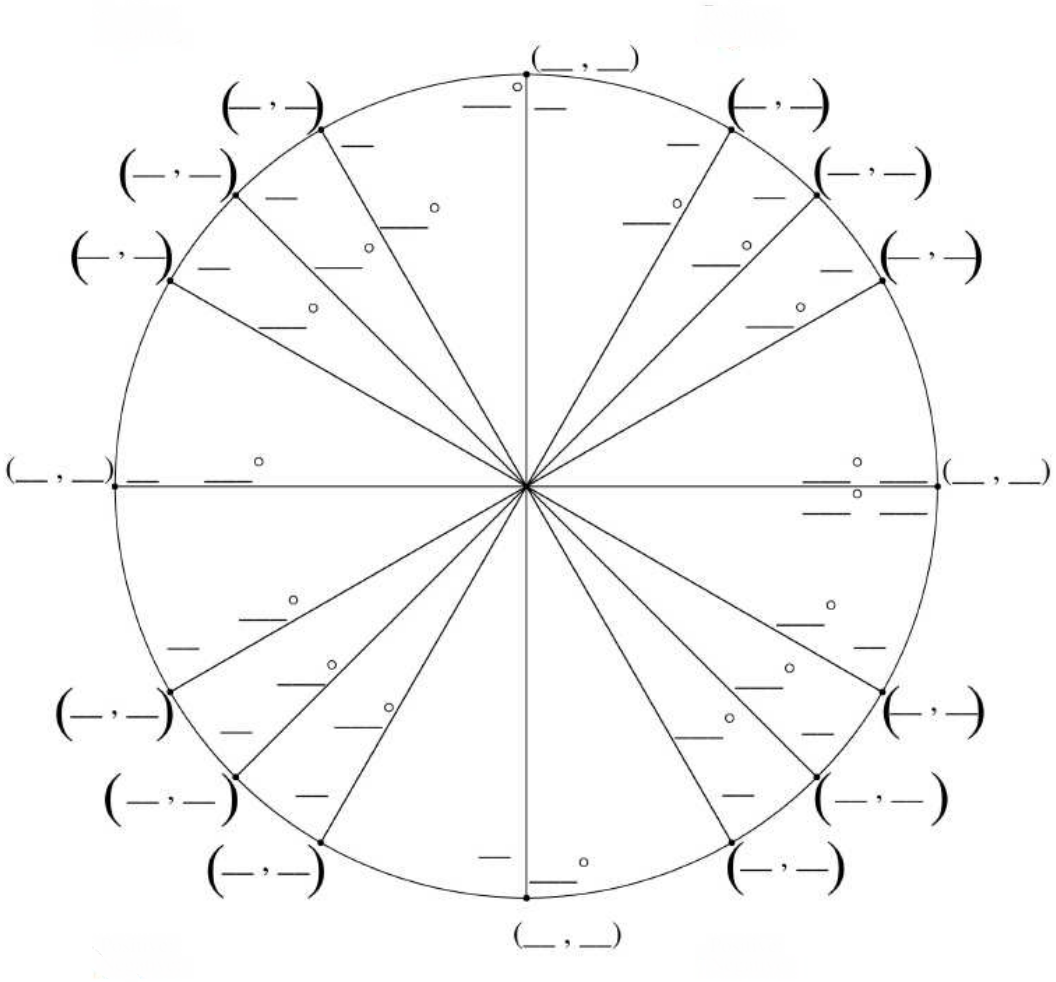
Exam 2 Review PART 1: No calculator allowed. You should be able to do #1-12 without a calculator.

1. **Unit Circle: Label the degree and radian measures, and give the x and y values for each ordered pair.**



2. **When working with the unit circle, we call the trigonometric functions the <u>circular functions</u>.**
Give the definitions of the circular functions. HINT: r = _____

sin s =_____ cos s =_____ tan s =_____ csc s =_____ sec s =_____ cot s =_____

3. **List all of the angles on the unit circle that have the following reference angles. Use radian measure.**

Reference angle θ	Angles in radians with reference angle θ
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	

4. **Complete the following table with the exact values.** Simplify and rationalize your answers.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						

5. **Find the exact value of each expression:**

a) $\sin\frac{5\pi}{4}$ = _____ b) $\tan\left(-\frac{\pi}{6}\right)$ = _____ c) $\cos\frac{7\pi}{4}$ = _____

d) $\csc\left(-\frac{8\pi}{3}\right)$ = _____ e) $\sec(-5\pi)$ = _____ f) $\cot\frac{\pi}{2}$ = _____

6. **Find the exact value of s in the following intervals**

a) $[\pi, \frac{3\pi}{2}]$, if $\tan s = 1$ b) $[0, 2\pi)$, if $\cos s = -\frac{\sqrt{3}}{2}$ c) $[0, 2\pi)$, if $\sin s = \frac{1}{2}$

s = _____ s = _____ s = _____

7. **Without graphing, give the amplitude, period, vertical translation, and phase shift of each of the following functions.**

a) $y = -2 + 5\sin\frac{1}{2}x$

Period:

Phase shift:

Amplitude:

Vertical shift:

b) $y = 1 - 3\cos(2x - \pi)$

Period:

Phase shift:

Amplitude:

Vertical shift:

c) $y = -\frac{1}{2}\sin\left[4\left(x + \frac{\pi}{2}\right)\right]$

Period:

Phase shift:

Amplitude:

Vertical shift:

d) $y = 4 + \cos(x - \pi)$

Period:

Phase shift:

Amplitude:

Vertical shift:

e) $y = -3 + \tan(2x - \pi)$

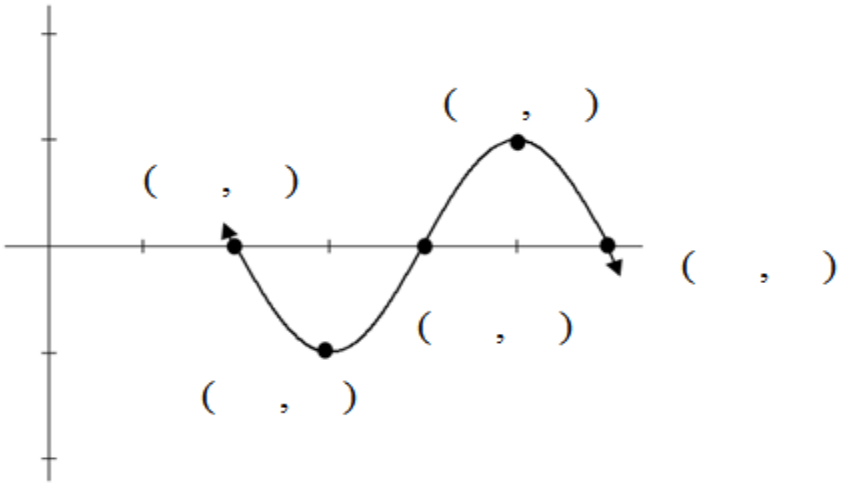
Period:

Phase shift:

Vertical shift:

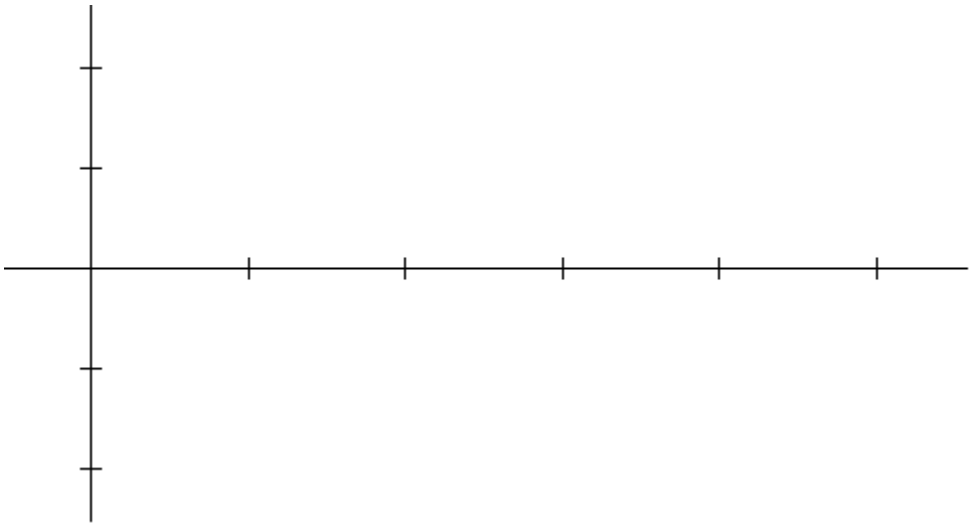
8. Using the given equation, label the five points with the exact x-values and y-values used to form the graph.

$$y = -\frac{1}{2}\sin\left[4\left(x - \frac{\pi}{4}\right)\right]$$



9. **Basic Shapes of the six trigonometric functions.** Graph each function over a one period interval. Clearly label all x and y values used to make the graph.

a) $y = \sin x$



Domain:

Range:

Period:

Amplitude:

By definition of an odd function,
$\mathbf{sin}(-\boldsymbol{x}) =$_____

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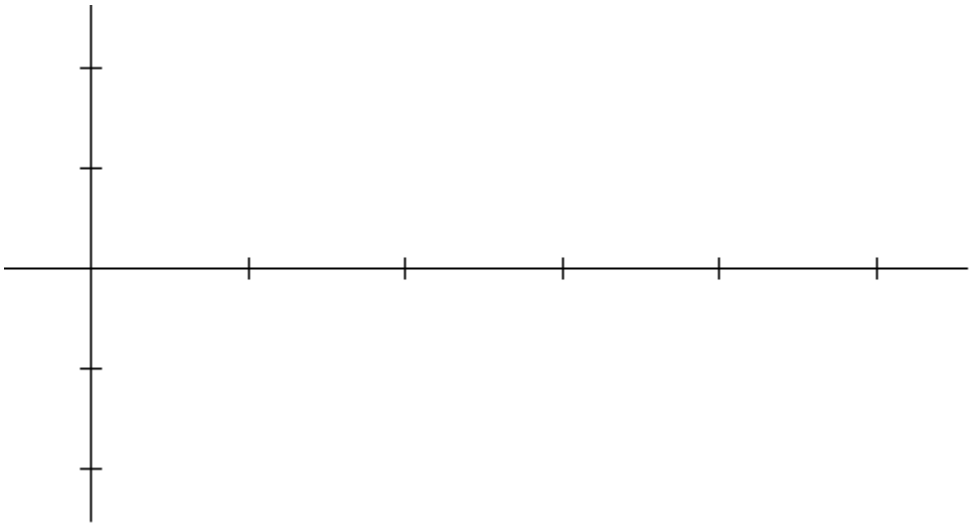
b) $y = \cos x$



Domain:

Range:

Period:

Amplitude:

By definition of an even function,
$\mathbf{cos}(-\boldsymbol{x}) =$_____

c) $y = \sec x$

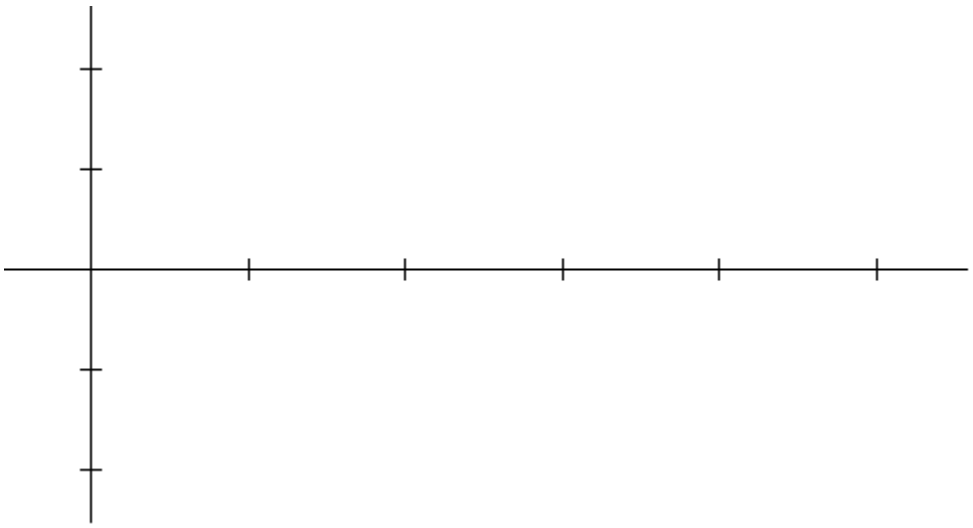
Domain:

Range:

Period:

Asymptotes:

By definition of an even function,
$\mathbf{sec}(-\boldsymbol{x}) =$_____



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d) $y = \csc x$

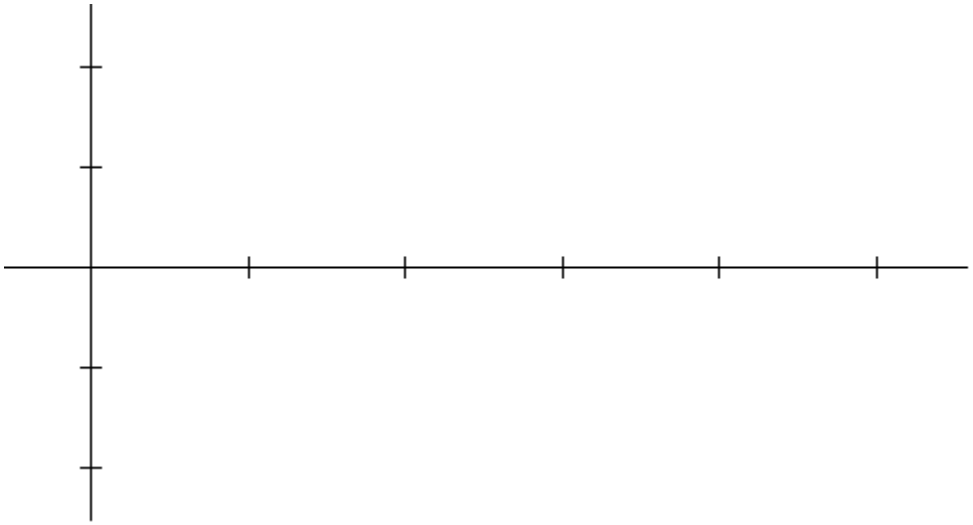
Domain:

Range:

Period:

Asymptotes:

By definition of an odd function,
$\mathbf{csc}(-\boldsymbol{x}) =$_____



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d) $y = \tan x$

Domain:

Range:

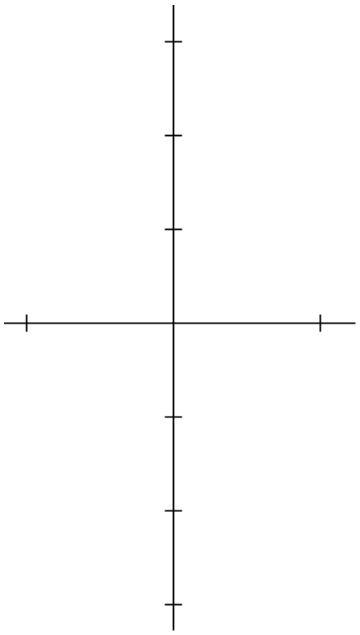
Period:

Asymptotes:

By definition of an

odd function,

$\mathbf{tan}(-\boldsymbol{x}) =$_____



e) $y = \cot x$

Domain:

Range:

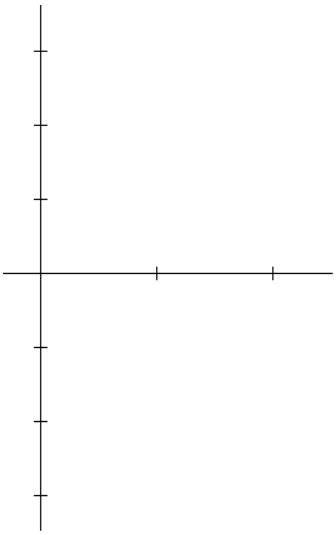
Period:

Asymptotes:

By definition of an

odd function,

$\mathbf{cot}(-\boldsymbol{x}) =$_____



10. Graph each function over a one period interval. Clearly label all x and y values used to make the graph.

MAKE SURE THAT YOU KNOW THESE STEPS!!! (THESE WILL NOT BE GIVEN TO YOU ON THE EXAM)

Basic steps to graph one period of Sine and Cosine.

1) **Find the x-values.** Set up the inequality $\mathbf{0} \leq \boldsymbol{inside\ of\ function} \leq \mathbf{2\pi}$ then solve for x. **Find midpoints**. Procedure: $\frac{1}{2}(start + end)$

2) **Draw a dotted horizontal line for the middle of the graph.** (Vertical shift will determine the middle.)

3) **Draw in the basic shape** for sine or cosine **using the amplitude** to determine the vertical distance from the middle of the graph.

a) $y = \sin 2x$

b) $y = -3\cos x$

c) $y = -2 + \cos 4x$

d) $y = -2\sin\frac{1}{4}x$

e) $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$

f) $y = \sin\left[3\left(x + \frac{\pi}{4}\right)\right]$

g) $y = -5\sin \pi x$

h) $y = -2 + \cos(\frac{1}{4}x - \frac{\pi}{8})$

i) $y = 1 - 2\cos(2x - \pi)$

j) $y = -2 + 3\sin\left(x + \frac{\pi}{4}\right)$

11. Graph each function over a one period interval. Clearly label all x and y values used to make the graph.

MAKE SURE THAT YOU KNOW THESE STEPS. THESE WIL NOT BE GIVEN TO YOU ON THE EXAM.

Graphing $y = c + a\tan[b(x-d)]$ $\qquad$ **$y = c + a\cot[b(x-d)]$**

1) Determine the x-values at the start and end of the period, set up and solve the following inequality for x.
   -For tangent: $-\frac{\pi}{2} < inside < \frac{\pi}{2}$ $\qquad$ -For cotangent: $0 < inside < \pi$
   (The endpoints are the asymptotes.)
2) Find the midpoints.
3) "c" Vertical Shift: Draw a dotted horizontal line for the new middle.
4) "a" Stretch the graph vertically from the middle "a" units.
5) Draw the basic shape of tangent or cotangent within the asymptotes.

a) $y = 2\tan\left(x - \frac{\pi}{4}\right)$

b) $y = 1 + \cot 3x$

12. Graph over a one-period interval. (The guide function has already been graphed as a dashed curve.)

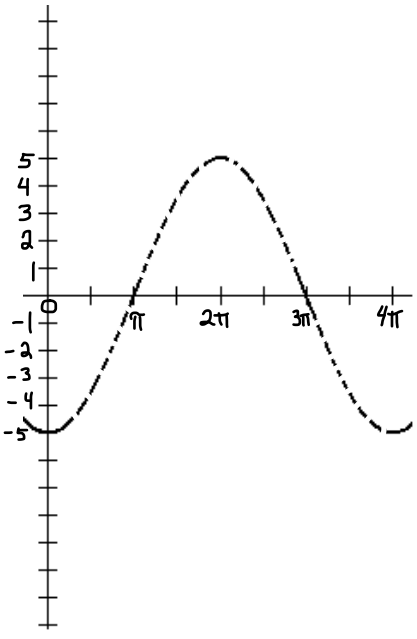
MAKE SURE THAT YOU KNOW THESE STEPS. THESE WIL NOT BE GIVEN TO YOU ON THE EXAM.

Graphing $y = c + a\csc[b(x-d)]$ $\qquad$ **$y = c + a\sec[b(x-d)]$**

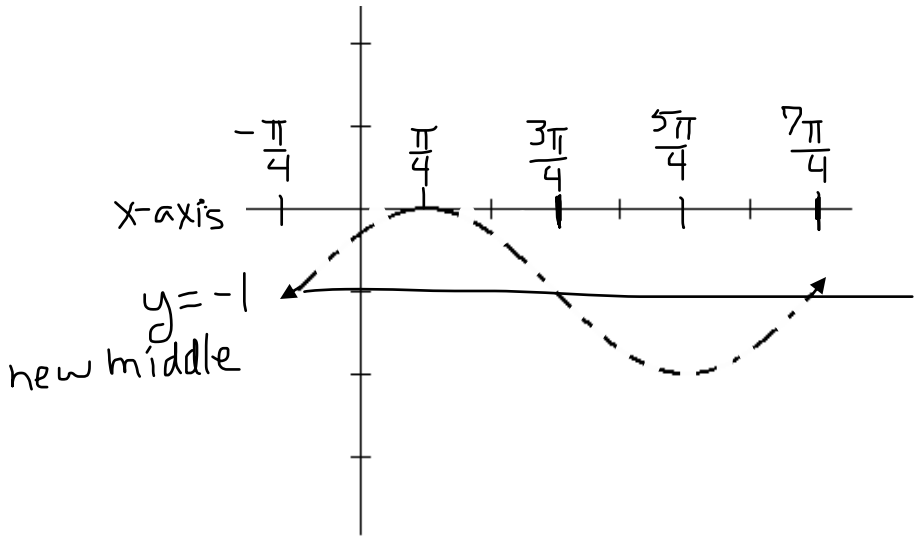
1) Graph the reciprocal function as a dashed curve to use as a guide.

2) Place asymptotes at the points that intersect at the "middle" line. Place points at the max and min values. Draw "U" shaped curves away from the guide function.

a) $y = -5\sec\frac{1}{2}x$



b) $y = -1 + \csc\left(x + \frac{\pi}{4}\right)$



13. Give the Even/Odd Identities for each function.

$\sin(-x) =$_____ $\csc(-x) =$_____

$\cos(-x) =$_____ $\sec(-x) =$_____

$\tan(-x) =$_____ $\cot(-x) =$_____

14. Give the Domain and Range for each function.

Domain of sin x :_____ Range of sin x: _____

Domain of cos x :_____ Range of cos x: _____

Domain of csc x :_____ Range of csc x: _____

Domain of sec x :_____ Range of sec x: _____

Domain of tan x :_____ Range of tan x: _____

Domain of cot x :_____ Range of cot x: _____

Exam 2 Review PART 2: Calculator allowed.

15. **Convert each degree measure to radians.** Show as an exact answer as multiple of π and give its decimal approximation to the nearest 2 decimal places, if needed.

a) $60°$ b) $-135°$ c) $325.7°$

16. **Convert each radian measure to decimal degrees. Round to the nearest 2 decimal places, if needed.**

a) 2.92 b) $\frac{\pi}{6}$ c) $\frac{13\pi}{3}$

17. **For the following angles, tell the quadrant where the terminal side lies.**

a) $\theta = 1$ a) $\theta = 2$ a) $\theta = 3.5$ a) $\theta = 4$

a) $\theta = 5$ a) $\theta = 6$ a) $\theta = -3$ a) $\theta = -.6234$

18. **Find the function values using a calculator.** Round to the nearest 4 decimal places.

a) $\cos(3.5) =$_____ b) $\cot(8.2345) =$_____

19. **Use a calculator to find the value of s in the interval** $[\ \mathbf{0},\ \frac{\pi}{2}\ ]$**.** Round to the nearest 4 decimal places.

a) $\tan s = 0.2126$ b) $\sec s = 1.0806$

$s =$ _____ $s =$ _____

20. **A circle has a radius of 18.2 cm. Find the length of the arc intercepted by a central angle of 144°**. Round to the nearest 2 decimal places.

21. **Find the distance in kilometers between each pairs of cities, assuming they lie on the same north-south line. Use the given latitudes and assume the earth has a radius of 6400 km.** Round to the nearest hundred.

New York City, New York 41°N and Lima, Peru 12°N

The following questions are examples of non partial credit questions:

SHORT ANSWER

22. To convert from radians to degrees we multiply by _____.

23. If no unit of angle measure is specified, then the angle is understood to be measured in _____.

24. The arc length equation is s = _____.

25. The period of $y = \sin x$ and $y = \cos x$ is _____.

26. The period of $y = \tan x$ is _____.

27. The period of $y = -3\cos \pi x$ is _____.

28. When dividing the interval $[\frac{\pi}{6}, \frac{5\pi}{6}]$ into four equal segments the x-values of the midpoints are _____, _____, and _____.

TRUE OR FALSE. If false, you must explain why to receive credit.

29. _____When using the arc length equation, θ can be in degrees.

30. _____ The terminal side of $\theta = 3 \ \ and \ \ \alpha = 100°$, both lie in the same quadrant.

31. _____ The terminal side of $\theta = 6 \ \ and \ \ \alpha = -100°$, both lie in the same quadrant.

32. _____ For $y = 1 - 3\cos(2x - \pi)$ the amplitude is considered to be 1.

33. _____ For $y = 1 - 3\cos(2x - \pi)$ the phase shift is considered to be $\frac{\pi}{2}$ units to the right.

34. _____ For y = cos $[5(x - \pi)]$ the phase shift is considered to be π units to the right.

35. _____ For y = sin(4x – π) the phase shift is considered to be π units to the right.