected to answer every question correctly. Rather, the expectation is for students to put forth their best effort, and work diligently through each problem.

To that end, you may wish to review notes from prior courses or on-line videos (www.KhanAcademy.com, www.glencoe.com, www.youtube.com) to refresh your memory on how to complete these problems. We recommend you circle any problems that cause you difficulty, and ask your teachers to review the respective questions when you return to school in September. Again, given that math builds on prior concepts, the purpose of this packet is to help prepare you for your upcoming math course by reviewing these prerequisite skills; therefore, the greater effort you put forth on this packet, the greater it will benefit you when you return to school.

Please bring your packet and completed work to the first day of class in September. Teachers will plan to review concepts from the summer packets in class and will also be available to answer questions during their extra help hours after school. Teachers may assess on the material in these summer packets after reviewing with the class.

If there are any questions, please do not hesitate to contact the Math Supervisors at the numbers noted below.

Enjoy your summer!

Ramapo High School
Michael Kaplan
mkaplan@rih.org
201-891-1500 x2255

Indian Hills High School
Amanda Zielenkiewicz
azielenkiewicz@rih.org
201-337-0100 x3355

Name _____

Algebra 1 Summer Review Packet
Ramapo Indian Hills

I. Writing Algebraic Expressions

In **algebraic expressions**, letters such as x and w are called variables. A variable is used to represent an unspecified number or value.

Practice: Write an algebraic expression for each verbal expression.

1. Four times a number decreased by twelve _____
2. Three more than the product of five and a number _____
3. The quotient of two more than a number and eight _____
4. Seven less than twice a number _____

II. Order of Operations

To evaluate numerical expressions containing more than one operation, use the rules for order of operations. The rules are often summarized using the expression **PEMDAS**

Examples:

Parentheses (Grouping Symbols)	$[(7 - 4)^2 + 3] + 15$	$\frac{(9-7)^2+6}{2}$
Exponents	$= [3^2 + 3] + 15$	$= \frac{11-6}{2}$
Multiply or Divide, from left to right	$= [9 + 3] + 15$	$= \frac{2^2+6}{2}$
Add or Subtract, from left to right	$= 12 + 15$	$= \frac{4+6}{2}$
		$= \frac{5}{2}$
		$= \frac{10}{2}$
		$= 5$

Practice: Evaluate each expression.

1. $250 \div [5(3 \cdot 7 + 4)]$

2. $\frac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}$

3. $\frac{1}{2} \cdot 26 - 3^2$

4. $8^2 \div (2 \cdot 8) + 2$

5. $5 + [30 - (6 - 1)^2]$

6. $\frac{2 \cdot 4^2 - 8 \div 2}{(5 + 2) \cdot 2}$

III. Evaluating Algebraic Expressions

To evaluate algebraic expressions, first replace the variables with their values. Then, use order of operations to calculate the value of the resulting numerical expression.

Example: Evaluate $x^2 - 5(x - y)$ if $x = 6$ and $y = 2$

$$\begin{aligned}x^2 - 5(x - y) &= (6)^2 - 5(6 - 2) \\ &= (6)^2 - 5(4) \\ &= 36 - 5(4) \\ &= 36 - 20 \\ &= 16\end{aligned}$$

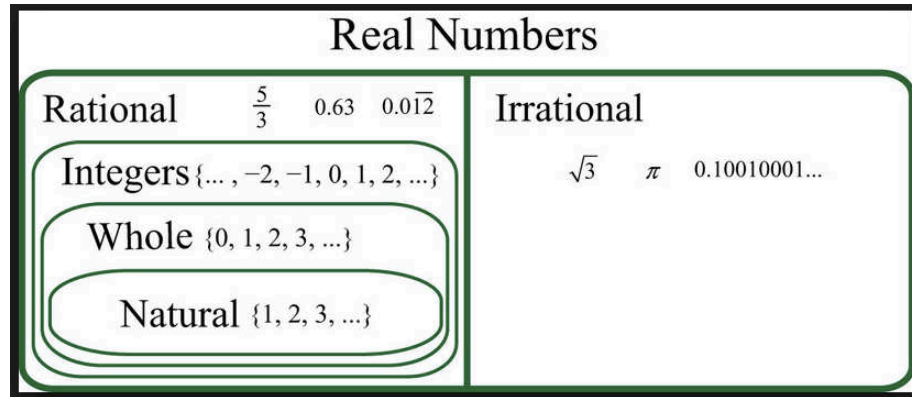
Practice: Evaluate each expression.

1. $5x^2 - y$ when $x = 4$ and $y = 24$

2. $\frac{3xy - 4}{7x}$ when $x = 2$ and $y = 3$

3. $(z \div x)^2 + \frac{4}{5}x$ when $x = 2$ and $z = 4$

4. $\frac{y^2 - 2z^2}{x + y - z}$ when $x = 12$, $y = 9$, and $z = 4$

IV. The Real Number System

The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers**.

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers**.

- **Real Numbers**- any number that can be represented on a number-line.
 - **Rational Numbers**- a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)

Examples: 2, -5, $\frac{-3}{2}$, $\frac{1}{3}$, 0.253, $0.\overline{3}$

 - **Integers**- positive and negative whole numbers and 0
Examples: -5, -3, 0, 8 ...
 - **Whole Numbers** - the counting numbers from 0 to infinity
Examples: { 0, 1, 2, 3, 4, ...}
 - **Natural Numbers**- the counting numbers from 1 to infinity
Examples: { 1, 2, 3, 4... }
 - **Irrational Numbers**- Non-terminating, non-repeating decimals (including π , and the square root of any number that is not a perfect square.)
Examples: 2π , $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111....

Practice: Name all the sets to which each number belongs.

1. -4.2 _____

4. 9 _____

2. $3\sqrt{5}$ _____

5. $\sqrt{16}$ _____

3. $\frac{5}{3}$ _____

6. $-\frac{8}{2}$ _____

V. Properties of Real Numbers

Following are properties of Real Numbers that are useful in evaluating and solving algebraic expressions.

Additive Identity	For any number a , $a + 0 = a$.
Multiplicative Identity	For any number a , $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a , $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a , $a = a$.
Symmetric Property	For any numbers a and b , if $a = b$, then $b = a$.
Transitive Property	For any numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.
Commutative Properties	For any numbers a and b , $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Properties	For any numbers a , b , and c , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Practice: Name the property illustrated in each equation.

1. $3 \cdot x = x \cdot 3$ _____

2. $3a + 0 = 3a$ _____

3. $2r + (3r + 4r) = (2r + 3r) + 4r$ _____

4. $5y \cdot \frac{1}{5y} = 1$ _____

5. $9a + (-9a) = 0$ _____

6. $(10b + 12b) + 7b = (12b + 10b) + 7b$ _____

7. $5x + 2 = 5x + 2$ _____

8. If $9 + 4 = 13$ and $13 = 2 + 11$ then $9 + 4 = 2 + 11$ _____

9. If $x = 7$ then $7 = x$ _____

10. $3 \cdot 1 = 3$ _____

VI. The Distributive Property

The Distributive Property states for any number a , b , and c :

1. $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$

2. $a(b - c) = ab - ac$ or $(b - c)a = ba - ca$

Practice: Rewrite each expression using the distributive property.

1. $7(h - 3)$

2. $-3(2x + 5)$

3. $(5x - 9)4$

4. $\frac{1}{2}(14 - 6y)$

5. $3(7x^2 - 3x + 2)$

6. $\frac{1}{4}(16x - 12y + 4z)$

7. $(9 - 2x + 3xy) \cdot -4$

8. $0.3(40a + 10b - 5)$

VII. Combining Like-Terms

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms**.

Like-terms have the same variables to the same power.

Example of like-terms: $5x^2$ and $-6x^2$

Example of terms that are **NOT** like-terms: $9x^2$ and $15x$

Although both terms have the variable x , they are not being raised to the same power

To combine like-terms using addition and subtraction, add or subtract the numerical factor

Example: Simplify the expression by combining like-terms

$$\begin{aligned} 8x^2 + 9x - 12x + 7x^2 &= (8+7)x^2 + (9-12)x \\ &= 15x^2 - 3x \\ &= 15x^2 - 3x \end{aligned}$$

Practice: Simplify each expression

1. $5x - 9x + 2$

2. $3q^2 + q - q^2$

3. $c^2 + 4d^2 - 7d^2$

4. $5x^2 + 6x - 12x^2 - 9x + 2$

5. $2(3x - 4y) + 5(x + 3y)$

6. $10xy - 4(xy + 2x^2y)$

VIII. Solving Equations with Variables on One-Side

To solve an equation means to **find the value** of the variable. We solve equations by isolating the variable using opposite operations.

Example:

Solve.

$$\begin{array}{r} 3x - 2 = 10 \\ + 2 \quad + 2 \end{array}$$

Isolate $3x$ by adding 2 to each side.

$$\frac{3x}{3} = \frac{12}{3}$$

Simplify

Isolate x by dividing each side by 3.

$$x = 4$$

Simplify

Check your answer.

$$\begin{array}{r} 3(4) - 2 = 10 \\ 12 - 2 = 10 \\ 10 = 10 \end{array}$$

Substitute the value in for the variable.

Simplify

Is the equation true? If yes, you solved it correctly!

Opposite Operations:
Addition (+) & Subtraction (-)
Multiplication (x) & Division (÷)

Please remember...
to do the same step on
each side of the equation.

**Always check your
work by substitution!**

Practice: Solve each equation.

1. $98 = b + 34$

2. $-14 + y = -2$

3. $8k = -64$

4. $\frac{2}{5}x = 6$

5. $14n - 8 = 34$

6. $8 + \frac{n}{12} = 13$

7. $\frac{3k-7}{5} = 16$

8. $-\frac{d}{6} + 12 = -7$

IX. Solving Equations with Variables on Each-Side:

To solve an equation with the same variable on each side, write an equivalent equation that has the variable on just one side of the equation. Then solve.

Example	Solve $4(2a - 1) = -10(a - 5)$.
$4(2a - 1) = -10(a - 5)$	Original equation
$8a - 4 = -10a + 50$	Distributive Property
$8a - 4 + 10a = -10a + 50 + 10a$	Add $10a$ to each side.
$18a - 4 = 50$	Simplify.
$18a - 4 + 4 = 50 + 4$	Add 4 to each side.
$18a = 54$	Simplify.
$\frac{18a}{18} = \frac{54}{18}$	Divide each side by 18.
$a = 3$	Simplify.

The solution is 3.

Practice: Solve each equation.

1. $5 + 3r = 5r - 19$

2. $8x + 12 = 4(3 + 2x)$

3. $-5x - 10 = 2 - (x + 4)$

4. $6(-3m + 1) = 5(-2m - 2)$

5. $3(d - 8) - 5 = 9(d + 2) + 1$

X. Ratios and Proportions

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y , $x:y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.

$$\frac{24}{36} = \frac{2}{3} \text{ when expressed in simplest form.}$$

$$\frac{12}{18} = \frac{2}{3} \text{ when expressed in simplest form.}$$

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.

$$\frac{10}{18} \stackrel{?}{=} \frac{25}{45} \quad \text{Write the proportion.}$$

$$10(45) \stackrel{?}{=} 18(25) \quad \text{Cross products}$$

$$450 = 450 \quad \text{Simplify.}$$

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Practice: Determine whether each pair of ratios forms a proportion.

1. $\frac{12}{32}, \frac{3}{16}$

4. 5 to 9, 25 to 45

2. $\frac{15}{20}, \frac{9}{12}$

5. 0.1 to 0.2, 0.45 to 1.35

3. $\frac{1.5}{2}, \frac{6}{8}$

6. 100:75, 44:33

X. Ratios and Proportions (Continued)

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes**. They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of ProportionsFor any numbers a , b , c , and d , if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 1:

$$\frac{x}{5} = \frac{10}{13}$$

$$x \cdot 13 = 5 \cdot 10$$

$$13x = 50$$

$$\frac{13x}{13} = \frac{50}{13}$$

$$x = \frac{50}{13}$$

Example 2:

$$\frac{x+1}{4} = \frac{3}{4}$$

$$4(x+1) = 3 \cdot 4$$

$$4x + 4 = 12$$

$$-4 \quad -4$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Practice: Solve each proportion.

1. $\frac{x}{21} = \frac{3}{63}$

4. $\frac{9}{y+1} = \frac{18}{54}$

2. $\frac{-3}{x} = \frac{2}{8}$

5. $\frac{a-8}{12} = \frac{15}{3}$

3. $\frac{0.1}{2} = \frac{0.5}{x}$

6. $\frac{3+y}{4} = \frac{-y}{8}$

XI. Percent of Change

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.

original: 48

new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$$\frac{12}{48} = \frac{r}{100} \quad \text{Percent proportion}$$

$$12(100) = 48(r) \quad \text{Cross products}$$

$$1200 = 48r \quad \text{Simplify.}$$

$$\frac{1200}{48} = \frac{48r}{48} \quad \text{Divide each side by 48.}$$

$$25 = r \quad \text{Simplify.}$$

The percent of increase is 25%.

Example 2

Find the percent of decrease.

original: 30

new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$$\frac{8}{30} = \frac{r}{100} \quad \text{Percent proportion}$$

$$8(100) = 30(r) \quad \text{Cross products}$$

$$800 = 30r \quad \text{Simplify.}$$

$$\frac{800}{30} = \frac{30r}{30} \quad \text{Divide each side by 30.}$$

$$26\frac{2}{3} = r \quad \text{Simplify.}$$

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Practice: State whether each percent of change is a percent of increase or percent of decrease. Then find each percent of change.

1. original: 50
new: 80

3. original: 27.5
new: 25

2. original: 14.5
new: 10

4. original 250:
new: 500

XI. Percent of Change (Continued)

Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example A coat is on sale for 25% off the original price. If the original price of the coat is \$75, what is the discounted price?

The discount is 25% of the original price.

$$\begin{aligned} 25\% \text{ of } \$75 &= 0.25 \times 75 & 25\% &= 0.25 \\ &= 18.75 & \text{Use a calculator.} & \end{aligned}$$

Subtract \$18.75 from the original price.

$$\$75 - \$18.75 = \$56.25$$

The discounted price of the coat is \$56.25.

Practice: Find the final price of each item. When a discount and sales tax are listed, compute the discount price before computing the tax.

- | | |
|-------------------------------------------------------------|-------------------------------------------------------|
| 1. Two concert tickets: \$28
Student discount: 28% | 4. Celebrity calendar: \$10.95
Sales tax: 7.5% |
| 2. Airline ticket: \$248.00
Frequent Flyer discount: 33% | 5. Camera: \$110.95
Discount: 20%
Sales tax: 5% |
| 3. CD player: \$142.00
Sales tax: 5.5% | 6. Ipod: \$89.00
Discount: 17%
Tax: 5% |

XII. Solving For a Specific Variable

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V , w , and h , then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ . If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 Solve $2x - 4y = 8$ for y .

$$\begin{aligned} 2x - 4y &= 8 \\ 2x - 4y - 2x &= 8 - 2x \\ -4y &= 8 - 2x \\ \frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\ y &= \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4} \end{aligned}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 Solve $3m - n = km - 8$ for m .

$$\begin{aligned} 3m - n &= km - 8 \\ 3m - n - km &= km - 8 - km \\ 3m - n - km &= -8 \\ 3m - n - km + n &= -8 + n \\ 3m - km &= -8 + n \\ m(3 - k) &= -8 + n \\ \frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \\ m &= \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k} \end{aligned}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Practice: Solve each equation or formula for the variable specified.

1. $15x + 1 = y$ for x

3. $7x + 3y = m$ for y

2. $x(4 - k) = p$ for k

4. $P = 2l + 2w$ for w