International General Certificate of Secondary Education

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

MATHEMATICS   **0580/4, 0581/4**

PAPER 4

Wednesday   **9 JUNE 1999**   Afternoon   2 hours 30 minutes

Additional materials:
    Answer paper
    Electronic calculator
    Geometrical instruments
    Graph paper (2 sheets)
    Mathematical tables (optional)

TIME   2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/ answer booklet.

Answer **all** questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 130.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142.

[Turn over

1 A football club asks all its members to vote 'Yes' or 'No' for a new stadium.
   They receive 48 790 'Yes' votes.
   The ratio   'Yes' votes : 'No' votes   is 7 : 5.

   (a) How many members voted?   [2]

   (b) There were 14 760 members who did not vote.
   What percentage of members did not vote?   [2]

   (c) To build the new stadium, 50% of the total number of members have to vote 'Yes'.
   Will the new stadium be built? **Show working** to explain your answer.   [3]

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2 For a certain type of tree, $C = 2.5y$ where C is the circumference in centimetres and y is the age of the tree in years.

   [The cross-section of the tree trunk is a circle. For π, use either your calculator value or 3.142.]

   (a) Estimate the age of a tree with a circumference of 100 cm.   [1]

   (b) Find the radius of the trunk of a 20 year old tree.   [3]

   (c) The cross-sectional area of a tree trunk is 1200 cm^2.

   Find

      (i)   the radius of the tree,   [2]

     (ii)   the age of the tree, to the nearest year.   [2]

   (d) A three year old tree was planted in 1971.
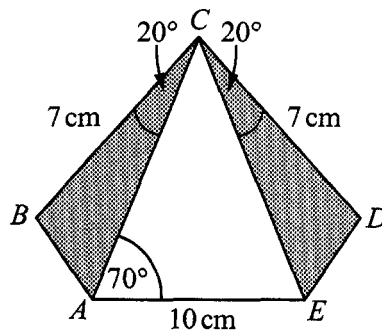   Calculate the year in which the diameter of its trunk will be one metre.   [4]

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3 **(a)**
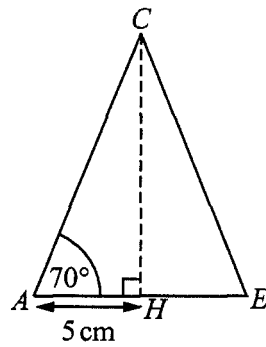
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ACE is an isosceles triangle.
CH is perpendicular to *AE*. Angle *CAH* = 70° and *AH* = 5 cm.
Calculate the length of *AC* correct to 5 significant figures.
Show that it rounds to 14.62 cm. [3]

(b)

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Pentagon *ABCDE* is formed from the isosceles triangle *ACE* together with congruent triangles *ABC* and *EDC*.
BC = 7 cm, angle *BCA* = angle *ECD* = 20° and angle *CAE* = 70°.

(i) Use *AC* = 14.62 cm and the cosine rule to calculate the length of *BA*. [4]

(ii) Find the area of triangle *ABC*. [3]

(c) Triangles *ABC* and *CDE* are folded over onto triangle *ACE*, as shown on the diagram below.
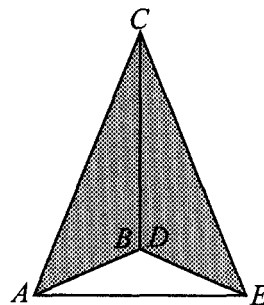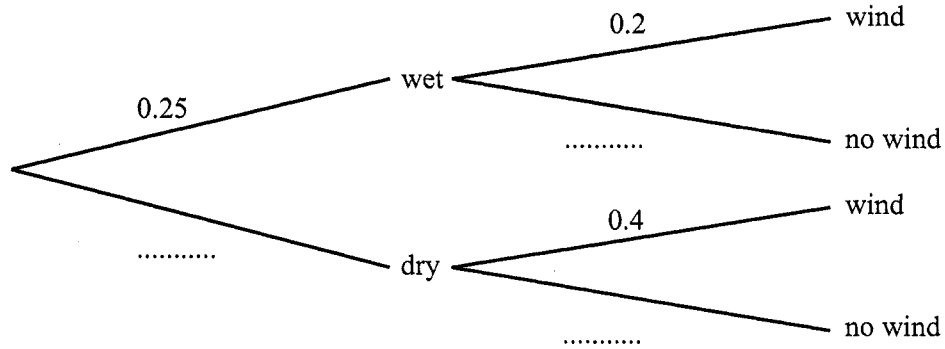

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Calculate the **unshaded** area. [5]

[Turn over

4   In summer the probability of a wet day is 0.25.
    On a wet day, the probability of wind is 0.2.
    On a dry day, the probability of wind is 0.4.
    You may assume that the weather each day is independent of the weather the day before.

(a) **Copy and complete** the tree diagram below.

[3]

(b) When it is wet **and** windy, concerts have to be postponed until the next day.
    Find the probability that Monday's concert

   (i) has to be postponed,   [2]

   (ii) takes place on Tuesday.   [3]

(c) Sailing boats can only sail on a windy day.
    Find the probability that they cannot sail on Monday.   [3]

(d) On a dry day with no wind, the probability that the temperature is more than 30 °C is 0.9.
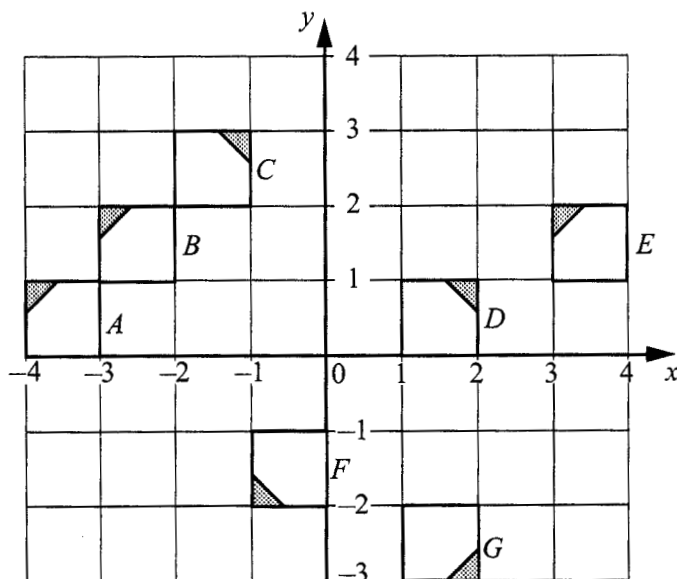   On a wet **or** windy day this will not happen.

   Find

   (i) the probability that the temperature is more than 30 °C on Monday,   [2]

   (ii) the probability that on Monday, Tuesday **and** Wednesday the temperature is more than 30 °C.   [2]

5

On the grid above there are seven identical shapes, labelled A to G.
Use these letters to answer the questions below.

(a) Which two shapes are a reflection of each other in the line $x = -1$? [2]

(b) Which two shapes are a reflection of each other in the line $x + y = 0$? [2]

(c) Which shape is a rotation of the shape D by 90° clockwise?
Write down the coordinates of the centre of rotation. [3]

(d) Which two shapes are a translation of each other by a vector with magnitude exactly 6?
Give the column vector of this translation. [3]

(e) The transformation with matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ maps the shape D onto another shape H.

    (i) Find the coordinates of the 4 vertices of the shape H. [3]

    (ii) Describe fully this single transformation. [3]

[Turn over

6 **Answer the whole of this question on a sheet of graph paper.**

(a) $$f(x) = x^3.$$

Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$. Use a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 10 units on the y-axis. [6]

(b) Use your graph to solve
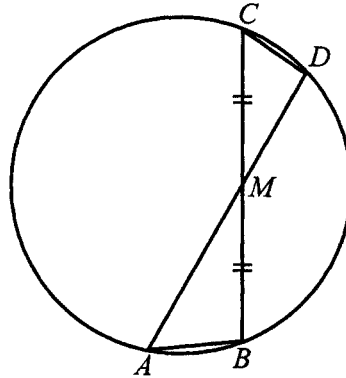
   (i) $f(x) = -20,$ [1]

   (ii) $f^{-1}(x) = 1.7.$ [2]

(c) The equation $x^3 - 5x - 1 = 0$ can be solved by drawing one straight line on your graph.

   (i) Write down the equation of this straight line. [1]

   (ii) Draw the line and write down the three solutions of $x^3 - 5x - 1 = 0$. [5]

7

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In the circle, the chords AD and BC meet at M.

(a) **(i)** Show that triangles AMB and CMD are similar. [3]

   (ii) $AM = 10$ cm and $MD = 4$ cm. If $CM = MB = x$ cm, calculate the value of x. [2]

(b) $CM = MB$ and $AM = \dfrac{5}{2} MD$.

$\overrightarrow{MC} = \mathbf{p}$ and $\overrightarrow{MD} = \mathbf{q}$.

Write the following vectors in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

   (i) $\overrightarrow{BM},$ [1]

   (ii) $\overrightarrow{MA},$ [1]

   (iii) $\overrightarrow{BA},$ [1]

   (iv) $\overrightarrow{DC}.$ [1]

(c) Use your answers to **(b)(iii)** and **(b)(iv)** to explain why BA is not parallel to DC. [1]

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8 Pedro and Anna measure the circumference (C) of 100 trees.
Their results are shown in the table and the cumulative frequency diagram below.

| Circumference (C) in cm	$C \leqslant 20$	$20 < C \leqslant 40$	$40 < C \leqslant 70$	$70 < C \leqslant 100$	$100 < C \leqslant 120$
Frequency	0	26	30	33	11

Circumference in cm
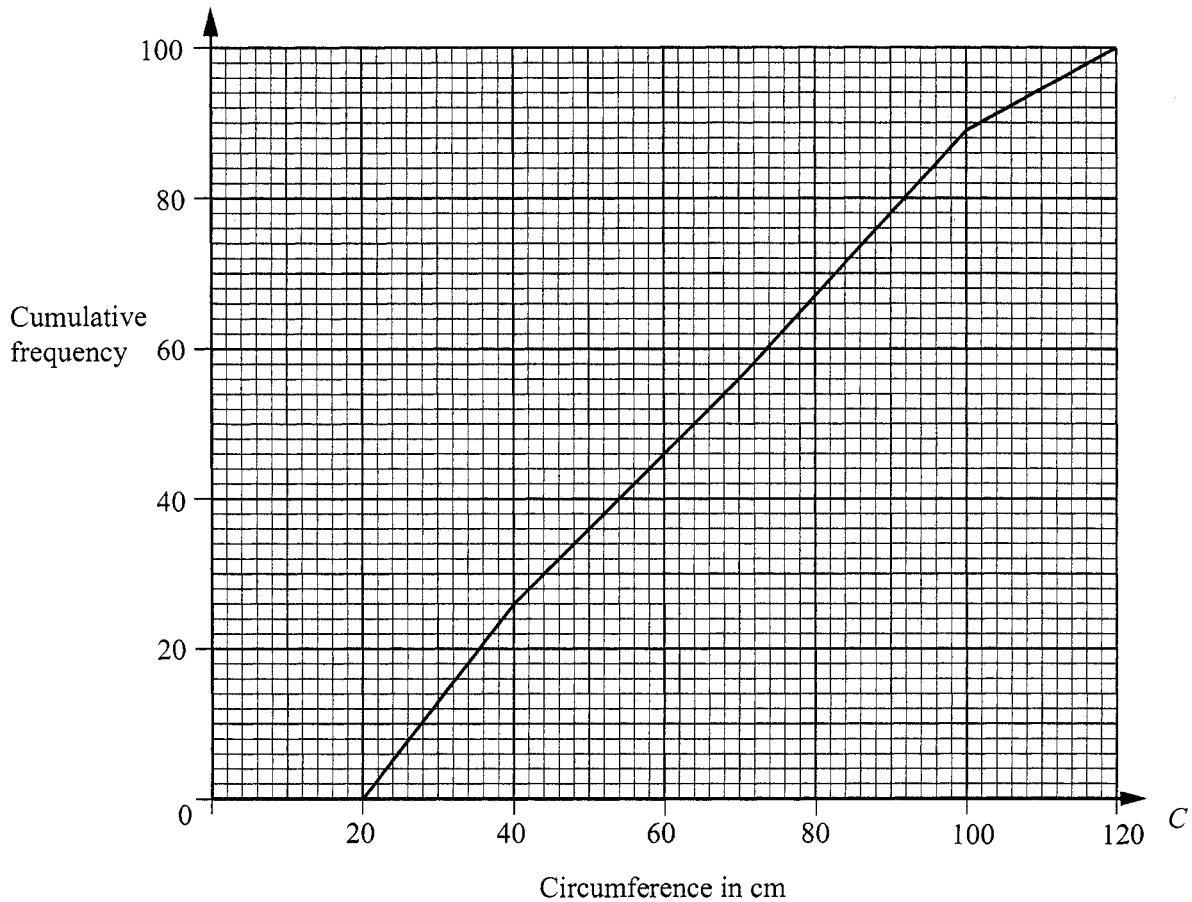
(a) **(i)** Estimate the number of trees whose circumferences are between 60 cm and 80 cm. [2]

(ii) Use the cumulative frequency graph to find the median, the quartiles and the interquartile range. [4]

(iii) Calculate an estimate of the mean circumference. [4]

(iv) Write down the modal class. [1]

(b) Anna wants to construct a histogram. She makes a table to show the heights of the bars she will draw, using a scale of 1 cm to represent 10 cm on the horizontal axis and 1 cm^2 to represent 1 tree.

Circumference (C) in cm	$20 < C \leqslant 40$	$40 < C \leqslant 70$	$70 < C \leqslant 100$	$100 < C \leqslant 120$
Height of bar in cm	x	10	y	z

(i) Explain why the height of the bar for the $40 < C \leqslant 70$ class interval is 10 cm. [1]

(ii) Find the values of x, y and z. **Do NOT draw a histogram.** [3]

[Turn over

9
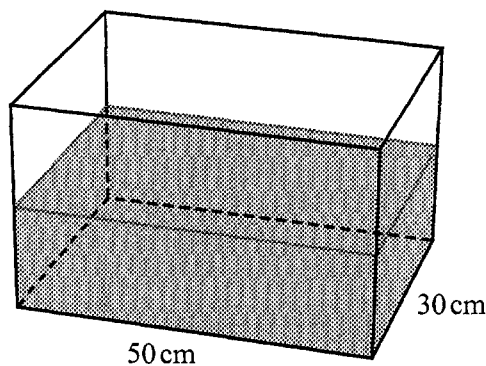
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30 cm

50 cm

(a) A rectangular tank with length 50 cm and width 30 cm contains 36 litres of water.
Show by calculation that the water is 24 cm deep. [2]

(b) A heavy rectangular block is 5 cm high and x cm wide.
Its length is 5 cm more than its width.
Write down an expression for the volume of the block in terms of x. [2]

(c) The block is placed in the tank and the water level rises by 1 cm.

(i) Write down an equation in x and show that it simplifies to

$$x^2 + 5x - 300 = 0.$$ [4]

(ii) Solve the equation $x^2 + 5x - 300 = 0$. [4]

(iii) Write down the width and length of the block. [1]

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10 **Answer the whole of this question on a sheet of graph paper.**

Alberto, Bernard and Carlos sell houses.

Alberto charges $600 whatever the selling price.

Bernard charges 1% of the selling price.

Carlos charges $200 for selling prices up to $30 000.
For selling prices more than $30 000, he charges $200 **and** $1\frac{1}{2}$% of the value over $30 000.
For example, when the selling price is $50 000, Carlos charges

$$\$200 + 1\tfrac{1}{2}\% \text{ of } (\$50\,000 - \$30\,000) = \$500.$$

(a) Use a scale of 2 cm to represent a selling price of $10 000 on the horizontal axis and 2 cm to represent a charge of $100 on the vertical axis.
Draw on the same grid the three graphs to show the charges made by Alberto, Bernard and Carlos for selling prices up to $80 000.
Label your graphs clearly. [7]

(b) (i) For which selling price is Alberto's charge the same as Bernard's? [1]

(ii) For what range of selling prices does Carlos charge the least? [2]

(iii) For which selling price, less than $50 000, does Bernard charge $50 less than Carlos? [2]