Name: ___________________________________

Pre-Calculus Notes: Chapter 6 – Graphs of Trigonometric Functions

Section 1 – Angles and Radian Measure

Angles can be measured in both degrees and radians. Radian measure is based on the circumference of a unit circle (a circle with a radius of 1).

Since the circumference of the unit circle is 2π, $360^o = 2\pi$ radians. It is often helpful to convert between degrees and radians.

Example 1

a. Change 115^o to exact (leave π in the answer) radian measure.

b. Change $-\frac{7\pi}{8}$ to degrees.

c. Change 215^o to exact radian measure.

d. Change 3π to degrees.

Example 2

Use the unit circle (no calculator) to evaluate.

a. $\sin\frac{2\pi}{3}$ b. $\cos\left(-\frac{\pi}{2}\right)$ c. $\cos 30^o$ d. $\sin 225^o$

e. $\tan\frac{\pi}{4}$ f. $\cos\frac{5\pi}{6}$ g. $\sin\frac{13\pi}{6}$ h. $\tan\frac{25\pi}{6}$

Both degree and radian measure can be used to calculate arc length and area of a sector.

	Degrees	Radians
Arc Length	$s = \frac{x^o}{360^o} \cdot 2\pi r$	
Area of a Sector	$A = \frac{x^o}{360^o} \cdot \pi r^2$	

Example 3

a. Given a central angle of 125^o, find the length of its intercepted arc in a circle of radius 7 centimeters. Round to the nearest tenth.

b. A pendulum with length of 1.4 meters swings through an angle of 30^o. How far does the bob at the end of the pendulum travel as it goes from left to right?

Example 4

a. Find the area of a sector if the central angle measures $\frac{3\pi}{7}$ and the radius of the circle is 11 centimeters. Round to the nearest tenth.

b. A sector has area of 15 square inches and central angle of 0.2 radians. Find the radius of the circle. Find the arc length of the sector.

Section 3 – Graphing Sine and Cosine Functions

Periodic Function ____

____

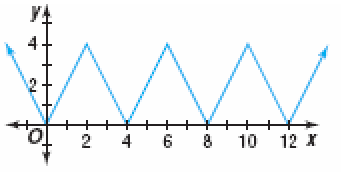
Period ____

____

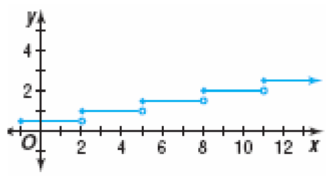
Example 1

Determine if each function is periodic. If so, state the period.

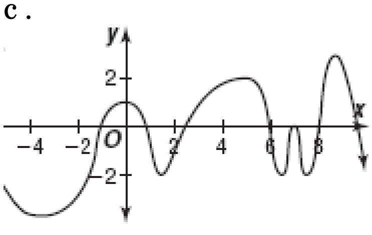
a.



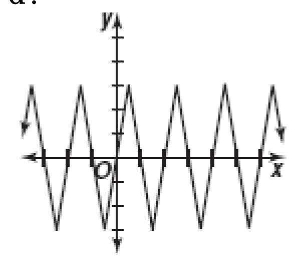
b.



c.

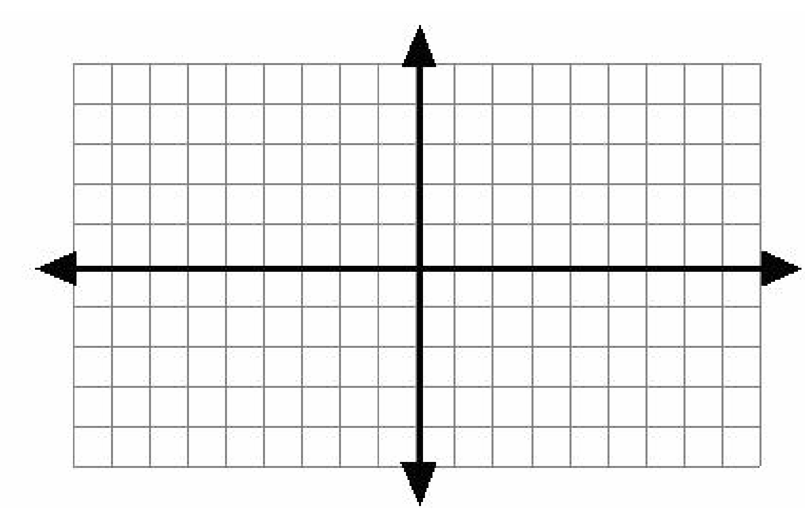


d.



Graphing the Cosine Function: $y = \cos\theta$

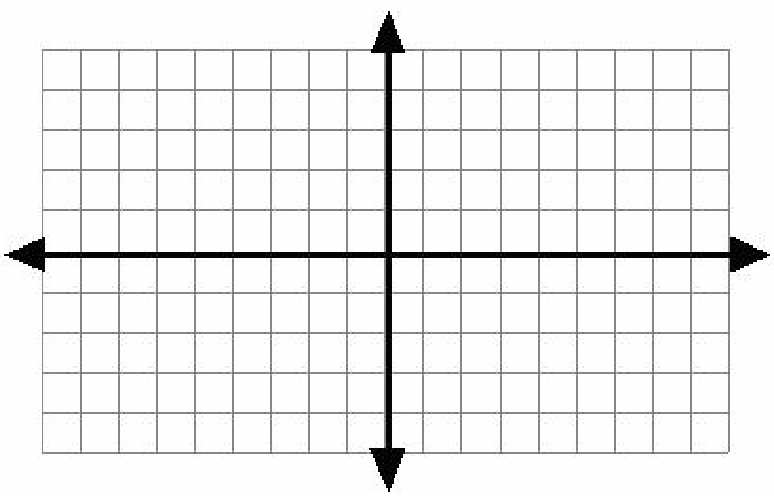
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$															



Domain:	Range:
Maximum:	Minimum:
y-intercept	x-intercept(s)

Graphing the Sine Function: $y = \sin\theta$

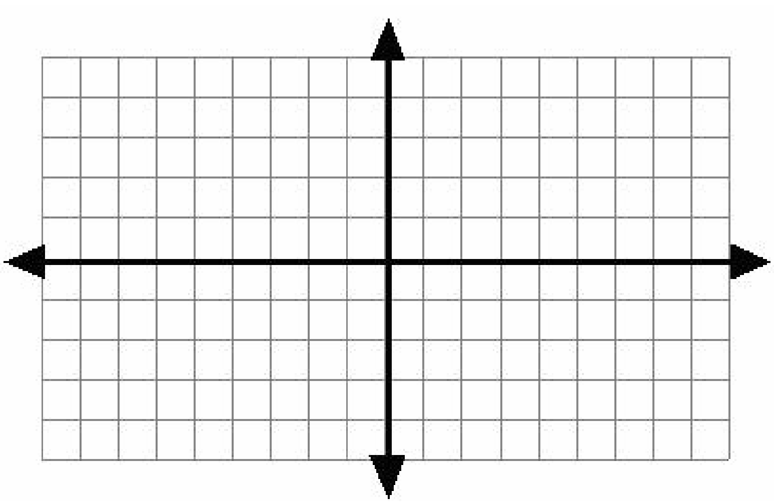
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$																	



Domain:	Range:
Maximum:	Minimum:
y-intercept	x-intercept(s)

Graphing the Tangent Function: $y = \tan\theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\tan\theta$																	



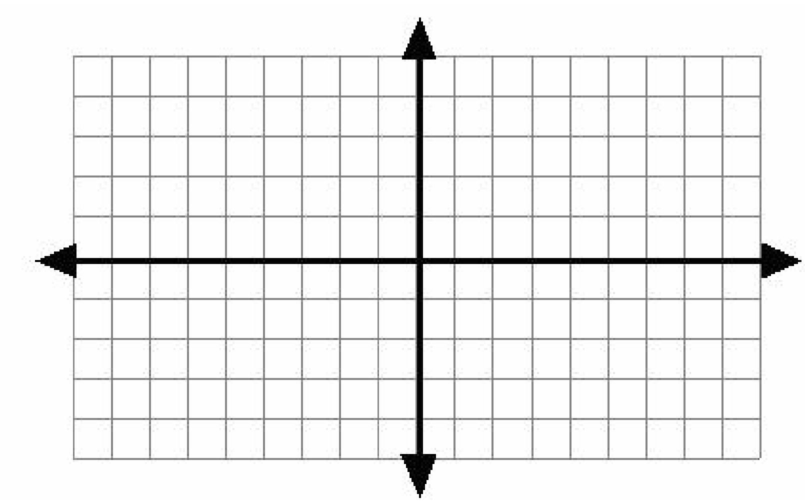
Domain:	Range:
Maximum:	Minimum:
y-intercept	x-intercept(s)

Example 2

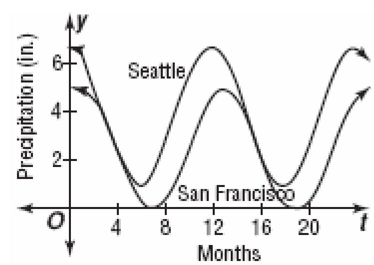
Find $\sin 3\pi$ by referring to the graph of the sine function.

Example 3
Find the value of θ for which $\sin\theta = -1$ is true.

Example 4
Graph $y = \sin x$ for $2\pi \le x \le 4\pi$.



Example 5
The graph at the right shows the average monthly precipitation (in inches) for Seattle, Washington, and San Francisco, California, with January represented as 1.



Model for Seattle's precipitation: $y = 3.45 + 2.55\cos\left(\frac{\pi}{6}t + 0.18\right)$

Model for San Francisco's precipitation: $y = 2.2 + 2.2\cos\left(\frac{\pi}{6}t - 0.41\right)$

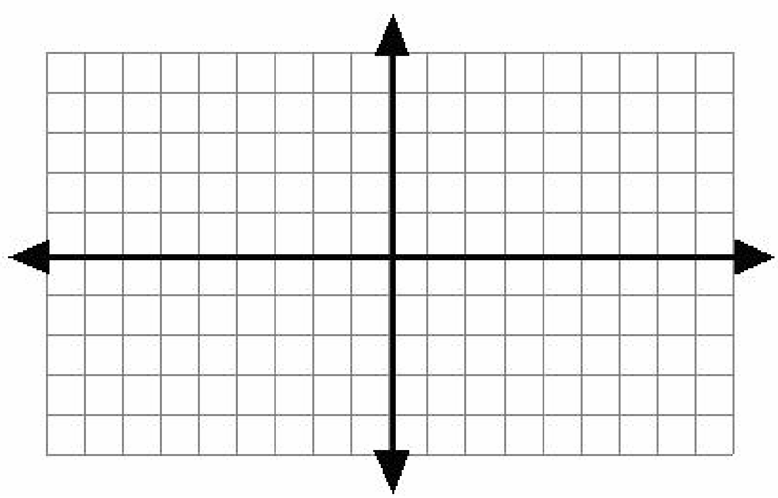
a. What is the average precipitation for each city for month 13?

b. Which city has the greater fluctuation in precipitation? Explain.

Section 4 – Amplitude and Period of Sine and Cosine Functions

Sketch the following functions on the axes below. Set the window of your graphing calculator to: xmin = -2π, xmax = 2π, xscl = $\frac{\pi}{2}$, ymin = -6, ymax = 6, yscl = 1.
After graphing each equation, fill in the table below.

	$y=\sin x$	$y=5\sin x$	$y=\frac{1}{2}\sin x$
Period			
Minimum			
Maximum			
Range			
y-intercept			
x-intercept(s)			

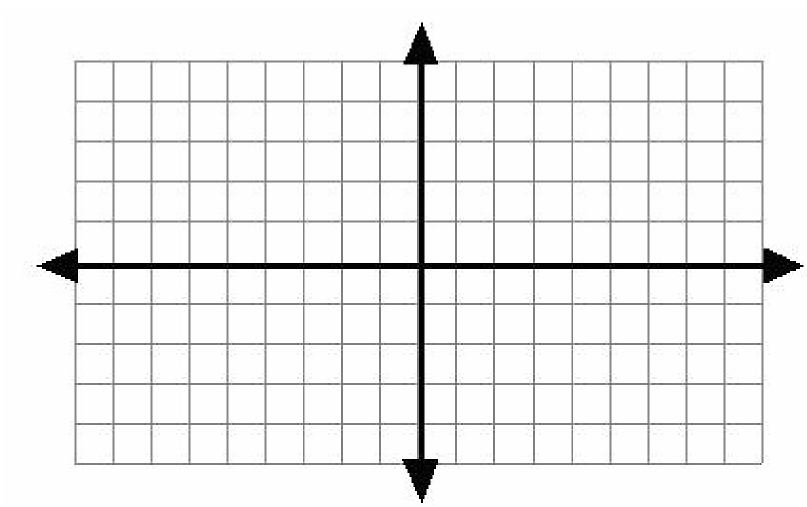


Amplitude ____

____

Sketch the following functions on the axes below. Set the window of your graphing calculator to: xmin = -6π, xmax = 6π, xscl = π, ymin = -2, ymax = 2, yscl = 1.
After graphing each equation, fill in the table below.

	$y = \cos x$	$y = \cos 2x$	$y = \cos\left(\frac{x}{3}\right)$
Period			
Minimum			
Maximum			
Range			
y-intercept			
x-intercept(s)			



Period of Sine and Cosine Functions ____

____

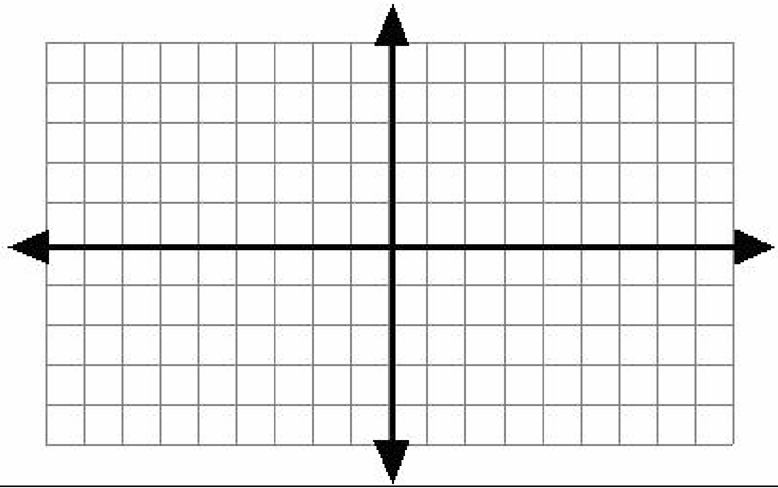
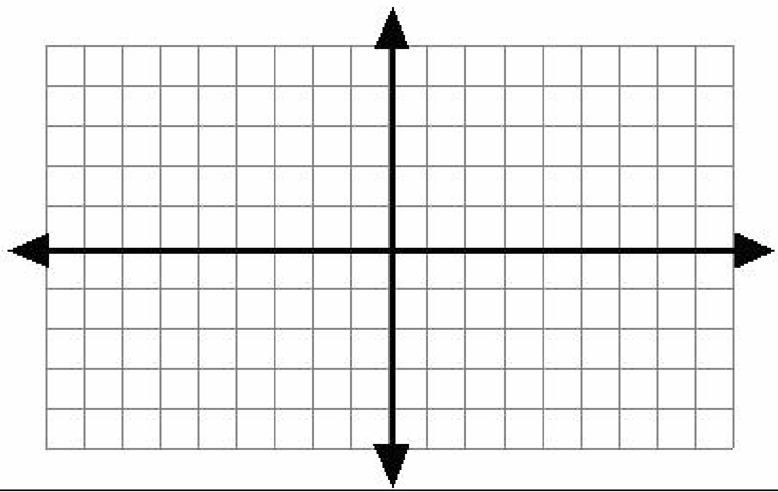
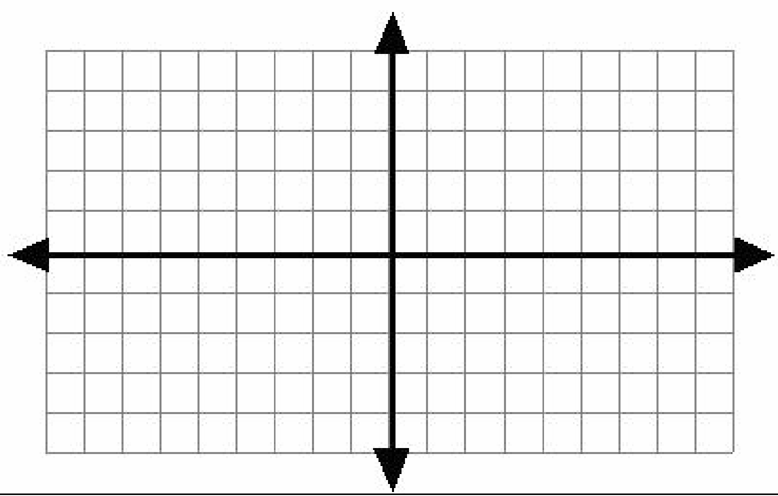
Frequency ____

Determine the amplitude and period of each equation.

	Amplitude	Period
a.) $y = \frac{1}{4}\sin(2x)$		
b.) $y = 3\cos\left(\frac{x}{2}\right)$		
c.) $y = 5\sin\left(\frac{\pi x}{4}\right)$		
d.) $y = \frac{1}{2}\cos(3x)$		

Example 1

Graph each function.

a. $y=\cos\frac{\theta}{4}$	
b. $y=-2\sin\theta$	
c. $y=5\cos 2\theta$	

Example 2

Write an equation of the sine function with amplitude 2 and period $\frac{\pi}{2}$.

Example 3

A pendulum swings a total distance of 0.30 meter. The center point is zero. It completes a cycle every 2 seconds.

a. Assuming that the pendulum is at the center point and heading right at t = 0, find an equation for the motion of the pendulum

b. Determine the distance from a center at 1 second, 1.5 seconds, 1.75 seconds, and 2 seconds.

Example 4

The Sears Building in Chicago sways back and forth at a vibration frequency of about 0.1 Hz. On average, it sways 6 inches from true center. Write an equation of the sine function that represents this behavior.

<u>Section 5 – Translations of Sine and Cosine Functions</u>

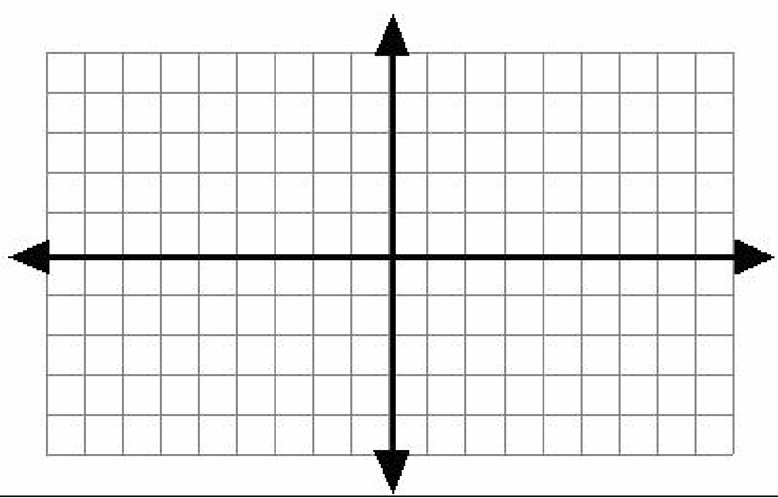
A horizontal translation or shift of a trigonometric function is called a **phase shift**.

Given the general form, $y = A\cos(k\theta - c) + h$ or $y = A\sin(k\theta - c) + h$:

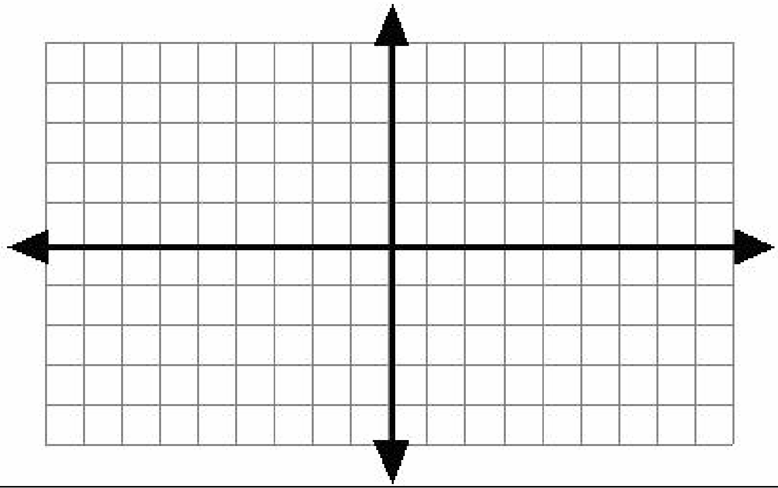
Amplitude __________ Period __________

Phase Shift __________ Vertical Shift __________

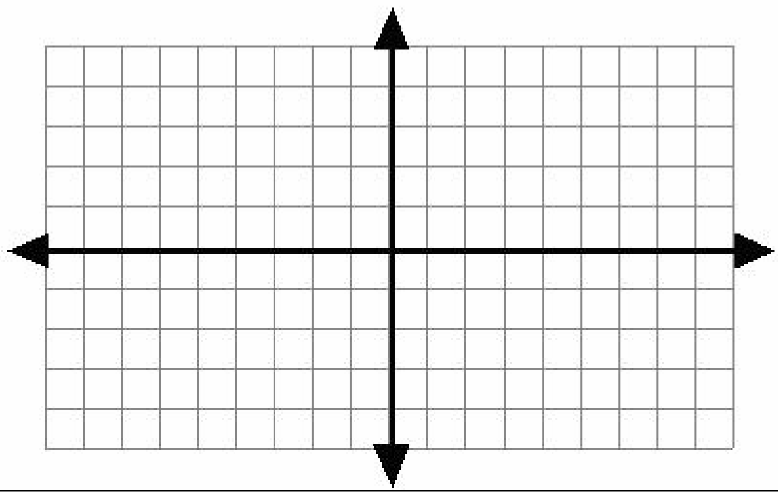
Examples: Graph each equation.

<table>
<tr><td>1. $y = \cos(\theta + \pi)$</td><td rowspan="5"></td></tr>
<tr><td>Amplitude</td></tr>
<tr><td>Period</td></tr>
<tr><td>Phase Shift</td></tr>
<tr><td>Vertical Shift</td></tr>
</table>

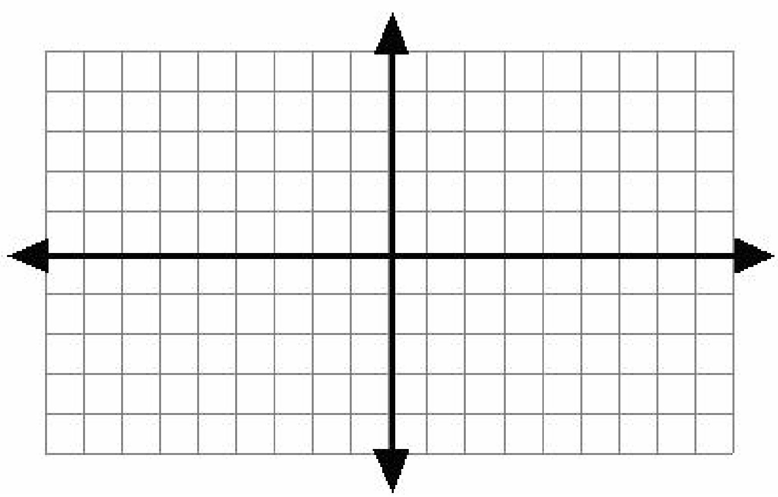
2. $y = \sin(4\theta - \pi)$
Amplitude
Period
Phase Shift
Vertical Shift

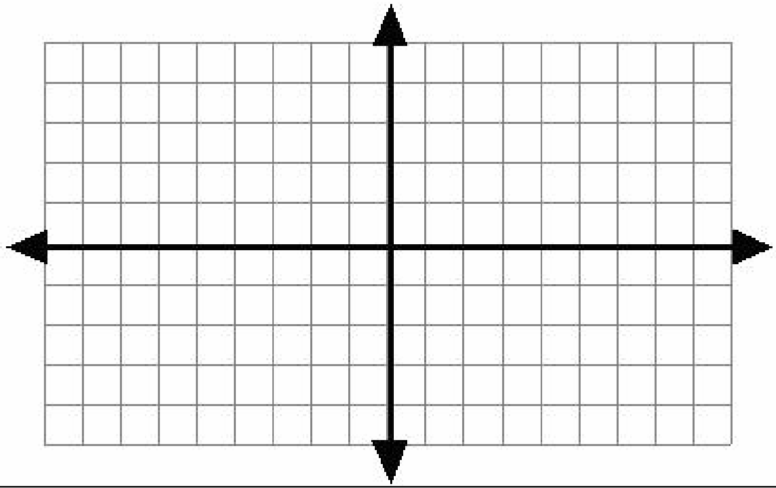


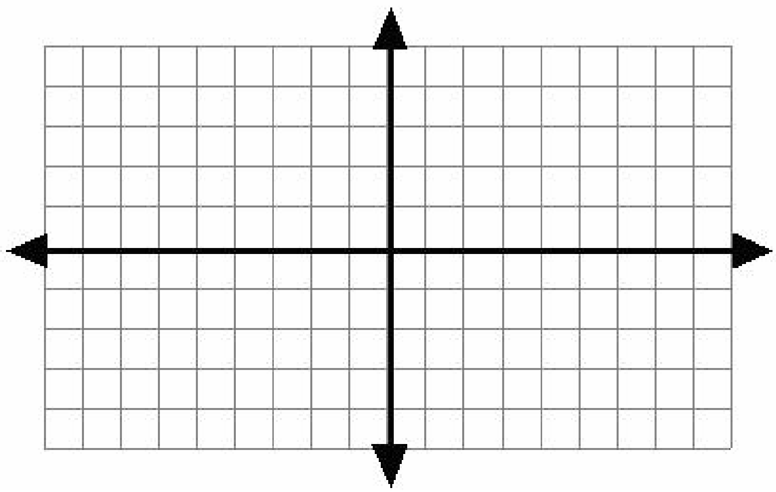
3. $y = 3\sin\theta + 2$
Amplitude
Period
Phase Shift
Vertical Shift



4. $y = 2\cos\left(\frac{\theta}{4} + \pi\right) - 1$
Amplitude
Period
Phase Shift
Vertical Shift



5. $y = \cos(\theta + \pi)$	
Amplitude	
Period	
Phase Shift	
Vertical Shift	

6. $y = 2\cos\left(\theta + \frac{\pi}{2}\right) - 3$	
Amplitude	
Period	
Phase Shift	
Vertical Shift	

Examples: Use the given information to write an equation.

7. Write an equation of a cosine function with amplitude 5, period 2π, phase shift $\frac{\pi}{8}$, and vertical shift -2.

8. Write an equation of a sine function with amplitude 7, period 3π, phase shift π, and vertical shift 7.

Section 6 – Modeling Real-World Data with Sinusoidal Functions

Example 1

An average seated adult breathes in and out every 4 seconds. The average minimum amount of air in the lungs is 0.08 liter, and the average maximum amount of air in the lungs is 0.82 liter. Suppose the lungs have a minimum amount of air at $t = 0$, where t is time in seconds.

a. Write a function that models the amount of air in the lungs.

b. Determine the amount of air in the lungs at 5.5 seconds.

Example 2

The tide in a coastal city peaks every 11.6 hours. The tide ranges from 3.9 meters to 3.3 meters. Suppose that the low tide is at $t = 0$, where t is time in hours.

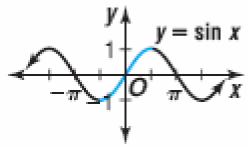
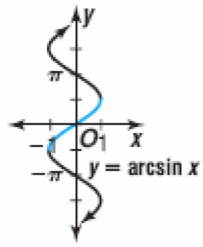
a. Write a function that models the height of the tide.

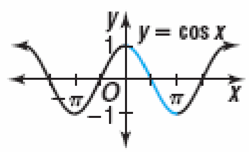
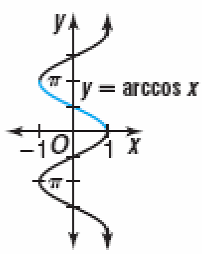
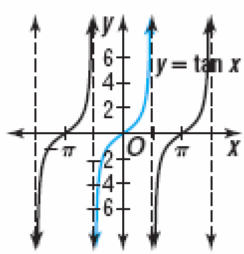
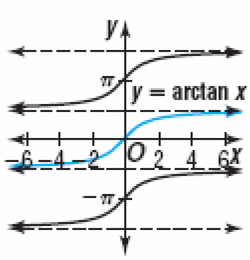
b. Determine the height of the tide at 6.2 hours.

Example 3

The average monthly temperatures for the city of Seattle, Washington, are given below. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
41°	44°	47°	50°	56°	61°	65°	66°	61°	54°	46°	42°

Relation	Ordered Pairs	Graph	Domain	Range
$y = \sin x$	$(x, \sin x)$		all real numbers	$-1 \leq y \leq 1$
$y = \arcsin x$	$(\sin x, x)$		$-1 \leq x \leq 1$	all real numbers

Relation	Ordered Pairs	Graph	Domain	Range
$y = \cos x$	$(x, \cos x)$		all real numbers	$-1 \leq y \leq 1$
$y = \arccos x$	$(\cos x, x)$		$-1 \leq x \leq 1$	all real numbers
$y = \tan x$	$(x, \tan x)$		all real numbers except $\frac{\pi}{2}n$, where n is an odd integer	all real numbers
$y = \arctan x$	$(\tan x, x)$		all real numbers	all real numbers except $\frac{\pi}{2}n$, where n is an odd integer

Because we want to focus on a part of the inverse that is a function, we determine a restricted domain for working with inverses of sine, cosine, and tangent.

Function	Domain	Range
$y = \text{Sin } x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
$y = \text{Arcsin } x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \text{Cos } x$	$0 \le x \le \pi$	$-1 \le y \le 1$
$y = \text{Arccos } x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \text{Tan } x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	all real numbers
$y = \text{Arctan}$	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Examples – Find each value.

1. $Arc\sin\left(\frac{1}{2}\right)$

2. $Cos^{-1}\frac{\sqrt{3}}{2}$

3. $Sin^{-1}0$

4. $Arc\sin\left(-\frac{1}{2}\right)$

5. $Sin^{-1}\left(\cos\frac{\pi}{2}\right)$

6. $Tan^{-1}\left(\sin\frac{\pi}{4}\right)$

7. $\cos\left(Arc\tan\sqrt{3} - Arc\sin\frac{\sqrt{3}}{2}\right)$

8. $\sin\left(Tan^{-1}1 - Sin^{-1}1\right)$

9. Determine whether $\text{Sin}^{-1}(\sin x) = x$ is true or false for all values of x. If false, give a counterexample.