MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 5: Ampere's Law

OBJECTIVES

1. To learn how to use Ampere's Law for calculating magnetic fields from symmetric current distributions
2. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius a and outer radius b using Ampere's Law.
3. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: Section 9-3, 8.02 Course Notes.

Problem-Solving Strategy for Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the closed loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

Step 1: Identify the 'symmetry' properties of the charge distribution.

Step 2: Determine the direction of the magnetic field

Step 3: Decide how many different spatial regions the current distribution determines

For each region of space…

Step 4: Choose an Amperian loop along each part of which the magnetic field is either constant or zero

Step 5: Calculate the current through the Amperian Loop

Step 6: Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the closed loop.

Step 7: Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

Example 1 : Magnetic Field of a Cylindrical Shell

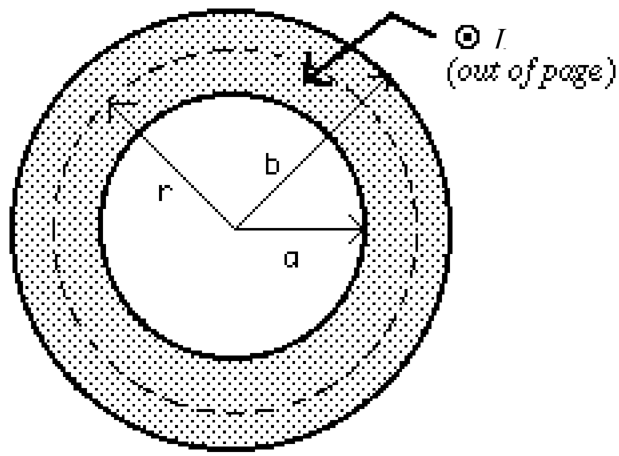
We now apply this strategy to the following problem. Consider the cylindrical conductor with a hollow center and copper walls of thickness $b - a$ as shown . The radii of the inner and outer walls are a and b respectively, and the current I is uniformly spread over the cross section of the copper (shaded region). We want to calculate the magnetic field in the region $a < r < b$.

Problem Solving Strategy Step
(1) <u>Identify Symmetry</u>
Either circular or rectangular

(2) <u>Determine Direction</u>
Clockwise or ccw?

(3) <u>How many regions?</u>
Three: r<a; a<r<b; r>b

(4) <u>Draw Amperian Loop</u>:
Here we take a loop that is a circle of radius r with $a < r < b$ (see figure).



(5) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop.
There are typically two ways to do this. One way is to simply calculate it as a fraction of the total current. The second is to first calculate the current density J (current per unit area) and then multiply by the area enclosed. You should use both methods and compare.

Question 1 ***(write your answer on the tear-sheet at the end)*****:** What is the magnitude of the current per unit area J in the region $a < r < b$? Remember we are assuming that the current I is uniformly spread over the area $a < r < b$, and also remember that current density J is defined as the current per unit area. Once you have the current density, calculate the total current enclosed by the Amperian loop.

Question 2 ***(write your answer on the tear-sheet at the end)*****:** What is the fraction of the total area that is enclosed by the Amperian Loop? What is the total current it encloses?

Question 3 ***(write your answer on the tear-sheet at the end)*****:** Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

(6): Calculate Line Integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$:

Question 4 ***(write your answer on the tear-sheet at the end)***: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

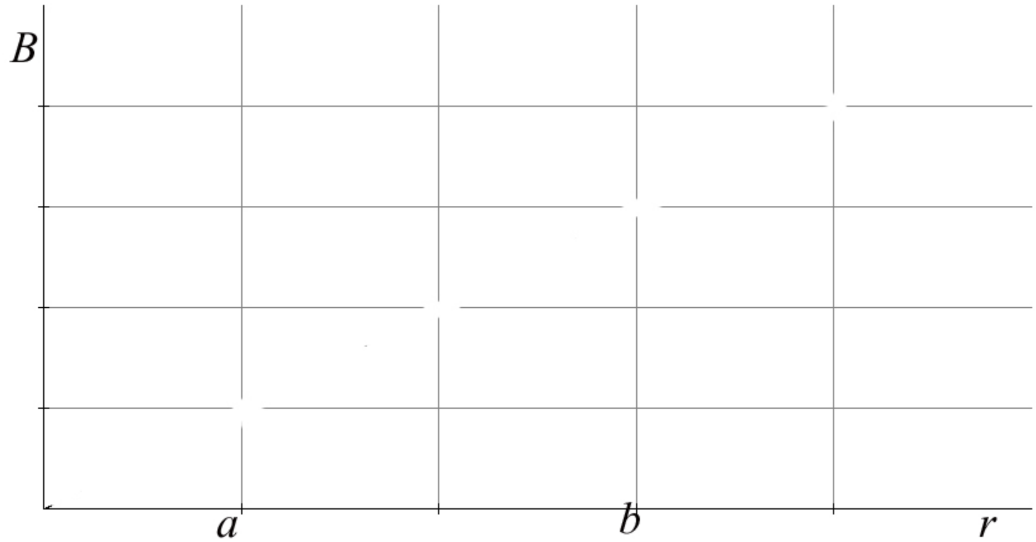
(7): Solve for $\vec{\mathbf{B}}$:

Question 5 (*write your answer on the tear-sheet at the end*): If you equate your answer to Question 4 to your answer to Question 2 times μ_o (i.e. use Ampere's Law), what do you get for the magnetic field in the region $a < r < b$?

Question 6 (*write your answer on the tear-sheet at the end*): Repeat the steps above to find the magnetic field in the region $r < a$.

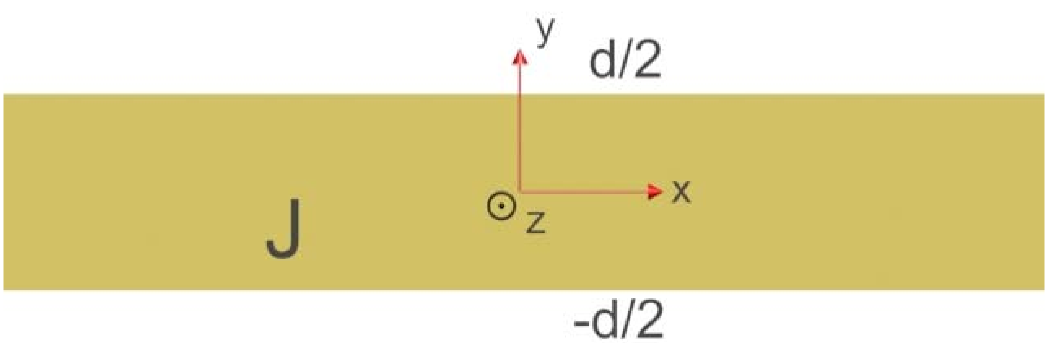
Question 7 (*write your answer on the tear-sheet at the end*): Repeat the steps above to find the magnetic field in the region $r > b$.

Question 8 ***(put your answer on the tear-sheet at the end)*:** Plot B on the graph below.



Example 2: Magnetic Field of a Slab of Current

We want to find the magnetic field $\vec{\mathbf{B}}$ due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{\mathbf{J}} = 2J_e\,|y|/d\,\hat{\mathbf{z}}$, where units of J_e are amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y-direction.



Question 9 (*write your answer on the tear-sheet at the end*): What is the magnetic field at $y = 0$, where $y = 0$ is the exact center of the slab?

Problem Solving Strategy Step
(1) <u>Identify Symmetry</u>
Either circular or rectangular. Which is it?

(2) <u>Determine Direction</u>
Make sure you determine the direction in all regions? Sketch on tear sheet figure of Q10.

(3) <u>How many regions?</u>
Two for this problem: in the slab and above it (we won't do below the slab).

(4) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 9 the magnetic field at $y = 0$. Therefore….

Question 10 (*write your answer on the tear-sheet at the end*): What Amperian loop do you take to find the magnetic field for $y > d/2$? Draw it on the figure above and also on the tear-sheet at the end, and indicate its dimensions.

(5) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is the integral of the current density over the enclosed area.

Question 11 (*write your answer on the tear-sheet at the end*): What is the total current enclosed by your Amperian loop from Question 10?

(6): <u>Calculate Line Integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$</u> :

Question 12 (*write your answer on the tear-sheet at the end*): What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

(7): <u>Solve for B:</u>

Question 13 (*write your answer on the tear-sheet at the end*): If you equate your answers in Question 12 to your answer in Question 11 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

We now want to find the magnetic field in the region $0 < y < d/2$.

(4) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 9 that the magnetic field at $y = 0$. Therefore…

Question 14 (*write your answer on the tear-sheet at the end*): What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and on the tear-sheet at the end, and indicate its dimensions.

(5) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop.

Question 15 (*write your answer on the tear-sheet at the end*): What is the total current enclosed by your Amperian loop from Question 14?

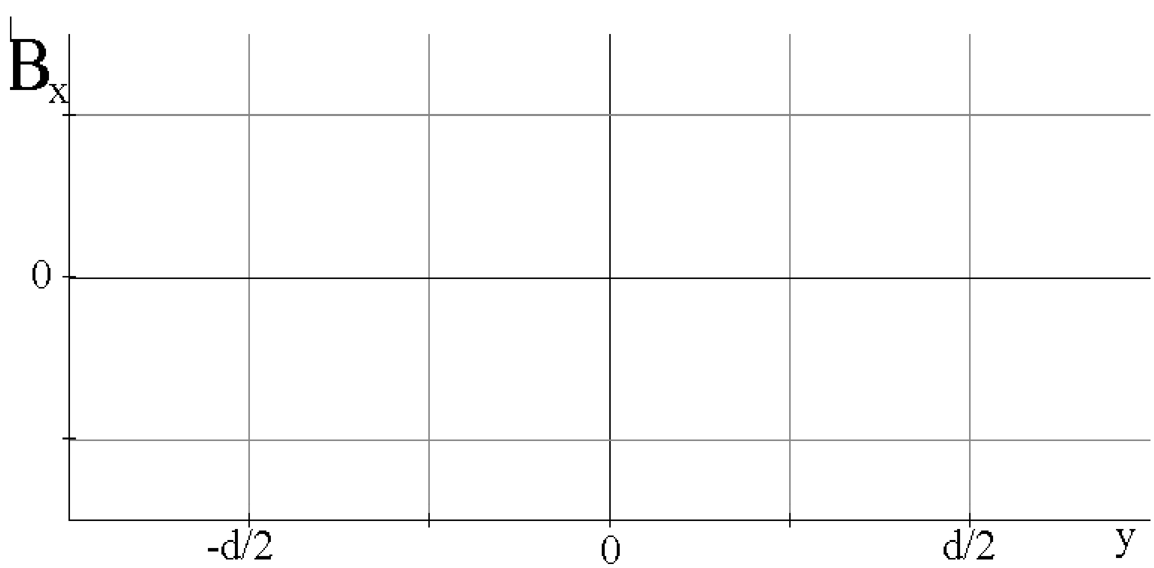
(6) <u>Calculate Line Integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$:</u>

Question 16 (*write your answer on the tear-sheet at the end*): What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

(7) <u>Solve for B:</u>

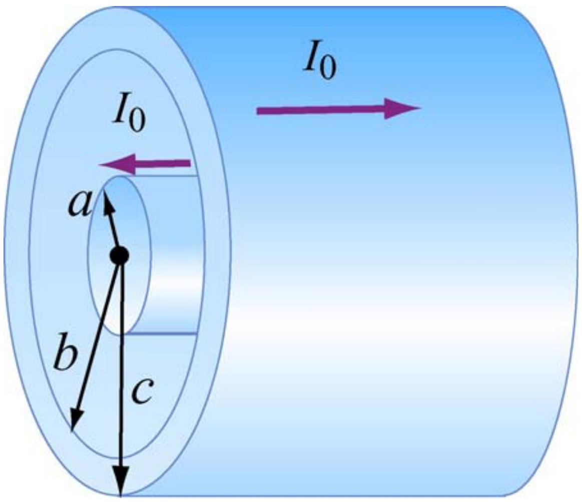
Question 17 (*write your answer on the tear-sheet at the end*): If you equate you answers in Question 16 to your answer in Question 15 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

Question 18 *(put your answer on the tear-sheet at the end)*: Plot B_x on the graph below. Use symmetry to determine B for y<0. Label the y-axis



Sample Exam Question (If time, try to do this by yourself, closed notes)

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections*. Determine the magnetic field at a distance r from the axis for the following ranges of radii. On the figure below, draw the Amperian loop you use in each case.

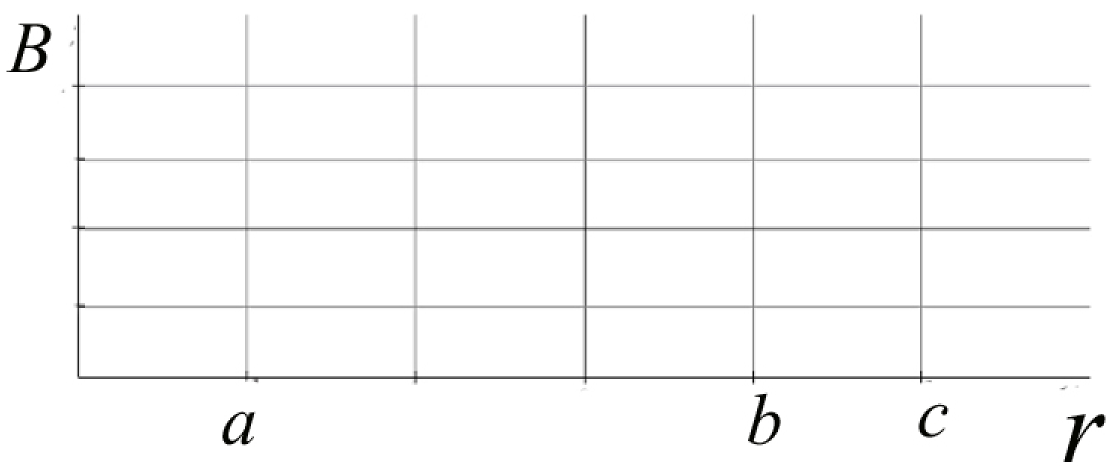


(a) $r < a$

(b) $a < r < b$

(c) $b < r < c$

(d) Plot your answers for the magnitude of B above on the graph below. Label your vertical axis or you will lose points.



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Tear off this page and turn it in at the end of class !

Note:
Writing in the name of a student who is not present is a COD offense.

Problem Solving 6: Ampere's Law

Group ______________________________ **(e.g. L02 6A Please Fill Out)**

Names ______________________________

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Question 1: What is J in the region $a < r < b$? What then is the current enclosed?

Question 2: What fraction of the current does your loop enclose? What is the current enclosed?

Question 3: Is your answer above zero when $r = a$ and I when $r = b$.

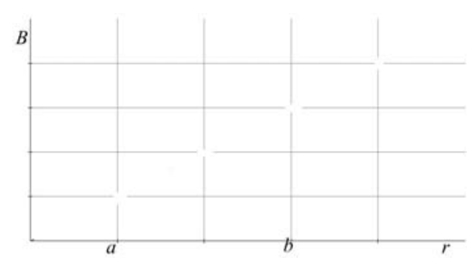
Question 4: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 5: What do you get for the magnetic field in the region $a < r < b$?

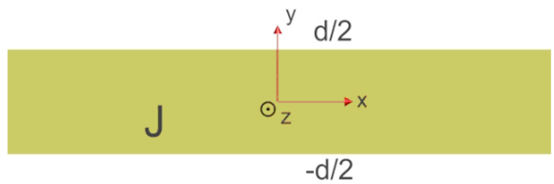
Question 6: What is the magnetic field in the region $r < a$.

Question 7: What is the magnetic field in the region $r > b$.

Question 8: Plot B on the graph.



Example 2: Magnetic Field of a Slab of Current



Question 9: What is the magnetic field at $y = 0$?

Question 10: Draw your Amperian loop for $y > d/2$ on the figure above, and indicate dimensions.

Question 11: What is the total current enclosed by your Amperian loop from Question 10?

Question 12: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 13: What is the magnetic field $\mathbf{B}(y > d/2)$?

Question 14: Draw your Amperian loop for $0 < y < d/2$ on the figure above.

Question 15: What is the total current enclosed by your Amperian loop from Question 14?

Question 16: What is the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for your loop?

Question 17: What is the magnetic field $\mathbf{B}(0 < y < d/2)$?

Question 18: Plot B_x on the graph.