

Normal Distributions

Common Core Math Standards

The student is expected to:

COMMON CORE S-ID.A.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Mathematical Practices

COMMON CORE MP.1 Problem Solving

Language Objective

Work with a partner to sketch and label the standard deviations above and below the mean in two different normal distributions.

ENGAGE

Essential Question: How do you find percents of data and probabilities of events associated with normal distributions?

If a given data value is 1, 2, or 3 standard deviations from the mean, use the fact that 68% of the data are within 1 standard deviation of the mean, 95% of the data are within 2 standard deviations of the mean, and 99.7% of the data are within 3 standard deviations of the mean. If the data value is not 1, 2, or 3 standard deviations from the mean, calculate z-scores and use the standard normal table.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and how a census can provide information that can be used to calculate a probability. For example, if you know how many people older than 18 are married, you can select 10 people over 18 at random and find the probability that all 10 of them would be married. Then preview the Lesson Performance Task.

Name _____ Class _____ Date _____

23.2 Normal Distributions

Essential Question: How do you find percents of data and probabilities of events associated with normal distributions?



Resource Locker

Explore 1 Substituting a Normal Curve for a Symmetric Histogram

The table below gives the mass (in kilograms) of 20 babies at birth. You know that there are 20 babies, because that is the sum of the frequencies. You also know that the masses are normally distributed because the mass with the greatest frequency occurs at the center of the distribution and the other frequencies taper off symmetrically from that center. The mean of the data is 3.5 kg, and the standard deviation is 0.14 kg.

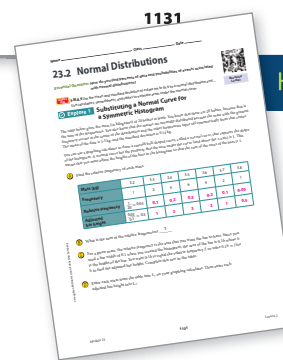
You can use a graphing calculator to draw a smooth bell-shaped curve, called a *normal curve*, that captures the shape of the histogram. A normal curve has the property that the area under the curve (and above the *x*-axis) is 1. This means that you must adjust the heights of the bars in the histogram so that the sum of the areas of the bars is 1.

- (A) Find the relative frequency of each mass.

Mass (kg)	3.2	3.3	3.4	3.5	3.6	3.7	3.8
Frequency	1	2	4	6	4	2	1
Relative frequency	$\frac{1}{20} = 0.05$	0.1	0.2	0.3	0.2	0.1	0.05
Adjusted bar height	$\frac{0.05}{0.1} = 0.5$	1	2	3	2	1	0.5

- (B) What is the sum of the relative frequencies? 1
- (C) For a given mass, the relative frequency is the area that you want the bar to have. Since you used a bar width of 0.1 when you created the histogram, the area of the bar is $0.1h$ where h is the height of the bar. You want $0.1h$ to equal the relative frequency f , so solve $0.1h = f$ for h to find the adjusted bar height. Complete this row in the table.
- (D) Enter each mass from the table into L_1 on your graphing calculator. Then enter each adjusted bar height into L_2 .

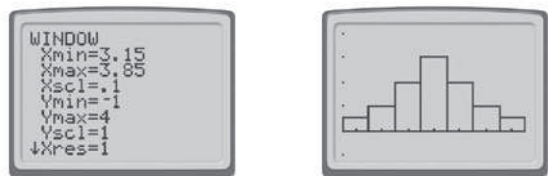
© Houghton Mifflin Harcourt Publishing Company



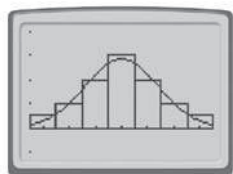
HARDCOVER PAGES 829–836

Turn to these pages to find this lesson in the hardcover student edition.

- E Turn on a statistics plot and select the histogram option. For Xlist, enter L₁. For Freq, enter L₂. Set the graphing window as shown. Then press **GRAPH**.



- F Your calculator has a built-in function called a *normal probability density function*, which you can access by pressing **2nd** **VARs** and selecting the first choice from the DISTR (distribution) menu. When entering this function to be graphed, you must include the mean and standard deviation of the distribution by entering **normalpdf(X, 3.5, 0.14)**. When you press **GRAPH**, the calculator will draw a normal curve that fits the histogram.



Reflect

- Describe the end behavior of the normal probability density function.
 $f(x)$ approaches 0 as x increases or decreases without bound.
- Discussion** If the area under the normal curve is 1, then what is the area under the curve to the left of the mean, 3.5? Describe how to obtain this area using the bars in the histogram. Show that your method gives the correct result.
0.5; add the areas of the first three bars plus half the area of the fourth bar:
 $0.05 + 0.1 + \frac{1}{2}(0.3) = 0.5$ (since bar area = relative frequency)
- Explain how you can use the bars in the histogram to estimate the area under the curve within 1 standard deviation of the mean, which is the interval from $3.5 - 0.14 = 3.36$ to $3.5 + 0.14 = 3.64$ on the x -axis. Then find the estimate.
The interval "captures" the middle three bars, so the sum of the areas of those bars should give an estimate of the area under the curve: $0.2 + 0.3 + 0.2 = 0.7$
- Explain how you can use the bars in the histogram to estimate the area under the curve within 2 standard deviations of the mean, which is the interval from $3.5 - 2(0.14) = 3.22$ to $3.5 + 2(0.14) = 3.78$ on the x -axis. Then find the estimate.
The interval "captures" the middle five bars, so the sum of the areas of those bars gives an estimate of the area under the curve: $0.1 + 0.2 + 0.3 + 0.2 + 0.1 = 0.9$

EXPLORE

Substituting a Normal Curve for a Symmetric Histogram

INTEGRATE TECHNOLOGY

Students have the option of completing the Explore activity either in the book or online.

QUESTIONING STRATEGIES

- ?** Why is the sum of the relative frequencies 1? **The sum is 1 because the sum represents the whole sample or, in terms of percent, 100% of the sample.**
- ?** Why were adjustments made to the heights of the bars? **They were adjusted in order to make the sum of all the areas of the bars in the histogram 1.**
- ?** What value does the area under the normal curve between two specific x -values represent? **It represents the probability that the outcome will be between those two x -values.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Help students see that a normal curve is a graphic representation of the concepts of mean and standard deviation. Have students identify those elements for the curves pictured in the lesson.

PROFESSIONAL DEVELOPMENT

Math Background

The general shape of a normal probability distribution, also called a Gaussian distribution, is that of a bell. The exact shape of the bell is determined by the distribution's mean and standard deviation. The graph of a normal probability distribution is called a normal curve and is given by the equation

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean and σ is the standard deviation. The mean and standard deviation can be found from the data. Students should be able to draw a histogram from data and sketch a smooth, "best-fit" normal curve by hand.

EXPLAIN 1

Finding Areas Under a Normal Curve

QUESTIONING STRATEGIES

? What does the 68-95-99.7 rule tell you about a set of data that is normally distributed with mean μ ? **It says that 68% of the data falls within one standard deviation of μ , 95% falls within 2 standard deviations of μ , and 99.7% falls within 3 standard deviations of μ .**

? How can you use the 68-95-99.7 rule to determine the percent of the data that falls more than 3 standard deviations above the mean? **Since 99.7% of the data falls within 3 standard deviations of the mean, subtract 99.7 from 100, and divide the difference by 2 (since half of this percentage represents the data that fall below 3 standard deviations below the mean).**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Ask students to use their own words to describe what the normal curve and the 68-95-99.7 rule indicate about a set of data in which the mean is 30 and the standard deviation is 5.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

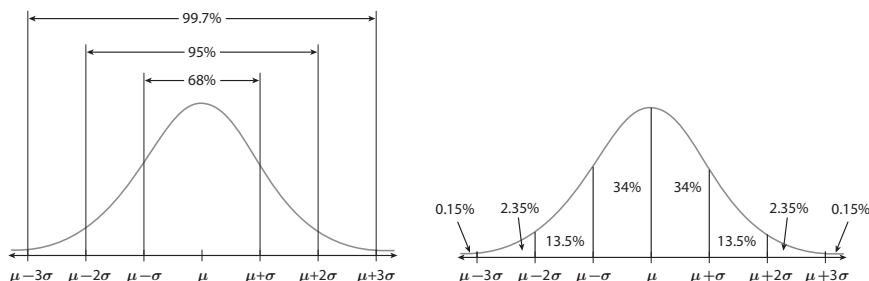
MP.2 Discuss with students how the 68-95-99.7 rule helps to describe how “typical” a data value is for a normally distributed data set. Help them to understand the relationship between the percents and the probability distribution of the data.

Explain 1 Finding Areas Under a Normal Curve

All normal curves have the following properties, sometimes collectively called the 68–95–99.7 rule:

- 68% of the data fall within 1 standard deviation of the mean.
- 95% of the data fall within 2 standard deviations of the mean.
- 99.7% of the data fall within 3 standard deviations of the mean.

A normal curve’s symmetry allows you to separate the area under the curve into eight parts and know the percent of the data in each part.



Example 1 Suppose the heights (in inches) of men (ages 20–29) years old in the United States are normally distributed with a mean of 69.3 inches and a standard deviation of 2.92 inches. Find each of the following.

- A** The percent of men who are between 63.46 inches and 75.14 inches tall.
- 63.46 inches is 5.84 inches, or 2 standard deviations, below the mean.
- 75.14 inches is 5.84 inches, or 2 standard deviations, above the mean.
- 95% of the data in a normal distribution fall within 2 standard deviations of the mean.
- So, 95% of males are between 63.46 inches and 75.14 inches tall.
- B** The percent of men who are taller than 72.22 inches.
- 72.22 inches is **2.92** inches, or **1** standard deviation(s), above the mean.
- When the area under a normal curve is separated into eight parts, the parts that satisfy the condition that the height will be greater than 72.22 inches have percents of **13.5%, 2.35%, and 0.15%**.
- The sum of these percents is **16**%.
- So, **16**% of males are taller than 72.22 inches.

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Instruct each pair to use the data in Example 2 to create a quiz containing 4 problems similar to those in the Example, and an answer key for their quiz. Have pairs exchange and take each other’s quizzes. Then have the students return their quizzes to the original pair for grading, and reconcile any discrepancies.

Your Turn

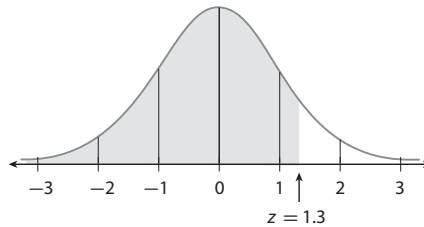
Suppose the heights (in inches) of men (ages 20–29) in the United States are normally distributed with a mean of 69.3 inches and a standard deviation of 2.92 inches. Find each of the following.

- The percent of men who are between 60.54 inches and 78.06 inches tall.
60.54 and 78.06 are both 3 standard deviations from the mean; 99.7%
- The percent of men who are shorter than 60.54 inches.
60.54 is 3 standard deviations below the mean; 0.15%.

Explain 2 Using the Standard Normal Distribution

The **standard normal distribution** has a mean of 0 and a standard deviation of 1. A data value x from a normal distribution with mean μ and standard deviation σ can be standardized by finding its **z-score** using the formula $z = \frac{x - \mu}{\sigma}$.

Areas under the standard normal curve to the left of a given z-score have been computed and appear in the standard normal table below. This table allows you to find a greater range of percents and probabilities than you can using μ and multiples of σ . For example, the area under the curve to the left of the z-score 1.3 is 0.9032. (In the table, “0.0000+” means slightly more than 0, and “1.0000–” means slightly less than 1.)



Standard Normal Table										
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-3	0.0013	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000+
-2	0.0228	0.0179	0.0139	0.0107	0.0082	0.0062	0.0047	0.0035	0.0026	0.0019
-1	0.1587	0.1357	0.1151	0.0968	0.0808	0.0668	0.0548	0.0446	0.0359	0.0287
-0	0.5000	0.4602	0.4207	0.3821	0.3446	0.3085	0.2743	0.2420	0.2119	0.1841
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000–

Example 2 Suppose the heights (in inches) of women (ages 20–29) in the United States are normally distributed with a mean of 64.1 inches and a standard deviation of 2.75 inches. Find the percent of women who are no more than 65 inches tall and the probability that a randomly chosen woman is between 60 inches and 63 inches tall.

Analyze Information

The mean of the heights is **64.1** inches.

The standard deviation of the heights is **2.75** inches.

EXPLAIN 2

Using the Standard Normal Distribution

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Discuss with students that no distribution fits a normal distribution perfectly but many are close to normal, such as heights of males or females, and standardized test scores. Discuss the importance of standardizing. By standardizing, you can compare values from groups with different means and standard deviations.

AVOID COMMON ERRORS

When finding a z-score, students may subtract the x -value from the mean instead of the mean from the x -value. Explain that any value greater than the mean must have a positive z-score, and any value less than the mean must have a negative z-score. Keeping this in mind will help students notice when they have made this error.

QUESTIONING STRATEGIES

? What is the meaning of the number read from the standard normal table for a given z-score? **The number is the probability that a randomly selected data value will have a z-score less than or equal to the given z-score.**

? How do you find the probability that a data value exceeds a given z-score? **Subtract the probability obtained from the standard normal table for the given z-score from 1.**

DIFFERENTIATE INSTRUCTION

Multiple Representations

To help students who may be having difficulty understanding how the normal curve relates to a set of data, have them take a specific set of normally distributed data with a given mean and standard deviation, draw a sketch of the normal curve, and beneath μ , $\mu + \sigma$, $\mu + 2\sigma$, etc., write the values that correspond to these quantities. Have them label the sections of the graph with the percentages defined by the 64-95-99.7 rule. Then have them plot the actual data values from the data set along the horizontal axis to see how the graph relates to the spread of the data.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Have students consider a normal distribution with mean μ and standard deviation σ , and ask them to find the z -score that corresponds to μ . Have them justify their results and explain the related percent from the standard normal table. Then have them make a conjecture about the z -score for $\mu + \sigma$, test their conjectures, and generalize their findings.

SUMMARIZE THE LESSON

? What is the normal curve, and how does it, in conjunction with the 68-95-99.7 rule, describe a set of normally distributed data? **It is the bell-shaped curve approximated by the histogram for the probability distribution for a set of normally distributed data. It shows how the data clusters about the mean, with 68% of the data falling within 1 standard deviation of the mean, 95% of the data falling within 2 standard deviations of the mean, and 99.7% of the data falling within 3 standard deviations of the mean.**

Formulate a Plan

Convert the heights to **z -scores** and use the **standard normal table** to find the probabilities needed.

Solve

First, find the percent of women who are no more than 65 inches tall.

$$\text{Convert 65 to a } z\text{-score (to the nearest tenth): } z_{65} = \frac{65 - \mu}{\sigma} = \frac{65 - 64.1}{2.75} \approx 0.3$$

Recognize that the phrase “no more than 65 inches” means that $z \leq z_{65}$. Read the decimal from the appropriate row and column of the standard normal table: **0.6179**

Write the decimal as a percent: **61.79%**

Next, find the probability that a randomly chosen woman is between 60 inches and 63 inches tall.

$$\text{Convert 60 to a } z\text{-score: } z_{60} = \frac{60 - \mu}{\sigma} = \frac{60 - 64.1}{2.75} \approx -1.5$$

$$\text{Convert 63 to a } z\text{-score: } z_{63} = \frac{63 - \mu}{\sigma} = \frac{63 - 64.1}{2.75} \approx -0.4$$

Because the standard normal table gives areas under the standard normal curve to the left of a given z -score, you find $P(z_{60} \leq z \leq z_{63})$ by subtracting $P(z \leq z_{60})$ from $P(z \leq z_{63})$. Complete the following calculation using the appropriate values from the table:

$$P(z_{60} \leq z \leq z_{63}) = P(z \leq z_{63}) - P(z \leq z_{60}) = 0.3446 - 0.0668 = 0.2778$$

Write the decimal as a percent: **27.78%**

Justify and Evaluate

65 inches is slightly **greater** than the mean height. Since all the heights up to the mean represent **50%** of the data, it makes sense that all the heights up to 65 inches are slightly **more** than 50%.

Heights from 60–63 inches are within the area for heights less than 65 inches but there are fewer, so it makes sense that the probability is **less** than 50%.

Your Turn

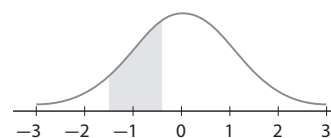
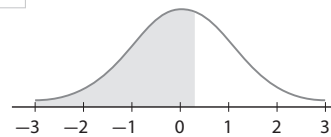
Suppose the heights (in inches) of adult females (ages 20–29) in the United States are normally distributed with a mean of 64.1 inches and a standard deviation of 2.75 inches.

7. Find the percent of women who are at least 66 inches tall.

$$z_{66} = \frac{66 - 64.1}{2.75} \approx 0.7; P(z \geq z_{66}) = 1 - P(z \leq z_{66}) = 1 - 0.7580 = 0.242, \text{ or about } 24.2\%$$

8. Find the percent of women who are less than or equal to 61.6 inches tall.

$$z_{61.6} = \frac{61.6 - 64.1}{2.75} \approx -0.9; P(z \leq z_{61.6}) = 0.1841, \text{ or about } 18.41\%$$



LANGUAGE SUPPORT **EL**

Communicate Math

Have students work in pairs to discuss what two different normal distributions might look like, and have one student sketch a narrow one and the other sketch a wide one. The pairs should talk about how to show what 1 standard deviation, 2 standard deviations, and 3 standard deviations from the mean are in each sketch, and the percentages attached to each.

Elaborate

9. Explain how you know that the area under a normal curve between $\mu + \sigma$ and $\mu + 2\sigma$ represents 13.5% of the data if you know that the percent of the data within 1 standard deviation of the mean is 68% and the percent of the data within 2 standard deviations of the mean is 95%.

The difference between 95% and 68% is 27%; the symmetry of the curve tells you that this difference is split evenly between the interval from $\mu - 2\sigma$ to $\mu - \sigma$ and the interval from $\mu + \sigma$ to $\mu + 2\sigma$, so each interval represents 13.5% of the data.

10. How can you use the 68-95-99.7 rule and the symmetry of a normal curve to find the percent of normally distributed data that are less than 1 standard deviation above the mean?

The symmetry of a normal curve tells you that 50% of the data are below the mean.

Knowing that 68% of the data are within that 1 standard deviation of the mean tells you that 34% of the data fall between the mean and 1 standard deviation above the mean.

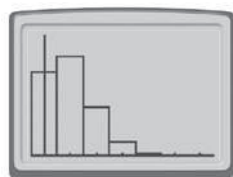
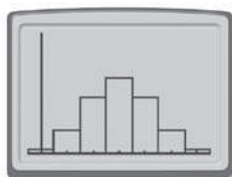
Then $50\% + 34\% = 84\%$ of the data are less than 1 standard deviation above the mean.

11. **Essential Question Check-In** Explain what a z-score is and how it's used.

A z-score represents the number of standard deviations that a data value is above or below the mean. z-scores are used to find probabilities based on the areas under the standard normal curve given in the standard normal table.

Evaluate: Homework and Practice

1. The first calculator screen shows the probability distribution when 6 coins are flipped and the number of heads is counted. The second screen shows the probability distribution of the number of correct answers given by a group of people on a 6-question quiz. For which distribution is it reasonable to use a normal curve as an approximation? Why?



It is reasonable to use a normal curve for the distribution of heads when 6 coins are flipped because the distribution is symmetric and follows a bell-shaped curve.



- Online Homework
- Hints and Help
- Extra Practice

EVALUATE



ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Substituting a Normal Curve for a Symmetric Histogram	Exercise 1
Example 1 Finding Areas Under a Normal Curve	Exercises 2–9
Example 2 Using the Standard Normal Distribution	Exercises 10–22

QUESTIONING STRATEGIES

? Why is the area to the left of a negative z-score always less than 0.5? **A negative z-score corresponds to a data value that lies below the mean. Since the area to the left of the mean is 0.5, the area to the left of any number less than the mean will be less than 0.5.**

? Why do you have to subtract to find the area between two z-scores? **The table gives the percentages below each of the z-scores, so you have to subtract the lesser percent from the greater percent to find the difference between them.**

VISUAL CUES

Suggest that students draw and shade normal curves to model each exercise. They can then use their models to check their answers for reasonableness.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE	Mathematical Practices
1	1 Recall of Information		MP.3 Logic
2–9	2 Skills/Concepts		MP.4 Modeling
10–15	2 Skills/Concepts		MP.6 Precision
16–22	2 Skills/Concepts		MP.5 Using Tools
23–24	3 Strategic Thinking H.O.T.		MP.3 Logic
25	3 Strategic Thinking H.O.T.		MP.2 Reasoning

AVOID COMMON ERRORS

Students may make calculation errors based on the faulty thinking that the percent of the data that lies between the mean and 3 standard deviations above (or below) the mean is 50%. Correct this thinking by pointing out the tails of the curve that extend beyond the points $\mu + 3\sigma$ and $\mu - 3\sigma$, and emphasizing that these unbounded regions each make up a small percentage of the area that lies above or below the mean.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Discuss with students how standardization and z -scores make it possible to compare two sets of related data that may have different means and standard deviations. Use an example, such as the heights of male basketball players and the heights of female basketball players, to illustrate this concept.

A college-entrance exam is designed so that scores are normally distributed with a mean of 500 and a standard deviation of 100.

- What percent of exam scores are between 400 and 600?
Both 400 and 600 are 1 standard deviation from the mean, so 68%.
- What is the probability that a randomly chosen exam score is above 600?
600 is 1 standard deviation above the mean. When the area under a normal curve is separated into eight parts, the parts that satisfy the condition that the score will be greater than 600 have percents of 13.5%, 2.35%, and 0.15%. The sum of these percents is 16%, or 0.16.
- What is the probability that a randomly chosen exam score is less than 300 or greater than 700?
300 is 2 standard deviations below the mean and 700 is 2 standard deviations above the mean. 95% of the data fall within 2 standard deviations of the mean, so the probability that an exam score is less than 300 or greater than 700 is $100\% - 95\% = 5\%$, or 0.05.
- What is the probability that a randomly chosen exam score is above 300?
 $1 - (0.0015 + 0.0235) = 0.975$

Flight 202's arrival time is normally distributed with a mean arrival time of 4:30 p.m. and a standard deviation of 15 minutes. Find the probability that a randomly chosen arrival time is within the given time period.

- After 4:45 p.m.
 $0.135 + 0.0235 + 0.0015 = 0.16$
- Between 4:15 p.m. and 5:00 p.m.
 $0.68 + 0.135 = 0.815$
- Between 3:45 p.m. and 4:30 p.m.
 $0.5000 - 0.0015 = 0.4985$
- By 4:45 p.m.
 $0.50 + 0.34 = 0.84$

Suppose the scores on a test given to all juniors in a school district are normally distributed with a mean of 74 and a standard deviation of 8. Find each of the following using the standard normal table.

- Find the percent of juniors whose score is no more than 90.
 $z_{90} = \frac{90 - \mu}{\sigma} = \frac{90 - 74}{8} = 2$; 0.9772, or about 98%
- Find the percent of juniors whose score is between 58 and 74.
 $z_{58} = \frac{58 - \mu}{\sigma} = \frac{58 - 74}{8} = -2$; $z_{74} = \frac{74 - \mu}{\sigma} = \frac{74 - 74}{8} = 0$; $0.5000 - 0.0228 = 0.4772$, or about 48%
- Find the percent of juniors whose score is at least 74.
74 is the mean, so the percent is 50%.
- Find the probability that a randomly chosen junior has a score above 82.
 $z_{82} = \frac{82 - \mu}{\sigma} = \frac{82 - 74}{8} = 1$; $1 - 0.8413 = 0.1587$
- Find the probability that a randomly chosen junior has a score between 66 and 90.
 $z_{66} = \frac{66 - \mu}{\sigma} = \frac{66 - 74}{8} = -1$; $z_{90} = \frac{90 - \mu}{\sigma} = \frac{90 - 74}{8} = 2$; $0.9772 - 0.1587 = 0.8185$
- Find the probability that a randomly chosen junior has a score below 74.
74 is the mean, so the probability is 0.5.

Graphing Calculator On a graphing calculator, you can use the function $\text{normalcdf}(\text{lower bound}, \text{upper bound}, \mu, \sigma)$ on the DISTR menu to find the area under a normal curve for values of x between a specified lower bound and a specified upper bound. You can use $-1E99$ as the lower bound to represent negative infinity and $1E99$ as the upper bound to represent positive infinity. Suppose that cans of lemonade mix have amounts of lemonade mix that are normally distributed with a mean of 350 grams and a standard deviation of 4 grams. Use this information and a graphing calculator to answer each question.

16. What percent of cans have less than 338 grams of lemonade mix?
 $\text{normalcdf}(-1E99, 338, 350, 4) \approx 0.001$
17. What is the probability that a randomly chosen can has between 342 grams and 350 grams of lemonade mix?
 $\text{normalcdf}(342, 350, 350, 4) \approx 0.477$
18. What is the probability that a randomly chosen can has less than 342 grams or more than 346 grams of lemonade mix?
 $\text{normalcdf}(-1E99, 342, 350, 4) \approx 0.023$; $\text{normalcdf}(346, 1E99, 350, 4) \approx 0.841$;
 $0.841 + 0.023 = 0.864$

Spreadsheet In a spreadsheet, you can use the function $\text{NORM DIST}(\text{upper bound}, \mu, \sigma, \text{TRUE})$ to find the area under a normal curve for values of x less than or equal to a specified upper bound. Suppose the heights of all the children in a state are normally distributed with a mean of 45 inches and a standard deviation of 6 inches. Use this information and a spreadsheet to answer each question.



19. What is the probability that a randomly chosen child is less than 40 inches tall?
 $\text{NORM DIST}(40, 45, 6, \text{TRUE}) \approx 0.2023$
20. What is the probability that a randomly chosen child is greater than 47 inches tall?
 $\text{NORM DIST}(47, 45, 6, \text{TRUE}) \approx 0.6306$; $1 - 0.6306 = 0.3694$
21. What percent of children are between 50 and 53 inches tall?
 $\text{NORM DIST}(50, 45, 6, \text{TRUE}) \approx 0.7977$; $\text{NORM DIST}(53, 45, 6, \text{TRUE}) \approx 0.9088$;
 $0.9088 - 0.7977 = 0.1111$; About 11.11% of children are between 50 and 53 inches tall.
22. What is the probability that a randomly chosen child is less than 38 inches tall or more than 51 inches tall?
 $\text{NORM DIST}(38, 45, 6, \text{TRUE}) \approx 0.1217$; $\text{NORM DIST}(51, 45, 6, \text{TRUE}) \approx 0.8413$;
 $1 - 0.8413 = 0.1587$; $0.1587 + 0.1217 = 0.2804$

© Houghton Mifflin Harcourt Publishing Company • Image Credits: © Diego Carvo/Shutterstock

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 The Central Limit Theorem states that the distribution of the means of random samples, rather than the actual data, are normally distributed for large sample sizes. So normal distributions can be used to describe the averages of data that do not necessarily have a normal distribution themselves.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner how, if you know the mean and standard deviation of a set of normally distributed data, you can find the data value that has a given z -score. Have students write a formula that can be used to find this value. **The formula for finding a z -score can be transformed to $x = z\sigma + \mu$, showing how the data value x can be calculated from the three given measures.**

JOURNAL

Have students explain how to use the standard normal table to find the probability of an event, given the mean and standard deviation of a normal distribution. Have them create an example to illustrate their explanations.

H.O.T. Focus on Higher Order Thinking

- 23. Explain the Error** A student was asked to describe the relationship between the area under a normal curve for all x -values less than a and the area under the normal curve for all x -values greater than a . The student's response was "Both areas are 0.5 because the curve is symmetric." Explain the student's error.

The student's answer is only correct if a is the mean. If a is not the mean, the area for the x -values less than a is p , and the area for x -values greater than a is $1 - p$.

- 24. Make a Conjecture** A local orchard packages apples in bags. When full, the bags weigh 5 pounds each and contain a whole number of apples. The weights are normally distributed with a mean of 5 pounds and a standard deviation of 0.25 pound. An inspector weighs each bag and rejects all bags that weigh less than 5 pounds. Describe the shape of the distribution of the weights of the bags that are not rejected.



The shape of the distribution will be the right half of a normal curve because all values below the mean are rejected.

- 25. Analyze Relationships** A biologist is measuring the lengths of frogs in a certain location. The lengths of 20 frogs are shown. If the mean is 7.4 centimeters and the standard deviation is 0.8 centimeter, do the data appear to be normally distributed? Complete the table and explain.

Length (cm)				
7.5	5.8	7.9	7.6	8.1
7.9	7.1	5.9	8.4	7.3
7.1	6.4	8.3	8.4	6.7
8.1	7.8	5.9	6.8	8.1

z	Area $\leq z$	x	Values $\leq x$	
			Projected	Actual
-2	0.02	5.8	$0.02 \cdot 20 = 0.4 \approx 0$	1
-1	0.16	6.6	$0.16 \cdot 20 = 3.2 \approx 3$	4
0	0.5	7.4	$0.5 \cdot 20 = 10$	9
1	0.84	8.2	$0.84 \cdot 20 = 16.8 \approx 17$	17
2	0.98	9	$0.98 \cdot 20 = 19.6 \approx 20$	20

The projected number of values that corresponds to each value of z is close to the actual number of data values. The data appear to be normally distributed.

© Houghton Mifflin Harcourt Publishing Company - Image Credits: © Irina Schmidt/Shutterstock

Lesson Performance Task

Suppose a nurse records the temperatures of healthy men and women. The table represents the means and standard deviations of the data.

	Women	Men
°F	98.4 ± 0.7 °F	98.1 ± 0.7 °F
°C	36.9 ± 0.4 °C	36.7 ± 0.4 °C



- A. What is the range of temperatures for 1, 2, and 3 standard deviations for both women and men? (Give temperatures in both degrees Fahrenheit and degrees Celsius.)

Women

1 standard deviation: 97.7 °F to 99.1 °F; 36.5 °C to 37.3 °C

2 standard deviations: 97 °F to 99.8 °F; 36.1 °C to 37.7 °C

3 standard deviations: 96.3 °F to 100.5 °F; 35.7 °C to 38.1 °C

Men

1 standard deviation: 97.4 °F to 98.8 °F; 36.3 °C to 37.1 °C

2 standard deviations: 96.7 °F to 99.5 °F; 35.9 °C to 37.5 °C

3 standard deviations: 96 °F to 100.2 °F; 35.5 °C to 37.9 °C

- B. Assuming a normal distribution, find the probability that a randomly selected man's temperature is between 96.7 °F and 97.4 °F.

97.4 is 1 standard deviation below the mean.

68% of the data fall between 97.4 °F and 98.8 °F.

96.7 is 2 standard deviations below the mean.

95% of the data fall between 96.7 °F and 99.5 °F.

The difference is 27%.

$$\frac{0.27}{2} = 0.135$$

The probability that a randomly selected man's temperature is between 96.7 °F and 97.4 °F is 0.135.

- C. A fever is considered medically significant if body temperature reaches 100.4 °F (38 °C). What can you observe about this temperature?

It is about 3 standard deviations above the mean for both men and women, which is higher than almost all (about 99.85% of) normal body temperatures.

© Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Gill/Jamie Gill/Blend Images/Corbis

EXTENSION ACTIVITY

Have students come up with an idea for a survey of students in the school that would collect data that should be normally distributed. Ask them to use the standard normal curve to predict the number of students who should fall within one standard deviation of the mean, two standard deviations of the mean, and so on. Then, if it is feasible, have students conduct the survey and find out whether their actual data is normally distributed.

CONNECT VOCABULARY **EL**

Describe how a person might *deviate* partly or completely from guidelines or from a plan. Connect this to the concept that data may deviate from the mean.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.1 Students may benefit by representing the mean temperatures and temperatures representing 1, 2, and 3 standard deviations from the mean on a number line, with the standard deviation ranges marked below the line and the relevant temperatures above the line. By using two number lines, one above the other, students can compare these temperatures for the genders.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.