

Pollen Count Levels and Allergies

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Objectives

This lesson will demonstrate to students the connection between mathematical concepts and the real world. The activity will enhance students' understanding of pollen counts and the related health problem of allergies, while providing a context through which to review linear, quadratic and graphing functions, as well as the applications of these functions. At the end of the project, students will have acquired the skills to develop mathematical models and use them to make predictions.

Reflection

This activity is based on information adapted from the Asthma and Allergy Foundation of America website (<http://www.aafa.org>), a reputable source. Today, millions of Americans suffer from seasonal allergies. This health problem is often overlooked, and sometimes may lead to other medical conditions. The activities allow students to apply linear and quadratic functions from the syllabus to connect two topics: pollen counts and health issues such as allergies and asthma.

The project is composed of two activities, each of which is based on the same reading material. Activity 1, covered in Handout #1, is based on linear functions. It involves understanding simple concepts such as slope and the equation of a straight line, and finding the values of a given function. This activity is introduced during the third week, when students have been taught slope and techniques of finding linear equations. It allows students to review the material and gauge their graphing skills and ability to find the value of a function from its graph.

In Activity 2, covered in Handout #2, quadratic functions are discussed. The activity involves understanding concepts such as graphs of nonlinear functions, interpretation of data, and using quadratic functions to find specific values. This activity is introduced during the sixth week. By comparing different mathematical models, students can clearly see that neither model is perfect, that neither can be used for predicting pollen counts throughout the year. Thus, they realize the limitations of mathematical models. The graphing of equations using Excel also gives students hands-on experience with computer software technology.

Activity Overview

Handout #1:

The project should be introduced after reviewing the following material: ordered pairs, plotting points in the coordinate plane, and graphing straight lines. The concept of dependent and independent variables can also be reviewed. The instructor can review the method of finding the equation of a line that passes through two points. Students are asked to find the equation of the regression line that represents the given data.

Math Topics

Linear and quadratic equations

Purpose

Review

When to Introduce

Week 3 and week 6

Activity Time Frame

One-hour class time and one week to complete each handout

Pollen Count Levels and Allergies

Step 1: Students are given handouts detailing the activity/project and are asked to read material on “Pollen and Mold Counts” available at <http://www.aafa.org/display.cfm?id=9&sub=19&cont=264>.

Step 2: During the second meeting, there is a brief discussion of the reading material and an overview of the assignment. The instructor reviews the concept of slope, finding the equation of a line, and using the equation to find more points on the line. The instructor can also show students how to graph curves of best fit using Excel.

Step 3: Students should be given about one week to complete their assignment. The instructor should allow time for small group discussion in class to help students with the assignment and review their progress on the project.

Step 4: After assessing the project, the instructor should provide feedback on common errors and clarify the concept of mathematical models. The instructor should check for mathematical as well as conceptual understanding errors.

Handout #2:

Step 1: The instructor recapitulates the main points of the reading material, initiating the discussion by asking students the following question: “Since the straight line equation does not work to predict the exact pollen levels for all times of the year, is there any other type of model that would work better?”

Step 2: The instructor provides an overview of the assignment. He/she can go over the method of plotting points and graphing curves, such as parabolas. Students can hand-draw the graph and use the equation provided by the instructor to answer questions in the assignment. If possible, the instructor should show students how to graph curves of best fit in Excel.

Step 3: Students should be given about one week to complete their assignment. The instructor should allow time for short class discussions (about 15 minutes) to encourage collaboration among students and to assess their progress on the project.

Materials and Resources:

- Handout #1: Linear Equations
- Handout #2: Quadratic Equations
- Pollen.com. *30 Day historic allergy and pollen levels*. Retrieved October 4, 2010, from <http://www.pollen.com/historic-pollen-levels.asp>
- Reading: Allergy and Asthma Foundation of America. *Pollen and mold counts*. Retrieved October 4, 2010, from <http://www.aafa.org/display.cfm?id=9&sub=19&cont=264>
- New York City Department of Health and Mental Hygiene Childhood Asthma Initiative. (2003, May). *Asthma Facts*. P. 18. Retrieved November 4, 2010 from <http://www.nyc.gov/html/doh/downloads/pdf/asthma/facts.pdf>

Handout#1: Linear Equations

Kothari | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Introduction

Straight lines are represented by linear equations, which are of the type $ax + by = c$ or $y = mx + b$. From statistical data given in the form of a chart, one can write the set of ordered pairs and plot them to create a graph. This graph of points is called a scatter plot. The curve of best fit can be obtained by connecting a maximum number of the points. Using algebraic techniques or graphing utilities, mathematicians can find the equations that represent best fit curves and use them to study the behavior of the data. One can also make predictions based on these graphs to determine additional data points that were not given in the original data. This analysis helps in understanding and interpreting the relationship between two physical quantities, namely x and y variables. Reading #1 will help you gain an understanding of pollen count and its importance to people who have pollen allergies.

Based on the reading, answer the questions below. Here is some additional information that may help you respond to these questions.

Pollen Level: The pollen forecast is usually given as low, moderate, high, or very high. Many factors such as weather, location, time, etc. affect the calculation of the pollen index.

Low:	Pollen levels between 0 and 2.4
Low Medium:	Pollen levels between 2.5 and 4.8
Medium:	Pollen levels between 4.9 and 7.2
Med-High:	Pollen levels between 7.3 and 9.6
High:	Pollen levels between 9.7 and 12.0

Source: Pollen.com. *30 Day historic allergy and pollen levels*. Retrieved October 4, 2010, from <http://www.pollen.com/historic-pollen-levels.asp>

1. What are the different types of pollen that can cause seasonal allergies?
2. What is the meaning of “pollen count?” Describe at least one method that researchers use to measure pollen levels.
3. List different factors that can cause changes in pollen count level.

Pollen Count Levels and Allergies

The data below show pollen count levels for Long Island City, Queens, N.Y. for 10 days between September 5, and September 17, 2010. Use the chart below to answer questions 4-7.

Pollen History

Date	Day	Pollen Count Level
9/5/10	1	9.5
9/7/10	3	8.6
9/9/10	5	7.8
9/11/10	7	6.5
9/12/10	8	2.8
9/13/10	9	5.2
9/14/10	10	6
9/15/10	11	5.7
9/16/10	12	3.3
9/17/10	13	1.3

Source: Pollen.com. *30 Day historic allergy and pollen levels*. Retrieved October 4, 2010, from <http://www.pollen.com/historic-pollen-levels.asp>

- Observe the pattern. On what day was the pollen count the highest? What was the level? On what day was the pollen count the lowest? What was the level?
- In the x - y coordinate plane, if x represents the Day and y represents the Pollen Count Level for that day, write the set of all ordered pairs from the chart. Plot these points to create a scatter plot. (Do not connect the points.) What relationship does the graph suggest between the day and the pollen count level?
- Choose points (1, 9.5) and (7, 6.5) to find the slope of the line passing through these points. What is the sign of slope value? How can you relate that to the pattern you observed?
- Find the equation of the line passing through these points in question 6. Put your answer in the form $y = mx + b$.

8. Use the equation found in your answer to question #7 to complete the chart below: (Hint: September 5 = Day 1, September 6= Day 2, September 7 = Day 3 & so on)

Date	9/13/10	9/16/10	9/24/10	9/26/10	9/30/10
Day # X	9	12	20	22	26
Pollen count level Y					

9. Observe the predicted values that you found for September 26 and September 30 in the chart above. Are these values meaningful?
10. Do you think a straight-line equation could predict the pollen count for any day of the year? Why or why not? Explain your answer.
11. Why should we watch pollen count levels? How do high pollen counts affect people with pollen allergies?
12. What can we do to prevent health problems related to pollen? List two actions that should be taken by anyone with pollen allergies.

Handout#2: Quadratic Equations

Kothari | Elementary Algebra – Problems and Issues in Public Health (MAT096)

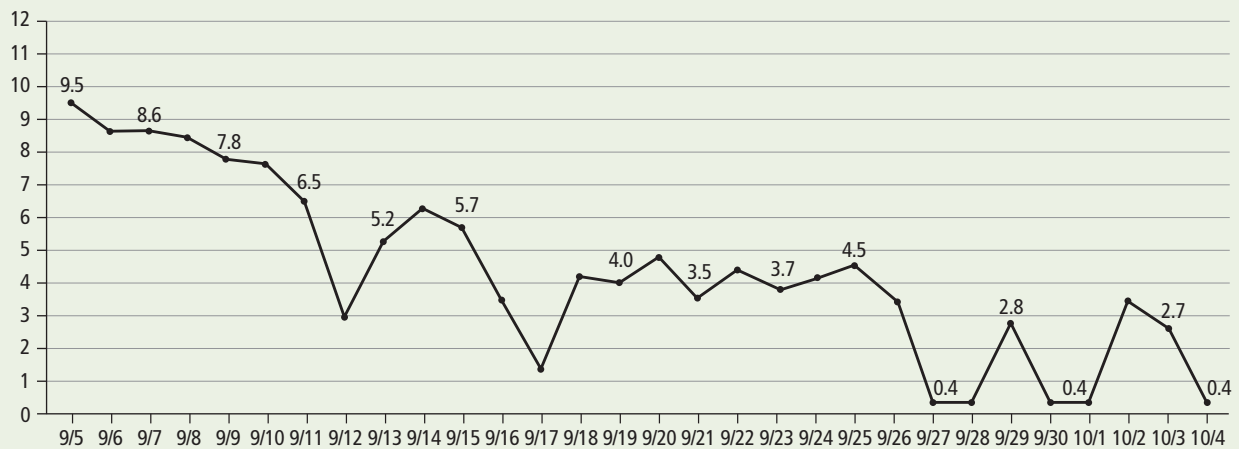
Introduction

Equations of the type $ax^2 + bx + c = 0$ are called quadratic models. From statistical data given in the form of a chart, one can graph the points to make a scatter plot. If we connect a maximum number of the points (note that at least three points are needed to fit a curve), we get an upside-down opening curve called a parabola. Using algebraic techniques or graphing utilities, we can find the equation that represents this curve. In question number 3 below, you are given the equation of the curve that best fits the given data. Please read the data carefully and answer the following questions. Also, keep your answers from Activity 1 handy for easy comparison.

Questions

The graph below is the pollen count history for Long Island City, NY, from September 5, 2010 to October 4, 2010.

30 Day History for Long Island City, NY



Source: Pollen.com. 30 Day historic allergy and pollen levels. Retrieved October 4, 2010, from <http://www.pollen.com/historic-pollen-levels.asp>

1. Refer to the graph above to answer the following questions:
 - a. On what day was the pollen count the highest?
 - b. On what day was the pollen count the lowest?
 - c. During what period of the month was the pollen count level between 6.5 and 9.5?
2. Plot the points given in the “Pollen History” chart in Activity 1. Join all the points from September 12, 2010 to September 17, 2010 to make a parabola. What form of the parabola do you observe? Is it opening upwards or downwards?

3. Suppose the function $f(x) = -0.6214x^2 + 12.664x - 58.6$ represents the equation of the graph you found in question 2 above. Calculate $f(5)$, $f(10)$, $f(12)$, and $f(15)$ to one decimal place. (Note that $f(5)$, $f(10)$, $f(12)$, and $f(15)$ and represent the pollen count levels for 9/9/10, 9/14/10, 9/16/10, and 9/19/10 respectively). Complete the following chart. Use your calculator to check the values you found.

Date	9/9/10	9/14/10	9/16/10	9/19/10
Day # X	5	10	12	15
Pollen Count Level Y $f(x) = -0.6214x^2 + 12.664x - 58.6$				

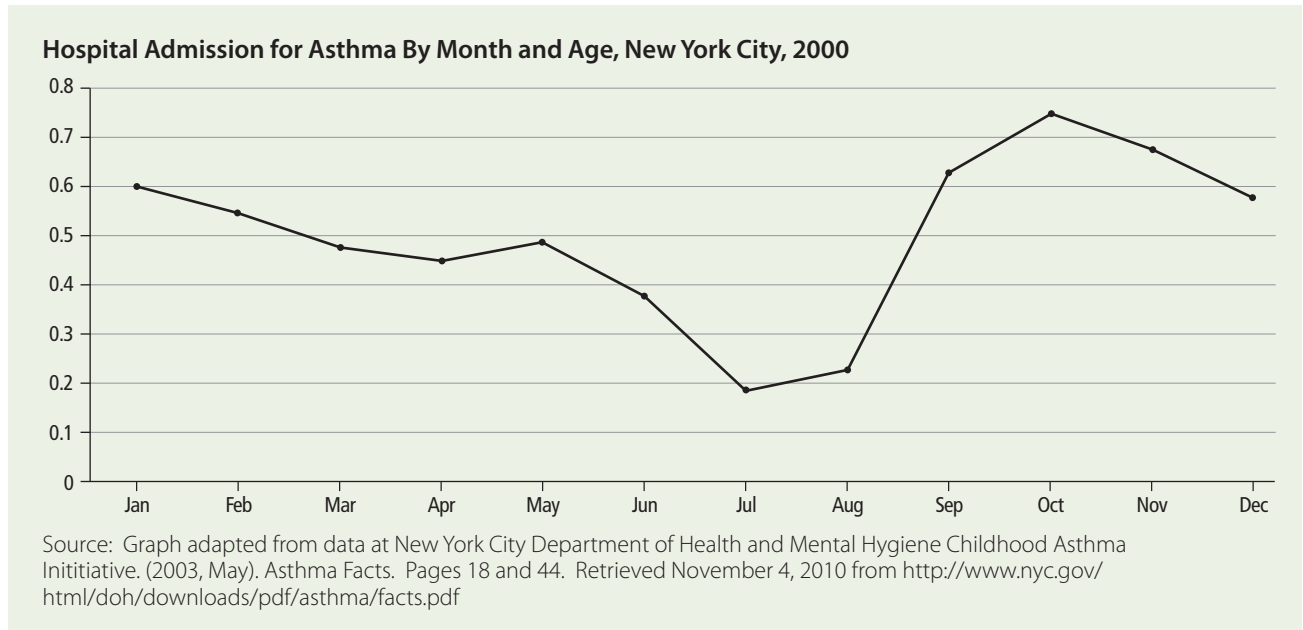
4. Are the values for Day 5 and Day 15 meaningful? Explain why or why not.
5. Do you think the above quadratic equation would work in predicting pollen levels for the month of October? Would it work better for finding pollen levels for any other time of the year? Explain your answer.
6. Compare these three sets of pollen counts using: the linear model in Handout #1; the quadratic model in Handout #2; and the values indicated on the graph in question 1 above. Which model is a better predictor of pollen counts? Write your comments. (Hint: Complete the chart below to interpret your observations.)

Date	9/9/19	9/13/10	9/16/10	9/19/10	9/26/10
Day # X	5	9	12	15	22
Pollen Count level (Using Linear Model)					
Pollen Count level (Using Quadratic Model)					
Pollen Count level (from graph)					

7. Do you think meteorologists can always depend on either the linear or quadratic model to predict the pollen count throughout the year? Is time of year the only factor we should use to predict pollen levels? Are there other factors that may affect pollen levels? Use information from the website to support your answer.

Pollen Count Levels and Allergies

8. We know that millions of people suffer from allergies and asthma each year. The following graph represents the incidence of asthma-related hospitalizations of New York City children aged 0-14 over the calendar year. Using the charts, what you have learned from our discussions, and the work you did on Handout #1 and Handout #2, answer the questions below.



- In which month was the rate of hospital admissions the highest?
 - How do asthma hospital admissions relate to the pollen count levels for particular times of the year shown in “30 Day History for Long Island City, NY”?
 - In which month was the rate of hospital admissions the lowest?
9. Writing Task: Use the information on the Allergy and Asthma Foundation of America website (<http://www.aafa.org/>) to learn about how high pollen levels can cause allergies that could trigger asthma attacks. Then, write a letter to a parent explaining the relationship between pollen, allergies and asthma. Make suggestions to parents about how to avoid or treat these problems.

Reading: Pollen and Mold Counts

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Source: Allergy and Asthma Foundation of America. *Pollen and mold counts*. Retrieved October 4, 2010, from <http://www.aafa.org/display.cfm?id=9&sub=19&cont=264>

A sure sign of spring (or summer or fall) in many regions of the United States is news media reports of pollen counts. These counts are of interest to some 35 million Americans who get hay fever because they are allergic to pollen.

People also look for counts of mold or fungus spores. These are another major cause of seasonal allergic reactions. Pollen and mold counts are important in helping many people with allergies plan their day.

WHAT IS THE POLLEN COUNT?

The pollen count tells us how many grains of plant pollen were in a certain amount of air (often one cubic meter) during a set period of time (usually 24 hours). Pollen is a very fine powder released by trees, weeds and grasses. It is carried to another plant of the same kind, to fertilize the forerunner of new seeds. This is called pollination.

The pollen of some plants is carried from plant to plant by bees and other insects. These plants usually have brightly colored flowers and sweet scents to attract insects. They seldom cause allergic reactions. Other plants rely on the wind to carry pollen from plant to plant. These plants have small, drab flowers and little scent. These are the plants that cause most allergic reactions, or hay fever.

When conditions are right, a plant starts to pollinate. Weather affects how much pollen is carried in the air each year, but it has less effect on when pollination occurs. As a rule, weeds pollinate in late summer and fall. The weed that causes 75 percent of all hay fever is ragweed which has numerous species. One ragweed plant is estimated to produce up to 1 billion pollen grains. Other weeds that cause allergic reactions are cocklebur, lamb's quarters, plantain, pigweed, tumbleweed or Russian thistle and sagebrush.

- Trees pollinate in late winter and spring. Ash, beech, birch, cedar, cottonwood, box, elder, elm, hickory, maple and oak pollen can trigger allergies.
- Grasses pollinate in late spring and summer. Those that cause allergic reactions include Kentucky bluegrass, timothy, Johnson, Bermuda, redtop, orchard, rye and sweet vernal grasses.

Much pollen is released early in the morning, shortly after dawn. This results in high counts near the source plants. Pollen travels best on warm, dry, breezy days and peaks in urban areas midday. Pollen counts are lowest during chilly, wet periods.

WHAT IS THE MOLD COUNT?

Mold and mildew are fungi. They differ from plants or animals in how they reproduce and grow. The “seeds,” called spores, are spread by the wind. Allergic reactions to mold are most common from July to late summer.

Although there are many types of molds, only a few dozen cause allergic reactions. *Alternaria*, *Cladosporium* (*Hormodendrum*), *Aspergillus*, *Penicillium*, *Helminthosporium*, *Epicoccum*, *Fusarium*, *Mucor*, *Rhizopus* and *Aureobasidium* (*pullularia*) are the major culprits. Some common spores can be identified when viewed under a microscope. Some form recognizable growth patterns, or colonies.

Many molds grow on rotting logs and fallen leaves, in compost piles and on grasses and grains. Unlike pollens, molds do not die with the first killing frost. Most outdoor molds become dormant during the winter. In the spring they grow on vegetation killed by the cold.

Mold counts are likely to change quickly, depending on the weather. Certain spore types reach peak levels in dry, breezy weather. Some need high humidity, fog or dew to release spores. This group is abundant at night and during rainy periods.

What Are the Symptoms for Hay Fever?

Pollen allergies cause sneezing, runny or stuffy nose, coughing, postnasal drip, itchy nose and throat, dark circles under the eyes, and swollen, watery and itchy eyes. For people with severe allergies, asthma attacks can occur.

Mold spores can contact the lining of the nose and cause hay fever symptoms. They also can reach the lungs, to cause asthma or another serious illness called allergic bronchopulmonary aspergillosis.

Pollen Count Levels and Allergies

HOW ARE POLLEN AND MOLD MEASURED?

To collect a sample of particulates in the air, a plastic rod or similar device is covered with a greasy substance. The device spins in the air at a controlled speed for a set amount of time— usually over a 24-hour period. At the end of that time, a trained analyst studies the surface under a microscope. Pollen and mold that have collected on the surface are identified by size and shape as well as other characteristics. A formula is then used to calculate that day's particle count.

The counts reported are always for a past time period and may not describe what is currently in the air. Some counts reflect poorly collected samples and poor analytical skills. Some monitoring services give "total pollen" counts. They may not break out the particular pollen or mold that causes your allergies. This means that allergy symptoms may not relate closely to the published count. But knowing the count can help you decide when to stay indoors.

HOW CAN I PREVENT A REACTION TO POLLEN OR MOLD?

Allergies cannot be cured. But the symptoms of the allergy can be reduced by avoiding contact with the allergen.

- Limit outdoor activity during pollination periods when the pollen or mold count is high. This will lessen the amount you inhale.
- The National Allergy Bureau (NAB) tracks pollen counts for different regions of the country. Contact the NAB through the American Academy of Allergy, Asthma and Immunology website.
- Pollen.com is also a reliable source of "pollen forecasts" in your zip code area, maintained by Surveillance Data Inc., a national monitor of medical and environmental statistics.
- Use central air conditioning set on "recirculate" which exclude much of the pollen and mold from the air in your home.
- Vacationing away from an area with a high concentration of the plants that cause your allergies may clear up symptoms. However, if you move to such an area, within a few years you are prone to develop allergies to plants and other offenders in the new location.

SOURCE: This information should not substitute for seeking responsible, professional medical care. First created 1995; fully updated 1998; most recently updated 2005.

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Swine Flu – H1N1

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Objectives

The goal of this activity is to have students apply their knowledge of quadratic functions and functional notation to propose a mathematical model of the H1N1 swine flu epidemic in the U.S. between April and May, 2009. Students will use that model to make predictions about swine flu, and will come to understand both the advantages and the limitations of mathematical models in predicting the behavior of a real life phenomenon such as swine flu.

Reflection

H1N1 is an influenza virus usually found in pigs, but which can also both infect and become infectious in humans. The virus can also change, or mutate. In the spring of 2009, cases of H1N1 flu virus were first discovered and confirmed in Mexico, and then in the United States. Since people were not immune to it, this virus spread very quickly. The number of people infected dropped in the summer of 2009; unfortunately, a new epidemic in the U.S. began in the fall of 2009. The data about the epidemic of the virus in the spring of 2009 is discussed at the website: [https://health.google.com/health/ref/H1N1+\(swine\)+influenza](https://health.google.com/health/ref/H1N1+(swine)+influenza).

In the introductory phase of the class activity, students learned how to manipulate a variable using whole numbers. The independent variable, days of spring in 2009, was converted from the date format to integers starting at zero, with April 23, 2009 as the reference day. Students were also introduced to the concept of mathematical (quadratic) modeling by studying the quadratic model obtained after correlating the data. Students observed that when comparing real data to data obtained from mathematical models, the results differed. They were asked to think about the possible reasons for that difference. They were also asked to analyze the quadratic equation that models the data. The sign of the quadratic coefficient and the missing term (the constant term of the quadratic equation) were discussed and explained.

Students' comments provided rewarding feedback:

"It never occurred to me that math is used in other areas of our daily lives."

"If I learn the rules and understand math better I will come to appreciate it."

"After I looked at and read the problem carefully it stopped blocking my mind and I could see that it is possible to solve math problems."

These comments are the best evidence of the power of such an activity to transform the students' apathy, fear and probably phobia of math into a positive force for learning.

Math Topics

Functional notation; quadratic function; zero factor property

Purpose

Motivation

Duration

About 90 minutes (50 minutes at home, 40 minutes in class discussion)

Activity Time Frame

Two weeks

Activity Overview

This activity should be introduced after teaching graphs of quadratic functions, and students have one week to complete it.

First lesson (30 minutes in class): The instructor distributes the handout and provides a brief overview of the issues related to swine flu and its impact on society. The class discusses Figure 1. The instructor explains how the data is summarized in Table 1 by shifting the independent variable. Then, the instructor describes the prediction equation and the shape of the function in Figure 2. All necessary information is included in the three page handout given to the students. Students should carefully read the handout at home.

Day 1 of the following week (10 minutes in class): Give the students the opportunity to ask questions arising from their careful reading of the handout at home. Remind them that office hours are available to answer additional questions. Emphasize the importance of completing the short reflection (on page 3 of the handout), as a means of processing the activity. The assignment is due the following day.

Materials and Resources

1. Handout
2. World Health Organization. www.g2weather.com/g2_weather/2009/05/swine-flu-graph-confirmed-us-cases-since-april-23rd.html. Last retrieved April 2, 2010. This site does not exist now. It contained the graph and numbers of flu cases in the U.S. The data and graph were derived from raw WHO data that can be found at <http://www.who.int/csr/disease/swineflu/updates/en/index.html>
3. Google Health. (2009, July 29.) *H1N1 (swine) influenza*. Retrieved Apr. 18, 2010, from [https://health.google.com/health/ref/H1N1+\(swine\)+influenza](https://health.google.com/health/ref/H1N1+(swine)+influenza).

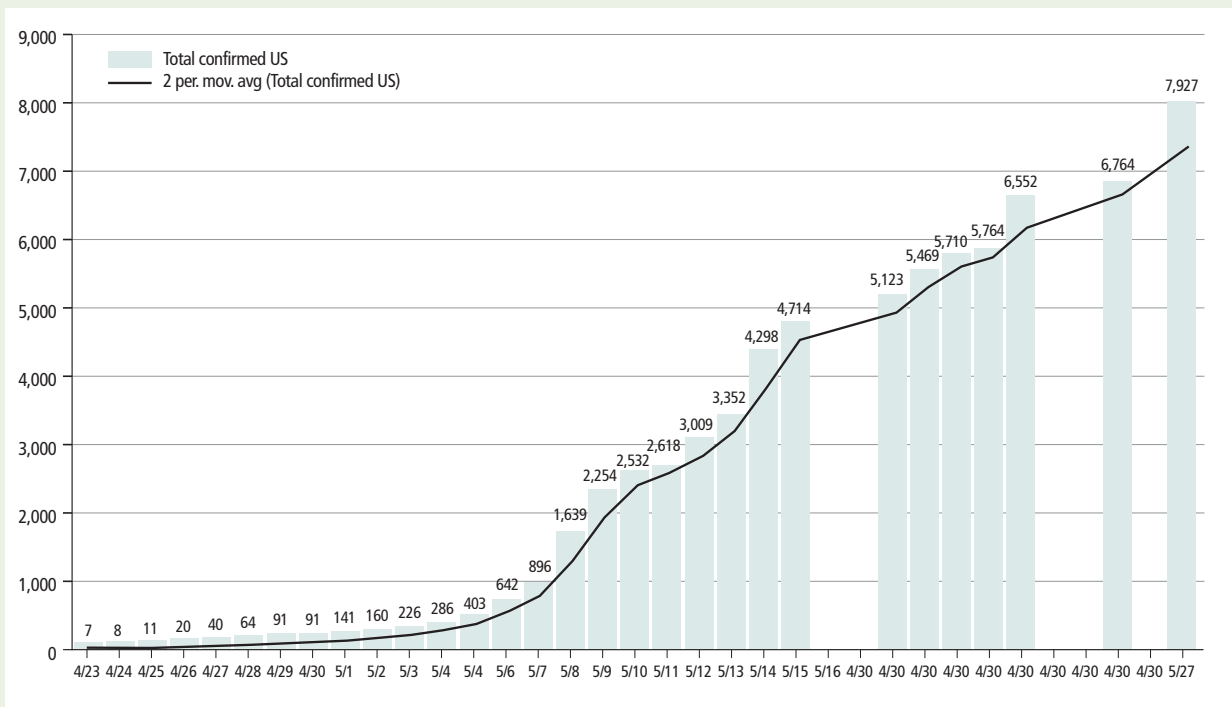
Handout

Przhebelskiy | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Objective

The goal of this activity is to apply algebra concepts to propose a quadratic function as a mathematical model of the swine flu cases in the U.S. and use that model to predict the number of swine flu cases.

Figure 1: Swine Flu confirmed U.S. cases as of May 27, 2009



Source: "Swine Flu Graph: 7927 Confirmed US Cases as of May 27, 2009" http://www.g2weather.com/g2_weather/2009/05/swine-flu-graph-confirmed-us-cases-since-april-23rd.html

Figure 1 presents the confirmed cases of swine flu in US as of May 27, 2009.

Swine Flu – H1N1

Table 1 presents the dates and the number of swine flu cases, where x is the number of days after April 23, 2009, and y is the number of flu cases on day x .

Note that the number of days is taken with the day April 23, 2009 as the reference date.

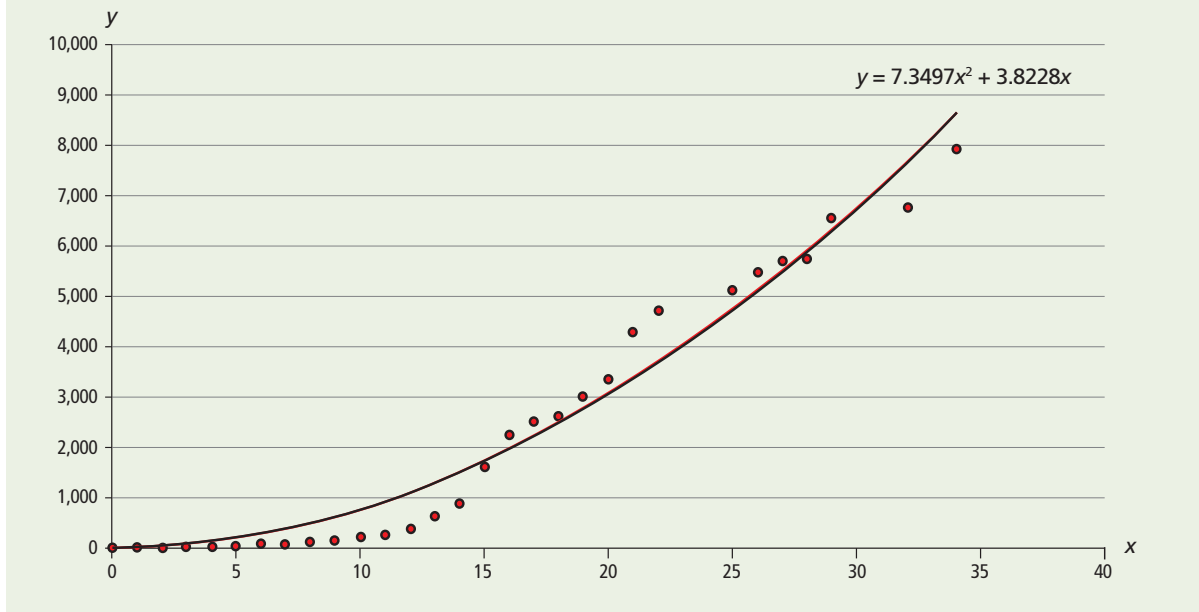
Points (x, y) are plotted in Figure 2. From Figure 2 it is apparent that the data does not follow a linear behavior. We shall use the quadratic function $y = ax^2+bx+c$ to approximate it.

Using Excel or a TI-83 calculator, the x and y data in Table 1 can be approximated to a quadratic model. The equation is $y = 7.3497x^2+3.8228x$. The plot of the quadratic function is also depicted in Figure 2.

Table 1: Swine flu data from Figure 1

Date	x = number of days after April 23	y = number of cases
23-Apr	0	7
24-Apr	1	8
25-Apr	2	11
26-Apr	3	20
27-Apr	4	40
28-Apr	5	64
29-Apr	6	91
30-Apr	7	91
1-May	8	141
2-May	9	160
3-May	10	226
4-May	11	286
5-May	12	403
6-May	13	642
7-May	14	896
8-May	15	1639
9-May	16	2254
10-May	17	2532
11-May	18	2618
12-May	19	3009
13-May	20	3352
14-May	21	4298
15-May	22	4714
18-May	25	5123
19-May	26	5469
20-May	27	5710
21-May	28	5764
22-May	29	6552
25-May	32	6764
27-May	34	7927

Figure 2: Real data and quadratic model proposed



Apply your knowledge about quadratic functions and functional notation to answer the following questions:

- (10 pts) Using the function $y = 7.3497x^2 + 3.8228x$, determine how many flu cases were reported on May 14, 2009. Hint: Remember that for this date $x=21$. Calculator required.
- (3 pts) Is the answer to #1 the same value in Table 1? _____ Yes _____ No
- (7 pts) Why is there a difference?
- (30 pts) According to the quadratic model $y = 7.3497x^2 + 3.8228x$, how many cases should be expected by June 1 2009? Calculator required.
- (30 pts) According to the modeling equation, at what values of x does the number of cases of swine flu equal zero? Give an explanation for each value obtained.
- (20 pts) Write a short summary of your experience with this activity: What mathematical skills did you use to solve this problem? Did this activity change how you think and feel about math? Did you ever think that swine flu and math might be related as they are in this activity? Why or why not?

References

- Google Health. (2009, July 29.) *H1N1 (swine) influenza*. Retrieved Apr. 18, 2010, from [https://health.google.com/health/ref/H1N1+\(swine\)+influenza](https://health.google.com/health/ref/H1N1+(swine)+influenza).

Exercise and Heart Rate

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Objectives

The objective of this project is to help students calculate their maximum and target heart rates, and to study the effect of exercise on heart rate. Using heart rate data obtained from the American Heart Association, students will learn to calculate slope, to construct a linear equation, and to use a linear equation to predict heart rates for subjects of different ages.

Reflection

This project on heart rate demonstrates to students that exercise is important for overall good health and for a healthy heart. Regular exercise reduces the risk of heart disease by reducing high blood pressure, stress, overweight and obesity.

The project incorporates a reading about heart rate, giving students the opportunity to understand and ask questions about main concepts and terms. Heart rate is defined and guidelines are provided for measuring heart rate, and keeping heart rate in the target zone during exercise. This project fosters excellent interaction in class, thereby contributing to students' growing awareness of medical problems related to heart rate. It may also potentially help students to diagnose cardiovascular disease at an early stage.

My personal communication with students has given me the impression that they enjoyed measuring their target heart rates and learning about strengthening the heart by exercising.

Since heart rate or pulse depends on one's emotional state, temperature, position or posture (sitting, standing, lying down), some students preferred to measure their pulse at home upon rising in the morning, as mentioned in the reading.

Most of the students did not understand how to determine function using slope, or predict heart rate using linear equations. I first showed them how to calculate the slope of each pair. If the slope of each pair is the same, then the equation of the line is a function; otherwise, it is not.

I also helped them find their heart rate using linear equations. Students learned to create graphs showing the relationship between heart rate and increasing age, to predict heart rates for subjects of different ages using estimated linear equations, and to use slopes to determine functions.

Finally, I guided them in writing a one-page reflection based on my question in Part 3.

Math Topic

Plotting ordered pairs, calculating slope, graphing linear equations with two variables, and functions and relations

Purpose

Review, synthesis

Comments

These exercises are based on calculating heart rate and building linear models using heart rate data for different age groups obtained from the American Heart Association.

When to Introduce

Week 2

Activity Time Frame

Two weeks

Activity Overview

Week one

Devote about 30 minutes of class time to a discussion of the Reading, “What You Should Know about Your Heart Rate or Pulse,” about measuring pulse or heart rate and calculating maximum heart rate based on age. You can begin by asking students what they already know about the main reading topics, as listed below, and then have them read again as necessary to correct their understanding or add any information to their understanding of these concepts.

- What is Heart Rate?
- How To Measure Your Pulse
- Reducing Your Heart Rate
- Target Heart Rate
- Recovery Heart Rate

Week Two

In Part 2 of this project, students are required to calculate slope and linear equations, to plot ordered pairs, construct and interpret graphs, and to determine domain, range, function and relation.

Students work out solutions to the given mathematical problems, using slope in each pair to determine function, and linear equations to predict heart rate. Both activities are done in class with instructor guidance (45 minutes in-class time).

Materials and Resources

- Handout
- Reading: National Emergency Medicine Association. (2003). *What you should know about your heart rate or pulse*. Retrieved from http://www.nemahealth.org/programs/healthcare/heart_rate_pulse.htm
- Optional links for further exploration:
 - Heart.com. (2009). *Articles of the heart*. Retrieved from <http://www.heart.com/the-heart.htm>
 - Heart.com (2009). *Heart rate chart*. Retrieved from <http://www.heart.com/heart-rate-chart.html>
 - ProHealth. (2011). *Target heart rate*. Retrieved from <http://www.prohealth.com/weight-loss/tools/target-heart-rate.cfm>

Handout

Rahman | Elementary Algebra – Issues in Public Health (MAT096)

The questions in Parts 1-3 are based on the Reading: “What you should know about your heart rate or pulse.” Please be sure to read the article before responding to the questions below.

Part 1

1. Why is a lower pulse rate good?
2. Why might an athlete have a lower pulse rate than a person who does not exercise regularly?
3. What are some of the factors that influence heart rate?
4. What can you do to reduce your heart rate (pulse)?
5. Calculate your maximum heart rate using the following formula:
Maximum heart rate=220-your age.
6. a. Using your maximum heart rate, calculate your personal target heart rate .
b. Compare your calculated personal target heart rate to those in Table 1 below.

Table 1: Target heart rates for different age groups

Age	Target Heart Rate Zone (50-85%)	Average Maximum Heart Rate (100%)
20 years	100-170 beats per minute	200 beats per minute
25 years	98-166 beats per minute	195 beats per minute
30 years	95-162 beats per minute	190 beats per minute
35 years	93-157 beats per minute	185 beats per minute
40 years	90-153 beats per minute	180 beats per minute
45 years	88-149 beats per minute	175 beats per minute
50 years	85-145 beats per minute	170 beats per minute
55 years	83-140 beats per minute	165 beats per minute
60 years	80-136 beats per minute	160 beats per minute
65 years	78-132 beats per minute	155 beats per minute
70 years	75-128 beats per minute	150 beats per minute

Source ProHealth. (2011). *Target heart rate*. Retrieved from <http://www.prohealth.com/weightloss/tools/target-heart-rate.cfm>

7. If the result you obtained from answering Question 6 doesn't fall within the suggested target heart rate zone closest to your age as indicated in Table 1, think about the possible reason(s) and describe in a paragraph how you can reach your target heart rate.

Exercise and Heart Rate

Part 2

The data in Table 2 below represent the maximum benefit to the heart from exercising, if the heart rate is in the target heart rate zone. In the graph, x represents different age groups and y represents the number of heart beats per minute.

Table 2: Maximum benefit to the heart from exercising

Age, x	Maximum Number of Heart Beats, y	Average (x, y)
20	140	
30	133	
40	126	
50	119	
60	112	
70	105	

Source: ProHealth. (2011). *Target heart rate*. Retrieved from <http://www.prohealth.com/weightloss/tools/target-heart-rate.cfm>

The American Heart Association's recommendation for achieving the maximum cardiac benefit when exercising is to start an exercise program at the lowest part of your heart rate zone and build up to your maximum heart rate. The lower limit of this zone is found by taking 50% of the difference between 220 and your age (maximum heart rate=220-age). The upper limit is found by taking 85%.

Therefore, looking at Tables 1 and 2 together, the connection is as follows:

To calculate the target heart rate zone for a 20 year-old:

$$220 - 20 = 200 \text{ beats per minute} = \text{Maximum Heart Rate}$$

From Table 1

$$\text{Lower Limit of Target Heart Rate Zone} = 200 \times 0.5 = 100$$

$$\text{Upper Limit of Target Heart Rate Zone} = 200 \times 0.85 = 170$$

From Table 2

The maximum number of heart beats for a 20 year old is in the target heart rate zone.

Now, write the data from Table 2 as ordered pairs, create a graph of the data, and then answer the questions or solve the problems below:

1. What kind of pattern can you observe from your graph?
2. What type of relationship appears to exist between the maximum number of heartbeats and age?
3. Select any two points and find an equation of the line containing the two points.
4. Calculate slope for every pair of points. If the slopes are the same, determine whether the function is linear or not.

5. Interpret the slope of the line you found in #5 above. How do you explain that particular negative value of the slope?
6. Using the equation of the line found in #5, predict the maximum number of heartbeats for ages 25, 55, 65, and 80
7. Using the equation of the line found in #5, what is the approximate age of a person whose maximum number of heart beats is 115?
8. Does the relationship defined by the set of ordered pairs (x, y) represent a function? If yes, write the domain and range.

Part 3

Reflection:

1. Write a paragraph explaining what math you have learned from this project.
2. Why does heart rate decrease as age increases? Explain your answer in writing, in terms of the slope of the equation of the line.
3. How have the quantitative calculations you've done on heart rate data enhanced your understanding of the seriousness of this health problem? Write a letter to a future MAT096 student sharing what you know as a result of this project. Include recommendations for a healthier heart.

Here are some optional links for further exploration:

- Heart.com. (2009). *Articles of the heart*. Retrieved from <http://www.heart.com/the-heart.htm>
- Heart.com (2009). *Heart rate chart*. Retrieved from <http://www.heart.com/heart-rate-chart.html>
- ProHealth. (2011). *Target heart rate*. Retrieved from <http://www.prohealth.com/weightloss/tools/target-heart-rate.cfm>

Reading: What you should know about your Heart Rate or Pulse

Rahman | Elementary Algebra – Issues in Public Health (MAT 096)

Source: National Emergency Medicine Association. (2003). *What you should know about your heart rate or pulse*. Retrieved from http://www.nemahealth.org/programs/healthcare/heart_rate_pulse.htm

Knowing how to measure your heart rate or pulse, can help you to learn about your own degree of fitness and can help to detect potential medical problems that should be brought to the attention of your physician.

What Is Heart Rate?

Very simply, your heart rate is the number of times your heart beats per minute. You can measure your heart rate by feeling your pulse - the rhythmic expansion and contraction (or throbbing) of an artery as blood is forced through it by the regular contractions of the heart. It is a measure of how hard your heart is working.

Your pulse can be felt at the wrist, neck, groin or top of the foot - areas where the artery is close to the skin. Most commonly, people measure their pulse in their wrist. This is called the radial pulse.

How To Measure Your Pulse

Taking your pulse is easy. It requires no special equipment, however, a watch with a second hand or digital second counter is very helpful.

1. Turn the palm side of your hand facing up.
2. Place your index and middle fingers of your opposite hand on your wrist, approximately 1 inch below the base of your hand.
3. Press your fingers down in the groove between your middle tendons and your outside bone. You should feel a throbbing - your pulse.
4. Count the number of beats for 10 seconds, then multiply this number by 6. This will give you your heart rate for a minute.

Example:

If you count **12 beats** in the span of **10 seconds**, multiply **12 X 6 = 72**. This means your Heart Rate or pulse, is **72** (or 72 beats per minute).

Another popular way to measure pulse rate is by measuring it at the neck (carotid pulse). This is especially convenient during exercise. The formula is the same as above, however, when taking the pulse at the neck, place your fingertips gently on one side of your neck, below your jawbone and halfway between your main neck muscles and windpipe.

Taking your pulse upon rising in the morning, or after sitting without activity for about 10 minutes, is known as your Resting Heart Rate.

What Is A Normal Heart Rate?

A Resting Heart Rate anywhere in the range of 60 - 90 is considered in the normal range. Your Heart Rate will fluctuate a lot depending on such factors as your activity level and stress level. If however, your pulse is consistently above 90, you should consult with your physician. This condition is called tachycardia (increased heart rate).

Many athletes have pulse rates in the 40 - 60 range, depending on how fit they are. In general, a lower pulse rate is good. Sometimes however, one's heart rate can be too low. This is known as bradycardia and can be dangerous, especially when blood pressure gets too low as well. Symptoms include weakness, loss of energy and fainting. If this situation applies, medical attention should be sought immediately.

If the pattern of beats or throbs you count is irregular (i. e. a beat is missed) take your pulse for a full minute. If you experience irregularities in your pulse on a consistent basis, you should consult with your personal physician.

Many factors influence heart rate. These include emotions, temperatures, your position or posture (sitting, standing, laying down), and your body size (if you are overweight for your size, your heart will have to work harder to supply energy to your body).

Reducing Your Heart Rate

A decrease in resting heart rate is one of the benefits of increased fitness due to exercise. Before starting into any exercise regimen, however, be sure to consult with your personal physician.

Your heart is a muscle and will respond just like any skeletal muscle in that it will become stronger through conditioning. If your heart muscles are stronger, then your heart rate will decrease. In other words, your heart will be putting out less effort to pump the same amount of blood.

Target Heart Rate

When undertaking an exercise program it is important to have a goal and a target range that you are trying to accomplish in each workout. To be of benefit, you want the workout to be neither too hard nor too easy. There is a simple formula to predict your maximum heart rate that is used in the fitness industry:

Take 220 and subtract your age.

This will give you a predicted maximum heart rate.

For example, if you are **42 years old**, subtract 42 from 220 (**220 - 42 = 178**). This means that your maximum physiological limit as to how fast your heart should beat is **178** beats per minute.

Most exercise programs suggest that when someone is just getting started that their heart rate during exercise should not exceed 60 - 70% of their maximum heart rate. Therefore, given the example above, 60% of 178 = 107 beats per minute. As you progress in your exercise, the percentage of your maximum heart rate to be set as a goal can be gradually increased.

Exercise and Heart Rate

Calculating a target heart rate zone is often desirable. To do so:

1. Start with your maximum heart rate as shown above.
2. Multiply your maximum heart rate by 0.8 to determine the upper limit of your target heart rate zone (divide this product by 6 to get the rate for a ten-second count).
3. Multiply your maximum heart rate by 0.6 to determine the lower limit of your target heart rate zone (divide this product by 6 to get the rate for a ten-second count).

Example:

For a person **42 years old**:

$$220 - 42 = \mathbf{178 \text{ Maximum Heart Rate}}$$

$$178 \times 0.8 = \mathbf{142 \text{ Upper Limit of Target Heart Zone}} \quad (142/6 = 24, 10 \text{ sec. count})$$

$$178 \times 0.6 = \mathbf{107 \text{ Lower Limit of Target Heart Zone}} \quad (107/6 = 18, 10 \text{ sec. count})$$

Note: Your maximum heart rate is the most your heart should reach after a strenuous workout.

Your Heart Rate should be measured during warm-up, halfway into your workout, at the end of your workout and at the end of your cool-down period. If during exercise you exceed your upper limit, decrease the intensity of your workout. Conversely, at the end of your workout if your heart rate is much lower than your target, you need to work harder next time.

Recovery Heart Rate

One way to determine if you are reaping the benefits from exercise is to calculate your Recovery Heart Rate, a measure of how quickly you return to your resting heart rate after a workout. To calculate your recovery heart rate:

1. Take your pulse ten seconds immediately after you have finished exercising. Write down the number.
2. One minute later, take your pulse again and write it down.
3. Subtract the number for the second pulse check from the number for the first pulse check.
This number is your Recovery Heart Rate. The greater the number, the better shape you are in!

A Final Word on Exercise Programs

Exercise programs help to increase the strength of the heart. Declines will be seen in resting heart rate, and hopefully, blood pressure, and stress levels as well. Overall body changes will also be experienced including weight loss and increase of lean body mass.

Remember, however, that it is important to check with your doctor and seek out a qualified exercise physiologist before you get started. An exercise stress test may be advised to help ensure the training parameters that are best for you.

The Mathematics of Mass Vaccination

Vladimir Przhabelskiy

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Objectives

The goal of this activity is to have students apply algebraic concepts in order to understand how vaccination helps manage and eradicate infectious diseases. Students will learn how the rate of infection is described mathematically by a basic reproduction number. Using linear function, they will calculate how the vaccination of a population slows the rate of infection and helps eradicate infectious diseases.

Motivated students may want to consult http://en.wikipedia.org/wiki/Infectious_disease to deepen their knowledge of infectious diseases, and <http://encyclopedia.thefreedictionary.com/Vaccination>, to further their understanding of the history and mechanism of vaccination.

Reflection

The recent outbreak of the H1N1 (swine) flu offers educators a perfect opportunity to introduce the mathematical modeling of vaccinations which combat infectious diseases.

While the biologist analyzes the composition of a particular virus and tries to create a vaccine to combat the resulting disease, it is the mathematician who allows scientists to understand how virulent that virus is (Du Sautoy, M. 2009).

The classic SIR model for the spread of an epidemic, representing the three primary states that any member of a population can occupy with respect to a disease: susceptible, infectious, and removed, requires knowledge of nonlinear differential equations (Smith, D. and Moore, L. 2001; and Wang, F. 2010).

This project, however, focuses only on the application of elementary algebra to the analysis of infectious diseases; in particular, on vaccination and its potential to control and eradicate infectious diseases. Adopting this material as a project in an elementary algebra course affords students the opportunity to learn mathematics through the examination of current and urgent public issues.

When surveyed about this project, students provided rewarding feedback. Their comments are quoted, verbatim, below:

- “This project helped me to analyze data carefully and logically. I understand now how math is applied in science, technology and that it is very commonly used to explain and demonstrate a variety of situations.”
- “This activity changed a lot my point of view about math. Cause I never realized that thank to math many diseases can be cured and less people are killed because of these diseases.”

Math Topic

Graphing linear equations, finding the slope of a line, and solving linear inequalities

Purpose

To improve student motivation and understanding by presenting mathematics through the context of its public health applications

Duration

About 90 minutes (40 minutes at home, 50 minute in- class discussion)

When to Introduce

Weeks 4–5

Activity Time Frame

2 weeks

The Mathematics of Mass Vaccination

- “This activity changed the way I used to think about math because with math we can find many solutions just like we did in this project.”
- “I used to hate math and now I did not hate it as much. This project helped me become more comfortable when solving equations and interpreting graphs.”
- “I now do see how math can be involved into anything. How you can put an everyday problem and involve it into math. “
- “The professor really tried to help me understand math, I will attempt to use math more in my day to day event.”

Activity Overview

By the end of Week 3, students will have learned different ways of graphing a line and working with its slope. This project can be introduced in week 4, and completed in two weeks.

Note: The effect of vaccination is described in the two sources below:

a. The article, “Reproductive Number” (2009, August 1), retrieved from the Ganfyd website: http://www.ganfyd.org/index.php?title=Reproductive_number), is required reading for students, and is part of the handout;

b. Lecture “Concepts for the prevention and control of microbial threats – 2”. Center for Infectious Disease Preparedness, UC Berkeley School of Public Health is recommended but not required reading

The **basic reproduction number** (R_0) refers to the number of secondary infectious cases that would be produced by a single infectious case introduced into a completely susceptible population with no control measures. For a given population, a communicable microbe has an expected R_0 . R_0 allows us to compare the potential for different microbes to cause outbreaks/epidemics in a population.

As an epidemic evolves, the average number of secondary cases produced by infectious cases generally declines as people die out or become immune to the disease. This is called the **effective reproduction number**.

In the presence of control measures, the effective reproduction number is called the **control reproduction number**. The goal in infectious disease control is to get the control reproduction number down to less than 1 as quickly and as cost effectively as possible.

The control reproduction number (R_c) is given by the following equation:

$R_c = R_0 (1 - h f)$ where **h** is the **vaccine efficacy** (the proportion of people vaccinated who will have complete protection), and **f** is the **vaccine coverage** (proportion of the population that has been vaccinated).

In this project, it is assumed for simplicity that vaccine efficacy $h = 1$, which results in Equation 2 in the handout: $R_c = R_0 - R_0 f$

Also, according to the Ganfyd website, ([http://www.ganfyd.org/index.php?title=Reproductive number](http://www.ganfyd.org/index.php?title=Reproductive_number)), if the goal of a vaccination program is to eliminate a disease (or, to put it another way, to ensure that there is “herd immunity”), the vaccine coverage that is required is given by the following inequality: Goal: $R_c < 1$

This inequality is used in the project to find the proportion of a population that must be immunized in order to eradicate a given communicable disease.

Lesson 1 (30 minutes in class):

Distribute the handout and provide an overview of the issues related to the basic reproduction number. Have students choose an infectious disease from Table 1 and calculate the average value of the basic reproduction number (R_0) for that disease. Then show students how to write the linear equation which demonstrates how the vaccination of a population changes the reproduction number R_0 to the new value R_c . Explain the work the students must do with the equation: find the slope of the line and graph the equation. At the end of the project, by solving the linear inequality, students will determine what percentage of the population needs to be vaccinated in order to immunize the entire population against a given infectious disease.

Lesson 2

The following week (20 minutes in class): Give students the opportunity to ask questions arising from their careful reading of the handout at home. Remind them that you are available during office hours to answer any additional questions. Emphasize the importance of completing the reflection (Step 5 of the Handout), as a means of processing the activity. The assignment is due the following day.

Materials and Resources

- Handout
- Reading: Reproductive number. (2009, August 1). Retrieved from the Ganfyd website: http://www.ganfyd.org/index.php?title=Reproductive_number

Additional reading materials:

- Basic reproduction number. Retrieved from The Free Dictionary website: <http://encyclopedia.thefreedictionary.com/Basic+reproduction+number>
- du Sautoy, M. (2009, June 17). Sexy maths: Arithmetic eases swine flu. *The Times*. http://technology.timesonline.co.uk/tol/news/tech_and_web/article6512745.ece
- Herd immunity. (2011, December 12). Retrieved from the Wikipedia website: http://en.wikipedia.org/wiki/Herd_immunity
- Reproductive number. (2009, August 1). Retrieved from the Ganfyd website: http://www.ganfyd.org/index.php?title=Reproductive_number
- Smith, D. and Moore, L. (2001, December). The SIR Model for spread of disease. *Journal of Online Mathematics and its Applications*. Retrieved from <http://www.sph.emory.edu/~cdckms/WinPepi/DESCRIBE.pdf>
- UC Berkeley School of Public Health, Center for Infectious Disease Preparedness. (2006, June). Concepts for the prevention and control of microbial threats – 2. Retrieved from: <http://www.idready.org/slides/01epiconceptsII-notes.pdf>
- Wang, F. (2010, March). Application of the Lambert W function to the SIR epidemic model. *The College Mathematical Journal*. 41 (2). Retrieved from <http://www.maplesoft.com/applications/view.aspx?SID=7088&view=html>

Handout: The Mathematics of Mass Vaccination

Przhebelskiy | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Objective

In this activity, you will apply algebraic concepts in order to discover how vaccination helps control and eradicate infectious diseases. You will perform mathematical calculations involving graphing lines, finding the slope of a line, and solving linear inequalities.

Directions

Step 1: Background Information

Read about the Reproduction Number in the article from the Ganfyd website provided below. This reading is also available at http://www.ganfyd.org/index.php?title=Reproductive_number.

When you read this material, you must make sure you understand the term “herd immunity.” “Herd immunity (or community immunity) describes a form of immunity that occurs when the vaccination of a significant portion of a population (or herd) provides a measure of protection for individuals who have not developed immunity.” For more information, see http://en.wikipedia.org/wiki/Herd_immunity.

To demonstrate your understanding of the following concepts, think of an infectious disease that you know about. For each of the terms below, write a phrase to define each term in the context of the infectious disease you selected:

- What is the meaning of “herd immunity”?
- What is the basic reproduction number?
- What is the effective reproduction number?
- What is the control reproduction number?

Step 2: Definition of the Basic Reproduction Number R_0

According to the Free Dictionary entry about the basic reproduction number, available at (<http://encyclopedia.thefreedictionary.com/Basic+reproduction+number>), each virus causing an infectious disease is assigned a number called the basic reproductive rate of infection, or basic reproduction number. The basic reproduction number, R_0 , measures how quickly the virus spreads, or, in other words, the number of individuals in an entirely susceptible population who will be infected by an infected person. Table 1 below shows the values of basic reproduction numbers for different infectious diseases.

These measures are useful because they help determine whether or not an infectious disease can spread through a population. When $R_0 < 1$, the infection will die out in the long run (provided infection rates are constant). But if $R_0 > 1$, the infection will spread throughout the population.

Table 1 Reproduction numbers, R_0 , for different infectious diseases

Disease	Transmission	R_0	Disease	Transmission	R_0
HIV/AIDS	Sexual contact	2–5	Pertussis	Airborne droplet	12–17
Diphtheria	Saliva	6–7	Polio	Fecal-oral route	5–7
Influenza (1918 pandemic strain)	Airborne droplet	2–3	Rubella	Airborne droplet	5–7
Measles	Airborne	12–18	SARS	Airborne droplet	2–5
Mumps	Airborne droplet	4–7	Smallpox	Social contact	6–7

Source: Basic reproduction number. Retrieved from The Free Dictionary website: <http://encyclopedia.thefreedictionary.com/Basic+reproduction+number>

Task

Choose an infectious disease from Table 1 and calculate the average value of the basic reproduction number, R_0 , for that disease. Use this value of R_0 to complete Step 3 below.

Step 3: How can the mass vaccination of a population reduce the danger of infectious disease?

Suppose that a proportion of the population, f , is immunized against a given infection with a basic reproduction number, R_0 . Being a proportion, f values range from 0 to 1, where $f = 0$ means that no one is vaccinated and $f = 1$ means that the entire population is vaccinated.

According to the “Reproduction Number” article on the Ganfyd website that you read, (http://www.ganfyd.org/index.php?title=Reproductive_number), the control reproduction number is given by the following equation:

$$\text{Equation 1: } R_c = R_0 (1 - h f)$$

where the control reproduction number (R_c) is the average number of secondary cases caused by each infectious case in the presence of control measures such as vaccination; h is the *vaccine efficacy* (the proportion of people vaccinated who will have complete protection), and f is the *vaccine coverage* (proportion of the population that has been vaccinated).

In this project, it is assumed for simplicity that vaccine efficacy $h=1$. Thus, the proportion of the population that is vaccinated, the original reproduction number and the control reproduction number can be related as follows:

$$\text{Equation 2: } R_c = R_0 - R_0 f$$

Step 3a: Substitute the vaccine efficacy $h = 1$ in Equation 1 and derive Equation 2.

Step 3b: Substitute the value of R_0 which you found in Step 2 into Equation 2, and write the resulting equation.

Step 3c: Now, change f to x and R_c to y in the equation set in Step 3b.

Step 3d: Find the intercepts of the equation obtained in Step 3c. Use the intercepts to sketch the line.

Step 3e: Identify the slope of the equation of the line set in Step 3c.

The Mathematics of Mass Vaccination

Step 4. Vaccine Coverage Level.

Remember, when R_c is less than 1,

$$\text{Inequality 1: } R_c < 1$$

the given disease will die out, and the population will achieve the herd immunity.

The inequality to find the required immunization of a population can be found by substituting the expression for R_c from the Equation 2 into Inequality 1:

$$\text{Inequality 2: } R_0 - R_0 f < 1$$

where: R_0 : basic reproduction number

f : proportion of population which must be vaccinated.

Step 4a: Now, solve linear Inequality 2 for f , to determine the proportion of the population that must be immunized by vaccination in order to eradicate the given infectious disease. Use the value of the reproduction number R_0 you used to answer questions in Step 3.

Step 5. Reflection

What mathematical skills did you use to solve this problem? Did this activity change the way you think and feel about learning and using math? Explain your answer in two paragraphs.

Reading: The Reproduction Number

Przhebel'skiy | Elementary Algebra – Problems and Issues in Public Health
(MAT 096)

Source: Reproductive number. (2009, August 1). Retrieved from the Ganfyd website: http://www.ganfyd.org/index.php?title=Reproductive_number

Introduction

The reproduction number is a concept in the epidemiology of infectious diseases. It is a measure of how infectious a disease is, and is required if you wish to calculate how many people you need to vaccinate if you are to achieve herd immunity.

When somebody gets an infectious disease, they may pass it on to nobody else, or they may infect 1, 2, or more other people (who become secondary cases). This can be displayed in a number of ways, including graphically. The reproduction number, R , is the average (mean) number of secondary cases caused by each case of an infectious disease, during the infectious period.

The R number will, of course, depend on a large number of factors, including:

- How the infectious organism is spread;
- Behaviours which affect the likelihood of spread (social mixing, sexual and feeding practices...);
- The level of susceptibility within the population - which will depend on factors such as:
 - prior immunity;
 - levels of nutrition and immune suppressions;
 - age
- R_0 , R , and R_c

The basic reproduction number – R_0

Basic reproductive rate (R_0 , basic reproduction number, basic reproductive ratio) is the expected number of secondary cases produced by a typical primary case in an entirely susceptible population. When $R_0 < 1$ the infection will die out but any value for $R_0 \geq 1$ implies it will spread (without control measures) and higher numbers are more likely to cause epidemics. When control measures are possible epidemiologists are more interested in the effective reproduction number (R).

The R number which would apply if nobody in the population had any immunity to the disease at all, in the absence of any control measures (such as when smallpox was first introduced to Pacific islands, or the American continents) is referred to as the *basic reproduction number* or R_0 (that's a zero or nought, not a letter "O"). R_0 gives a measure of the infectiousness of the organism *per se*, which tends to be relatively fixed, as it is not affected by e.g. the uptake of vaccine or immunity from previous epidemics of the disease.

The Mathematics of Mass Vaccination

R_0 is proportionate to:

- The length of time that the case remains infectious (*duration of infectiousness*)
- The number of contacts a case has with susceptible hosts per unit time (**the contact rate**)
- The chance of transmitting the infection during an encounter with a susceptible host (*the transmission probability*).

This is plain common sense, and can be expressed mathematically as:

$$R = c p d$$

where:

- c is the number of contacts per unit time,
- p is the transmission probability per contact, and
- d is the duration of infectiousness.

The effective reproduction number – R

Effective reproductive number (R) is the actual average number of secondary cases per primary case observed in a population with an infective disease. The value of R is typically smaller than the value of *basic reproductive rate* (R_0), and it reflects the impact of control measures and depletion of susceptible persons by the infection.

Examples

Early in a new infectious disease R will be close to R_0

- SARS
 - $R_0 = 3.6$ (95% CI 3.1–4.2) which was the same as R in early stages as this condition had no specific treatment²
 - $R = 0.7$ (95% CI: 0.7–0.8) obtained by intense control measures and allowed fairly rapid control once recognised as a highly infectious disease with respiratory transmission
- Swine influenza 2009
 - R_0 northern hemisphere summer 1.4 – 1.5 with delay strategy
 - Initial R from southern hemisphere winter 1.8 to 2.3 in community/school winter outbreaks before disease recognised and control measures emplaced³

The effective reproduction number will change as, for example, people become immune to the disease. During an epidemic R will typically start as >1 , fall to about 1 (at which stage the incidence of the disease will remain approximately static), or fall below 1, at which point the level the epidemic will cease – at least until the proportion of the population that is susceptible increases to levels at which another epidemic may arise. This explains the regular peaks and troughs in the incidence of e.g., Parvovirus B19 infection, or of most childhood illnesses prior to the introduction of vaccination.

The effective reproduction number – R is the basic reproduction number (R_0) times the fraction of the population that is susceptible to infection (x):

$$R = R_0 x$$

As an epidemic spreads, and people die or become immune to the disease, x decreases, and eventually becomes small enough that R drops below 1.

The control reproduction number – R_c

The control reproduction number (R_c) is the average number of secondary cases due to each case in the presence of control measures such as vaccination. The aim of control measures is to ensure that the disease is eliminated from a population, which will happen if R_c is less than 1

In the case of vaccination, the control reproduction number is given by the following equation:

$$R_c = R_0 (1 - h f)$$

where

- h is the vaccine efficacy (the proportion of people vaccinated who will have complete protection), and
- f is the vaccine coverage (proportion of the population that has been vaccinated).

(It's actually usually rather more complicated, as some of the population will have natural immunity.)

So, if the goal of a vaccination programme is to eliminate a disease (or, to put it another way, to ensure that there is herd immunity), the vaccine coverage that is required is given by the following equation:

$$\text{Goal: } R_c < 1$$

Vaccine coverage required:

$$f > \frac{1 - (1/R_0)}{h}$$

R_0 values and vaccine coverage levels for particular infectious diseases

R_0 values, and the vaccine coverage required to prevent them are given for selected disease in the following table:⁴

R_0 values, and the vaccine coverage required to prevent them

Disease	R_0	Vaccine coverage (course completed) required for herd immunity
Diphtheria	6–7	85%
Measles	12–18	83%–94%
Mumps	4–7	75%–86%
Pertussis	12–17	92%–94%
Polio	5–7	80%–86%
Rubella	6–7	83%–94%
Smallpox	5–7	80%–85%

References

1. Figure 2: Probable cases of severe acute respiratory syndrome, by reported source of infection – Singapore, February 25–April 30, 2003. From Leo YS, Chen M, Heng BH, Lee CC, Paton N, Ang B, et al. Severe Acute Respiratory Syndrome --- Singapore, 2003. *MMWR – Morbidity & Mortality Weekly Report* 2003;52(18):405–11.
2. Wallinga J, Teunis P. Different epidemic curves for severe acute respiratory syndrome reveal similar impacts of control measures. *American Journal of Epidemiology*. 2004 Sep 15; 160(6):509–16. (Link to article – subscription may be required.)
3. Planning Assumptions for the First Wave of Pandemic A(H1N1) 2009 in Europe ECDC 29 July 2009
4. Lecture “Concepts for the prevention and control of microbial threats – 2”. Center for Infectious Disease Preparedness, UC Berkeley School of Public Health.

Drug Concentration in the Blood (In-Class Activity)

Natalia Mosina

Department of Mathematics, Engineering, and Computer Science

Objectives

The goal of this PQL project is to help students learn to recognize and appreciate math in the everyday world. In this activity students will learn what pharmacokinetics is and will look at several mathematical models that can be used to calculate the concentration of a drug in a patient's blood. While doing exercises comparing different concentrations of drugs in the blood at various times, students will apply the skills of simplifying rational expressions and complex fractions. Finally, students will see that there are different ways (algebraic and graphical) to model drug/alcohol concentration, and will learn how both ways can be used to arrive at certain conclusions.

Reflection

I recommend doing this activity during the week when complex fractions are studied. When I have used this activity with my students, I have found that if all the project requirements are clearly explained and students are guided through the in-class activity step-by-step, they have no problem answering all the questions on Handout #2. For further study, I recommend that students also complete the follow up PQL activity "More on Blood Drug/Alcohol Concentration" at home. The take-home activity helps to ensure that students have mastered the math concepts used in both activities, giving them a chance to reflect more upon the consequences of drugs and alcohol, and providing additional opportunities to see how math is used in "real world" situations.

Either student responses to Handout #2 or the work they complete on the take-home "More on Blood Drug/Alcohol Concentration" activity can be deposited into the Assessment area of the ePortfolio system. It is best to demonstrate to students how to upload their projects into the Assessment area. The lab hour can be used for this purpose. Uploading should take about 5–10 minutes, providing there are no technology glitches.

Activity Overview

In order to help students see the connection between the math they're learning in class and the power of math to solve real-life problems, particularly issues involving public health, start with a discussion of Reading #1: Everything's A Math Problem. The following procedure is recommended for this PQL activity:

1. Have students work in groups to reflect on and discuss the ways in which they use math in their everyday lives. Then distribute Reading #1: Everything's A Math Problem and Handout #1 with the questions. Have students read and discuss the reading in class.

Math Topic

Rational expressions, complex fractions

Purpose

Motivation

When to Introduce

Week 7–8

Activity Time Frame

Approximately 1 hour of in-class time

Drug Concentration in the Blood

2. Distribute Reading #2: Math of Drugs and Bodies for students to read in class.
3. Discuss the reading with students, checking that they understand pharmacokinetics, what happens with drug concentration over time, what models can be used to describe the behavior of a drug in the patient's blood, etc. Emphasize how the topic connects to MAT096.
4. Have students answer questions 1–6 in Handout #2. Give students time to think, and to work independently. Encourage students to show their computations on the board.
5. Compare students' computations to yours and discuss any errors in students' computations. Make sure students understand the source of their errors.
6. Collect students' work at the end of the activity and distribute the Answer Key (Handout #3)

At this point, the instructor can give students the take-home PQL activity, "More on Blood Drug/Alcohol Concentration."

Materials and Resources:

- Handout #1 : Discussion Questions
- Handout #2: Drug Concentrations, Rational Expressions, and Complex Fractions
- Handout #3: Answer Key for questions in Handout #2
- Reading #1: Duval, Robert. (n.d.). Everything's a math problem. Retrieved from Article Sphere.com website: <http://www.articlesphere.com/Article/Everything-s-a-Math-Problem/183538>
- Reading #2: Excerpted from "Math of drugs and bodies (pharmacokinetics)." (2010, February 1). Retrieved and adapted from SquareCircleZ website: <http://www.squarecirclez.com/blog/math-of-drugs-and-bodies-pharmacokinetics/4098>

Handout#1: Discussion Questions

Mosina | Elementary Algebra – Problems and Issues in Public Health (MAT096)

The following discussion questions are based on Reading #1: Everything's a math problem, by Robert Duval. (Source: Duval, Robert. (n.d.). Everything's a math problem. Retrieved from Article Spere.com website: <http://www.articlesphere.com/Article/Everything-s-a-Math-Problem/183538>.)

Based on the reading, answer the questions below. Make sure you have read Reading #1 before you take a few minutes to write a response to each question. We will discuss these questions in class.

1. Explain the meaning of “everything's a math problem.”
2. How do you feel about learning math? Explain why you feel that way.
3. In your experience learning math, how often do you think your teachers showed you the power of math to solve real life problems? Did this change the way you felt about math?
4. Do you think a basic competency in math is essential for everyone? Explain why, or why not.
5. According to the article, why is the expression “math puzzle” preferable to “math problem?” Would it make you see math problems differently if they were called math puzzles? Explain why, or why not.
6. This project is about drug concentration in the blood. Before you look at Reading #2, use your intuition, and your own current knowledge and experience to answer the following question:

When you take a drug (for example, a cold medication), what happens to the concentration of this drug in your blood over time? Write your answer below.

Handout#2: Drug Concentrations, Rational Expressions, and Complex Fractions

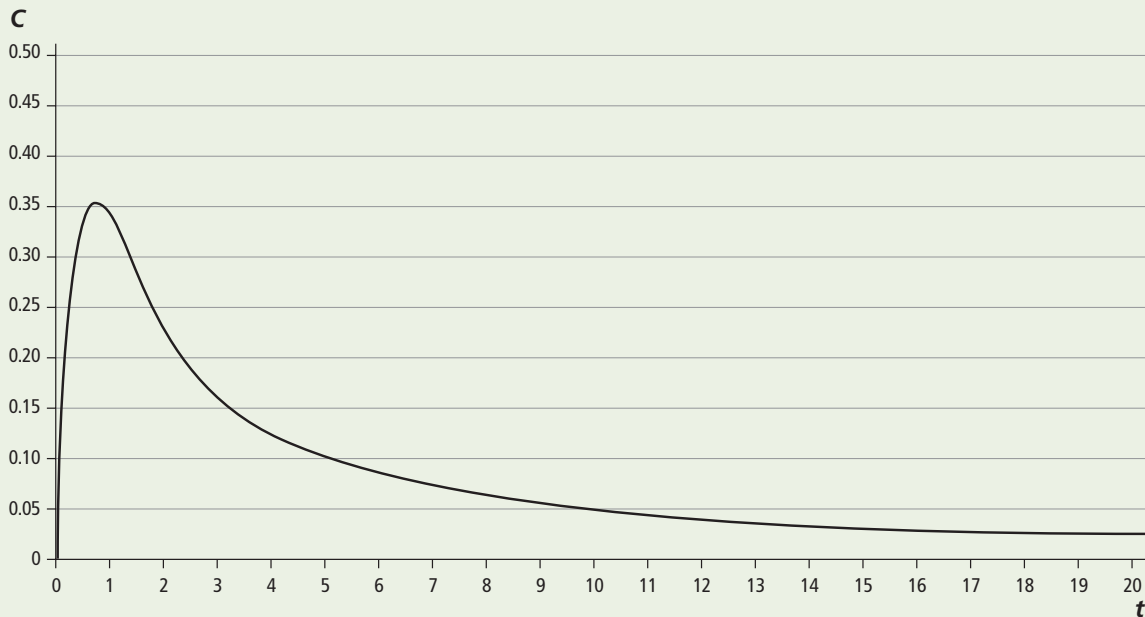
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In Reading #2, we saw one of the possible models for drug concentration at time t .

Sometimes, the concentration of a certain drug in a patient's bloodstream can be given as a rational expression, such as,

Equation (2): $C(t) = \frac{t}{2t^2 + 1}$ (measured in grams of drug /100ml blood, or %)

Figure 2: Drug concentration after t hours



The graph in Figure 2 was created using Maple software. Note: This shape is similar to the one you saw in Figure 1 on Reading #2.

Now, answer the following questions:

1. What is the concentration of the drug at time $t = 0$ hours in the model described by Equation (2)?
2. What is the concentration of the drug at time $t = \frac{1}{2}$ hour in the model described by Equation (2)?

3. Look at Figure 2 and estimate the time t (after injection) at which the concentration is highest.
4. What happens to the concentration of this drug over time? Find explanations in Reading#2.

Critical Thinking: Do both models, the graphical and the algebraic representations of the drug concentration in Figure 2, confirm your intuition and the inference you drew from the text?

In class, discuss whether each model confirms your intuition that $C(t)$ will decline as t increases.

5. Let $t_1 = 1$ and $t_2 = \frac{3}{2}$ hours. Find the ratio $\frac{C(t_1)}{C(t_2)}$
6. Let $t_1 = \frac{x}{2}$ and $t_2 = x$ hours. How many times greater is $C(t_1)$ than $C(t_2)$, assuming that $x > 1$? [In other words, find the ratio $\frac{C(t_1)}{C(t_2)}$.] Simplify your rational expression.

Handout#3: Solutions/Answers to Questions in Handout #2

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1. It is given that $t = 0$ hours. Plug it in Equation (2) and find what $C(0)$ is. In this way, we find the concentration of the drug at time $t = 0$ in the model described by Equation (2):

$$C(0) = \frac{0}{2(0^2) + 1} = \frac{0}{1} = 0\%$$

2. Now, $t = \frac{1}{2}$. Plug it in Equation (2):

$$C\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{2 \times \frac{1}{2} \times \frac{1}{2} + 1} = \frac{\frac{1}{2}}{\frac{1}{2} + 1} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{2} \times \frac{2}{3} \approx 0.33\%$$

Look at Figure 2 to see that the graph supports the answer you calculated algebraically.

3. The graph in Figure 2 shows that the concentration of the drug is the highest at the time $t \approx 0.8$ hours (after injection).
4. The concentration of a drug in the blood will level off as time goes by. In other words, $C(t)$ will eventually decrease. Indeed, as stated in Reading #2, “As time goes on, the drug concentration gets less and less and falls below a certain effective amount. Then it’s time to take some more pills.” The graphical representation of the drug concentration in Figure 2 confirms our intuition and the inference we drew from the Readings. Indeed, Figure 2 shows that $C(t)$ gradually decreases as time goes by. If you think about it, the algebraic model tells the same story: after some time, the bigger the value of t , the smaller the value of the rational expression for $C(t)$. Discuss this in class with your instructor.
5. If $t = 1$ hour, then

$$C(1) = \frac{1}{2(1^2) + 1} = \frac{1}{2 + 1} = \frac{1}{3}\%$$

If $t = \frac{3}{2}$ hours, then

$$C\left(\frac{3}{2}\right) = \frac{\frac{3}{2}}{2 \times \frac{3}{2} \times \frac{3}{2} + 1} = \frac{\frac{3}{2}}{\frac{9}{2} + 1} = \frac{\frac{3}{2}}{\frac{11}{2}} = \frac{3}{2} \times \frac{2}{11} = \frac{3}{11}\% \quad \text{and} \quad \frac{C(1)}{C\left(\frac{1}{3}\right)} = \frac{\frac{1}{3}}{\frac{11}{9}} = \frac{11}{9}$$

$$6. \quad C\left(\frac{x}{2}\right) = \frac{\frac{x}{2}}{2 \times \frac{x}{2} \times \frac{x}{2} + 1} = \frac{\frac{x}{2}}{\frac{x^2}{2} + 1} = \frac{\frac{x}{2}}{\frac{x^2 + 2}{2}} = \frac{x}{2} \times \frac{2}{x^2 + 2} = \frac{x}{x^2 + 2}, \quad C(x) = \frac{x}{2x^2 + 2}$$

$$\frac{C(t_1)}{C(t_2)} = \frac{\frac{x}{x^2 + 2}}{\frac{x}{2x^2 + 1}} = \frac{x}{x^2 + 2} \times \frac{2x^2 + 1}{x} = \frac{2x^2 + 1}{x^2 + 2}$$

Reading #1: Everything's A Math Problem

Mosina | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Source: Duval, Robert. (n.d.). Everything's a math problem. Retrieved from Article Sphere.com website: <http://www.articlesphere.com/Article/Everything-s-a-Math-Problem/183538>

There are probably an endless number of math problems. When you are studying math in elementary or high school, you have no idea of the huge world of math that exists at the college and post-college levels. Additionally, when you're studying elementary-level math, it's sometimes hard to make the connection between seemingly-insignificant math problems and the ultimate power that math has to solve problems in real life.

Think of medicine, for example. Students who started out the same as you and I, learning about square roots and fractions in elementary and junior high schools, have ended up using math to solve major health problems such as polio and tetanus. By turning health problems into math problems, collecting data and turning it into numbers, public health workers and epidemiologists figured out what was causing these diseases. Then, they solved the math problems and figured out how to get rid of the diseases. Without the beginning elements of addition, subtraction, algebra, geometry, calculus, and statistics, this could not have happened. Mastery of finding solutions to math problems allowed scientists to solve health problems and relieve human suffering. They can then analyze the effect of various solutions to those problems with controlled trials. All of this would be impossible without math.

At the college level, students usually see those seemingly meaningless math problems, like how much bread Joe can carry if his bicycle has a basket that is 1 foot by 1 foot, turn into real-life issues. If you study social science, you'll do research using math. You'll probably use software such as SPSS that makes the quantitative part of solving math problems easy. However, the student has to understand what the data is telling her/him and know how to input it into the program in order for it to work.

Improving your house is also another area where you will encounter math problems. If you want to repaint one or several walls, you have to figure out many issues. This is not a really advanced math problem, but it does show how math applies to everyday life. It's for this reason that everyone in the United States is required to achieve at least a basic competency in math. People who study education are aware that all aspects of our daily lives involve math in many ways.

When we think of solving math problems, the idea makes many of us cringe. Perhaps this is in part due to the misnomer of the word problems. It would be better if we called them "math puzzles" instead. Doesn't that make it sound more enticing? Math puzzles would be something fun, playful, or exciting. While calling them math problems makes them sound bad, calling them math puzzles makes you feel that you're accomplishing something if you put the puzzle together. That's really more accurate as to what math is all about anyway.

In math, we are given a question, such as how much, when, how long, to what degree, etc. That is the mystery part and provides the frame for the puzzle. In real life we always have some elements of the puzzle, like the speed of a vehicle and how far the vehicle is going to go. These are the puzzle pieces. Then we take the information and the framework to put the pieces together and put it into the frame. This is what math problems really are.

Reading #2: Math of Drugs and Bodies (Pharmacokinetics)

Mosina | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Source: Excerpted from “Math of drugs and bodies (pharmacokinetics).” (2010, February 1). Retrieved and adapted from SquareCircleZ website: <http://www.squarecirclez.com/blog/math-of-drugs-and-bodies-pharmacokinetics/4098>

Pharmacokinetics is a branch of pharmacology that studies processes by which substances (like food and drugs [alcohol, in particular] are ingested into the body (via mouth or needles) and processed. We will concentrate on drugs.



The process of pharmacokinetics has 5 steps:

- **Liberation** – the drug is released from the formulation
- **Absorption** – the drug enters the body
- **Distribution** – the drug is dispersed throughout the body
- **Metabolism** – the drug is broken down by the body
- **Excretion** – the drug is eliminated from the body

Of course, each drug needs to act on the body in a different way. Some drugs need to be absorbed quickly (like nitroglycerin, if we are having a heart attack) and preferably eliminated quickly (otherwise toxins build up in the blood). However, slow absorption is necessary for other drugs so that we derive maximum benefit and don't lose a lot of the drug through excretion.

So when your doctor prescribes (say) “take 2 tablets every meal time”, this is based on the desirable levels of drug concentration and known levels of distribution, metabolism and excretion in the body.

What's the math?

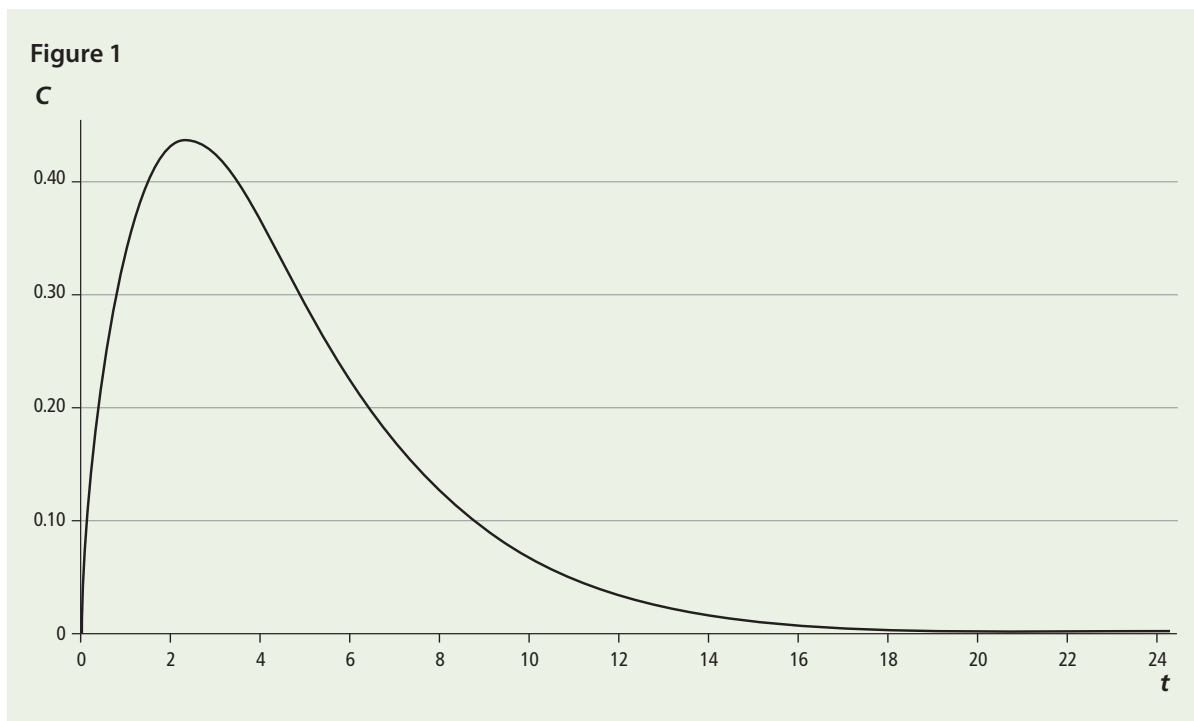
When a nurse first administers a drug, the concentration of the drug in the blood stream is zero. As the drug moves around the body and is metabolized, the concentration of the drug increases. There comes a point when the concentration no longer increases and begins to decrease. This is the period when the drug is fully distributed and metabolism is taking place. As time goes on, the drug concentration decreases more and more, until it falls below a certain effective level. At that point, it's time to take some more pills.

It is possible to model such a situation. One of the possible models for drug concentration at time t is, for example,

Equation (1): $C(t) = 533.3(e^{-0.4t} - e^{-0.5t})$, where $C(t)$ = concentration of the drug at time t .

NOTE: $e = 2.718281828459\dots$ is an irrational number

Drug Concentration in the Blood



We can use one of the computer algebra systems to generate a visual aid (graph) representing the drug concentration in a given model in Equation (1) above.

For the sake of simplicity, let us avoid units of measurement for now.

We can see in the graph the portion where the concentration increases (up to around $t = 3$) and levels off. The concentration then decreases to almost zero at $t = 24$.

Indeed, pharmacokinetics is yet another interesting “real life” application of math.

More on Blood Drug/Alcohol Concentration (Take-Home Activity)

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Department of Mathematics, Engineering, and Computer Science

PREREQUISITE:

Drug Concentration in the Blood (In-Class Activity)

Objectives

The goal of this PQL project is to follow up on the in-class PQL activity, “Drug Concentration in the Blood.” In this take-home activity, students will work with a mathematical model involving rational expressions that can be used to calculate the concentration of alcohol in a patient’s blood. Students will also learn what factors affect alcohol absorption and elimination as well as the consequences of alcohol abuse and alcoholism. They will also practice drawing conclusions based on given graphs and pie charts.

Reflection

I recommend assigning this activity during the week when complex fractions are explained, after the in-class PQL activity “Drug Concentration in the Blood” is completed. It is advisable to give students one week to complete this take-home project. When I gave this activity to my students, I found that if I explained all the project requirements clearly, students had no problem completing it at home. The completed work can be uploaded into the Assessment area of the ePortfolio system. Uploading should take about 5-10 minutes, provided there are no technology glitches.

Activity Overview

The activity consists of two parts: Part I provides an immediate follow up to the in-class PQL Activity, “Drug Concentration in the Blood.” Students work with a given drug concentration model, performing computations with a given rational expression using the skills they developed while studying complex fractions. In Part II, students read about the factors affecting blood alcohol concentration (BAC), and learn about the cost of alcohol abuse in our society. After reading these materials, students respond to questions.

Make sure that students have completed and understood the in-class PQL activity “Drug Concentration in the Blood,” which is a prerequisite for the current activity.

The following procedure is recommended for this take-home PQL project: After you distribute the handout, take a few minutes to discuss the introduction and observation section of the Handout and how these relate to the “Drug Concentration in the Blood” PQL activity. Then, distribute Readings Passages A and B.

Working independently at home, students answer all the questions in the Handout.

Math Topic

Rational expressions, complex fractions, graphs, pie charts, and percentages

Purpose

Motivation, application of math skills working with fractions and rational expressions

When to Introduce

Weeks 7–8

Activity Time Frame

One week at home

Materials and Resources

- Handout
- Reading: Passage A and Passage B from National Institutes of Health/National Institute on Alcohol Abuse and Alcoholism. (n.d.). Understanding alcohol. Retrieved from <http://science-education.nih.gov/supplements/nih3/alcohol/guide/info-alcohol.htm#BAC>
- Also see: Reading #2 from the prerequisite PQL activity, “Drug Concentration in the Blood.” Reading excerpted from “Math of drugs and bodies (pharmacokinetics).” Retrieved and adapted from the SquareCircleZ.com website: <http://www.squarecirclez.com/blog/math-of-drugs-and-bodies-pharmacokinetics/4098>

Handout: Project Tasks

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Introduction and Observation:

Refer to Reading #2 and Handout #2 from the in-class PQL activity entitled “Drug Concentration in the Blood.”

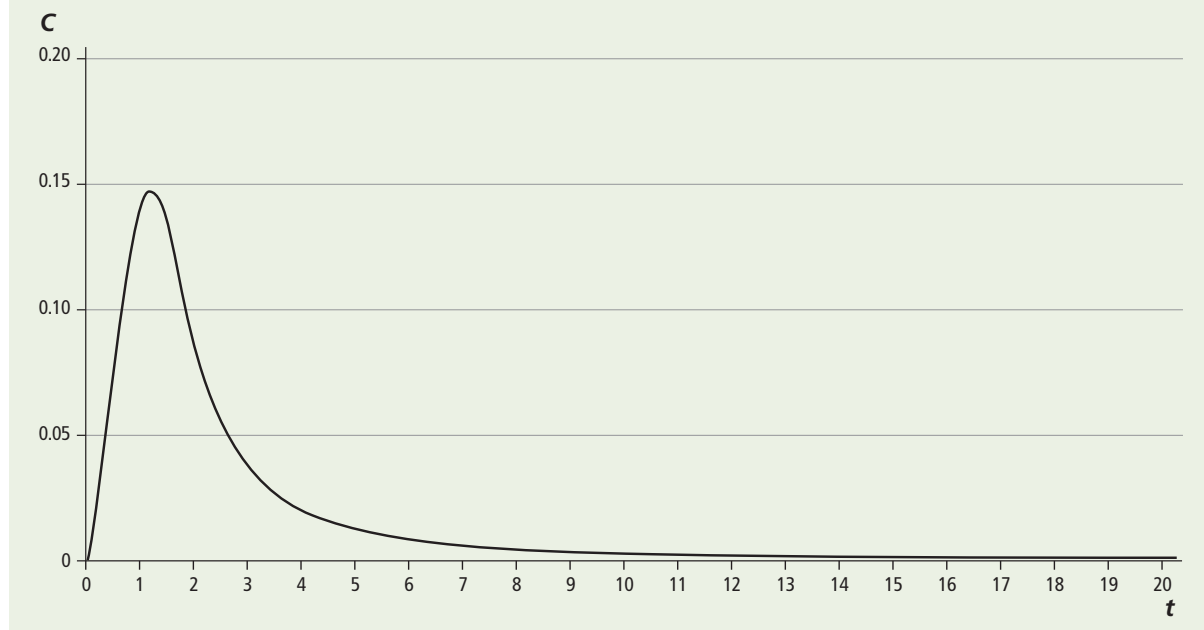
Given Model (X): The concentration of alcohol in a person’s bloodstream is given as a rational expression

$$C(t) = \frac{t^2 + t}{(4t^4 + 10)(t + 1)} \quad \text{where } C(t) = \text{concentration of the drug/alcohol (grams of alcohol/100ml blood or \%)} \text{ at time } t \text{ (hours).}$$

The following graph of $C(t)$ will help you visualize the model:

We can use the Maple computer algebra system to generate a graph representing the drug/alcohol concentration in a given Model (X).

Figure A: Graph of $C(t)$ – Blood Alcohol Concentration (BAC) after t hours



Observe the typical shape of the graph. Compare it with Figure 1, in Reading #2 from the in-class PQL activity you completed, “Drug Concentration in the Blood.”

More on Blood Drug/Alcohol Concentration

Task 1

Use what you have learned from your work on the “Drug Concentration in the Blood” activity, and what you have read to answer the questions below:

1. Simplify the rational expression (X) that models the alcohol concentration in a person’s bloodstream after t hours:

$$C(t) = \frac{t^2 + t}{(4t^4 + 10)(t + 1)}$$

Hint: Factor the numerator and reduce the rational expression.

2. What is the concentration of the drug at time $t = 0$ in the given model (X)? Does your answer confirm what you see in the graph of $C(t)$?
3. What is the concentration of the drug at time $t = \frac{1}{2}$ hour in the model described by Equation (X)?
4. Look at Figure A and estimate the time t (after drinking) at which the blood alcohol concentration (BAC) is the highest.
5. Let $t_1 = x$ hours and $t_2 = 2x$ hours. How many times greater is $C(t_1)$ than $C(t_2)$, assuming that $x > 2$? (In other words, find the ratio $\frac{C(t_1)}{C(t_2)}$.) Simplify your answer.

Task 2

Read the following two excerpts from the NIH/NIAAA website, (retrieved February 25, 2011 from <http://science-education.nih.gov/supplements/nih3/alcohol/guide/info-alcohol.htm#BAC>, National Institute of Health.) Answer the questions for each.

After you have read Reading Passage A, answer the following questions in paragraph form:

1. Think about the factors that influence how quickly alcohol is absorbed into the blood. Explain the role of one factor of your choice.
2. Look at Figure B in Reading Passage A :
 - What does Figure B show? How does it support the statement that “Absorption of alcohol is faster when the stomach is empty”?
 - From Figure B, approximate BAC after a person drank alcohol following an overnight fast and immediately after breakfast at time $t = 1$ hour. Which one is greater?
3. Explain how gender influences BAC.

After you have read Reading Passage B, answer the following questions in paragraph form:

1. How much did alcohol abuse and alcoholism cost our society in 1998?
2. In 1998, what percent of the total cost of alcohol abuse was borne by private insurance companies? Calculate the exact dollar amount. Use the text and the pie chart in Figure C in Reading Passage B to answer this question.

Reading Passage A: Factors affecting alcohol absorption and elimination

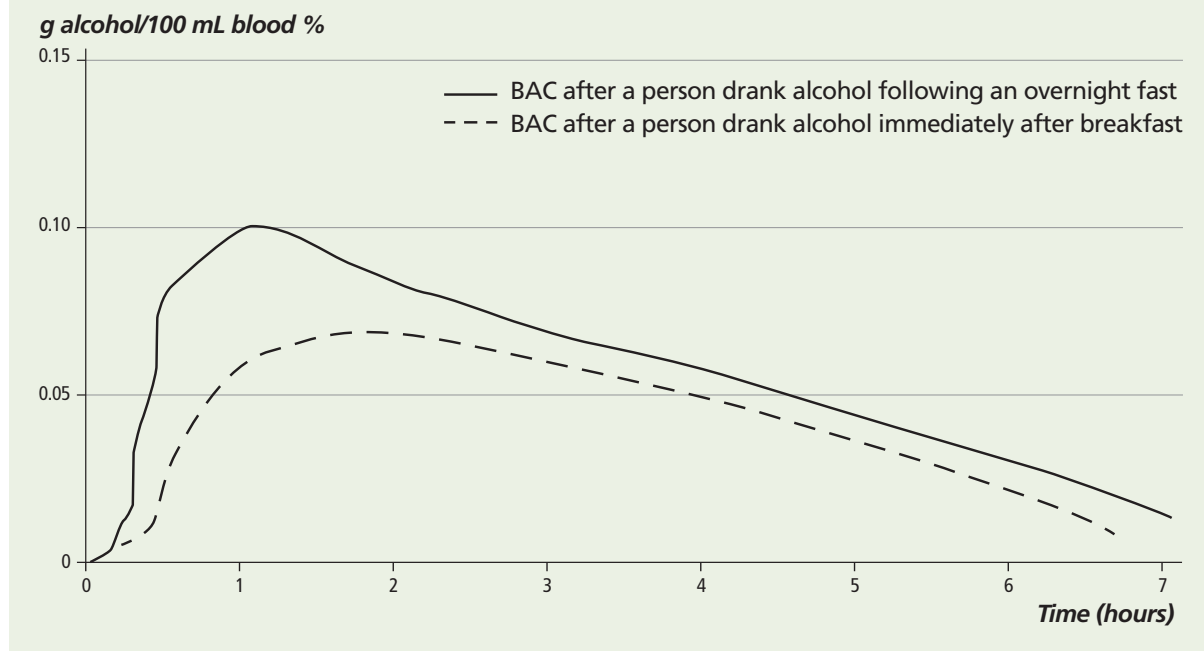
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Source: National Institutes of Health/National Institute on Alcohol Abuse and Alcoholism. (n.d.). Understanding alcohol. Retrieved from <http://science-education.nih.gov/supplements/nih3/alcohol/guide/info-alcohol.htm#BAC>

Alcohol impairs the functions of the mind and body. These impairments depend on the amount of alcohol in the blood, as measured by the blood alcohol concentration. Factors that influence the BAC include the number of drinks and the time period over which they are consumed, as well as the drinker's gender and body weight. The body breaks down, or metabolizes, alcohol at a relatively constant rate, regardless of the rate at which it is consumed. Humans vary widely in their ability to absorb and eliminate alcohol. Here we describe some of the important factors that influence how quickly alcohol is absorbed into the blood.

Food. Absorption of alcohol is faster when the stomach is empty; the empty stomach allows rapid passage of the alcohol into the small intestine, where absorption is most efficient. (see Figure B). The rate of alcohol absorption depends not only on the presence or absence of food, but also on the type of food present. Eating fatty foods will allow alcohol absorption to take place over a longer time.

Figure B. The effect of food on blood alcohol concentration



More on Blood Drug/Alcohol Concentration

Body weight and build. Greater body weight provides a greater volume in which alcohol can be distributed. This means a larger person will be less affected by a given amount of alcohol than a smaller person would be. Alcohol is more soluble in water than in fat. This means that tissues rich in water, like muscle, take up more alcohol than do tissues rich in fat. A leaner person with a greater muscle mass (and less fat) provides a larger volume for alcohol to be distributed in compared with a person who weighs the same but has a higher percentage of body fat.

In summary, if you compare two people of equal size but who differ in amount of body fat, the effects of alcohol will be different in them.

Gender. Females, on average, have a smaller body mass and a higher proportion of body fat than do males. These characteristics mean that, on average, females have a lower proportion of total body water in which to distribute alcohol. Females also may have a lower activity of the alcohol-metabolizing enzyme **alcohol dehydrogenase (ADH)** in the stomach; therefore, more of the ingested alcohol reaches the blood. These factors mean that females generally exhibit higher BACs than do males after consuming the same amount of alcohol, and are more vulnerable to alcohol's effects.

Reading Passage B: Consequences of Alcohol Abuse and Alcoholism: The costs to society

Mosina | Elementary Algebra – Problems and Issues in Public Health (MAT096)

Source: National Institutes of Health/National Institute on Alcohol Abuse and Alcoholism. (n.d.). Understanding alcohol. Retrieved from <http://science-education.nih.gov/supplements/nih3/alcohol/guide/info-alcohol.htm#BAC>

Alcohol abuse and alcoholism have a large economic impact on our society. In 1998, alcohol abuse and alcoholism cost an estimated \$185 billion in lost productivity, illness, premature death, and healthcare expenditures. For 1995, these costs were estimated to be over \$166 billion, and in 1992, they were \$148 billion. A large portion of these costs is borne, in various ways, by non-abusers (see Figure C). While 45 percent of the costs of alcohol abuse fall on the abusers themselves and their families, 38 percent falls on government (in the form of lost or reduced tax revenue). Additional costs to non-abusers include, but are not limited to, the economic costs of the criminal justice system and higher insurance premiums, as well as the social costs of alcohol-related crimes and trauma.

Figure C. Distribution of the cost of alcohol abuse in the United States in 1998

