NAME: $\qquad$
ME 270 - Summer 2013
Examination No. Final Exam
Please review the following statement:
I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

## Signature:

$\qquad$

## INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, use the white lined paper provided to you.

Work on one side of each sheet only, with only one problem on a sheet.
Each problem is worth 25 points.
Please remember that for you to obtain maximum credit for a problem, it must be clearly presented, i.e.

- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.

If the solution does not follow a logical thought process, it will be assumed in error.
When handing in the test, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded.

Instructor's Name and Section:
Section 1: J. Jones 9:50-10:50 am

Problem 1 $\qquad$

Problem 2 $\qquad$

Problem 3 $\qquad$

Problem 4 $\qquad$

Total $\qquad$

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PROBLEM 1 ( 25 points) - Prob. 1 questions are all or nothing.

## PROBLEM 1A. (5 points)

FIND: Massless beam OC is loaded as shown and is held in static equilibrium by the fixed support at O . Determine the reactions at the fixed support.


| $\mathrm{O}_{\mathrm{x}}=$ | $(1 \mathrm{pt})$ |
| :--- | :--- |
| $\mathrm{O}_{\mathrm{y}}=$ | $(1 \mathrm{pt})$ |
| $\mathrm{C}_{\mathrm{o}}=$ | $(3 \mathrm{pts})$ |

## PROBLEM 1B. (5 points)

FIND: For the massless beam shown, determine the equivalent force-couple system acting at A.


$$
\begin{align*}
& \overline{\mathrm{F}}_{\mathrm{A}}= \\
& \overline{\mathrm{C}}_{\mathrm{A}}=
\end{align*}
$$

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PROBLEM 1C. (5 points)
FIND: For the truss shown in static equilibrium, determine the loads in members 34 \& 46 and indicate whether they are in tension or compression. Hint: This can be done without solving for the reaction forces at joints 1 and 8.


$$
\begin{aligned}
& \mathrm{F}_{34}= \\
& \mathrm{F}_{46}=
\end{aligned}
$$

Tension or Compression
Tension or Compression

PROBLEM 1D. (5 points)
FIND: Force P is separately applied to a ) the 60 lb block and b ) the 100 lb block. For each case, determine the magnitude of $P$ required to
 initiate motion.

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## PROBLEM 1E. (5 points)

FIND: The frame shown supports a 3 kN load at an angle of $\theta=30^{\circ}$ and is in static equilibrium. Sketch the free-body diagrams of members $A B C$ and $B D$ on the sketches provided and determine the force at $B$ on member $A B C$ in vector form.


$$
\overline{\mathrm{F}}_{\mathrm{B}}=
$$

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## PROBLEM 2. (25 points)

GIVEN: Boom AB of negligible weight is loaded with a downward acting 30 kN force as shown and is held in static equilibrium by a ball-and-socket support at $A$ and cables $T_{B C}$ and $T_{F D}$.

FIND:
a) Complete the free-body diagram of the boom on the sketch provided below. (4 pts)
b) Express the tensions in cables $\overline{\mathrm{T}}_{\mathrm{BC}}$ and $\overline{\mathrm{T}}_{\mathrm{FD}}$ in terms of their known unit vectors and their unknown magnitudes. (4 pts)
c) Determine the magnitude of the tensions in cables $T_{B C}$ and $T_{F D}$. (8 pts)
d) Determine the vector reaction at the ball-and-socket support at A. (9 pts)

a) Free-body diagram

b) $\overline{\mathrm{T}}_{\mathrm{BC}}=$

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Problem 2C Determine the magnitude of the tensions in cables $\mathrm{T}_{\mathrm{BC}}$ and $\mathrm{T}_{\mathrm{FD}}$. (8 pts)

c) | $\mathrm{T}_{\mathrm{BC}}=$ | (4 pts) |
| :--- | :--- |
| $\mathrm{T}_{\mathrm{FD}}=$ | (4 pts) |

Problem 2D Determine the vector reaction at the ball-and-socket support at A. (9 pts)

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## PROBLEM 3A. (5 points)

FIND: For the shaded right triangle shown, use the method of integration to determine $\mathrm{I}_{\mathrm{x}}$ about the x -axis shown (NOT about the centroid). Knowing the centroid of the shaded triangle is at $\bar{y}=\frac{2}{3} a$, determine $I_{x_{0}}$ using the Parallel Axis Theorem.


$$
\begin{align*}
& I_{x}= \\
& I_{x_{0}}= \tag{3pts}
\end{align*}
$$

## Problem 3B (5 Points)

FIND: Determine the second moment of area in the x -direction $\mathrm{I}_{\mathrm{x}_{\mathrm{o}}}$ of the z -section about its centroid $\left(\mathrm{x}_{\mathrm{o}}, \mathrm{y}_{0}\right)$. By observation, is the second moment of area it the $y$-direction $\left(I_{y_{0}}\right)$ going to be larger, smaller or the same as $I_{x_{0}}$.


$$
\begin{align*}
& \mathrm{I}_{\mathrm{x}_{0}}=  \tag{3pts}\\
& \mathrm{I}_{\mathrm{y}_{0}}=\text { Larger or Smaller or Same (Circle One) }
\end{align*}
$$(2 pts)

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## Problem 3C (5 Points)

FIND: Beam $A B$ is loaded with two 6 kN loads as shown and is held in static equilibrium by a 20 mm - diameter pin support at $A$ and a 30 mm -diameter pin attached to cable BC. A static equilibrium analysis demonstrates that $\mathrm{A}_{\mathrm{x}}=\mathbf{6 . 9 6} \mathbf{k N}, \mathrm{A}_{\mathrm{y}}=\mathbf{8} \mathbf{k N}$, and $\mathrm{T}_{\mathrm{BC}}=\mathbf{8} \mathbf{k N}$. Making special note of the different pin
 designs at $A$ and $B$ (see the inset pictures), determine the average shear stress at each support.

$$
\begin{aligned}
& \left(\tau_{\mathrm{A}}\right)_{\mathrm{AVG}}= \\
& \left(\tau_{\mathrm{B}}\right)_{\mathrm{AVG}}=
\end{aligned}
$$

## Problem 3D (5 Points)

FIND: Two steel rods (AB and CD) are used to support 6,000 lb (or 6 kip) load as shown. The cross-sectional area of each rod is given in the figure. Determine the axial stress and strain in rod $A B$ assuming $E=20 \times 10^{3} \mathrm{ksi}$.


$$
\sigma_{\mathrm{AB}}=
$$

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## Problem 3E (5 Points)

FIND: Two closely-spaced wrenches are used to tighten pipe AC which has an outer diameter of 50 mm and an inner diameter of 40 mm . If $P=300 \mathrm{~N}$ is applied to each wrench, determine the maximum torsional shear stress developed in the pipe in section $B C$ and $A B$ (between the wrenches).


$$
\begin{aligned}
& \left(\tau_{\mathrm{BC}}\right)_{\mathrm{MAX}}= \\
& \left(\tau_{\mathrm{AB}}\right)_{\mathrm{MAX}}=
\end{aligned}
$$

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## PROBLEM 4 (25 points)

GIVEN: Rectangular beam AD is loaded as shown and is held in static equilibrium by a pin support at $B$ and a roller support at $C$. The beam cross-section is rectangular with dimensions of 0.05 m wide and 0.1 m high.

FIND: a) Evaluate and sketch the shear-force and bending-moment diagrams for the beam shown, indicating the magnitudes of these at each section of the beam. (16 pts)
b) Identify the section(s) of the beam where pure bending occurs and determine the maximum normal stress due to bending. (6 pts)
c) Where across the beam cross-section does the maximum tensile stress occur (Top, middle or bottom)? (3 pts)

b) $\sigma_{\mathrm{MAX}}=$
c) Max Tension occurs: Top or Middle or Bottom (Circle One)

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4A Cont.
$10 \mathrm{~N} / \mathrm{m}$


Normal Stress and Strain
$\sigma_{\mathrm{x}}=\frac{\mathrm{F}_{\mathrm{n}}}{\mathrm{A}}$
$\sigma_{x}(y)=\frac{-\mathrm{My}}{\mathrm{I}}$
$\varepsilon_{\mathrm{x}}=\frac{\sigma_{\mathrm{x}}}{\mathrm{E}}=\frac{\Delta \mathrm{L}}{\mathrm{L}}$
$\varepsilon_{\mathrm{y}}=\varepsilon_{\mathrm{z}}=-\vartheta \varepsilon_{\mathrm{x}}$
$\varepsilon_{x}(y)=\frac{-y}{\rho}$
$\mathrm{FS}=\frac{\sigma_{\text {fail }}}{\sigma_{\text {allow }}}$
Shear Stress and Strain
$\tau=\frac{V}{A}$
$\tau(\rho)=\frac{T \rho}{J}$
$\tau=G \gamma$
$G=\frac{E}{2(1+\vartheta)}$
$\gamma=\frac{\delta_{\mathrm{s}}}{\mathrm{L}_{\mathrm{s}}}=\frac{\pi}{2}-\theta$
For a rectangular crosssection,
$\tau(\mathrm{y})=\frac{6 \mathrm{~V}}{\mathrm{Ah}}\left(\frac{\mathrm{h}^{2}}{4}-\mathrm{y}^{2}\right)$
$\tau_{\max }=\frac{3 \mathrm{~V}}{2 \mathrm{~A}}$

Second Area Moment
$I=\int_{A} y^{2} d A$
$\mathrm{I}=\frac{1}{12} \mathrm{bh}^{3} \quad$ Rectangle
$I=\frac{\pi}{4} r^{4} \quad$ Circle
$\mathrm{I}_{\mathrm{B}}=\mathrm{I}_{\mathrm{O}}+\mathrm{Ad}_{\mathrm{OB}}{ }^{2}$

## Polar Area Moment

$\mathrm{J}=\frac{\pi}{2}\left(\mathrm{r}_{\mathrm{o}}^{4}-\mathrm{r}_{\mathrm{i}}^{4}\right)$ Tube

## Shear Force and Bending

 Moment$\mathrm{V}(\mathrm{x})=\mathrm{V}(0)+\int_{0}^{\mathrm{x}} \mathrm{p}(\epsilon) \mathrm{d} \epsilon$
$M(x)=M(0)+\int_{0}^{x} V(\epsilon) d \epsilon$

## Buoyancy

$\mathrm{F}_{\mathrm{B}}=\rho \mathrm{gV}$

Fluid Statics
$\mathrm{p}=\rho \mathrm{gh}$
$\mathrm{F}_{\text {eq }}=\mathrm{p}_{\mathrm{avg}}(\mathrm{Lw})$

## Belt Friction

$$
\frac{T_{L}}{T_{S}}=e^{\mu \beta}
$$

Distributed Loads
$\mathrm{F}_{\text {eq }}=\int_{0}^{\mathrm{L}} \mathrm{w}(\mathrm{x}) \mathrm{dx}$
$\bar{x}_{\text {eq }}=\int_{0}^{L} \mathrm{xw}(\mathrm{x}) \mathrm{dx}$

## Centroids

$$
\begin{aligned}
& \bar{x}=\frac{\int x_{c} d A}{\int d A} \\
& \bar{y}=\frac{\int y_{c} d A}{\int d A}
\end{aligned}
$$

$\bar{x}=\frac{\sum_{i} x_{c i} A_{i}}{\sum_{i} A_{i}} \quad \bar{y}=\frac{\sum_{i} y_{c i} A_{i}}{\sum_{i} A_{i}}$
In 3D, $\bar{x}=\frac{\sum_{i} x_{c i} V_{i}}{\sum_{i} V_{i}}$

## Centers of Mass

$$
\begin{aligned}
& \tilde{x}=\frac{\int x_{c m} \rho d A}{\int \rho d A} \tilde{y}=\frac{\int y_{c m} \rho d A}{\int \rho d A} \\
& \tilde{x}=\frac{\sum_{i} x_{c m i} \rho_{i} A_{i}}{\sum_{i} \rho_{i} A_{i}}
\end{aligned}
$$

$$
\tilde{y}=\frac{\sum_{i} y_{c m i} \rho_{i} A_{i}}{\sum_{i} \rho_{i} A_{i}}
$$

## Summer 2013 ME 270 Final Exam Solutions

1A. $\mathrm{O}_{\mathrm{x}}=1.5 \mathrm{kN}$
$\mathrm{O}_{\mathrm{y}}=1.2 \mathrm{kN}$
$\mathrm{C}_{\mathrm{o}}=-4.21 \mathrm{kN}-\mathrm{m}$
1B. $\overline{\mathrm{F}}_{\mathrm{A}}=-3 \overline{\mathrm{j}} \mathrm{kN}$ $\overline{\mathrm{C}}_{\mathrm{A}}=-13 \overline{\mathrm{k}} \mathrm{kN}-\mathrm{m}$

1c. $\mathrm{F}_{34}=7.5 \mathrm{kN}$ Tension $\mathrm{F}_{46}=12.5 \mathrm{kN}$ Compression
1D. $\mathrm{P}(\mathrm{a})=19.2 \mathrm{lbs}$
$\mathrm{P}(\mathrm{b})=19.2 \mathrm{lbs}$

1E. Free Body Diagram

$$
\overline{\mathrm{F}}_{\mathrm{B}}=-3.9 \overline{\mathrm{i}}+5.2 \overline{\mathrm{j}} \mathrm{kN}
$$

## 2A Free Body Diagram

2B. $\overline{\mathrm{T}}_{\mathrm{BC}}=\mathrm{T}_{\mathrm{BC}}(-0.388 \overline{\mathrm{i}}-0.873 \overline{\mathrm{j}}+0.291 \overline{\mathrm{k}})$

$$
\overline{\mathrm{T}}_{\mathrm{FD}}=\mathrm{T}_{\mathrm{FD}} \overline{\mathrm{i}}
$$

2C. $\mathrm{T}_{\mathrm{BC}}=45.8 \mathrm{kN}$
$\mathrm{T}_{\mathrm{FD}}=26.6 \mathrm{kN}$
2D. $\overline{\mathrm{A}}=-8.83 \overline{\mathrm{i}}+40.0 \overline{\mathrm{j}}+16.7 \overline{\mathrm{k}}$
3A. $I_{x}=\frac{a^{4}}{4}$
$I_{x_{0}}=\frac{a^{4}}{36}$
3B. $I_{x_{0}}=1,613$ in $^{4}$
$\mathrm{I}_{\mathrm{y}_{\mathrm{o}}}=$ Smaller
3c. $\left(\tau_{\mathrm{A}}\right)_{\mathrm{AVG}}=16.9 \mathrm{MPa}$
$\left(\tau_{\mathrm{B}}\right)_{\mathrm{AVG}}=11.3 \mathrm{MPa}$
3D. $\tau_{\mathrm{AB}}=2000 \mathrm{psi}$
$\varepsilon_{\mathrm{AB}}=0.0001 \mathrm{in} / \mathrm{in}$ or $1 \times 10^{-4} \mathrm{in} / \mathrm{in}$
3E. $\left(\tau_{\mathrm{BC}}\right)_{\mathrm{MAX}}=10.4 \mathrm{MPa}$
$\left(\tau_{\mathrm{AB}}\right)_{\mathrm{MAX}}=5.18 \mathrm{MPa}$
4A. Sketch
4B. $\sigma_{\mathrm{MAX}}=240 \mathrm{kPa}$
4C. Top

