Economic Studies 100

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Education, Pensions, and Demography

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Abstract

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This dissertation comprises three essays on demography and intergenerational transfers.

Essay 1 investigates the general equilibrium effects of a fertility shock under different intergenerational transfer schemes; where the workers provide for the young and the retired. The analysis concerns the closed economy where fertility fluctuations affect factor prices, besides intergenerational flows. How savings, factor prices, and growth evolve does not only differ quantitatively but could also differ qualitatively, depending on intergenerational transfers. How sensitive the results are to different assumptions about intergenerational transfers depends on how education investments affect future productivity.

Essay 2 compares alternative designs of an unfunded pension system. The objective is to maximize the expected *ex-ante* welfare under stochastic fertility. The model is a three-period CGE framework where the financing of education and effects on factor prices are accounted for. Factor prices depend on the degree of capital mobility. For low degrees of capital mobility, it is optimal to have a fixed benefit rate in the pension system. But for the small open economy, a fixed contribution rate is optimal if the education system has a fixed benefit rate. In this case individuals in the small open economy are unaffected by fertility fluctuations.

Essay 3 considers how to design an unfunded pension system with respect to longevity uncertainty. The aim is to find the optimal design of *behind the veil of ignorance*. The model is a computable overlapping generations model where the effects on labor supply and human capital are accounted for. Individuals decision to enter and exit the labor force is endogenous. Results show that it is important to be able to alter the retirement time in response to a longevity shock. When this is possible then there is no crucial difference between the different pension designs. If it is not possible to alter the retirement time then the fixed benefit rate is preferred. This means that pensions should not change when old age dependency changes but that taxes should adjust instead. In this case the design of the pension system will also have an impact on the labor supply.

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Acknowledgements

The most challenging, terrifying, and enjoyable part during this work was when I started exploring fertility matters hands on. Once discovered it is hard to leave fertility matters aside, and there seem to be almost endless possibilities for rediscoveries. I managed to rediscover it twice and it must be admitted, that the meaning of intergenerational transfers and dependency would be hard to grasp without these discoveries.

Although this theses abstracts from altruism and familial transfers, this should not be viewed as a personal disbelief in these issues. On the contrary, I regard these so highly, that only injustice would be done if squeezing these matters into a mathematical formula. I can think of no better than my parents, though closely rivaled by my grandparents, to teach me the true meaning of intergenerational altruism and familial support. That perfect altruism can be an intragenerational matter I know with certainty through my brother. Here, I also want to thank the Swedish government for truly helping me realize how important familial transfers are.

Trying to unravel the series of events that made me embark on this journey is off course an impossible mission, due to Newton's third law. With this being said, however, rest assure that were it not for Yngve Andersson, you would not be reading this. The time he devoted to a mere undergraduate student was exceptional; it seemed as if he were interested in what was being said. As time continued, he was always ready to assist.

Well onboard I had the great fortune of getting Nils Gottfries and Thomas Lindh as supervisors. Both have insisted on clarity, which thankfully removed all risk of finding "GAYG, PAYG, and Demography" instead of the current theses title. If you find something clear and concise, they are to be thanked; what you find blurry and inconsistent is my shortcoming. All that I know about revising papers I owe to Nils, and considering the amount of revisions made I owe him plenty. Thomas always made things look less burdensome and sometimes, I almost got the impression that a smile was affordable. Above all, both of them showed genuine concern about more than just the theses.

My fellows at the department - thanks for the excellent time - without you this would not have lasted. Especially thanks to my classmates: Martin Å, a grand vocalist whose skills will not be forgotten among Serbian basketball fans, Jonas, one victory is all I need (and got), Martin S, for turning much of the first year into a stocker thriller. My office companion Pär, I did my best to slow you down, but off course this was an impossible task. Guidance from fellows above, I received from Lars and Nikolay. Lars has helped me with many unimaginable things over these years; including the layout package for this theses. Nikolay, the first among us to deal with fertility, has guided me through these matters in an excellent way. Luca, thanks for giving us northeners some of the southern flair. Also let us not forget the administrative staff who, at times, has done more than duty requires.

I also want to thank the people at the Department of Demography at U.C. Berkeley for an excellent time during my visit. What I know about demography I owe to them and especially to Ronald Lee. The essays have also been improved by suggestions from: Annika Sundén, Conny Olovsson, Panu Poutvaara, Andreas Wagener, Young Jun Chun, and Hans Fehr. I am further grateful for financial support from: Swedish Social Insurance Agency, Institute for Future Studies, Sweden-America Foundation, Swedish Council for Working Life and Social Research,

Needless to say, the one truly responsible for all is *Malin*. Without you I would have accomplished nothing, or perhaps even worse, a theses in business. Nikola, Nadja, and Vera make this theses almost redundant. From them I know with certainty that a boom creates an immeasurable gain for the parents, and let us just hope that we can pass it on downwards.

Jovan Žamac
Uppsala, December 2006

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Introduction

This theses comprises three self-contained essays dealing with how the economy is affected by changes to the age distribution. In particular, I investigate the impact of demographic changes under different adjustment mechanisms for intergenerational transfers. The first two essays investigate the impact of fertility disturbances while the third analyzes the effect of changes to the old age mortality. This summary chapter aims to give a basic understanding of the issue at hand, and to explain how these three essays are interrelated, and how they are related to previous literature.

Demographic concern in retrospect

Since 1950 the human population has increased from 2.5 billion to 6.5 billion in 2005, and is projected to increase to 9.1 billion by 2050 (United Nations, Population Division 2005). Considering that sharing is not mankind's defining attribute, adding 34 million persons annually could pose a difficulty. If the available resources do not expand in the same manner, this population increase must result in less resources per capita. This basic mechanism of how demography affects the economy is an old insight. Tertullian eloquently stated:

> What most frequently meets our view (and occasions complaint), is our teeming population: our numbers are burdensome to the world, which can hardly supply us from its natural elements; our wants grow more and more keen, and our complaints more bitter in all mouths, whilst Nature fails in affording us her usual sustenance.
>
> *Tertullian, De Anima*

Though eloquently, this was stated almost two millennia ago when the world population was about 0.19 billion (Kremer 1993). From the time of Tertullian around 200 A.D. to Malthus's *An Essay on the Principle of Population* (1798), world population managed to more than quadruple, indicating that the resources managed to expand. Malthus realized this and thus his principle did not have a fixed resource constraint, but instead the resources where growing linearly. However, since he also stated that population if unchecked grows exponentially it implies that a population might temporarily outgrow its resources. When this happens the economy affects demography by reducing the population size back to subsidence level through misery and vice. War, famine and disease, i.e. misery, would increase mortality and thus reduce the population size. This is what Malthus called the positive check. Fertility could also decrease during economic recessions by vices such as contraceptives and prostitution; the latter being a result of postponed marriage.[1] This is what Malthus called the preventive check. Population increase is thus restrained by the resource constraint either trough increased mortality or decreased fertility, i.e. population increase is either positively checked or preventively checked.

Besides population growth being bound by the slow expansion of the resource constraint, the theory also predicted a zero growth in GDP per capita. Any technological progress that increases the available resources would not increase the standard of living but would instead result in a population increase.

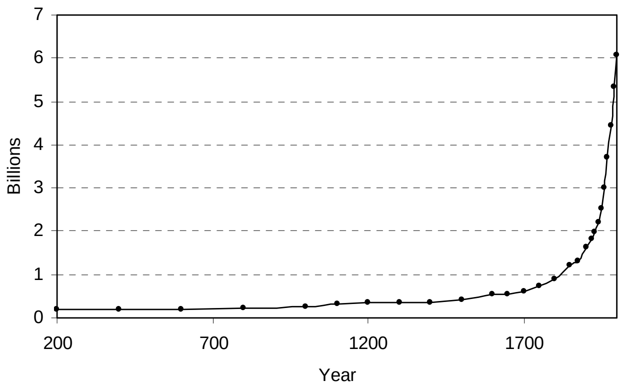
The interdependence between demography and the economy was central for the early economists. The Malthusian negative view about expanding population was shared by many, but there was an opposing view also.[2] Adam Smith, for instance, stated that large populations cause efficiency gains due to labor division, specialization; which ultimately enhances prosperity.

At that time, Malthus seems to have made a correct assessment of past events, at least for Europe. Population growth was modest and restrained by the resource expansion, and little or no improvements in standards of living was made over time. (e.g. Livi-Bacci 1997, Galor and Weil 1999, Lee 2003). The theory, however, did not prove successful in explaining future events to come. Around 1800 the first countries

[1] In the second edition from 1803, and following editions, he added that postponed marriage does not necessarily lead to vice if moral restraint was implemented. This distinction is however not crucial since both vice and moral restraint affect fertility. For Malthus however this was not a trivial distinction since he viewed moral restraint as the least evil, although he regarded it as unlikely.

[2] Among others David Ricardo shared the Malthusian view, and stated in *The principles of Political Economy* (1817):"... for the land being limited in quantity, and differing in quality, with every increased portion of capital employed on it there will be a decreased rate of production, whilst the power of population continues always the same." I'm indebted to Livi-Bacci (1997) for this citation.

Figure 1: World population 200 to 2000 A.D..



Source: Kremer (1993) and United Nations, Population Division (2005).

begun the demographic transition. This transition was unprecedented and altered completely the world demography. It is the demographic transition that is the cause of the rapid population increase during the last two centuries.

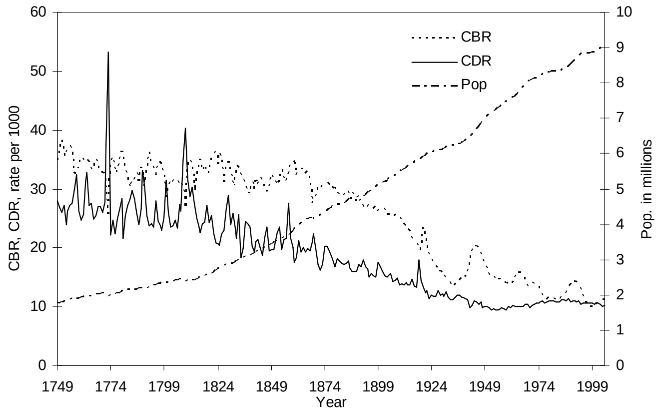
The demographic transition

Figure 1, shows how world population has evolved from 200 to 2000. During a very long period population growth was modest. This pre-transition stage, lasted until around 1800, and was characterized by high mortality and high fertility, which weakly balanced each other. This made the population grow at a slow rate.

The transition started in northwest Europe with a decline in mortality. Several factors contributed to the mortality decline: improved nutrition, better hygiene, and reductions in infectious diseases, among others. Irrespective of the cause there is no doubt that mortality decreased. The mortality decline was followed by decreasing fertility, but not immediately. The Swedish experience, presented in figure 2, illustrates how the time gap between mortality reduction and fertility reduction caused a period of high population growth.

About 1810 mortality started to decline in Sweden while the decline in fertility did not start until about 60 years later. During this period population size increased rapidly. When fertility started to decrease the gap between fertility and mortality remained, because fertility did not fall as much as mortality. Therefore population growth continued until recently. The final stage of the demographic transition occurs when fertility approaches the low level at which mortality has stabilized. This is a

Figure 2: Crude birth rate, and crude death rate, per 1000 inhabitants on the left axis, and population size in millions on the right axis, for Sweden 1749-2005.



Source: Statistics Sweden.

recent event with low population growth, because mortality and fertility once again weakly balance each other. The crucial difference between the pre-transition and the post-transition is that mortality and fertility rates are low instead of high. As we will see, this will affect the age distribution of the population.

The demographic transition started at different times around the world, with Sweden being among the first. Thus even if Sweden's mortality and fertility rates once again weakly balance each other, the gap remains for most other countries. Because of this, world population is still growing at a high rate, but this should only last until all countries have reached the final stage of the demographic transition. According to Lee (2003), this is projected to occur around 2100.

The countries that have made the transition have managed to increase their population size without being bound by the Malthusian limit. This does not imply that the period has been without misery; numerous wars are proofs of the contrary. It simply implies that the resources have been able to increase at a pace that Malthus did not envision.

It seems impossible to say if future world population growth will be accompanied by the necessary resource expansion. This is debated between neo-Malthusians (e.g Ehrlich 1968, Ehrlich and Lui 1997) and those who believe that the world can provide a practically limitless abundance of natural resources, i.e. cornucopians (e.g. Simon 1996).[3] Instead of choosing side in this controversy I will argue that

[3]Cornucopia refers to the *horn of plenty* from Greek mythology which magically provided its owners with endless food and drink.

irrespective of which view we take, we should expect countries to reach what I described as the final stage of the demographic transition.

The countries that have reached, or are closest to reach, the final stage of the transition seem to find a new equilibrium without being bound by the Malthusian limit during the transition. Judging from past experience, it thus seems that even if the cornucopians are right that the population expansion will eventually come close to a complete halt, with low mortality and low fertility. In this case the halt will however not depend on the resource constraint.

If the neo-Malthusians are right, the same halt is awaiting, but sooner and more brutal. The objection might be raised that the Malthusian positive check does not decrease fertility but instead raises mortality, and thus implies a reversion back to the pre-transition stage. This might occur in the short run, but, in the long run I view it as unlikely. Given that the knowledge exists how to reduce mortality, it does not seem plausible that humanity would choose the higher level. It is possible to reduce mortality without increasing population size, if fertility is reduced first, or at the same time. This would not follow the classical demographic transition but it would still not be an unprecedented scenario. In the past, at least, France and the U.S. have reached post-transition levels of low fertility and low mortality, with fertility moving first (Chesnais 1997).

After the transition, population growth is once again modest. This seems to remove the classical concern about overpopulation, but new issues arise. Low fertility and low mortality creates a population with an old age distribution, and long lives. This is a radical change from the pre-transition state, with a young age distribution and short lives. The focus on the age distribution and its effects on the economy is fairly new, at least if compared to the overpopulation concerns.[4] To understand why the age distribution is important one needs to be familiar with the economic life-cycle, which I turn to next.

[4] As pointed out by Feen (1996), mechanisms according to the Malthusian principle can be found in the early Babylonian epic of Athrasis (1700 B.C.) which states that barrenness, stillbirth, and natural disasters, were all part of the cosmic order to balance humankind's numbers with the land's.

The economic life-cycle

I think I may fairly make two postulata: First, that consumption goods cannot be produced without labor. Secondly, that labor input is zero at certain ages. These two laws ever since we have had any knowledge of mankind, appear to have been fixed laws of nature.[5]

Given that the above postulates are granted, and that Malthus first postulate is not refuted, it is inevitable that we at certain age consume more than our labor product. Let us define these occurrences as periods of dependency, and let us label it's opposite as working periods.

At birth we enter the first period of dependency, which lasts until around the age of 20. This duration seems to be quite stable for different societies such as hunter-gatherer, agriculture and modern societies (Lee 2000, Lee, Lee, and Mason 2006). Much of what is consumed during this period enhances future productivity, i.e. future human capital. As human capital is accumulated it enables us to leave the initial dependency phase and to continue to the working period, where more is produced than consumed.

In modern societies there is also a second period of dependency in the form of retirement. We usually say that this period starts at the age 65 and lasts until death, although there is great variation regarding the starting age over countries and time.[6] There are thus two periods of dependency and between these two periods there is a working period. Henceforth, I will refer to the first and the second dependency period as childhood dependency and old age dependency, respectively.

The economic life-cycle implies that several factors in the economy are affected by the dependency ratio. What has received much attention is how output per capita varies with the age distribution. Output per capita is affected since the population comprises a fraction that work while the remaining are dependent. Thus even if the amount of capital, land, or for that matter anything else that might be used in the production of output, is abundant, the available resources are constrained by the number of individuals that are in their working period. This means that output per capita depends on the dependency ratio.

[5]As probably noticed, there is a striking resemblance between this paragraph and Malthus writing from 1798: "I think I may fairly make two postulata: First, that food is necessary to the existence of man. Secondly, that the passion between the sexes is necessary and will remain nearly in its present state."

[6]It can be mentioned that this last period of dependency does not seem to have existed for hunter-gatherer societies (Kaplan 1994). In this case individuals continued to produce more than they consumed from the age of 20 something until death.

For the individual, the problem arises how to transfer some of the output from the working period to the dependency periods. One way to accomplish this is through intergenerational transfers. This means that the current working population transfers a part of their labor income to the current dependent population. The total amount that the working population contributes must equal the total amount that the dependent population receives. This makes these transfers sensitive to changes in the dependency ratio, i.e. the ratio between the dependent population and the working population.

An alternative to intergenerational transfers is to reallocate physical assets. This would imply negative savings during periods of dependency and positive savings during the working period. The aggregate saving in the economy is thus affected by the dependency ratio. If the economy is closed, the dependency ratio will affect the wage and the interest rate, and if the economy is open it will affect the current account.

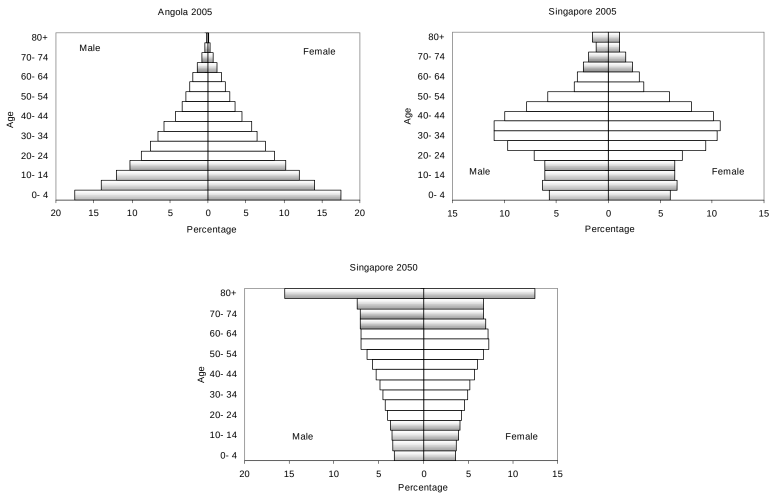
Age distribution

Consider the age distributions presented in figure 3. The two upper sub-figures are the actual age distributions by sex for Angola and Singapore in 2005, the lower one is the projected distribution for Singapore in 2050. These specific distributions capture the essence of the demographic transition. The Angola 2005 figure represents the pre-transition stage, Singapore 2005 is the intermediate stage, while Singapore 2050 represents the post-transition distribution. The white bars in the figure represent the population of working age.

The pre-transition distribution, illustrated by Angola 2005, has a high dependency ratio but this is mainly caused by the high child dependency ratio. Such high dependency ratios imply that a relatively small amount is invested in each child's education. Also, the high mortality implies that far from all children survive to adult age, which further raises the dependency ratio.

The demographic transition starts by reducing mortality at younger ages which initially increases the number of children. Eventually, however, more children survive to adult age. As time pases, fertility is reduced, leading to fewer children. This creates an age distribution similar to Singapore 2005, where the dependency ratio is small. The large work force in relation to the dependent population implies that resources expand more then the total population. This situation is referred to as the demographic dividend. According to Bloom and Williamson (1998) this effect is a major explanation for the rapid growth in the East Asian economies during

Figure 3: Age distribution by sex for Angola 2005, Singapore 2005 and Singapore 2050 (projection). White bars mark the ages 20 to 65.



Source: U.S. Census Bureau, International Data Base.

the last four decades of the 20^{th} century. The rapid growth of the work force can however have some disadvantages in the form of less physical and human capital per worker. Essay 1 identifies under what circumstances a large work force will enhance the per capita growth. There I show that the design of the education system and how education affects future human capital is crucial for the growth in per capita output.

This demographic dividend is transitory. The large workforce eventually enters the retirement period and is replaced by a smaller cohort, due to low fertility. The resulting age distribution is represented by projections for Singapore 2050. Here the dependency ratio is higher compared to Singapore 2005, but total dependency does not differ much compared to Angola 2005. However, in this case it is mainly the old age dependency that contributes to the total dependency ratio.

The developed countries have already high proportions of elderly, but this proportion is expected to increase even further until the middle of the century. One extreme projection is for Italy that in 2000 had an old age dependency ratio around 26, while the projection for 2050 is 68. Other high projections are for Spain at 74 and for Japan at 71 (United Nations, Population Division 2001). The increase in the old age dependency ratio has received considerable attention. In particular there has been great anxiety regarding the financial viability of pension systems and many studies investigate how to reform these.

This theses investigates how changes to the age distribution affect the economy under different intergenerational transfer schemes. Below I describe how different intergenerational schemes react to changes in the dependency ratios.

Intergenerational transfers

There are both public and private channels through which intergenerational transfers flow from the active to the dependent population.[7] Two major public channels that support the old are, the unfunded pension transfers and the medicare system. In this theses, I focus on the pension system although similar reasoning and methodology could be applied for the medicare system, or any other transfer that the old receive. The public pension system is an explicit intergenerational contract, since it has a stated benefit formula regulated by law. This formula specifies what the benefits/contributions will be under different circumstances. Specifically, it determines how the pension system reacts to changes in the old age dependency. Considering changes to old age dependency, two possible extreme responses are either to change the contributions per worker and keep the benefit per retired fixed, or vice versa. From the pension literature it is well known that these two different responses will have different distributional effects (e.g. Hassler and Lindbeck 1997, Thøgersen 1998, Lindbeck 2000, Bohn 2001, Wagener 2003).

As noted by Bommier, Lee, Miller, and Zuber (2004), education is usually considered to be an private investment in human capital and is seldom viewed as an intergenerational transfer. However, they argue against this common characterization and note that education was the largest public transfer program in the U.S. in 2000; education expenses amounted to 4.5 percent of GNP, followed by OASI (pensions and survivors benefits) at 3.7 percent, and Medicare at 2.1 percent. Moreover, it is clear from Lee, Lee, and Mason (2006) that spending on education is mostly publicly financed in the U.S. and that it is mainly consumed by age groups 5 to 25. This means that the lion's share of education is provided by intergenerational transfers rather than by asset reallocation over the life-cycle. Even though there is no explicitly stated intergenerational formula for the education system, it still must adjust when the child dependency changes. The extreme responses are either to keep the education benefit per child fixed and to alter the contribution per worker, or vice versa. How the education system responds to changes to child dependency ratio has received little attention. The lack of an explicitly stated adjustment mechanism might explain why this is so.

[7] See Lee, Lee, and Mason (2006) for empirical estimates of both public and private flows.

Essay 1, investigates three different pension schemes, the fixed benefit rate (FB), the fixed contribution rate (FC), and the fixed replacement rate (FR). An FB scheme implies that the pension benefit is always a fixed fraction of the current labor income. The FR scheme implies that the pension benefit is a fixed fraction of *previous* income instead of current income, i.e. the income from one's own active life. The FC scheme has a fixed tax rate for financing pensions, implying that pension benefits fluctuate with the old age dependency ratio. For the education system two different schemes are analyzes, the FB scheme and the FC scheme. An FB education scheme implies that the children are always promised a certain amount of current income, independent of how many they are in relation to the working population. With an FC education scheme the workers are promised to pay a fixed fraction of their income to finance education, irrespective of how many the children are.

Essay 2, allows for convex combinations between the FB and the FC schemes, implying that both the contributions and the benefits can adjust when the dependency ratio changes. How much the contributions and how much the benefits adjust depends on the specific convex combination, i.e. design. Essay 3, also allows for convex pension schemes but it does not investigate different education schemes.

Essay 1

Demographic shock under different intergenerational transfer schemes, this essay analyzes the general equilibrium effects of a demographic shock. Typically, simulation methods are required to analyze the effects of a changing age distribution. Most of these studies, however, do not investigate different types of intergenerational transfer systems. At most, some studies investigate different types of pension systems but with respect to intergenerational transfers to the young, they do not employ the same systematic treatment.[8]

This essay investigates how the closed economy is affected by a fertility shock, illustrated by a baby boom, under different intergenerational transfer schemes. The novelty of this essay consists of explicitly treating the education system as an intergenerational transfer system that can respond to a demographic change in different ways. This is of interest since the results of many simulation models are usually based on one type of education scheme.

A demographic shock which creates a gain in the financing of the education system will create a burden for the pension system (though not in the same period).

[8] See, for instance, Auerbach and Kotlikoff (1987) Docquier and Michel (1999), Fougère and Mérette (1999), Pecchenino and Utendorf (1999), Bouzahzah, Croix, and Docquier (2002) and Pecchenino and Pollard (2002).

Moreover, in both cases, this is a distributional matter between the same two generations; the parent generation and the shock generation. Which generation will be burdened and which will receive the gain depends on the education scheme and the pension scheme. There will also be an effect on capital intensity which affects factor prices. Thus, there are three main effects. For a baby boom shock, there is first the cost of raising and educating this generation when young. This will, in turn, affect the baby boom's human capital when it enters into the work force. When entering into the work force, it will also suffer from capital dilution, which suppresses its wage and raises the interest rate. However, at the same time, there will be a positive effect since old age dependency is reduced.

I find that the transition path for macro variables such as savings, wages, and interest rate can be highly dependent on the underlying assumption regarding education scheme. The trajectory for these variables after a fertility disturbance can differ qualitatively between the different educational schemes. I also find that the growth rate in per capita output can be negative in periods when the work force as a share of the total population increases. If this occurs or not depends on how education affects future human capital. Numerical studies aiming at explaining how the economy evolves after demographic changes should thus carefully model the adjustment mechanism for the education system and how it affects future human capital.

Essay 2

Pension design when fertility fluctuates: the role of education and capital mobility, is an extension of essay 1 and focuses, as the title suggests, on pension design. As mentioned, the increase in old age dependency ratio has motivated a large literature on how to reform the unfunded pension systems. One way to deal with the demographic impact on the pension system is to switch to a fully funded system, as for instance Chile has done. It is by now, however, widely recognized that a complete switch is very costly for countries that already have unfunded pension systems in place. Such a switch requires that the currently active workers either make double contributions, paying for those currently retired and also setting aside assets for their own retirement, or that the benefits to the currently retired are abolished. Moreover, there are studies showing beneficial risk-sharing and diversifications motives for introducing an unfunded pension systems (e.g. Enders and Lapan 1982, Merton 1983, Hauenschild 1999). The focus of the debate is thus not on how to abolish unfunded pension schemes but on how to design unfunded systems. Numerous studies have investigated these issues (e.g. Smith 1982, Hassler and Lindbeck 1997, Thøgersen 1998, Lindbeck 2000, Bohn 2001, Wagener 2003).

This theses contributes to this debate, and investigates what unfunded pension scheme is preferred considering changes of the old age dependency ratio. Changes of the old age dependency ratios have mainly two sources. Fertility changes, including the baby boom after the second world war, have been one major driving force. As mentioned previously these changes will affect the child dependency ratio also, and thus it is not possible to view the pension system in isolation from the education system. Further, since capital is not abundant in the real world, changes to the work force will affect the capital intensity in the economy. This is true except if the economy is small and completely open. For this reason it is important to account for the effects on factor prices.

The aim is to find the preferred pension *ex ante*, i.e. before knowing if there will be a positive or a negative shock to fertility. I allow for varying degrees of openness, which implies that the effect on factor prices varies. I also allow for convex combinations for the intergenerational transfers. The model used is thus similar to the one used in essay 1 except for the degree of openness and for the convex combinations. Essay 2 also extends the analysis by investigating the market outcome that would arise if children could enter into efficient contracts with adults, in line with Becker and Murphy (1988), Rangel (2003), and Boldrin and Montes (2005).

The main result is that the preferred pension design crucially depends on the degree of capital mobility. With limited capital mobility every retiree should be guaranteed a fixed fraction of output. After capital mobility has reached a threshold level, the preferred pension design gradually moves towards fixing the wage tax rate used to finance the pension system. A completely fixed tax rate is only preferred for the small open economy.

In the case of limited capital mobility, a large working generation creates a gain in the pension system, by decreasing the old age dependency ratio, and depresses the wage due to capital dilution. Having a fixed share of output that is given to each retiree implies that the tax rate can be lowered. The lower tax rate compensates the large cohort for the lower wage by allocating the gain in the pension system to them.

For a small open economy, factor prices are unaffected by fertility changes. In this case it is optimal to have a fixed tax rate to finance the pension system if the education system is such that each pupil is guaranteed a fixed fraction of output. With these designs, the individuals in the small open economy are unaffected by fertility changes. What happens in this case is that the preceding cohort that invests more heavily to educate the large cohort are compensated for this by higher pensions.

Essay 3

Pension design and longevity, is similar in spirit as essay 2, but investigates what happens when old age mortality changes. In the U.S., life expectancy has increased by about 30 years from 48 in 1900 to 77 years today. In the beginning, this increase was caused mainly by mortality improvements at pre-retirement ages. Now, however, 70 percent of the gain in life expectancy comes from mortality reductions after age 65 (e.g. Lee and Tuljapurkar 1997). This implies that unless the retirement age is raised substantially, the old age dependency will increase.

As showed by Gruber and Wise (2005), retirement choice depends on how the pension system is designed. Given that life expectancy is steadily increasing, it is infeasible to have a fixed retirement age. Thus, the new Swedish pension system has no fixed retirement age; the hope is that longevity increases will lead to an increase of the working period. If the working period expands so that the relation between the working period and the retirement period is unaltered, then there will be no increase in the old age dependency ratio. But adjustment of the working period is based on life expectancy which may differ from the actual life length. The question is then how to design the pension system considering that there can be unexpected changes to longevity.

Essay 3 deals with this issue. It investigates which pension system that is preferred when there are shocks to longevity. It tries to find the preferred pension system *ex ante*, i.e. before knowing the longevity shock.

An important issue arises: when is information about longevity shocks revealed? Suppose, for instance, that life-expectancy at birth is equal for all generations and constant over time, at 75 years. It is highly unlikely that a shock that changes the life-expectancy for a generation that has reached the age 74 will affect retirement decision of this generations. But if the life-expectancy for a generation that is of age 50 changes, then it is more likely that they will alter their retirement decision. We can view this difference in terms of prior notice about the shock or not. If the shock affects a generation after it has retired it could be said that it has not been given any prior notice about the shock. If the longevity shock is revealed prior to retirement then it could be said that they have received prior notice, or head notice.

One finding is that prior notice about the shock is very important. If the generations are able to adjust to the shock then there is little difference between the pension schemes. In this case, the pension design will have negligible impact on the retirement decision. The resulting *ex ante* welfare difference is also negligible and the difference in labor distortions induced by the different pension designs is quite small.

If the old cannot adjust their labor supply then the preferred pension design is to not reduce pensions. This design implies that the generation that cannot adjust is sheltered from longevity shocks and that future generations that are more able to adjust are involved in the risk-sharing. The retirement choice will also differ between the designs. Keeping the pensions unaltered leads to a variable tax rate which in turn affects the entry into and the exit from the labor force.

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Essay 1

Demographic Shock Under Different Intergenerational Transfer Schemes

1 Introduction

In most OECD countries, the old age dependency ratio is projected to increase dramatically during the first half of this century. This has lead to considerable anxiety regarding the financial viability of the social security programs. Less attention has been devoted to the fact that fertility has been the principal source of the changing demographic structure;[1] implying that changes to the old age dependency ratio are predated by changes in the young age dependency ratio. Just as old age dependency is crucial for social security financing, young age dependency is crucial for the financing of education. When analyzing the distributional effects between generations, it is necessary to account for both young age dependency and old age dependency. It must also be considered which types of intergenerational transfers are in place, since it is well known from the pension literature that different schemes have very different distributional properties.

The pension literature identifies the main distinctions between different intergenerational transfer schemes, and investigates if one type of system dominates the other. The schemes respond differently to demographic and productivity disturbances.[2] Unfortunately, the pension literature does not investigate the general equilibrium effects of a demographic change. Factor prices are treated exogenously

[1] See OECD (1988) on the relative importance among fertility, mortality, and migration.

[2] See for instance Hassler and Lindbeck (1997), Thøgersen (1998), Lindbeck (2000), and Wagener (2003).

and changes in young age dependency are seldom accounted for. The exclusion of young age dependency may be misleading and the *ceteris paribus* assumption about factor prices is also most likely incorrect, unless the Feldstein-Horioka puzzle disappears altogether. To account for these factors, it seems necessary to use a general equilibrium framework.

Typically, simulation methods are required to analyze demographic effects in a general equilibrium framework. The work by Auerbach and Kotlikoff (1987), which created a tool for investigating the macroeconomic effects of demographic changes, was pioneering in this direction. Most of these studies, however, do not investigate different types of intergenerational transfer systems. At most, some studies investigate different types of pension systems but with respect to intergenerational transfers to the young, they do not employ the same systematic treatment.[3]

This paper investigates how the effects from a fertility shock, illustrated by a baby boom, vary with different types of intergenerational transfer systems. This is done from a theoretical perspective by using a three-period overlapping generations (OLG) model. I focus on the intergenerational transfer schemes, as in the pension literature, while incorporating both young age dependency and the effect on factor prices, as in the computable general equilibrium literature. The novelty of this paper consists of explicitly treating young age dependency as an intergenerational transfer system that can respond to a demographic change in different ways.

The aim of this paper is twofold. One part is to investigate how savings, factor prices, and growth rate in per capita output, evolve after a fertility shock under the different intergenerational transfer schemes. This is of interest since the results of many simulation models are usually based on one type of transfers to the young. The second par is to determine the distributional properties for the various transfer schemes. Will the results regarding the preferred pension scheme hold when including young age dependency?

A demographic shock which creates a gain in the financing of young age dependency will create a burden in the old age dependency system (though not in the same period). Moreover, in both cases, this is a distributional matter between the same two generations; the parent generation and the shock generation. Which generation will be burdened and which will receive the gain depends on the type of transfer systems. There will also be an effect on capital intensity which affects factor prices.

[3]See, for instance, Auerbach and Kotlikoff (1985) and Blomquist and Wijkander (1994) for models with no human capital. Some studies that include human capital are Docquier and Michel (1999), Fougère and Mérette (1999), Pecchenino and Utendorf (1999), Bouzahzah, Croix, and Docquier (2002) and Pecchenino and Pollard (2002). Bohn (2001) studies analytically the general equilibrium effects under different pension schemes but he does not consider different adjustment possibilities for transfers to the young.

Thus, there are three main effects. For a baby boom shock, there is first the cost of raising and educating this generation when young. This will, in turn, affect the baby boom's human capital when it enters into the work force. When entering into the work force, it will also suffer from capital dilution, which suppresses its wage and raises the interest rate. However, at the same time, there will be a positive effect since old age dependency is reduced.

The impact on human capital from reduced education is determined by the production function for human capital. As noted by Bouzahzah, Croix, and Docquier (2002), there is no consensus on how to view the production of human capital. In computable models, this creates difficulties when it comes to calibration. For this reason, the sensitivity to the calibration of human capital is evaluated.

I find that the transition path for macro variables can be highly dependent on the underlying assumption regarding transfers to the young. The trajectory for these variables after a fertility disturbance can differ qualitatively between the different educational schemes. I also find that the growth rate in per capita output can be negative in periods when the work force as a share of the total population increases. If this occurs or not depends on the returns to scale in the production of human capital. With small returns to scale, however, the resulting outcome for macro variables is not that sensitive to different educational schemes. Numerical studies aiming at explaining how the economy evolves after demographic changes should thus carefully model the adjustment mechanism for the education system, especially if high returns to scale are used.

Regarding the relative outcome for different generations, this is highly dependent on the transfer schemes, even with small returns to scale. Using an *ax ante* approach to compare the different schemes, based on an utilitarian social welfare function, I find that the transfers that flow to the old should not be adjusted to achieve a fixed tax rate. This is in contrast to what Thøgersen (1998) finds, but supports what Wagener (2003) finds with an *ex post* approach. The reason for this is that a variable tax scheme will counter the factor price effect. That the factor price effect dominates the old age dependency effect has previously also been shown by Blomquist and Wijkander (1994) and Bohn (2001). This analysis shows that this result holds, even when young age dependency is included.

The remainder of this paper is organized as follows. In section 2, the different schemes for intergenerational transfers are presented. Section 3 presents the general equilibrium model. In section 4, the model is calibrated and the steady state results are presented. Section 5 presents the results while section 6 concludes the paper.

2 Intergenerational transfers

Intergenerational transfers go in many directions but, on average, in modern societies the net receivers consist of the young and the old (Lee, Lee, and Mason 2006). There are both formal and informal channels through which these intergenerational transfers flow from the active to the inactive population. For the elderly, the formal channels are dominant in the developed world, so it is quite natural to view this as a separate system, since it is more or less an explicit intergenerational contract between the active population and the elderly. The transfers to the children have strong formal channels such as mandatory education, but even the informal channels can be viewed as an implicit intergenerational transfer due to customs and traditions. It seems reasonable to view these transfers as two separate systems. Similar to other applications which suffer from a time inconsistency problem, it is desirable that these systems or institutions are governed by laws which seldom change (Kotlikoff, Persson, and Svensson 1988). This paper investigates different types of laws that can govern these transfers.

In developed countries, a large portion of the transfers to the old consist of unfunded pensions. For this reason, I will restrict the analysis to the pension system, and investigate different pension schemes. The same principle would apply if one were to choose to include other transfers to the elderly, for instance medicare.

Since a large portion of the intergenerational transfers to the young consists of education, it is natural to refer to this system as the education system. Certainly, this is only one part of the transfers that go to the young. But the analysis should also be applicable to overall transfers. Many of the goods used when raising children will affect their future human capital, and these goods are seldom non-rival.

What is characteristic of the pension system is that when entering the system (i.e. when entering the active labor age), individuals start by paying into the system and then later, when retired, they will receive. The education system is the opposite from the pension system, in the sense that when entering the education system (i.e. at birth) individuals start by receiving and then later when joining the work force, they will pay to the system. This, seemingly trivial, difference is crucial for the analysis of a demographic shock.

2.1 Modelling the transfers

The simplest way of capturing both the education and the pension system is to use a three-period OLG model. The OLG model consists of one period when young, one period when working, and one period when retired. The young receive contributions from the working population via the education system, and the retired receive contributions from the working population via the pension system. For the systems to be pure intergenerational transfers, it is necessary that the budgets are balanced in each period. Thus, assuming a period-by-period balanced budget for each system separately makes it possible to state the transfers in period t as:[4]

$$b_{E,t}N_t = d_{E,t}N_{t-1}, \tag{1}$$

$$b_{P,t}N_{t-2} = d_{P,t}N_{t-1}, \tag{2}$$

where $b_{E,t}$ denotes the per child benefit from the education system, $d_{E,t}$ is the contribution per worker to the education system, $b_{P,t}$ is the benefit per retired from the pension system, and $d_{P,t}$ denotes the contribution per worker to the pension system. These are indexed with subscript t to denote that the transfer occurs in period t. The size of each generation is denoted by N, where subscript t indicates in which period the generation was born. In period t the number of children is N_t, while the number of workers is N_{t-1}, and the number of retirees is N_{t-2}.

Suppose that each worker in period t has n_t children. Then the young age dependency ratio in period t, N_t/N_{t-1}, is denoted by n_t and hence, the old age dependency ratio, N_{t-2}/N_{t-1}, equals n_{t-1}^{-1}. From the balanced budget restrictions in equations (1) and (2), the impact of changes in the dependency ratios can immediately be seen. Demographic changes will either change the received benefits or the contributions, or both.[5]

Above, the contributions and benefits were not related to the level of income in society. In a world with growing income over time, it would not make sense to have fixed benefits/contributions over time. It is reasonable to relate benefits/contributions to income, where income refers to the mean income of the working generation.

Let $\tilde{w}_t$ denote the mean labor income of the workers in period t, and let $\tau_{E,t}$ and $\tau_{P,t}$ denote the *contribution rate* devoted to financing the education and the pension system, respectively. The contribution from the workers, $d_{i,t}$, where $i = E, P$, can then be stated as:

$$d_{i,t} = \tilde{w}_t \tau_{i,t}. \tag{3}$$

The received benefits, $b_{i,t}$, can also be related to the income level of the working

[4]The assumption regarding two separate systems is mainly based on the fact the existing social security programs have a very weak connection to the education system, if any. If the period-by-period balanced budget assumption were loosened, there would be other financing opportunities for an open economy.

[5]Which it changes depends on the transfer schemes; this is explained in subsection 2.2, however.

population according to:

$$b_{i,t} = \tilde{w}_t \gamma_{i,t}, \tag{4}$$

where $\gamma_{i,t}$ are the *benefit rates* in the transfer systems. The benefit rates are the fraction of active workers income that each child/retired receives.[6]

The period-by-period balanced budget constraints for the two transfer systems can then be rewritten as:

$$\gamma_{E,t} = \tau_{E,t}/n_t, \tag{5}$$

$$\gamma_{P,t} = \tau_{P,t} n_{t-1}. \tag{6}$$

Changes in the dependency ratios will either affect the contribution rate or the benefit rate. By inserting equations (5) and (6) into equations (3) and (4), it is clear that the benefits/contributions will not only depend on demographic changes, but also on how income changes. Only in steady state when $n_t = n$ is it possible to fix both the contribution rate and the benefit rate.

2.2 Different schemes

The various intergenerational transfer schemes differ in how the benefits and the contributions respond to changes in demography and income. The difference between the schemes can be understood from the balanced budget restrictions.

From equations (5) and (6), two simple schemes emerge. Either the benefit rate is fixed, $\gamma_{i,t} = \gamma_i$, or the contribution rate is fixed, $\tau_{i,t} = \tau_i$. These schemes will simply be referred to as *fixed benefit rate, FB,* and *fixed contribution rate, FC*.[7] It is, however, possible to have a fixed benefit rate in the education system, while having a fixed contribution rate in the pension system since the systems operate independent of each other.

One more scheme will be considered for the pension system. This scheme will be labeled *fixed replacement rate, FR*. In this case, the benefits received in the pension system are related to *previous* income instead of current income, i.e. the income from one's own active life. For the education system the benefits will always be related to current income, since individuals have no previous income when entering the education system .

[6]The term benefit rate is, to my knowledge, not used in the literature. This is not to be confused with the term *replacement rate* which is used in the pension literature, and which will be described further on. The benefit rate is a theoretical abstraction and is also used in Lindbeck (2000), although using different term.

[7]It is possible to let both the contribution rate and the benefit rate vary, this is a convex combination of these two extreme cases that will not be explored in this paper, but is instead left for essay 2.

The motivation for investigating the FC and the FR scheme is that existing pension systems often belong to one of these schemes. The motivation for the FB scheme is that from a theoretical viewpoint, this scheme is the opposite of the FC scheme, according to equations (5) and (6). Moreover, when investigating the education system, this is the only natural alternative to the FC scheme.

Below, the different schemes are presented and distinguished according to their benefit formula.[8]

Fixed benefit rate, FB

A fixed benefit rate in either the education or the pension system, i.e. $\gamma_{i,t} = \gamma_i$, gives the following benefit formula in period t:

$$b_{i,t}(\tilde{w}_t) = \gamma_i \tilde{w}_t. \tag{7}$$

In this case, the benefit in period t only depends on current income. How the dependency ratio evolves over time does not directly affect the benefit. With respect to demographic changes, it is the workers' contribution that is altered to fulfill the budget restriction. The retired and/or the children are always promised a certain amount of current income, independent of how many they are in relation to the working population.

Fixed contribution rate, FC

In this case the workers are promised to pay a certain fraction, τ_i, of their income to the young and/or the pension system. This will result in the following benefits in the education and pension systems:

$$b_{E,t}(\tilde{w}_t, n_t) = \tau_E \tilde{w}_t / n_t, \tag{8}$$

$$b_{P,t}(\tilde{w}_t, n_{t-1}) = \tau_P \tilde{w}_t n_{t-1}. \tag{9}$$

Benefits will not only fluctuate with income, but also with demographic fluctuations. On the other hand, the contributions from workers will only fluctuate with current income.

[8]Alternatively, the contribution formula could be used. Which is used is of no importance, if the benefit formula is known then the contribution formula is given via the balanced budget restrictions. Here the benefit formula is used since the common approach in the pension literature is to identify the pension formula.

Fixed replacement rate, FR

Benefits are a fraction of the retired individual's own income while working and this fraction is referred to as the *replacement rate*.[9] Let $\tilde{\gamma}$ denote the replacement rate, which implies that the benefit rate can be stated as, $\gamma_{P,t} = \tilde{\gamma}/\theta_t$ where $\theta_t = \tilde{w}_t/\tilde{w}_{t-1}$. In this case, the benefit formula can be stated as:

$$b_{P,t}(\tilde{w}_{t-1}) = \tilde{\gamma}\tilde{w}_{t-1}. \tag{10}$$

With respect to demographic shocks, this benefit formula is similar to the benefit formula in the FB scheme. In both cases, the benefit is independent of the dependency ratios. The difference is that past income instead of current income determines the benefit. After income realization, the workers know what their future retirement benefit will be, irrespective of future wages and demographic structure.[10] In the previous schemes, the contributors and the beneficiaries shared the income uncertainty; in this case the workers bear the full cost of both demography and income uncertainty.

For transfers to the children, i.e. the education system, it is not reasonable to assume such a benefit formula since they have no past earnings.

2.3 Implicit interest rates

Samuelson (1958) demonstrated how an unfunded pension system generates a rate of return from the population growth, albeit the transfers being instantaneous. This was extended by Aaron (1966) who showed that the unfunded pension system is preferred over the funded alternative, if the growth rate of the tax base exceeds the rate of return on capital, and vice versa. Below I derive the implicit interest in the different schemes, in the traditional way according to Samuelson. The pension system will have its own implicit interest rate, as will the education system. However, with two intergenerational transfers systems in place simultaneously, there is an other possibility to define the implicit interest rate. Instead of viewing each system separately it is possible to define the implicit interest over both systems. This implicit interest rate is derived for the different education and pension scheme combinations.

[9] In the literature, it sometimes occurs that the replacement rate refers to the fraction of current income (what is referred to as the benefit rate in this paper). This is, however, conceptually obscure since the benefits of the present pensioners do not replace the wages of present workers. Augusztinovics (2000), among others, has also pointed at this misuse in the literature.

[10] This holds under the assumption that the feasibility constraint is not violated.

The traditional approach

In the education system, the generations start by receiving benefits which will implicitly be repaid in the next period when working. The implicit gross interest rate on intergenerational loans between period t and $t+1$ will be denoted $R_{E,t+1}$ where subscript E indicates the education system. It is thus generation t that must pay the implicit interest rate $R_{E,t+1}$. Since each generation implicitly borrows in the education system, it wants this interest rate to be as low as possible.

In the pension system, the generations start by making contributions when working and then they receive benefits when retired. Thus, there is an implicit rate of return on the contributions made. The interest rate received by generation t is denoted $R_{P,t+2}$, where the subscript indicates that it is an implicit rate of return on investment made between period $t+1$ and $t+2$. Since it is an implicit investment, each generation wants the interest rate in the pension system to be as high as possible. By definition, the implicit interest rates in the education system and the pension system, for generation t, can be stated as:

$$R_{E,t+1} = d_{E,t+1}/b_{E,t}, \tag{11}$$

$$R_{P,t+2} = b_{P,t+2}/d_{P,t+1}. \tag{12}$$

The implicit interest rate for generation t under the different transfers schemes is presented in table 1. From table 1, it is clear that if there were no changes to

Table 1: Implicit interest rate for generation t.

	Education $R_{E,t+1}$	Pension $R_{P,t+2}$
FR		$\theta_{t+1}n_t$
FB	$\theta_{t+1}n_{t+1}$	$\theta_{t+2}n_t$
FC	$\theta_{t+1}n_t$	$\theta_{t+2}n_{t+1}$

income development or population growth, the schemes would be identical. All schemes would have an implicit interest rate equal to the growth rate of the tax base, in line with Aaron (1966).

Increased population growth (or productivity growth) implies a burden in the education system due to a higher interest rate on "loans"; while it implies a gain in the pension system, due to a higher interest rate on "investments". What also emerges is that the education and the pension systems respond in the opposite way after a demographic shock. In the education system, an FB scheme implies that the implicit interest rate for generation t is determined by population growth between generation t and its children. If the education system is an FC scheme, then the implicit interest rate for generation t is determined by the population growth between generation t and its parents.

The interest rate in the pension system of FR or FB type is determined by the population growth between generation t and its parents, while if it is an FC type, the interest rate is given by the population growth between generation t and its children. From a policy perspective, this could be interesting since generation t has more control over the population growth between itself and its children, than that between itself and its parents.

Regarding income growth, the interest rate in the education system is always determined by the income growth between generation t and its parents. The interest rate in the pension system is dependent on the income growth between generation t and its children. The exception is the pension system of FR type, where the interest rate depends on the growth between generation t and its parents. Incentives to enhance the income of future generations seem to be absent with the FR pension scheme.

Defining the implicit interest rate for each system separately implies that they are sensitive to demographic fluctuations. This is often used as an argument against unfunded pension systems. Moreover, it is argued that funded pension systems should be implemented for dynamically efficient economies, since the implicit interest rate is below the rate of return on capital.[11] This, however, means that the education system and the pension system cannot both be financed by intergenerational transfers. If the economy is dynamically efficient, the education system should be financed by intergenerational transfers and the pension system should be funded. If dynamic inefficiency applies, then the education system should be financed via capital markets and the pension system should be unfunded. In reality, however, we observe that both systems have large transfer components.

The implicit interest rate over both systems

Instead of viewing the implicit interest rate of each system separately, it is possible to define the implicit interest rate over both systems, in line with Boldrin and Montes (2005). Let us view the pension benefits received by generation $t-1$ as an implicit debt repayment for the education contributions they it made in period t. In this

[11] Hauenschild (1999) shows that this conclusion no longer holds when uncertainty is included. Then the unfunded pension system has a role as a means of risk diversification, albeit its rate of return is lower than the interest rate.

case, the implicit interest is defined according to:

$$R_{H,t+1} = \frac{b_{P,t+1}}{d_{E,t}}. \tag{13}$$

I denote this implicit interest rate with subscript H to emphasize that the contributions made to the education system are considered as an implicit human capital investment.

The implicit rate of return for the different cases is presented in table 2, where the variables without time subscripts indicate their steady state value. In steady state, the implicit interest rate is the same for all scheme combinations; however, it does not equal the growth rate of the tax base. What is crucial for the return is the ratio between the pension benefit rate over the education benefit rate. In this case, it is possible to motivate that the pension system and the education system are financed by intergenerational transfers, even without uncertainty. By adjusting the rates, it is possible to adjust the implicit return and thus, if the economy is dynamically efficient or not will be of no importance for the choice between the funded and unfunded system. What also emerges is that the steady state population growth depresses the implicit rate of return, which is a clear contrast as compared to Samuelson (1958).

Table 2: Implicit rate of return on human capital investments, $R_{H,t+1}$.

Pension	Education	
	FB	FC
FR	$\frac{\gamma_p \theta}{\gamma_E n_t}$	$\frac{\gamma_P \theta}{\gamma_E n}$
FB	$\frac{\gamma_P \theta_{t+1}}{\gamma_E n_t}$	$\frac{\gamma_P \theta_{t+1}}{\gamma_E n}$
FC	$\frac{\gamma_P \theta_{t+1}}{\gamma_E n}$	$\frac{\gamma_P n_t \theta_{t+1}}{\gamma_E n^2}$

After a fertility disturbance, the scheme combination is important. We see that opposite schemes for the education system and the pension system imply that the implicit interest rate is not directly affected by fertility fluctuations. There is an indirect effect through the effect on wage growth.

The combination with a fixed replacement rate in the pension system and a fixed contribution rate in the education system gives a constant return, irrespective of fertility and productivity fluctuations. This does not imply that this scheme combination is preferred in a general equilibrium setting. In general equilibrium, the individuals are affected by changes in factor prices and the design combination that can counter this effect may be a better choice.

3 The model

The general equilibrium model adds a production function and capital accumulation to the three-period OLG model. The model consists of three components: individuals that maximize their lifetime utility, firms that maximize their profit, and intergenerational transfer systems. The transfer systems are exogenous and permanent and they can operate according to the above schemes. Agents know under which scheme the systems operate. Except for the exogenous intergenerational contract (i.e. the transfer systems), there is no altruism between generations. The model is a simpler version of Pecchenino and Pollard (2002) who include altruism and uncertainty about time of death.[12]

3.1 Individuals

Individuals live for three periods. In young age, children invest all their time (one unit) in human capital accumulation, from which they all receive the same utility. Children's time input is combined with education benefits, provided by the workers, to develop their human capital which will be used when working. Any difference in the per child education benefit will thus not affect the utility in the first period of life, but will instead alter the human capital. In the next period, when working, they all inelastically supply their effective labor, the product of their one unit of time and their human capital to firms and receive wage income. A fraction of this wage income will finance the education and pension systems; the remaining part will be divided between savings and consumption. In the third and final period, individuals are retired and consume their own savings and income from the pension system.

Since all generations gain the same utility when young, this period is suppressed. The lifetime utility of an individual, belonging to generation $t-1$, is assumed to be additively separable according to:

$$U_{t-1} = \ln c_{w,t} + \beta \ln c_{r,t+1}, \tag{14}$$

where β is the subjective discount factor and thus, a measure of the individual's impatience to consume. Consumption per worker in period t is denoted by $c_{w,t}$, while consumption per retired in period t is denoted by $c_{r,t}$.

Denote by h_t the human capital for generation $t-1$ while at work. This is a

[12]Pecchenino and Utendorf (1999) show that using intergenerational loans for education financing instead of including altruism does not alter their results in any significant way.

product of the benefits from the education system in period $t-1$, i.e.:

$$h_t = b^{\sigma}_{E,t-1}, \tag{15}$$

where $\sigma \in (0,1]$ measures the returns to scale in the production of human capital. The human capital determines the effective labor supply for each individual in period t. The individuals take their human capital, wages, the interest rate, the tax rate, and the benefits in the pension system, as given. Their only decision variable is savings, which they choose so as to maximize the lifetime utility, according to equation (14), subject to the following budget constraints:

$$c_{w,t} = (1-\tau_t)\, w_t h_t - s_t, \tag{16}$$

$$c_{r,t+1} = R_{t+1} s_t + \gamma_{P,t+1} \tilde{w}_{t+1}, \tag{17}$$

where s_t denotes the per worker savings in period t, w_t is the wage for one unit of effective labor, and R_{t+1} denotes the gross interest rate on savings between period t and $t+1$. As before, τ_t denotes the total tax rate used in the financing of the education and the pension systems, $\gamma_{P,t+1}$ is the benefit rate received when retired, and $\tilde{w}_t = w_t h_t$.

Maximizing the objective function (14) under constraints (16) and (17) yields the familiar intertemporal Euler equation:

$$c_{r,t+1} = \beta R_{t+1} c_{w,t}. \tag{18}$$

3.2 Production

The aggregate production function in the economy is assumed to be of Cobb-Douglas type and homogeneous of degree 1. Production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where L_t is aggregate effective labor, i.e. $L_t = h_t N_{t-1}$, K_t is the aggregate capital stock in period t, and A is a scaling parameter. The capital stock K_t fully depreciates during the production process. Defining production in terms of output per worker yields:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}, \tag{19}$$

where $y_t = Y_t/N_{t-1}$, and $k_t = K_t/N_{t-1}$.

The prices of factor inputs are obtained from the firms' maximization problem and since perfect competitive factor markets are assumed, these prices equal their marginal product, that is:

$$R_t = A\alpha k_t^{\alpha-1} h_t^{1-\alpha}, \tag{20}$$

$$w_t = A\left(1-\alpha\right)k_t^{\alpha}h_t^{-\alpha}, \tag{21}$$

where R_t is the price of physical capital, and w_t is the price per unit of human capital, both in period t.

3.3 Market clearing

All markets are assumed to be perfectly competitive and the following condition must be satisfied for the goods market to clear:

$$y_t = s_t + c_{w,t} + c_{r,t}/n_{t-1} + b_{E,t}n_t,$$

which states that the supply of goods must equal demand, which comprises consumption, savings, and education expenditures.

Using firms' and individuals' first-order conditions together with the balanced budget restriction for the transfer systems, this condition can be reduced to:

$$k_{t+1} = s_t/n_t. \tag{22}$$

The capital labor ratio of the next period is determined by current savings and workforce growth. If n_t increases without an equivalent increase in savings, there will be capital dilution in the next period.

3.4 Equilibrium

Given the initial capital stock, $k_0 > 0$, the initial human capital stock, $h_0 > 0$, and the population growth, $\{n_t\}_{t=0}^{\infty}$, a competitive equilibrium for this economy is a sequence of: prices $\{w_t, R_t\}_{t=0}^{\infty}$, allocations $\{c_{w,t}, c_{r,t}, s_t\}_{t=0}^{\infty}$, human and physical capital stocks $\{k_t, h_t\}_{t=0}^{\infty}$, and benefit rates and tax rates $\{\gamma_{E,t}, \gamma_{P,t}, \tau_{E,t}, \tau_{P,t}\}_{t=0}^{\infty}$, such that the individuals maximize their utility, firms maximize their profits, markets clear, and the budgets of the transfer systems are balanced.

Individual saving decisions fully characterize the equilibrium, since they define the equilibrium trajectory for $\{k_t\}_{t=0}^{\infty}$ via eq. (22). Eqs. (16)-(18) and (20)-(22) yield the following saving function in equilibrium:

$$s_t = \frac{\beta\alpha\left(1-\tau_t\right)w_t h_t}{\lambda_t}, \tag{23}$$

where $\lambda_t = \alpha\left(1+\beta\right) + \left(1-\alpha\right)\gamma_{P,t+1}/n_t$. Saving is a fraction of disposable income and independent of the interest rate in the economy; this is a result from the utility function which has an intertemporal elasticity of substitution equal to unity.

As expected, the savings respond negatively to an increase in the future pension benefit rate, since these two are substitutes.

The steady state

There are two different types of steady state equilibria, depending on whether the production function for human capital exhibits diminishing returns or not. If there are diminishing returns, i.e. $\sigma < 1$, then there is a stationary equilibrium with no growth in the per capita variables. If there are constant returns, i.e. $\sigma = 1$, then there is a balanced growth equilibrium such that the per capita variables $\{y_t, k_t, h_t\}$ grow at a constant gross rate equal to:

$$\theta = A\left(1-\alpha\right)\gamma_E^{(1-\alpha)}\left[\frac{\beta\alpha\left(1-\tau\right)}{\lambda n}\right]^{\alpha}, \tag{24}$$

where $\theta_t = y_t/y_{t-1}$ and in steady state $\theta_t = \theta \; \forall \; t$.

4 Calibration

4.1 Demographic shock

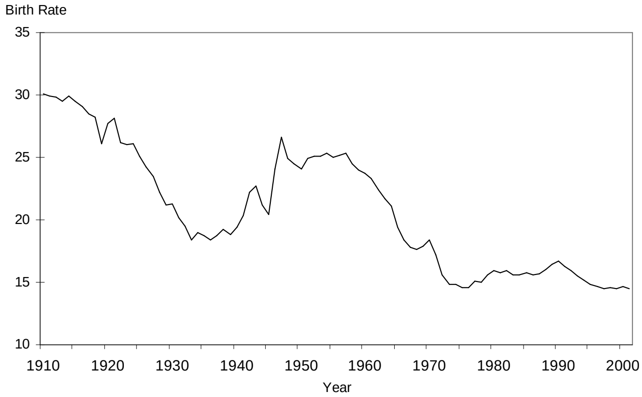
The baby boom shock under consideration can be stated as $n_{t+j} = n \; \forall j \neq 0$ and $n_t > n$. To get an estimate of the shock, the U.S. experience will be used. In figure 1, the birth rates per 1000 inhabitants for the U.S. between the period 1910 to 2001 are presented.

A shock is by definition, a sudden deviation from expectations. To estimate the size of the shock, one needs to know what the expectations were, and obviously the outcome. To avoid historic researching of what the expectations actually were, figure 1 can be used to assess what the expectations might have been. Moreover, knowing that the official years of the U.S. baby boom generation are 1946 to 1964 makes it possible to at least view this period as a shock period.

In the model, every period roughly represents 27 years. Using a 27 year period length, while trying to assess the magnitude of the shock, does not seem to yield estimates lower than 20 percent, according to figure 1.[13] For this reason, the magnitude of the shock used will be 20 percent, i.e. $n_t = 1.2n$. Regarding the steady state gross population growth n this will be set to 1.3, based on the annual average for the U.S. between 1910-2001.[14]

[13] If a period length of 19 years is instead used to fit the official years of the baby boom, the estimates of the shock are around 30 percent depending on the specification of the expectations.
[14] The annual average is approximately 1.01, which implies that per period $n = 1.01^{27}$.

Figure 1: Birth Rate per 1000 inhabitants for the U.S. between 1910 to 2001.



Source: Vital Statistics of the United States, 2001, Volume I, Natality.

The demographic structure used in the simulation can be stated as $n_{t+j} = 1.3 \; \forall j \neq 0$ and $n_t = 1.56$. The boom generation is thus relatively large as compared to its parent generation, but its relative size as compared to its child generation is unaltered.

4.2 Preferences

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one-year estimate from Auerbach and Kotlikoff (1987) of 0.98 translates to $\beta = 0.6$, since every period represents about 27 years.

4.3 Production

There are two parameters in the production function that need to be calibrated, α and A. The share of capital income in the national product, α, is calibrated to one third. The scale parameter A can in the benchmark simulation be freely chosen since it will not alter the relative outcome in any significant way. However, when allowing for endogenous growth in the sensitivity analysis, i.e. $\sigma = 1$, the growth rate of the economy will depend on A. Since A can be freely chosen when $\sigma < 1$, but not when $\sigma = 1$, I will let the latter decide the value for A; which will be chosen to yield an annual growth rate per worker of 2.5%, which corresponds to U.S. historical rates in the balanced growth case.[15]

[15] There are many empirical studies that try to estimate this growth rate. A short review is given in Pecchenino and Utendorf (1999), which finds 2.5% to be the best compromise between the different estimates.

4.4 Intergenerational transfers

Calibrating the education and the pension system amounts to calibrating the benefit rates in steady state, $\gamma_{i,ss}$. For the pension system, it is possible to use the existing pension systems as a guideline. According to Pecchenino and Utendorf (1999) the benefit ratio, i.e. the benefit over the average wage ratio in the same period, is 0.42. In reality, however, the ratio between working years and years of retirement is almost 2, while in this three-period model, it is 1. For this reason, the benefit rate in the pension system is chosen such that $\gamma_{P,ss} = 0.21$. When the pension system operates under the FR scheme, it is the replacement rate, $\tilde{\gamma}$, that is fixed. The replacement rate is calibrated such that the same benefit rate is obtained in steady state, i.e. $\tilde{\gamma} = 0.21\theta_{ss}$.

An efficiency requirement from Boldrin and Montes (2005) is that the rate of return on human capital equals the rate of return on physical capital. From table 2, we know that the rate of return on human capital is determined by the ratio between the two benefit rates. I will set the ratio between γ_P and γ_E so as to fulfill this requirement. Given the choice of γ_P, this implies $\gamma_E = 0.6$, and hence $\tau_E = 0.08$.

Is this a reasonable calibration of the education system? The answer seems to be yes when comparing it to the GDP share devoted to education. According to Rangazas (2002), the U.S. GDP share for primary and secondary school spending has been approximately 4 percent during the last three decades and the GDP share for higher education is close to 3 percent. The share of GDP spent on education thus amounts to 7 percent, which is close to my calibration.

4.5 Human capital

The production function of human capital only has one exogenous parameter, σ, but this parameter is the most difficult to calibrate. This difficulty is well known from Bouzahzah, Croix, and Docquier (2002) and one aim of the paper is to investigate how sensitive the results are to this parameter.

It is, however, useful to consider what σ might be. There are several ways in which this could be done. One way would be to translate the empirical estimates from the education literature. Card and Krueger (1992) investigate how the pupil-teacher ratio affects future productivity. Translating their results, via assumptions on how spending per pupil is related to the pupil-teacher ratio, would yield $\sigma = 0.17$.

A different way of finding the value for σ would be to relate it to the benefit rates in the education system and the pension system, which have already been calibrated. However, to find the relation between these variables one needs to make the intergenerational transfer systems endogenous. This is done in essay 2 where I derive the transfer that would arise if children could enter into efficient contracts with adults. The result is that the intergenerational flows will be such as to equalize the return on human capital with the return on physical capital. In this case, if one has chosen the benefit rates one would obtain σ. Calibrating σ in this way implies that the exogenously chosen benefit rates correspond to those that would arise in the market outcome when children and adults enter into contracts. Using the already calibrated benefit rates from above implies $\sigma = 0.16$. We see that this is very close to the translated estimate from Card and Krueger (1992). Since both these approaches yield almost the same value, I will use $\sigma = 0.16$ as the benchmark calibration.

However, we cannot be sure that this is the proper value for σ. This would require that the actual benefit rates we observe are optimal; and however, we cannot say whether this is the case or not.[16] Previous studies by Chakrabarti, Lord, and Rangazas (1993) and Pecchenino and Utendorf (1999) that use the same production function for human capital have calibrated σ to 0.6 and 1, respectively. This shows that there is no consensus on how to calibrate the production of human capital. Certainly, in a non empirical study, it would not be fruitful to dwell on which estimate is more correct. Instead, I will simply investigate what happens to the results when varying σ. I choose a high value for σ equal to 0.8, which is a compromise between the two mentioned studies.

The benchmark calibration is thus $\sigma = 0.16$, which I compare with the alternative $\sigma = 0.8$. The sensitivity analysis in the appendix shows how the results change for other values for σ.

Table 3: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.6
Share of capital income	α	1/3
Steady state benefit rate in the pension system	γ_P	0.21
Steady state benefit rate in the education system	γ_E	0.06
Population gross growth rate	n	1.3
Baby Boom shock	n_t/n	1.2
Steady state gross growth rate	θ	1
Total factor productivity	A	34
Efficiency in human capital production		
benchmark	σ	0.16
alternative	σ	0.8

[16]There is some evidence that this might be the case at least for Spain (Boldrin and Montes 2005).

4.6 Steady state

Before the model is used to study the effects of demographic changes, it is useful to report the steady-state values for some key variables, according to the calibration in table 3. In steady state, all cases are identical and it is possible to obtain analytical results. To obtain the numerical results, presented in table 4, it is enough to plug in the parameter values from table 3 into the analytical solution.

Table 4: Steady state values according to calibration in table 3.

Cons. per worker relative output	c_w/y	0.35
Cons. per retired relative output	c_r/y	0.58
Gross interest rate for capital	R	2.74
Saving rate	S/Y	3.7%
Capital output ratio	k/y	0.12

To see how the model fits stylized facts, the last three variables from table 4 are of most interest. The magnitude of the interest rate is quite realistic when adjusting for the time length in the model.[17] The saving ratio in life-cycle models with no bequests has notorious difficulties in fitting empirical facts.[18] As for other similar models, the saving rate is considerably below the comparable U.S. rate, which is around 6.7 percent. This should not cause any large problems as long as the capital output ratio is within a reasonable range. From table 4, the capital output ratio is 0.12, which on an annual basis becomes 3.2 times yearly GDP. This is slightly higher than the comparable U.S. ratio, but still within reason.

Regarding the consumption, we see that it is increasing over the life-cycle. Such a consumption profile is consistent with observed patterns in the U.S. (Lee, Lee, and Mason 2006).

5 Results

The issue at hand is to investigate how macro variables and distribution between generations vary with different intergenerational transfer systems. I will focus on the following macro variables: savings, interest rate and the growth rate of GDP per capita.[19] Besides the macro variables, the distributional impact between generations

[17] The reported interest rate is the compounded interest rate over 27 years, which on an annual basis becomes 3.8%.

[18] See Kotlikoff and Summers (1981).

[19] In the appendix, I also present the results for the efficient wage and the net discounted lifetime income for the generations.

is of interest. For this reason, I will present the compensating variation for each generation following a baby boom shock.

5.1 Savings

Aggregate savings by the workers will determine the capital labor ratio in the next period which, in turn, determines the factor prices. To offset the capital dilution of the 20 percent larger workforce in period $t+1$, and onwards, the aggregate savings by the workers need to increase accordingly. Clearly, the change in aggregate savings in period t is not even close to what would be needed to compensate for the coming increase in the workforce; on the contrary, in two of the cases, the change in savings will aggravate capital dilution.

When the education system is of FB type, the parent generation will be burdened with a higher education tax, leaving it with less disposable income which reduces its savings. How much it reduces its savings depends on the design of the pension system. If the pension system is of FB type, the parent generation will increase its savings to compensate for the future reduction in pension benefits, arising from the lower wage of the baby boom generations. Since the benefit rate is fixed, the wage decrease in the next period will punish its benefits without any cushioning. Thus, it will increase its savings to compensate for future lower benefits, which will mitigate the boom generation's capital dilution.

If the pension system is of FC type, then the demographic benefit from the pension system, i.e. the higher benefit rate due to the increased worker/retiree ratio, will compensate the parent generation for the lower wage in the next period. In this case, the parent generation will not alter its savings due to future events. Thus, if the pension system is of FC type, the parent generation will not increase their savings since its retirement income has increased. For the parent generation to increase its savings, it is necessary that the pension system does not have a fixed contribution rate (this can be shown analytically).

Under the FR pension scheme, the parent generation will not receive the demographic benefit in the pension system, as it did in the FC scheme. It will, however, receive a higher benefit rate, since the lower wage the boom generation receives will not affect its pensions.

The boom generation's capital dilution is thus mitigated most under case 2 (pFB eFC). However, even in this case, the increase in savings is only about 3 percent, far from the needed 20 percent. The factor price movements are aggravated most in case 3 (pFC eFB), when the parent generation must pay for the burden in the education system, and when retirement income increases. How the parent generation adjusts its savings does not depend on σ. The only exception is case 6 (pFR eFC) but the difference is marginal.

When the boom generation enters the work force, aggregate saving increases but it will not be enough to restore the capital intensity for coming generations. In this case, we see that σ is important. For benchmark σ, the difference between the schemes is 7 percentage points at most. For the alternative $\sigma = 0.8$, however, we see that the education scheme is really of importance. In this case, it is almost a 15 percentage point difference between the highest and the lowest saving by the boom generation.

5.2 Factor prices

Since the wage and the interest move in the opposite direction, the discussion will focus on the outcome for the interest rate, which is presented in figure 3. The difference between the schemes is considerably smaller under the benchmark σ than for the alternative σ.

For the alternative σ, the interest rate only reacts marginally if the education system has a fixed contribution rate. The reason for this is that the reduction in educational spending per child results in an almost equal reduction in future human capital with high returns to scale. This keeps the ratio between capital intensity and human capital almost intact. Under an FB education scheme, the human capital does not change at all, while capital intensity is reduced by between 14 and 18 percent. Further, we see that the outcome two periods after the shock differs qualitatively between the cases.

Under the benchmark, the parent generation obtains between a 9 and a 14 percent higher return on its savings. As expected, the interest rate deviates the least under case 2 (pFB eFC) which was the case that resulted in least capital dilution for the boom generation.

Note that since the wage moves in the opposite direction, it means that the boom generation receives a lower wage than under the steady state. How much lower it is depends on σ and the transfer schemes. The boom generation will thus be negatively affected by the lower wage while the parent generation will gain from the higher interest rate. How large this effect is depends on the schemes and how much the schemes differ in outcomes depends on σ.

When trying to find the factor price trajectories after fertility changes, it is important to consider how the education system is financed, and how it will affect future human capital. The effects on factor prices are not that sensitive to the different pension schemes, however.

5.3 Growth

It is well known that the age structure of the economy will affect the growth rate. When a baby boom generation is born it will alter the GDP per capita in the economy, due to its numbers. Later when it enters productive age, it is expected that the growth rate will increase. From 4, we see that the boom generation reduces GDP per capita when it is are born, just as expected. What happens when it enters into productive age is, however, highly dependent on the production of human capital and the financing of education.

Under the benchmark calibration, there is no significant difference between the schemes. The growth rate initially turns negative and then increases and becomes positive when the boom generation enters productive age. The reason why the GDP per capita turns positive when the boom generation enters productive age is due to the demographic dividend of having a large working share of the population, as compared to the previous period.

Under the alternative σ, the initial reduction is the same as in the benchmark, and there are no differences between the schemes. But when the boom generation enters working age, the growth rate will be highly dependent on the educational scheme. With FC education, the growth rate will still be negative, implying that GDP per capita continues to decrease even after the boom has entered working age. Even if the number of workers increases proportionally more than the young and the old, it cannot compensate for the lower human capital and the lower capital intensity per worker. Under the FB education scheme, the growth rate becomes positive just as it did for the benchmark case, because the human capital is not reduced.

That the growth rate can remain negative even after the boom generation has entered productive age is somewhat unexpected. Empirical studies find that increasing the working-age population relative to the total population favors growth (Bloom and Williamson 1998). The results in figure 4 indicate that this relationship might be influenced by the process of human capital accumulation and how the education system is financed.

5.4 Distributional effects

To see how the generations are affected under the different schemes, the compensating variation is presented in figure 5. Compensating variation measures the net revenue of a planner who must compensate the generation after the change has occurred to bring the generation back to its original utility level. It can be considered as the negative of the amount that the generation would be willing to accept for the shock to occur. Thus, if the compensating variation is positive, the generation is better off after the shock than before, and vice versa.

When it comes to the distributional effects, we see that the difference between the schemes is substantial even for the benchmark case. Case 4 (pFC eFC) implies an almost 16 percentage point difference between the parent generation and the boom generation, while case 1 (pFB eFB) yields less than a 5 percentage point difference between the generations. Under the benchmark case, it is important not to have an FC pension scheme if the aim is to minimize the distributional effects. Under the alternative σ, the education scheme is what matters the most, and an FB education minimizes the distributional effects. However, within each education scheme, we see that the FB pension scheme minimizes the distributional effects.

Looking at the alternative σ in figure 5, we see that for every pension scheme, the parent generation would be better off financing the education burden. For the FC and the FB, it is understandable that this could be the case, since the parent generation's pension depends on the boom generation's income. That this can be the case for the FR pension scheme is somewhat more difficult to understand. In this case, there is no relation between the parent generations' pensions and the boom generations' income. All incentives to invest in future generations' human capital seem to be gone. However, since the FB education scheme increases the interest rate, while it is almost unaltered in the FC education scheme, the parent generation is still better off. Investing in the future generation does not increase its pension benefit but it will increase its returns on savings.

Figure 2: Aggregate savings during the working period as a percentage deviation from steady state outcome.

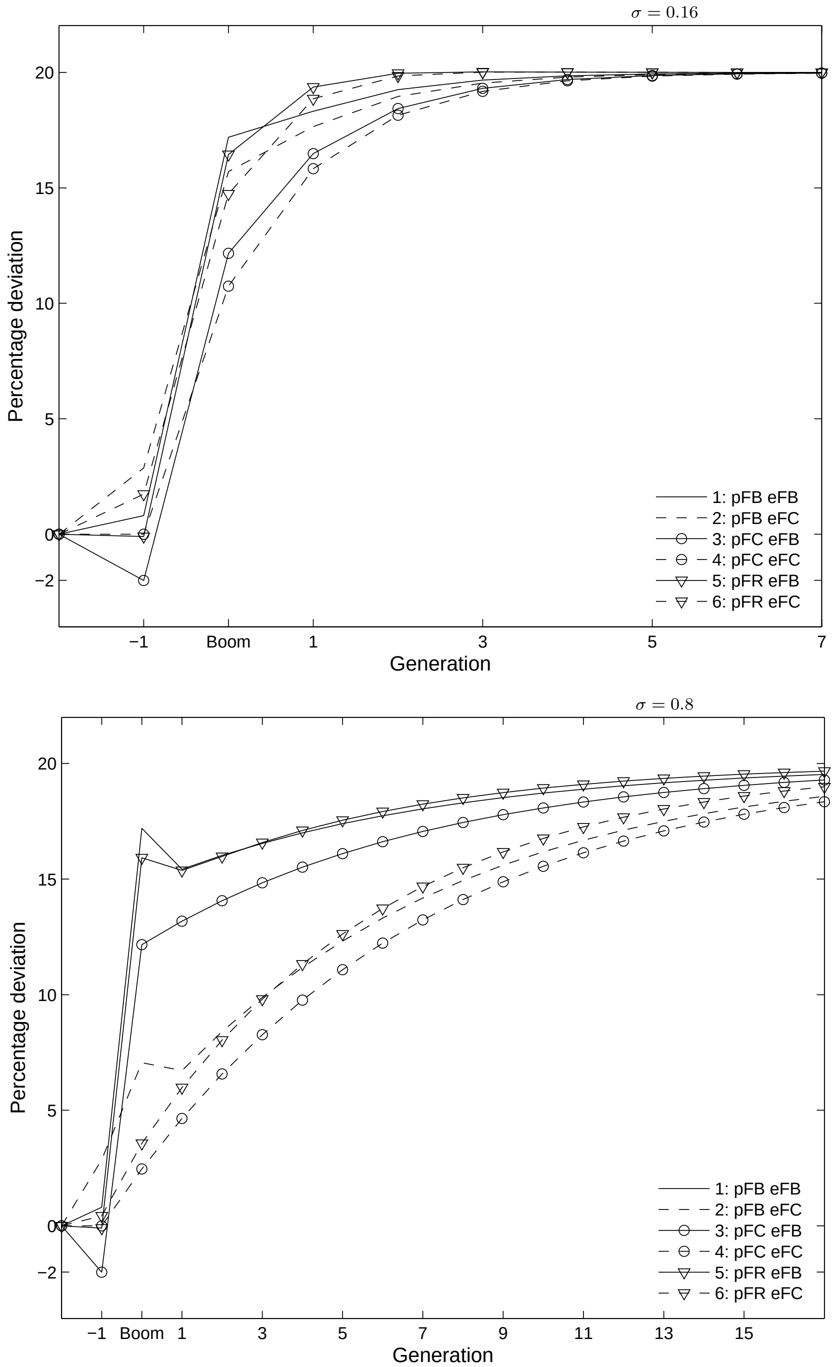


Figure 3: Percentage change for the interest rate.

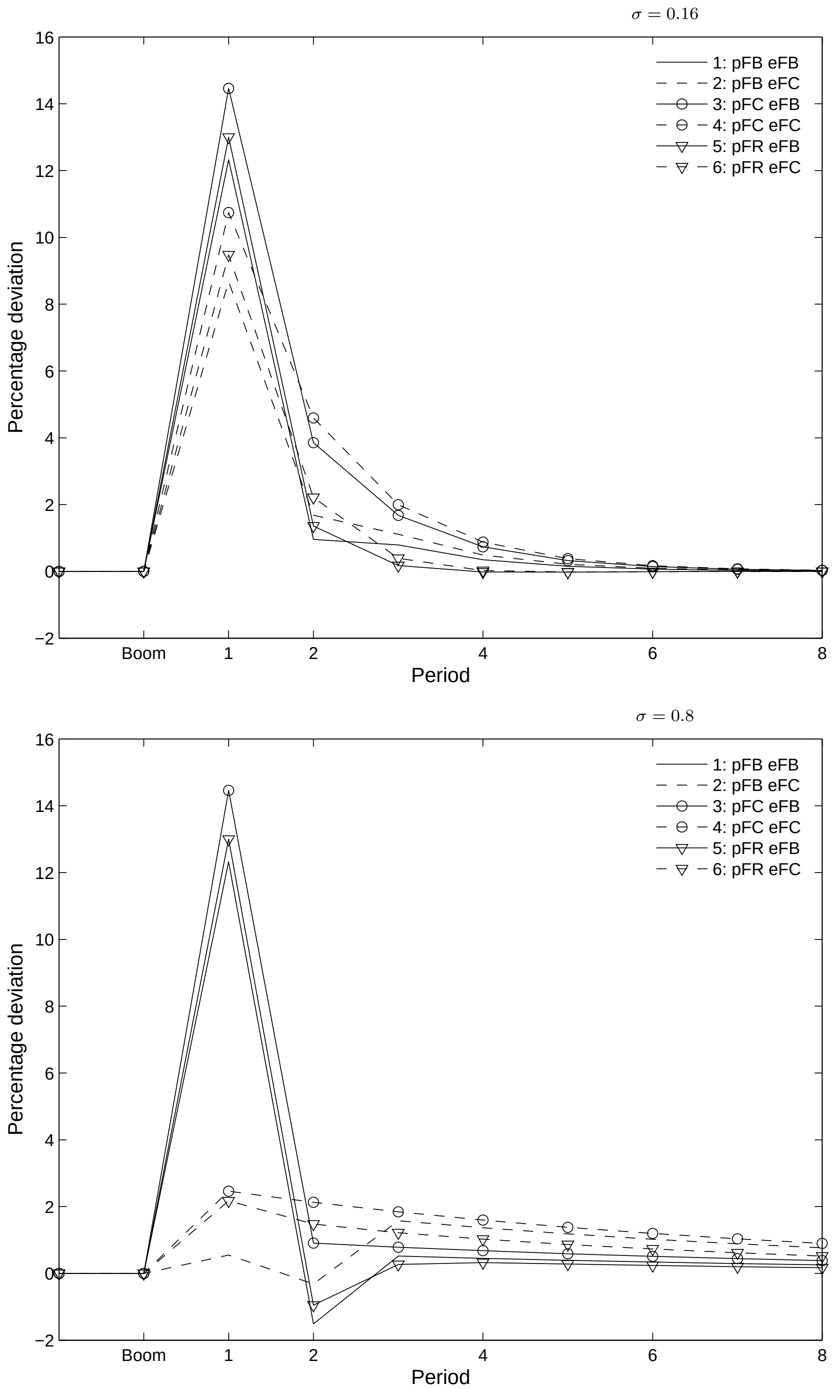


Figure 4: Growth rate in GDP per capita, between t and $t + 1$.

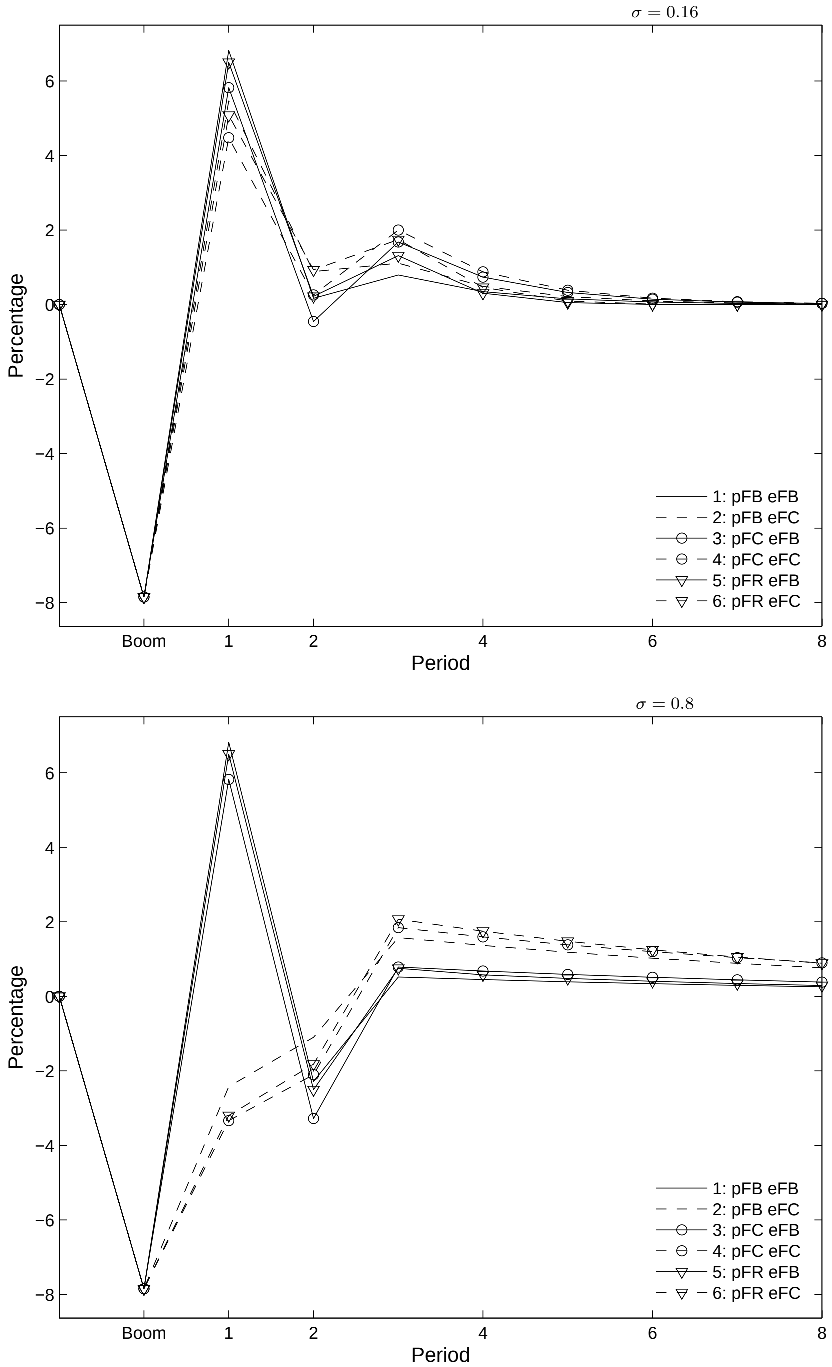
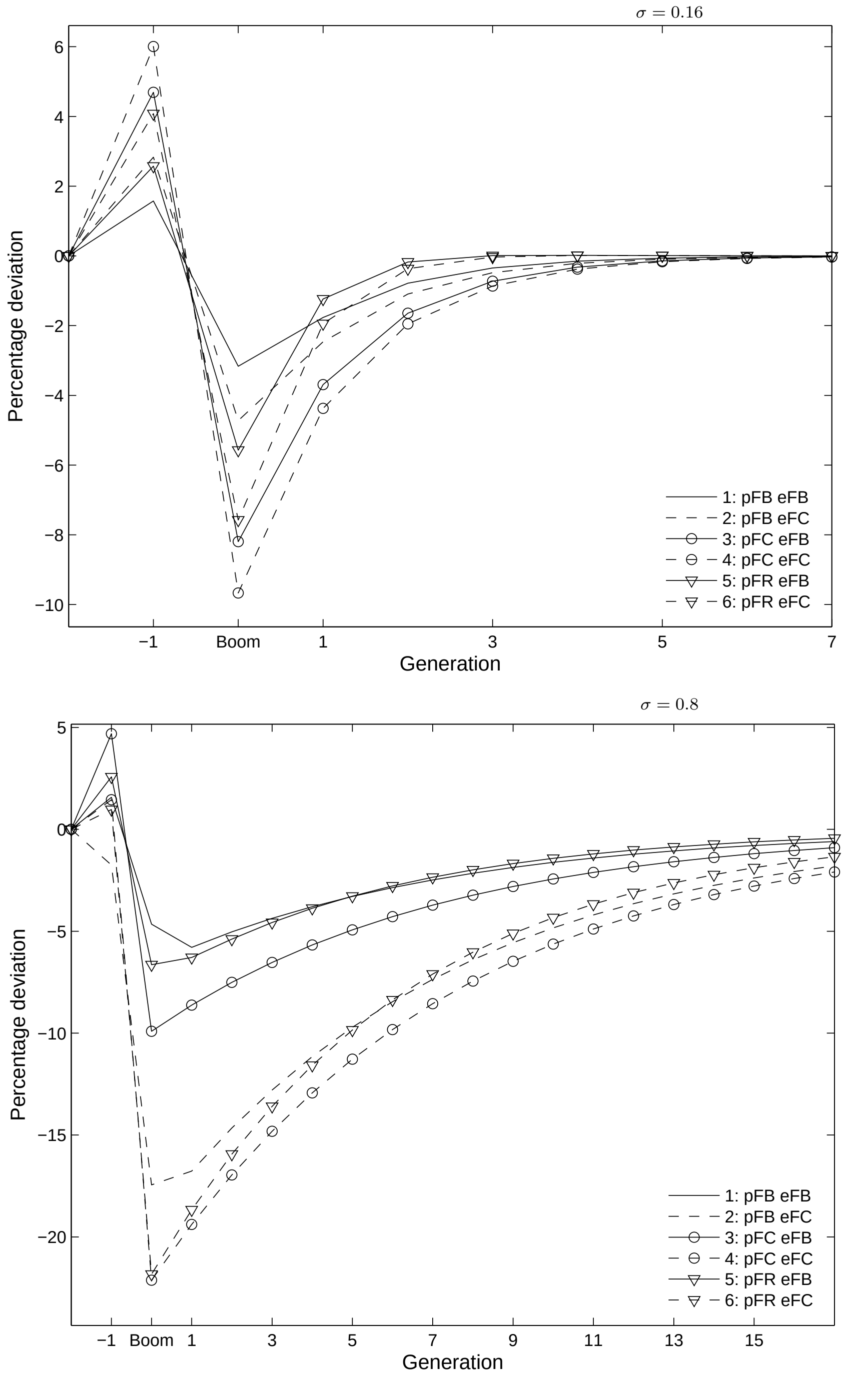


Figure 5: Compensating variation as a percentage of first-period consumption in steady state.



5.5 Intergenerational welfare

The above results showed that intergenerational distribution is highly dependent on the transfers schemes. It also emerged that the generations would rank the cases in a different manner. How they rank the different pension schemes was not dependent on the calibration of σ, and for a given education system, the parent generation would prefer pFC while the boom generation would prefer pFB. For the benchmark calibration, the generations would disagree about the preferred education system as well.

To obtain a compact measure of how all generations are affected by a fertility shock, a welfare function is defined according to:

$$W = \sum_{t=1}^{\infty} \phi_t U_t. \tag{25}$$

This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.[20] The separability assumption made above is standard, but a comment on the weighting factor, ϕ, is in order.

There are different views on how the per capita lifetime utility of generation t should be weighted. The question is if the utility should be weighted by generation size, and/or by a social discount factor. Not to dwell to much on this issue, it seems more or less necessary to account for the generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will also be included which allows for sensitivity analysis when varying this parameter. The weighting factor used will be the following:

$$\phi_t / \phi_{t-1} = \beta_s n_t, \tag{26}$$

where β_s is the social discount rate. The social discount rate will be set equal to the individuals discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth, then the discount rate should not exceed the inverse of population growth; if it does, then future generations will get an ever increasing impact on the welfare function, due to their larger number.[21] The appendix illustrates how the results change with respect to the social discount factor.

[20] Choosing a general utilitarian welfare function with aversion towards inequality between generations utility would strengthen the results.

[21] See for instance Blanchet and Kessler (1991) and Boadway, Marchand, and Pestieau (1991) for a short comment concerning the weighting problem.

Table 5 presents how the cases rank according to social welfare. That case 1 is ranked first is what would be predicted from figure 5. This case reduces the burden of the boom generation the most. Second in rank is case 5, which is not that surprising when it comes to the alternative σ value. However, under the benchmark σ, case 2 reduces the boom generation's burden more than does case 5. The reason why case 5 ranks higher than case 2 is that the boom generations progeny is better off under case 5.

The FB pension scheme is preferred for both education schemes irrespective of the human capital calibration. How important the education system is and which education scheme is preferred depends on the returns to scale in the production of human capital.

Table 5: Ranking based on the social welfare function.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.16$	1	3	5	6	2	4
$\sigma = 0.8$	1	4	3	6	2	5

Note: Population scaling is included. The case that is ranked 1 yields the highest welfare, while the case ranked 6 yields the lowest welfare.

Expected Intergenerational welfare

The baby boom was used to illustrate the importance of accounting for different types of intergenerational transfers. It is, however, problematic to use it when trying to rank the cases. For a baby bust shock, there will be an opposite reaction, i.e. a benefit in the education system, capital labor deepening, and a burden in the pension system. This would result in an opposite ranking of the cases by the generations.

If trying to choose between the cases, one would want to adopt an *ex ante* approach, not an *ex post* approach. Assume that there is a fifty fifty probability of a positive and negative fertility shock, such that $E(n_t = 1.2n) = 0.5$ and $E(n_t = 0.8n) = 0.5$. Which case yields the highest expected welfare? This is answered in table 6.

For the baby boom shock, it was understandable why the cases ranked as they did. The reason why the same result is obtained from equal probability of a positive and a negative shock is because the utilities do not respond symmetrically; since the marginal benefit from consumption is decreasing.

Table 6: Ranking based on expected social welfare, $E[W]$.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.16$	3	1	6	5	4	2
$\sigma = 0.8$	2	4	3	6	1	5

Note: Population scaling is included. The case that is ranked 1 yields the highest welfare while the case ranked 6 yields the lowest welfare.

The ranking according to expected welfare gives almost the same conclusion. For a given education system, the pension system should not be FC, irrespective of σ. The difference here is that the FR pension scheme can be ranked higher than the FB scheme. Note, however, that the FB and the FR scheme respond in the same way to a demographic shock. Regarding the education system, we see that σ completely determines if the FB or FC scheme is preferred.

The result regarding the pension system is the opposite of what Thøgersen (1998) found but supports the findings of Wagener (2003). However, both only investigate the FC contra the FR scheme under income uncertainty, while not analyzing the FB scheme. Wagener (2003) finds that the FR scheme is preferred over the FC scheme under an *ex post* comparison, while neither dominated the other from an *ex ante* perspective. Thøgersen (1998), however, finds that the FC scheme is strictly preferred from an *ex ante* perspective. The result in this paper indicates that the pension system should not be of FC type. Note that whatever risk-sharing feature the FC scheme could have with respect to wage uncertainty, the FB scheme analyzed here has the same feature.

6 Conclusion

The main purpose of this paper was to investigate how different intergenerational transfer schemes affect macro variables and the utilities of different generations, after a fertility disturbance. The large body of literature that has analyzed the effects of varying age distribution has not devoted much attention to the adjustment mechanism in the education system. Some studies have investigated the consequences of ad hoc changes to the education system, but hardly any attempts have been made to use the same approach as in the pension literature. Numerical studies often assume that a fixed fraction of output is devoted to the young and thus, implicitly assume a fixed contribution scheme. The question is how sensitive these results are with respect to the implicit assumption of the education scheme.

This study shows that macro variables, such as interest rate, wages, and growth rate per capita, can be sensitive to assumptions about the education scheme. Whether the results are sensitive depends on the returns to scale in the production of human capital. With higher returns to scale (however still diminishing), the difference in outcome between assuming an FB and an FC scheme is considerable. It is under the FC education scheme that the results are sensitive to the returns to scale. In this case, the growth rate per capita can be negative even when there is a high share of workers. To avoid this, it would be necessary to have an FB education system. This means that a higher share of the output should be devoted to education when young age dependency is high. The interest rate and the wages may also differ qualitatively over the transition path, depending on the education scheme. Numerical results based on relatively high returns to scale should thus carefully model the adjustment mechanism in the education system. Using the FB education scheme is more robust to the returns of scale in the production of human capital.

With small returns to scale, there are only small differences between the schemes with respect to the macro variables. In this case, the exclusion of different education schemes in numerical analysis is not that restrictive. However, the question whether these studies have chosen the right specification for the production of human capital remains; an open question which is not answered within this paper.

The relative outcome between different generations is, however, highly dependent on the transfer schemes, irrespective of the returns to scale. A baby boom first creates a young age dependency burden, and later an old age dependency gain. The allocation of these is a distributional matter between the parent generation and the boom generation. When including the effect on factor prices, I find that there is a strong case for the pension system not being financed with a fixed contribution rate. The boom generation should obtain the gain in the pension system, since it suffers from the capital dilution cost. How the education system should be financed crucially depends on the returns to scale in the production process of human capital.

How the different schemes yield an implicit rate of return was also presented. When viewing each system separately, i.e. calculating the difference between what one generations pays into the system as compared to what it receives, the standard implicit rate of return according to Samuelson (1958) was obtained. When abandoning this standard approach and instead defining the implicit rate of return over both systems, there is a role for unfunded pensions in a dynamically efficient economy, even without uncertainty. Moreover, it is possible to obtain a constant implicit rate of return on human capital under uncertainty. This is obtained with a fixed contribution rate in the education system and a fixed replacement rate in the pension system.

Although this essay does not analyze how society makes decision regarding the transfer systems, it can be mentioned that the parent generation will prefer to finance the burden in education if the returns to scale in the production of human capital are relatively high. It is not hard to realize that this could be the case if the retiree' pension is a fraction of current workers income. Then, the retirees certainly have an incentive to uphold future generations' human capital, and thus the wages. More surprising was the fact that the parent generation could be better off financing the education burden under the fixed replacement rate in the pension system. Since the pension benefit is in this case unrelated to the productivity of future generations, it would seem as if all incentives to invest in future generations are gone. However, it turns out that investing in human capital increases the rate of return on physical capital, which more than compensates the parent generation for the education burden.

Many countries, including Sweden, have reformed their pension systems from an FR scheme to an FC scheme. One argument for the transformation was based on the risk-sharing properties regarding income uncertainty. However, the FB scheme within this paper responds to income disturbances in the same way as the FC scheme. Another argument for the FC transition is that it will not lead to higher payroll tax when the old dependency ratio increases. This could have beneficiary effects on labor supply which are not accounted for in this analysis.

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A Sensitivity analysis

Steady state

The steady state results relative to output do not change for $\sigma < 1$, except for the interest rate and the wage which have the same level. For $\sigma = 1$ the level for the interest rate goes up to 5.3. What also happens when growth is introduced is that the capital output ratio goes down to 0.06 and the saving ratio goes up to 0.09. The reduction in the capital output ratio leads to the increase in the interest rate. That these changes occur is not strange since the model exhibits two different steady states, one for $\sigma < 1$ and one for $\sigma = 1$.

Savings

Here it is worth nothing that under case 4 (eFC pFC) and case 6 (eFC pFR), there is no effect on the aggregate saving by the workers when $\sigma = 1$, according to table A.1. This will imply that there is no effect on factor prices, presented in table A.2. That this is the case can be shown analytically. What happens is that the physical capital to human capital ratio remains intact, if the education system has a fixed contribution rate, when $\sigma = 1$. This means that the per worker income that the boom generation receives will vary inversely with the demographic shock and thus, does not affect the aggregate savings by the boom generation. Further, if the pension system is then of an FC or an FR type, $\sigma = 1$ will imply that the denominator, i.e. λ_t, in the savings function equation (23) will not change.

Lifetime consumption

Here, I also present the results regarding discounted lifetime consumption. The interesting results about the generation's lifetime consumption are that the boom generation could be better off than the parent generation. From table A.1, it emerges that this result holds even when varying σ. Another interesting result is that a baby boom shock implies that there are no winners in terms of net discounted lifetime income. At most, the parent generation can remain unaffected; however, this implies a large burden on future generations.

Social welfare

From tables A.4 and A.5, it emerges that the results about social welfare vary as expected with σ, and that the results are fairly robust to changes in β_S.

Table A.1: Percentage deviation from steady state outcome, for variables C_{t-1} and $S_{w,t}$, t periods after the shock, when varying σ.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$
$\sigma = 0.16$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-3.28	0.80	-1.30	2.87	-2.01	-2.01	0.00	0.00	-2.87	-0.10	-0.79	1.74
1	-2.34	17.20	-3.57	15.71	-6.53	12.16	-7.71	10.74	-3.99	16.45	-5.55	14.74
2	-1.40	18.32	-1.95	17.66	-2.93	16.48	-3.47	15.83	-0.84	19.37	-1.36	18.86
3	-0.62	19.26	-0.86	18.96	-1.30	18.44	-1.54	18.15	-0.10	19.96	-0.24	19.85
15	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
25	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 0.34$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-3.28	0.80	-1.30	2.87	-2.01	-2.01	0.00	0.00	-2.87	-0.10	-0.63	1.38
1	-2.34	17.20	-5.66	13.21	-6.53	12.16	-9.71	8.35	-3.93	16.30	-8.10	11.64
2	-2.08	17.50	-3.96	15.25	-3.71	15.55	-5.56	13.33	-1.76	18.29	-3.69	16.28
3	-1.17	18.59	-2.24	17.31	-2.10	17.49	-3.15	16.22	-0.78	19.23	-1.65	18.35
15	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
25	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 0.6$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-3.28	0.80	-1.30	2.87	-2.01	-2.01	0.00	0.00	-2.87	-0.10	-0.38	0.84
1	-2.34	17.20	-8.59	9.69	-6.53	12.16	-12.52	4.98	-3.85	16.09	-11.61	7.10
2	-3.06	16.32	-7.66	10.81	-4.83	14.20	-9.34	8.79	-3.06	16.66	-8.06	11.08
3	-2.26	17.29	-5.67	13.19	-3.57	15.72	-6.94	11.67	-2.14	17.64	-5.58	13.84
15	-0.06	19.93	-0.14	19.83	-0.09	19.89	-0.17	19.79	-0.02	19.98	-0.06	19.93
25	0.00	20.00	-0.01	19.99	0.00	20.00	-0.01	19.99	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 0.8$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-3.28	0.80	-1.30	2.87	-2.01	-2.01	0.00	0.00	-2.87	-0.10	-0.19	0.43
1	-2.34	17.20	-10.79	7.06	-6.53	12.16	-14.62	2.46	-3.79	15.92	-14.19	3.56
2	-3.81	15.42	-11.07	6.72	-5.68	13.18	-12.80	4.64	-4.03	15.37	-12.12	5.98
3	-3.31	16.02	-9.67	8.40	-4.95	14.07	-11.19	6.57	-3.49	15.98	-10.34	8.04
15	-0.60	19.28	-1.81	17.83	-0.91	18.91	-2.11	17.47	-0.47	19.46	-1.44	18.34
25	-0.14	19.83	-0.44	19.48	-0.22	19.74	-0.51	19.39	-0.09	19.90	-0.27	19.69
50	0.00	20.00	-0.01	19.99	-0.01	19.99	-0.01	19.98	0.00	20.00	0.00	20.00
$\sigma = 1$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-3.28	0.80	-1.30	2.87	-2.01	-2.01	0.00	0.00	-2.87	-0.10	0.00	0.00
1	-2.34	17.20	-12.93	4.49	-6.53	12.16	-16.67	0.00	-3.73	15.76	-16.67	0.00
2	-4.56	14.53	-14.91	2.11	-6.53	12.16	-16.67	0.00	-4.97	14.05	-16.67	0.00
3	-4.56	14.53	-14.91	2.11	-6.53	12.16	-16.67	0.00	-5.07	13.92	-16.67	0.00
15	-4.56	14.53	-14.91	2.11	-6.53	12.16	-16.67	0.00	-5.07	13.91	-16.67	0.00
25	-4.56	14.53	-14.91	2.11	-6.53	12.16	-16.67	0.00	-5.07	13.91	-16.67	0.00
50	-4.56	14.53	-14.91	2.11	-6.53	12.16	-16.67	0.00	-5.07	13.91	-16.67	0.00

Table A.2: Percentage deviation from steady state outcome, for variables R_t and w_t, t periods after the shock, when varying σ.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t
$\sigma = 0.16$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	12.32	-5.65	8.68	-4.07	14.46	-6.53	10.74	-4.97	13.00	-5.93	9.48	-4.43
2	0.96	-0.48	1.68	-0.83	3.85	-1.87	4.60	-2.22	1.36	-0.67	2.22	-1.09
3	0.79	-0.39	1.11	-0.55	1.68	-0.83	2.00	-0.98	0.17	-0.09	0.39	-0.19
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.34$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	12.32	-5.65	6.33	-3.02	14.46	-6.53	8.35	-3.93	13.00	-5.93	7.37	-3.49
2	0.26	-0.13	1.80	-0.89	3.02	-1.47	4.59	-2.22	0.70	-0.35	2.64	-1.29
3	0.93	-0.46	1.79	-0.89	1.68	-0.83	2.55	-1.25	0.41	-0.20	1.06	-0.53
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.6$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	12.32	-5.65	3.02	-1.48	14.46	-6.53	4.98	-2.40	13.00	-5.93	4.39	-2.13
2	-0.75	0.37	1.02	-0.51	1.82	-0.90	3.63	-1.77	-0.23	0.12	2.37	-1.17
3	0.83	-0.41	2.15	-1.06	1.33	-0.66	2.65	-1.30	0.46	-0.23	1.55	-0.77
15	0.02	-0.01	0.05	-0.03	0.03	-0.02	0.06	-0.03	0.01	0.00	0.02	-0.01
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.8$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	12.32	-5.65	0.55	-0.27	14.46	-6.53	2.46	-1.21	13.00	-5.93	2.17	-1.07
2	-1.51	0.76	-0.32	0.16	0.90	-0.45	2.13	-1.05	-0.95	0.48	1.48	-0.73
3	0.52	-0.26	1.58	-0.78	0.78	-0.39	1.84	-0.91	0.27	-0.13	1.22	-0.60
15	0.09	-0.05	0.28	-0.14	0.14	-0.07	0.33	-0.16	0.05	-0.03	0.16	-0.08
25	0.02	-0.01	0.07	-0.03	0.03	-0.02	0.08	-0.04	0.01	0.00	0.03	-0.01
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 1$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	12.32	-5.65	-1.87	0.95	14.46	-6.53	0.00	0.00	13.00	-5.93	0.00	0.00
2	-2.27	1.16	-2.27	1.16	0.00	0.00	0.00	0.00	-1.66	0.84	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12	0.06	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.3: Compensating variation as a percentage of first-period consumption, when varying σ.

t	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.16$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	1.57	2.83	4.69	6.01	2.57	4.08
1	-3.17	-4.72	-8.20	-9.67	-5.57	-7.58
2	-1.77	-2.46	-3.69	-4.38	-1.24	-1.94
3	-0.78	-1.09	-1.64	-1.95	-0.18	-0.37
15	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.34$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	1.57	1.56	4.69	4.74	2.57	3.21
1	-3.58	-7.98	-8.67	-12.86	-5.87	-11.40
2	-2.78	-5.27	-4.94	-7.39	-2.57	-5.27
3	-1.56	-2.98	-2.79	-4.20	-1.13	-2.37
15	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.6$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	1.57	-0.30	4.69	2.89	2.57	1.95
1	-4.19	-13.14	-9.37	-17.90	-6.31	-17.18
2	-4.41	-10.98	-6.94	-13.38	-4.62	-11.98
3	-3.25	-8.15	-5.13	-9.95	-3.20	-8.31
15	-0.08	-0.20	-0.13	-0.25	-0.03	-0.09
25	0.00	-0.01	-0.01	-0.01	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.8$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	1.57	-1.75	4.69	1.45	2.57	0.97
1	-4.66	-17.45	-9.91	-22.13	-6.64	-21.83
2	-5.79	-16.77	-8.63	-19.39	-6.29	-18.67
3	-5.03	-14.65	-7.51	-16.96	-5.39	-15.94
15	-0.92	-2.75	-1.38	-3.20	-0.73	-2.23
25	-0.22	-0.66	-0.33	-0.77	-0.14	-0.42
50	-0.01	-0.02	-0.01	-0.02	0.00	-0.01
$\sigma = 1$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	1.57	-3.22	4.69	0.00	2.57	0.00
1	-5.12	-22.07	-10.45	-26.67	-6.98	-26.67
2	-7.29	-23.85	-10.45	-26.67	-8.03	-26.67
3	-7.29	-23.85	-10.45	-26.67	-8.11	-26.67
15	-7.29	-23.85	-10.45	-26.67	-8.12	-26.67
25	-7.29	-23.85	-10.45	-26.67	-8.12	-26.67
50	-7.29	-23.85	-10.45	-26.67	-8.12	-26.67

Table A.4: Ranking according to social welfare, with different social discount factors when varying σ.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.17$						
$\beta_s = \beta$	3	1	6	5	4	2
$\beta_s = 1/n$	4	2	6	5	3	1
$\beta_s = 0.5\beta$	5	2	4	1	6	3
$\sigma = 0.34$						
$\beta_s = \beta$	1	3	5	6	2	4
$\beta_s = 1/n$	2	4	5	6	1	3
$\beta_s = 0.5\beta$	4	1	6	2	5	3
$\sigma = 0.6$						
$\beta_s = \beta$	1	4	3	6	2	5
$\beta_s = 1/n$	2	5	3	6	1	4
$\beta_s = 0.5\beta$	1	4	3	6	2	5
$\sigma = 0.8$						
$\beta_s = \beta$	1	4	3	6	2	5
$\beta_s = 1/n$	2	5	3	6	1	4
$\beta_s = 0.5\beta$	1	4	3	6	2	5
$\sigma = 1$						
$\beta_s = \beta$	1	4	3	6	2	5
$\beta_s = 1/n$	1	4	3	6	2	5
$\beta_s = 0.5\beta$	1	4	3	6	2	5

Table A.5: Ranking according to expected social welfare, $E[W]$, with different social discount factors, when varying σ.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.16$						
$\beta_s = \beta$	3	1	6	5	4	2
$\beta_s = 1/n$	3	4	5	6	1	2
$\beta_s = 0.5\beta$	3	1	6	5	4	2
$\sigma = 0.32$						
$\beta_s = \beta$	2	3	5	6	1	4
$\beta_s = 1/n$	2	5	4	6	1	3
$\beta_s = 0.5\beta$	2	3	5	6	1	4
$\sigma = 0.6$						
$\beta_s = \beta$	2	5	3	6	1	4
$\beta_s = 1/n$	2	5	3	6	1	4
$\beta_s = 0.5\beta$	2	5	3	6	1	4
$\sigma = 0.8$						
$\beta_s = \beta$	2	4	3	6	1	5
$\beta_s = 1/n$	2	5	3	6	1	4
$\beta_s = 0.5\beta$	2	4	3	6	1	5
$\sigma = 1$						
$\beta_s = \beta$	2	4	3	6	1	5
$\beta_s = 1/n$	3	6	2	5	1	4
$\beta_s = 0.5\beta$	2	4	3	6	1	5

Essay 2

Pension design when fertility fluctuates: the role of education and capital mobility

1 Introduction

Unfunded pension systems are sensitive to changes in dependency ratios. Given the ongoing shift in the demographic structure, particularly the increase in old age dependency, several countries have re-designed their pension systems. While still mainly unfunded, they now have a *fixed contribution rate* instead of a *fixed replacement rate*. Is a *fixed contribution rate* the preferred design when considering changes in dependency ratios?

Changes in old age dependency ratios can arise from three main sources: fertility, mortality, and migration. This paper deals with changes arising from fertility fluctuations. Such fluctuations are a major source of a changing demographic structure.[1] Moreover, they affect economic conditions throughout the life-cycle, where changing old age dependency is merely one part.

Initially, fertility shocks alter the young age dependency ratio and thus, affect another important intergenerational transfer system, the education system. Just as old age dependency affects the pension system, young age dependency will affect the education system. Further down the life-cycle, fertility fluctuations affect factor prices via changes in the capital labor ratio. How much factor prices change depends on the degree of capital mobility. In the final stage of the life-cycle, the old age

[1] Among others, Cutler, Poterba, Sheiner, and Summers (1990) stress the fact that fertility fluctuations are important.

dependency ratio shifts, which affects the pension system. Thus, it is necessary to account for the effects on the education system and for the degree of capital mobility when trying to find the optimal pension design with respect to fertility fluctuations.

From the pension literature, it is known that alternative unfunded pension schemes have very different distributional properties.[2] Changes in the old age dependency ratio alter contributions from workers, benefits to retirees, or both. Which of these alternatives that occurs depends on how the pension system is designed. In this paper, I analyze convex combinations between a pure *fixed contribution rate* (FC scheme) and a pure *fixed benefit rate* (FB scheme). In the pure FC scheme, workers always pay a certain fraction of their income to the system, irrespective of the dependency ratio. In a pure FB scheme, retirees are guaranteed a certain fraction of current workers' income. Convex combinations between these two extremes imply that both workers and retirees are affected by changes in dependency ratios.

Similar issues arise in the education system since it is also mainly financed by intergenerational transfers. A changing young age dependency ratio alters the contributions from workers, the education received by children, or both. This is unavoidable as long as the education system is financed through intergenerational transfers. I use the same approach as in the pension literature to distinguish between the different adjustment possibilities. The distinction between a *fixed contribution rate* and a *fixed benefit rate* is thus also applied to the education system. An FB scheme for the education system implies that the benefit per child is independent of the number of children as compared to the number of workers. An FC education scheme implies that the contribution per worker is independent of the young age dependency ratio. As for the pension system, I will allow the education system to be a convex combination between these two extremes. Convex combinations between FC and FB for the education system determine how workers and children share the effects of varying young age dependency.

Assuming that the education system is a convex combination between these two extreme types is not as restrictive as it may seem. This way of modelling does not hinge on a particular rationale for why the education system is financed via intergenerational transfers. It simply uses the budget restriction, which must also be satisfied by any endogenously determined education system. What the analysis excludes are responses such that *total* spending on education falls, or that the benefit *per* student increases, when there are many to educate. It is hard to imagine any endogenous response that yields such an outcome. Even if we do not see any explicit

[2]See, for instance, Hassler and Lindbeck (1997), Thøgersen (1998), Lindbeck (2000) and Wagener (2003).

intergenerational contract for the education system, as we do for the pension system, it seems reasonable that the actual response is somewhere between the analyzed extremes.

How fertility affects factor prices is determined by capital mobility. If there is no capital mobility, then the interest rate and the wage will change with changes in capital intensity. This is also a distributional matter between generations. Changes in the interest rate, which affect the retired, and changes in the wages, which affect the workers, are of opposite signs. With perfect capital mobility, however, the capital intensity remains unchanged and thus, factor prices are unaltered. Allowing for different degrees of capital mobility makes it possible to investigate the intermediate cases.

The aim of this paper is to find the unfunded pension design that yields the highest expected *ex ante* welfare when fertility fluctuates, while taking account of the financing of education and the degree of capital mobility. The *ex ante* approach corresponds to finding the pension scheme the individuals would choose *behind the veil of ignorance*, i.e. before knowing whether they will belong to a large or a small generation. These results are then compared to the market outcome without state intervention, when children are capable of making contractual arrangements with adults.

The main result is that the preferred pension design crucially depends on the degree of capital mobility. A pure FB pension scheme maximizes the expected welfare for economies with limited capital mobility. After capital mobility has reached a threshold level, the preferred pension design gradually moves towards the pure FC scheme. The pure FC scheme is only preferred for the small open economy.

In the case of limited capital mobility, a large working generation creates a gain in the pension system, by decreasing the old age dependency ratio, and depresses the wage due to capital dilution. A pure FB pension scheme compensates the large generation for the lower wage by allocating the gain in the pension system to them. Put differently, the reason why the FB pension scheme is preferred is that the return on physical capital will be negatively correlated with the return on human capital.

For a small open economy, where factor prices are unaffected by fertility changes, the optimal pension design is a pure FC scheme if the education has a pure FB design. With these designs, the individuals in the small open economy are unaffected by fertility changes. The education system and the pension system are both sensitive to fertility fluctuations, but the two effects cancel each other out with these designs.

When analyzing public transfer systems, it is important to have in mind why these systems exist and what is their purpose. One rationale for having these systems is that the immaturity of children prevents efficient arrangements between them and the adults (e.g. Becker and Murphy, 1988; Rangel, 2003; Boldrin and Montes, 2005). Thus, it is of interest to compare the allocations that occur with exogenous systems with the market outcome that would arise if children could enter into efficient contracts with adults. Such analysis is conducted at the end of the paper. It is shown that the payments from the workers to the retired which arise in the market outcome resemble a *fixed contribution rate*. That is, the workers transfer a fixed share of their income to the retired. This holds for all degrees of openness.

Previous studies that have investigated preferred pension designs can be divided into two strands. The first includes studies focusing on fertility fluctuations in a closed economy setup (e.g. Smith, 1982; Blomquist and Wijkander, 1994; Bohn, 2001). These studies find that a large cohort faces less favorable factor prices as compared to surrounding generations. To counter this capital intensity effect, the pension system should have a varying contribution rate. Second, there are studies focusing on factor price uncertainty in a small open economy setup (e.g. Thøgersen, 1998; Wagener, 2003). The conclusion from these studies is inconclusive from an *ex ante* perspective.[3]

This paper differs from previous studies, since it allows for varying degrees of openness and takes into account the financing of education. I find that previously obtained results for the closed economy hold irrespective of how the education system is designed. Moreover, this study identifies the preferred *ex ante* pension design for all degrees of openness. The previous result that the FB pension scheme is preferred rests entirely on the closed economy assumption. Once we include the education system, it is possible to find the optimal pension design for the small open economy. This was previously not possible and the FC design turns out to be optimal for the small open economy.

This study is far from the first to explore the interaction between education and pensions (e.g. Burns, 1936; Pogue and Sgontz, 1977; Becker and Murphy, 1988; Bommier, Lee, Miller, and Zuber, 2004; Boldrin and Montes, 2005). The novelty consists of explicitly modelling the adjustment possibilities facing the education system when the young age dependency ratio changes. Moreover, this is done in the same way as previously done in the pension literature. The distinctions used in the pension literature can easily be adopted to the education system, since both are intergenerational transfer systems.

The remainder of this paper is organized as follows. Section 2 presents the gen-

[3]From an *ex post* perspective, when the uncertainty has been realized, Wagener (2003) shows that the *fixed replacement rate* is preferred.

eral equilibrium model. In section 3, the model is calibrated and the steady state results are presented. Section 4 presents the results, while section 5 compares these results to the market outcome, where the intergenerational contracts are endogenously determined. Section 6 contains some final remarks.

2 The model

The framework consists of a three-period overlapping generations model, with four main components: individuals who maximize their life-time utility, firms which maximize their profit, an international capital market, and the intergenerational transfer systems for education and pension. What will be considered is a one-period shock to fertility. There is also a standard intergenerational welfare function, similar to that applied in Boadway, Marchand, and Pestieau (1991), which will be used to identify the optimal pension design.

2.1 Intergenerational transfers

The aim of intergenerational transfers is to provide support during life-cycle periods with no labor activity. There are both public and private channels through which these intergenerational transfers flow from the active to the inactive population. Two major public channels to the elderly are the unfunded pension transfers and the medicare system. In this paper, I focus on the pension system although similar reasoning and methodology could be applied for the medicare system. The pension system is an explicit intergenerational contract, since it has a stated benefit formula regulated by law. This formula specifies what the benefits/contributions will be under different circumstances. For this reason, it is not surprising that economist have argued about the design, i.e. the formula.

As noted by Bommier, Lee, Miller, and Zuber (2004), education is usually considered to be an investment in human capital and is seldom viewed as intergenerational transfers. However, they argue against the common characterization and note that education was the largest public transfer program in the U.S. in 2000; education amounted to 4.5 percent of GNP, followed by OASI (pensions and survivors benefits) at 3.7 percent, and Medicare at 2.1 percent. Moreover, it is clear from Lee, Lee, and Mason (2006) that public spending on education dominates in the U.S. and that it is mainly consumed by age groups 5 to 25. This means that education is provided by intergenerational transfers and not by asset reallocation over the life-cycle. Even though the education system does not have an explicitly stated intergenerational formula, it is still based on intergenerational transfers. This implies that the education system must adjust when the young age dependency ratio changes. The lack of an explicitly stated adjustment mechanism might explain why this issue has received little attention.

Similar to other applications which suffer from a time inconsistency problem, it is desirable that these systems or institutions are governed by laws which seldom change; in the spirit of Kotlikoff, Persson, and Svensson (1988). This paper investigates different types of laws that can govern these transfers.

Modelling the transfers

The OLG model consists of three periods: one period when young, one when working, and one when retired. The young receive contributions from the working population via the education system, and the retired receive contributions from the working population via the pension system. For the systems to be pure intergenerational transfers, it is necessary that the budgets be balanced in each period. Assuming a period-by-period balanced budget for each system separately makes it possible to state the transfers in period t as:[4]

$$b_{E,t}N_t = d_{E,t}N_{t-1}, \tag{1}$$

$$b_{P,t}N_{t-2} = d_{P,t}N_{t-1}, \tag{2}$$

where $b_{E,t}$ denotes the per child benefit from the education system, $d_{E,t}$ is the contribution per worker to the education system, $b_{P,t}$ is the benefit per retired from the pension system, and $d_{P,t}$ denotes the contribution per worker to the pension system. These are indexed with subscript t to denote that the transfer occurs in period t. The size of each generation is denoted by N, where subscript t indicates in which period the generation is born.

So far, the contributions and benefits have not been related to the level of income in society. In a world with growing income over time, it would not make sense to have fixed benefits/contributions over time. It is reasonable to relate the benefits/contributions to income, where income refers to the labor income of the working generation.

Let $\tilde{w}_t$ denote the labor income of workers in period t, and let $\tau_{E,t}$ and $\tau_{P,t}$ denote the *contribution rate* devoted to financing the education and the pension system, respectively. The contribution from the workers, $d_{i,t}$, where $i = E, P$, can then be

[4]The assumption regarding two separate systems is mainly based on the fact that the existing social security programs have a very weak connection with the education system, if any.

stated as:

$$d_{i,t} = \tilde{w}_t \tau_{i,t}. \tag{3}$$

The received benefits, $b_{i,t}$, can also be related to the labor income of the working population according to:

$$b_{i,t} = \tilde{w}_t \gamma_{i,t}, \tag{4}$$

where $\gamma_{i,t}$ are the *benefit rates* in the transfer systems. The benefit rates are the fraction of active workers' income that each child/retired receives.[5]

The period-by-period balanced budget constraints for the two transfer systems can now be rewritten as:

$$\gamma_{E,t} = \tau_{E,t}/n_t, \tag{5}$$

$$\gamma_{P,t} = \tau_{P,t} n_{t-1}, \tag{6}$$

where n_t denotes the young age dependency ratio in period t, i.e. N_t/N_{t-1}, and hence, the old age dependency ratio, N_{t-2}/N_{t-1}, equals n_{t-1}^{-1}. Changes in dependency ratios must affect either the contribution rate or the benefit rate, or both.

Various intergenerational transfer schemes differ in how the benefits and the contributions respond to changes in dependency ratios. The difference between the schemes can be understood from the balanced budget restrictions. From equations (5) and (6), two simple schemes emerge. Either the benefit rate is fixed, $\gamma_{i,t} = \gamma_i$, or the contribution rate is fixed, $\tau_{i,t} = \tau_i$. These schemes will simply be referred to as *fixed benefit rate, FB,* and *fixed contribution rate, FC.* These are the two extreme cases. To allow for convex combinations between the extreme cases, the same approach as in Wagener (2004) will be used.[6] The benefit formula for the education and the pension system are stated as:

$$b_{E,t} = \tilde{w}_t \gamma_E(\phi_E + (1 - \phi_E)n/n_t), \tag{7}$$

$$b_{P,t} = \tilde{w}_t \gamma_P(\phi_P + (1 - \phi_P)n_{t-1}/n), \tag{8}$$

where n is the steady state population growth and $\phi_i \in (0, 1)$ indicates under which scheme the systems operate. The extreme cases are $\phi_i = 0$, which corresponds to a

[5]The term benefit rate is not to be confused with the term *replacement rate*. The benefit rate is a theoretical abstraction and is also used in Lindbeck (2000), although not using the same term. In the pension literature, it sometimes occurs that the replacement rate refers to the fraction of current income (what is referred to as the benefit rate in this paper). This is, however, conceptually obscure since the benefits of the present pensioners do not replace the wages of present workers. Augusztinovics (2000), among others, has also pointed at this misuse i the literature.

[6]There are some differences as compared to Wagener (2004). Since the pension system is a political process in his model, it means that it is not *a priori* fixed. Furthermore, he models the convex combination between the FC scheme and the fixed replacement rate.

pure FC scheme, and $\phi_i = 1$, which corresponds to a pure FB scheme. In the latter case, γ_i is simply the fixed benefit rate. When $\phi_i = 0$, $\gamma_E n$ represents the fixed contribution rate in the education system, and γ_E/n becomes the fixed contribution rate in the pension system. The intermediate cases, when $\phi_i \in [0,1]$, are convex combinations between the extreme cases. Parameter ϕ_i determines to which extent changes to dependency ratios affect contributors versus beneficiaries.

The extreme points are quite natural for the education system, but a note is warranted for the pension system. Usually, the *fixed replacement rate, FR*, is considered as the opposite to the FC scheme. In this case, the benefits received in the pension system are related to *previous* income instead of current income, i.e. the income from one's own active life. The main reason why the FR scheme is excluded is because it removes all incentives for workers to invest in the human capital of coming generations. This scheme could never mimic the market outcome.[7]

2.2 Individuals

Individuals live for three periods. During young age, children invest all their time (one unit) in human capital accumulation, from which they all receive the same utility. Children's time input is combined with education benefits, provided by the workers, to develop their human capital which will be used when working. Any difference in the per child education benefit will thus not affect the utility in the first period of life, but will instead alter the human capital. In the next period, when working, all individuals inelastically supply their effective labor, the product of their one unit of time and their human capital and receive labor income. A fraction of this income finances the education system and the pension system; the remaining part is divided between savings and consumption. In the third and final period, individuals are retired and consume their own savings and income from the pension system.

Since all generations gain the same utility when young, this period is suppressed. The life-time utility of an individual, belonging to generation $t-1$, is assumed to be additively separable according to:

$$U_{t-1} = \ln c_{w,t} + \beta \ln c_{r,t+1}, \tag{9}$$

[7]Considering shocks to productivity gives another reason for choosing the FB scheme. The FR scheme exposes individuals to the same productivity realization, both when working and when being retired. This is a disadvantage that is well known in the literature. The FB scheme, however, bases the retirement income on the succeeding productivity realization, in the same way as does the FC scheme. Note also that when leaving productivity uncertainty aside, there are only slight differences in the results between the FR scheme and the FB scheme, while the former is somewhat more complicated to analyze.

where β is the subjective discount factor and thus, a measure of the individual's impatience to consume. Consumption per worker in period t is denoted $c_{w,t}$, while consumption per retired in period t is denoted $c_{r,t}$.

Denote by h_t the human capital of generation $t-1$. This is a product of the benefits from the education system in period $t-1$, i.e.:

$$h_t = b_{E,t-1}^{\sigma}, \tag{10}$$

where $\sigma \in (0,1]$ measures the returns to scale in the production of human capital. The human capital determines the effective labor supply for each individual in period t. The individuals take their human capital, wages, the interest rate, the tax rate, and the benefits in the pension system, as given. Their only decision variable is savings, which they choose in order to maximize life-time utility, according to equation (9), subject to the following budget constraints:

$$c_{w,t} = (1-\tau_t)\, w_t h_t - s_t, \tag{11}$$

$$c_{r,t+1} = R_{t+1} s_t + b_{P,t+1}. \tag{12}$$

s_t denotes the per worker savings in period t, w_t is the wage for one unit of effective labor, and R_{t+1} denotes the gross interest rate on savings between period t and $t+1$. Further, τ_t denotes the total tax rate used in financing the education and the pension systems, that is $\tau_t = \tau_{P,t} + \tau_{E,t}$.

The individuals use their savings either for investments in domestic firms or to lend to the rest of the world (or borrow). So we have:

$$s_t = i_t + a_t, \tag{13}$$

where i_t is the investments made in domestic firms, and a_t is the amount lent to the rest of the world, which may be negative.

Maximizing the objective function (9) under the constraints (11) and (12) yields the familiar intertemporal Euler equation:

$$c_{r,t+1} = \beta R_{t+1} c_{w,t}. \tag{14}$$

2.3 Production

The aggregate production function in the economy is assumed to be of Cobb-Douglas type and homogeneous of degree 1. Production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where L_t is aggregate effective labor, i.e. $L_t = h_t N_{t-1}$, K_t is the aggregate capital stock at the beginning of period t, and A is a scaling parameter. The capital stock K_t fully depreciates during the production process. Defining production in terms of output per worker yields:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}, \tag{15}$$

where $y_t = Y_t/N_{t-1}$, and $k_t = K_t/N_{t-1}$.

The prices of factor inputs are obtained from the firms' maximization problem, and since perfectly competitive factor markets are assumed, prices of factors equal their marginal products, that is:

$$R_t = A\alpha k_t^{\alpha-1} h_t^{1-\alpha}, \tag{16}$$

$$w_t = A(1-\alpha) k_t^{\alpha} h_t^{-\alpha}, \tag{17}$$

where R_t is the price of physical capital, and w_t is the price per unit of human capital, both in period t.

2.4 Capital mobility

Capital mobility determines the effect of demographic changes on factor prices. The extreme cases are zero capital mobility, i.e. $a \equiv 0$, and full capital mobility such that a can fluctuate freely so as to keep the rate of return on capital constant. These cases correspond to the the closed economy and the small perfectly open economy, respectively. By allowing for imperfect capital mobility, it is possible to analyze the intermediate cases between a small perfectly open economy and the closed economy. This is of interest since not many economies are considered to be closed and there is ample evidence of the financial integration being imperfect (Obstfeld and Rogoff, 2000).

There are many possible ways of modelling imperfect capital mobility or, more correctly, why capital flows will be less than required to keep the interest rate constant. In this paper, limited capital mobility is modelled in terms of risk-premium, similar to the specification used in Schmitt-Grohé and Uribe (2003). The interest rate is given by:

$$R_t = R^w - \rho \frac{(1-\phi_K)}{\phi_K} a_{t-1}, \tag{18}$$

where R^w is the constant world interest rate, $\rho > 0$ is a scale parameter, and $\phi_K \in (0,1)$ is a measure of capital mobility, or degree of openness. When $\phi_K = 1$, then there is no risk-premium and a can vary freely in order to keep the marginal return on investment constant, and equal with the world interest rate, R^w. If $\phi_K = 0$, then the risk-premium is infinite which leads to zero capital mobility, i.e. $a = 0$.

In essence, there is a cost associated with deviating from the autarky interest rate, how large this cost is depends on ϕ_K. Here, the risk-premium argument was used to limit capital mobility, but other specifications would yield similar results.[8]

2.5 Equilibrium

Given the initial capital stock, $k_0 > 0$, the initial human capital stock, $h_0 > 0$, and population growth, $\{n_t\}_{t=0}^{\infty}$, a competitive equilibrium for this economy is a sequence of: prices $\{w_t, R_t\}_{t=0}^{\infty}$, allocations $\{c_{w,t}, c_{r,t}, i_t, a_t\}_{t=0}^{\infty}$, human and physical capital stocks $\{k_t, h_t\}_{t=0}^{\infty}$, and benefit rates and tax rates $\{\gamma_{E,t}, \gamma_{P,t}, \tau_{E,t}, \tau_{P,t}\}_{t=0}^{\infty}$, such that individuals maximize their utility, firms maximize their profits, the budgets of the transfer systems are balanced, and markets clear. The market clearing condition can be reduced to:

$$k_{t+1} = i_t / n_t, \tag{19}$$

where i_t must be such that equation (13) holds.

The saving decisions characterize the equilibrium, since they define the equilibrium trajectory for $\{k_t\}_{t=0}^{\infty}$ via eq. (19) and (13). Eqs. (11)-(14) and (16)-(19) yield the following saving function in equilibrium:

$$s_t = \frac{\beta\alpha\left(1-\tau_t\right)w_t h_t}{\lambda_t} + \frac{\left(1-\alpha\right)\gamma_{P,t+1}}{\lambda_t n_t} a_t, \tag{20}$$

where $\lambda_t = \alpha\left(1+\beta\right) + \left(1-\alpha\right)\gamma_{P,t+1}/n_t$. The savings are divided between investments, i_t, and capital transactions with the rest of the world, a_t, in order to equalize the marginal return on investments with the interest rate; implying that both equation (16) and equation (18) hold.

2.6 The intergenerational welfare function

To obtain a compact measure of how all generations are affected by a fertility shock, welfare is defined as:

$$W = E\left[\sum_{t=1}^{\infty} \psi_t U_t\right]. \tag{21}$$

This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.

There are different views on how the per capita life-time utility of generation t should be weighted. The question is if utility should be weighted by generation

[8]Trade costs as in Obstfeld and Rogoff (2000) would require a more elaborated model, while the outcome for factor prices would be similar.

size, and whether the utility of future generations should be discounted. It seems more or less necessary to account for generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will be included and the weighting factor will be the following:

$$\psi_t/\psi_{t-1} = \beta_s n_t, \tag{22}$$

where β_s is the social discount rate. In the simulation, the social discount rate will be set equal to the individual's discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth, the discount rate should not exceed the inverse of population growth; if it does, then future generations will get an ever increasing impact on the welfare function, due to their larger number.[9]

3 Simulation and calibration

3.1 Fertility rate

What is considered is a one-period shock to the fertility rate. It is, however, uncertain if the shock will be positive or negative. This is an *ex ante* analysis similar to the one applied in Ball and Mankiw (2001).

The fertility rate can be stated as $n_{t+j} = n \; \forall j \neq 0$ and $n_t = n(1+x)$, where x is either a positive or a negative disturbance. The steady state gross population growth, n, will be set to 1.3, based on the annual average for the U.S. between 1910-2001.[10] Note that each period corresponds to about 27 years. The size of the disturbance is of no importance and it will be set to 20 percent, i.e. $x = \{-0.2, 0.2\}$.[11] Changes in these numbers do not alter the qualitative results.

The demographic structure used in the simulation can be stated as $n_{t+j} = 1.3 \; \forall j \neq 0$ and $n_t = \{1.04, 1.56\}$. This implies that generation t is relatively larger/smaller compared to generation $t-1$ than what it would have been without the shock. The relative size between all other generations is unaltered.

[9] See, for instance, Blanchet and Kessler (1991) and Boadway, Marchand, and Pestieau (1991) for a short comment concerning the weighting problem.

[10] The annual average, from Vital Statistics of the United States, 2001, Volume I, Natality, is approximately 1.01. This implies that per period $n = 1.01^{27}$, since one period corresponds to 27 years.

[11] The size of the shock is well within reason, if one considers the U.S. experience.

3.2 Preferences

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one-year estimate from Auerbach and Kotlikoff (1987) of 0.98 translates to $\beta = 0.6$, since every period represents about 27 years.

3.3 Production

There are two parameters in the production function that need to be calibrated, α and A. The share of capital income in the national product, α, is set to one third. The scale parameter A can be freely chosen since it will not alter the relative outcome in any significant way. To make these results comparable to essay 1, A will be set to 21.6.[12]

The world interest rate is assumed to be equal to the steady state interest rate in autarky, implying that $a = 0$, in steady state. The parameter ρ in equation (18) is chosen such that the interest rate deviates half the distance towards the autarky interest rate after a disturbance when $\phi_K = 0.5$. This also implies that the debt deviates half the distance towards the fully open economy value after a disturbance when $\phi_K = 0.5$.

3.4 The benefit rates and human capital

Choosing the size of the education system and the pension system amounts to calibrating the benefit rates in steady state, γ_E, and γ_P. These are set in order to equalize the rate of return on human capital with the rate of return on physical capital in steady state. The benefit rates are thus set to reproduce the market outcome in steady state.

The rate of return on human capital is determined by σ. This implies that the efficient levels for γ_E and γ_P are functions of σ (and other parameters already calibrated). Since it is an exogenous parameter, one would ideally calibrate it and then infer the optimal values for γ_E and γ_P. Unfortunately, it is very hard to get an accurate measure for σ. Card and Krueger (1992) investigate how the pupil-teacher ratio affects future productivity. Translating their results, via assumptions on how the spending per pupil is related to the pupil-teacher ratio, would yield $\sigma = 0.17$.

Since it is hard to obtain a measure for σ an alternative is to choose one of the benefit rates from data, and then infer the σ that would equalize the return on human capital with the return on physical capital. It is possible to use the existing

[12] This value was chosen in essay 1 to fit the empirical growth rate of the U.S.; here, this is not an issue since only the stationary case is considered.

pension systems as a guideline. According to Pecchenino and Utendorf (1999) the benefit ratio, i.e. the benefit over the average wage ratio in the same period, is 0.42. In reality, however, the ratio between working years and years of retirement is almost 2, while in this three-period model, it is 1. This would lead to a benefit rate in the pension system such that $\gamma_P = 0.21$.

This value for γ_P would require that $\sigma = 0.16$ and $\gamma_E = 0.06$, to equalize the return on the two forms of capital. We see that σ is quite close to the translated estimate from Card and Krueger (1992). What can be said about the benefit rate in the education system? For the U.S., the GDP share for primary and secondary school spending has been approximately 4 percent during the last three decades and the GDP share for higher education is close to 3 percent.[13] The total share of GDP spent on education thus amounts to 7 percent. Note that a $\gamma_E = 0.06$ corresponds to $\tau_E = 0.08$, which is not that far from the reported estimate.

Table 1: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.6
Share of capital income	α	1/3
Efficiency in human capital production	σ	0.16
Steady state benefit rate in the pension system	γ_P	0.21
Steady state benefit rate in the education system	γ_E	0.06
Population gross growth rate	n	1.3
Fertility shock	x	$\pm$ 0.2
Total factor productivity	A	21.6
Constant risk-premium parameter	ρ	0.25

3.5 Steady state

Before the model is used to study the effects of fertility changes, it is useful to report the steady state values for some key variables, according to the calibration in table 1. The magnitude of the interest rate is quite realistic when adjusting for the time

Table 2: Steady state values according to calibration in table 1.

Gross interest rate for capital	R	2.74
Saving rate	S/Y	3.7%
Capital output ratio	k/y	0.12

[13] See Rangazas (2002) p. 947.

length in the model.[14] The saving ratio in life-cycle models with no bequests has notorious difficulties to fit empirical facts.[15] As for other similar models, the saving rate is considerably below the comparable U.S. rate, which is around 6.7 percent. This should not create any large problems as long as the capital output ratio is within a reasonable range. From table 2, the capital output ratio is 0.12, i.e. 3.2 times yearly GDP, which is not far from the comparable U.S. ratio.

4 Simulation Results

The aim is to find the pension design that maximizes the expected welfare according to equation (21). This is done for each possible education system and degree of capital mobility. However, before presenting the expected welfare results, I first study the life-time utilities. Hopefully, this will make it easier to understand the results concerning the expected welfare. Due to the large amount of cases, the presentation of life-time utilities will be limited to only a positive shock under the extreme cases for education, pension, and capital mobility.

4.1 Life-time utilities after a positive shock

Here, only a positive fertility shock is analyzed, so that $n_{t+j} = 1.3 \ \forall j \neq 0$ and $n_t = 1.56$. I will refer to generation t as the baby boom generation, and to generation $t-1$ as the parent generation. Note that the baby boom generation is not larger than generation $t+1$. The relative size between these two generations is unaltered, since the baby boom generation has the same fertility rate as in steady state. Here, where only the extreme cases are considered, we have that $\phi_E,\ \phi_P,\ \phi_K = \{0, 1\}$.

Small open economy, $\phi_K = 1$

In the small open economy, factor prices are unaltered and the generations are only affected through the education system and the pension system. The baby boom leads to a burden in the education system, due to increased young age dependency. Who bears this burden will be a distributional matter between the boom generation (generation t) and its parent generation (generation $t-1$). How the burden is divided between these generations is determined by the education scheme. With a pure FB education design, $\phi_E = 1$, the parents bear the whole burden, and with a pure FC education design, $\phi_E = 0$, the children bear the whole burden.

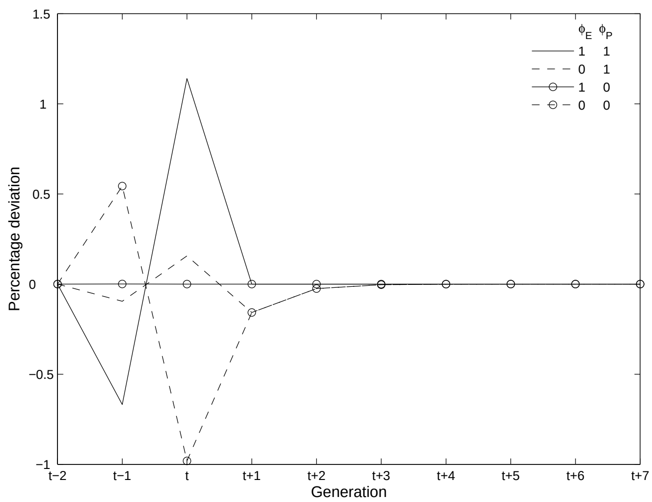
[14] The reported interest rate is the compounded interest rate over 27 years, which on an annual basis becomes 3.8%.

[15] See Kotlikoff and Summers (1981).

The effects in the pension system are similar but of opposite signs; instead of a burden, the baby boom creates a gain. This since there will be many workers as compared to the retirees in period $t+1$, i.e. a temporary decrease in old age dependency. The allocation of this gain is determined by the pension design, and it will also be a distributional matter between the baby boom generation and the parent generation. With a pure FB pension design, $\phi_P = 1$, the boom generation receives the gain, and with a pure FC pension design, $\phi_P = 0$, the parent generation receives the gain.

There are, thus, two effects of opposite signs which are both a distributional matter between the same two generations. If the education system and the pension system are of opposite types ($\phi_P \neq \phi_E$) the same generation will receive both the burden and the gain. Otherwise, one generation will obtain the gain while the other will obtain the burden. Opposite designs should thus lead to less distortion from steady state, which is precisely what emerges in figure 1.

Figure 1: Life-time utility as a percentage deviation from the steady state in the small open economy after a positive fertility shock.



Here, we see that opposite designs lead to much less fluctuations than if the education and the pension system have the same design. Moreover, we see that the combination of education FB and pension FC leads to no distortion. This is an important result which will be elaborated on later. Let me first comment on the fact that neither the parent nor the boom generation seem to prefer the opposite design combinations.

The parent generation prefers both systems to be of FB type, while the boom generation prefers both systems to be of FC type. This is not strange since only a positive shock was analyzed. It is what the generations prefer *ex post*. Both generations want to receive the gain from the pension system, while letting the other generation bear the burden from the education system. Note that for a negative fertility shock, the parent generation would prefer both systems to be of FC type, while the boom generation would prefer both systems to be of FB type.

Why does the FB education system and the FC pension system lead to no distortion? This question can be answered by investigating the implicit rate of return over both systems. Let us define this rate according to:

$$R^H_{t+1} = b_{P,t+1}/d_{E,t}. \tag{23}$$

Viewing education contributions as human capital investment and the pension benefits as repayments for this investment makes R^H_{t+1} the implicit gross return on human capital investments. For the small open economy, the only price that can vary is R^H_{t+1}, since R and w are fixed. Table 3 shows how R^H_{t+1} deviates after a fertility shock under the different extreme cases in the open economy. We see that

Table 3: R^H_{t+1}/R for the extreme cases in a fully open economy.

Pension	Education	
	$\phi_E = 0$	$\phi_E = 1$
$\phi_P = 0$	$(n_t/n)^{(1-\sigma)}$	1
$\phi_P = 1$	$(n/n_t)^{\sigma}$	n/n_t

the specific combination $\phi_E = 1$ and $\phi_P = 0$ keeps R^H_{t+1} constant. This combination implies that all prices are constant and hence, that individuals in the small open economy are unaffected by fertility fluctuations. The reason why this cannot be accomplished by the combination $\phi_E = 0$ and $\phi_P = 1$ depends on the non-linearity in the production of human capital.

Note that when viewed separately, the pension system and the education system are sensitive to fertility fluctuations. The design will determine which generation will be affected, but it will not be possible to avoid the effect. When viewed together, however, the effects cancel out for the right design combination. The additional burden the parent generation pays to educate the boom generation is exactly repaid in the next period via the pension system.[16] The parent generation's additional

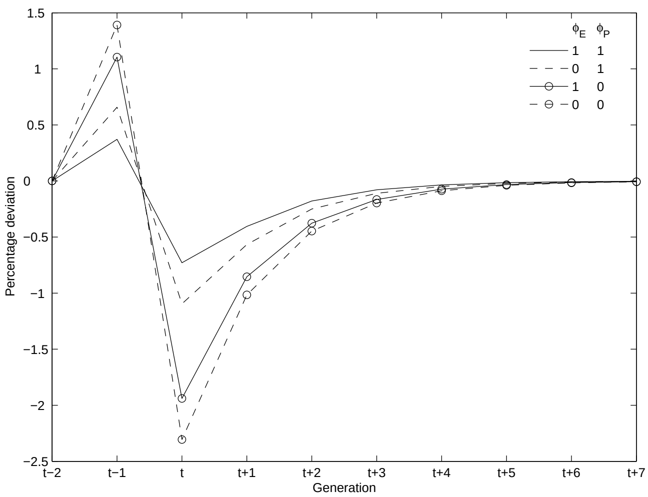
[16]This holds as long as the return to human capital equals the return to physical capital. That

education burden implies that these individuals increase their human capital investments and decrease their physical capital investments. Since these have the same rate of return, it will not affect the parent generation. The reduction in the parent generation's savings will not affect capital intensity due to perfect capital mobility.

Closed economy, $\phi_K = 0$

In this case, fertility changes affect factor prices, besides the effects on the education system and the pension system.[17] When the baby boom generation reaches productive age, there will be capital dilution, since the savings by the parent generation are not sufficient to equip the baby boom generation with the same capital per worker as before. Capital dilution burdens the baby boom through lower wages, while the parent generation receives a higher interest rate on its savings. Accounting for this affect gives life-time utilities according to figure 2.

Figure 2: Life-time utility as percentage deviation from steady state in the closed economy after a positive fertility shock.



Irrespective of how the education system and the pension system are designed, the baby boom generation is worse off, while the parent generation gains. The increased interest rate is more than sufficient to compensate the parent generation

these two should be equal is the efficiency requirement demonstrated in Boldrin and Montes (2005).

[17]The direct effects on the education and pension systems are the same as above. There are also indirect effects through income changes.

for the education burden. The wage decrease for the boom generation exceeds the possible pension gain, leaving this generation worse off.

Here, it is no longer true that opposite designs for the education system and the pension system lead to least deviation from steady state. Rather, it is important that the pension system is of FB type. The combination with education FB and pension FB leads to least deviation from steady state. This combination compensates for the factor price effect by letting the baby boom generation acquire the pension gain, while the parent generation bears the education burden.

From the baby boom analysis, it is not possible to make any conclusion about the preferred pension design. The analysis illustrated the distributional effects under different designs for the open and closed economy. It was possible to see what the different generations prefer *ex post*. Given that individuals are risk-averse, however, it is expected that the designs with least deviation from steady state should rank high from an *ex ante* perspective. To be able to rank the different designs, a welfare measure needs to be adopted, which I turn to next.

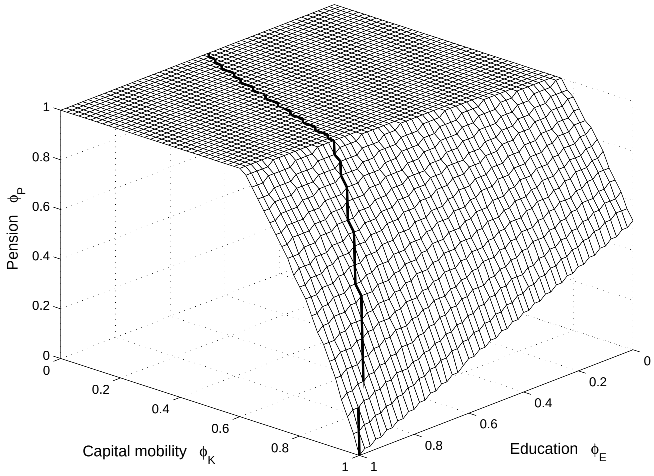
4.2 The *ex ante* pension choice

Here, the generations decide *ex ante*, *behind the veil of ignorance*, which pension design they will choose for a given degree of openness and education design. What must be decided is ϕ_P that maximizes the objective function in equation (21), for each combination of ϕ_E and ϕ_K. The results are presented in figure 3.

Previous studies have showed that the factor price effect dominates the gain in the pension system in a closed economy (e.g. Blomquist and Wijkander, 1994; Bohn, 2001). That is, the pension gain can only alleviate some of the wage decrease from which the baby boom generation suffers. Now, we see that this result holds irrespective of the education design, even with a considerable degree of capital mobility. From the baby boom analysis above, we know that even when including the education burden, it is not possible to reverse the factor price effect in the closed economy. Here, we see that even half the response in factor prices, compared to the closed economy response, is enough to motivate a pure FB pension design, irrespective of the education design. This indicates that the factor price effect is strong as compared to the education effect and the pension effect. The degree of capital mobility is thus crucial for determining the pension design.

The pure FC pension scheme is only preferred in a fully open economy, $\phi_K = 1$, when the education system operates under a pure FB scheme. As previously discussed, this specific combination implies that the generations are completely unaffected by fertility fluctuations. As shown by the black line in figure 3, this specific

Figure 3: Optimal pension combination for different education combinations and degrees of capital mobility.



Note: The black line indicates the ϕ_P and ϕ_E pair that yield the highest expected welfare, at each degree of capital mobility, ϕ_K.

combination gives the highest expected welfare for the small fully open economy.

The findings show that with respect to fertility fluctuations:

- Pure FC pension design can only be motivated for a small fully open economy if the education design is pure FB.
- For the small open economy, the pure FC pension design in combination with pure FB education design leaves the individuals unaffected.
- Pure FB pension design is motivated irrespective of the education design for economies with limited capital mobility.

5 The market outcome

An important motivation for state intervention is to correct for children's inability to enter into agreements with adults (Becker and Murphy, 1988; Rangel, 2003; Boldrin and Montes, 2005). One aim of public transfers would then be to mimic the market outcome that would occur if children were capable of and allowed to enter into contractual agreements. This section investigates what transactions would arise in the market outcome and compares these allocations with the transfers in the exogenous systems.

Children want to finance their human capital accumulation by borrowing from workers. Workers want to set aside assets for future consumption in a way yielding highest return. Workers now decide how much to invest in each asset, i.e. human capital and physical capital. Their maximization problem can be stated as:

$$\max_{s_t, d_{E,t}} U = \ln c_{w,t} + \beta \ln c_{r,t+1}, \tag{24}$$

subject to the following budget constraints:

$$c_{w,t} = w_t h_t - s_t - d_{E,t} - R_t^H b_{E,t-1}, \tag{25}$$

$$c_{r,t+1} = R_{t+1} s_t + R_{t+1}^H d_{E,t}, \tag{26}$$

which leads to the following first-order conditions:

$$c_{r,t+1} = \beta R_{t+1} c_{w,t}, \tag{27}$$

$$c_{r,t+1} = \beta R_{t+1}^H c_{w,t}. \tag{28}$$

From the above, we see that they will invest in the two assets in such a way as to equalize their return, i.e. $R_{t+1} = R_{t+1}^H$. The efficient level of human capital investment would equalize the rate of return on savings with the marginal rate of human capital.

The children (who have no opportunity cost of education, and who receive the same utility irrespective of the quality of their education) are interested in maximizing their future income. They want to maximize:

$$\max_{b_{E,t}} \pi = w_{t+1} h_{t+1} - R_{t+1}^H b_{E,t}, \tag{29}$$

and their first-order condition is:

$$R_{t+1}^H = \frac{\sigma \tilde{w}_{t+1}}{b_{E,t}}. \tag{30}$$

Could this market outcome be replicated by exogenous systems? To be able to analytically answer this question, only the extreme cases for capital mobility are considered. The aim is to find the corresponding benefit rates in the market outcome, for the closed economy and the small open economy.[18]

[18]Note that the benefit rates (and the contribution rates) can always be defined since these are just fractions of workers' income.

5.1 Rates in the "pension system"

Using equation (30) together with equation (23), and noting that the balanced budget restriction in equation (1) now states a market clearing condition, we obtain:

$$\gamma_{P,t+1} = \sigma n_t, \tag{31}$$

$$\tau_{P,t+1} = \sigma. \tag{32}$$

To replicate the market outcome, the pension system should operate under a pure FC pension scheme, i.e. $\phi_P = 0$. This holds irrespective of the degree of openness.

5.2 Rates in the "education system"

Closed economy

Solving for the workers' maximization problem and using the demand condition for human capital according to equation (30) makes it possible to find the respective investment level in physical and human capital, according to:

$$s_t = \frac{\beta\alpha\,(1-\sigma)\,\tilde{w}_t}{(1+\beta)\,(\alpha+\sigma\,(1-\alpha))}, \tag{33}$$

and

$$d_{E,t} = \frac{\sigma\,(1-\alpha)\,\beta\,(1-\sigma)\,\tilde{w}_t}{(1+\beta)\,(\alpha+\sigma\,(1-\alpha))}. \tag{34}$$

From equation (34), it is then possible to obtain the contribution rate according to:

$$\tau_E = \frac{\sigma\,(1-\alpha)\,\beta\,(1-\sigma)}{(1+\beta)\,(\alpha+\sigma\,(1-\alpha))}. \tag{35}$$

Since τ_E is constant, it means that the resulting transfers in the closed economy resemble an education system with a pure FC scheme, i.e. $\phi_E = 0$.

Small open economy

In this case, factor prices are constant, $w_t = w$ and $R_t = R$. From the workers' optimization problem, it then follows that $R^H_{t+1} = R$. Combining this with the children's first-order condition in equation (30) yields:

$$\gamma_{E,t} = \frac{\sigma h_{t+1}}{R h_t}. \tag{36}$$

This condition is satisfied with a constant benefit rate.[19] Assuming that the steady state interest rate for the closed economy equals the world interest rate makes it possible to state the contribution rate for the education system according to:

$$\tau_{E,t} = \frac{\sigma(1-\alpha)\beta\,(1-\sigma)\,n_t}{(1+\beta)\,(\alpha+\sigma\,(1-\alpha))\,n}. \tag{37}$$

Thus, in the small open economy, the market outcome resembles a pure FB education system.

5.3 Market outcome and the exogenous systems

It is possible for the exogenous education and pension systems to mimic the market outcome.[20] What is needed, for the closed economy and the small open economy, is to define the benefit rates as in the market outcome, as presented in table 4.

Table 4: Benefit rates in the market outcome.

	Open ec. $\phi_K = 1$	Closed ec. $\phi_K = 0$
$\gamma_{P,t}$	σn_{t-1}	σn_{t-1}
$\gamma_{E,t}$	$\frac{\sigma\beta(1-\alpha)(1-\sigma)}{(1+\beta)(\alpha+\sigma(1-\alpha))}$	$\frac{\sigma\beta(1-\alpha)(1-\sigma)}{(1+\beta)(\alpha+\sigma(1-\alpha))n_t}$

With these benefit rates, the systems will mimic the market outcome. Without having any explicit link between the contributions to the education system and the benefits in the pension system, the systems will still behave as if pension benefits were a return to human capital investments.

In the small open economy, the exogenous ϕ_E and ϕ_P pair that maximized expected welfare, is the same combination as in the market outcome. This is not the case for the closed economy where the pure FB pension design yields a higher expected welfare than the market outcome. This is no surprise since the market outcome did not allow for the generations to specify contracts *ex ante*, before the uncertainty was realized, for all future realizations.

Maximizing the welfare function *ex ante* incorporates the diversification effect offered by the pure FB pension scheme. For $n_t > n$, the change in the interest rate is such that $R_{t+1} > R$ (unless the economy is a small open one). From table 3, it is

[19] Note that the FB education design implies $h_t = h \forall t$ in the small open economy.

[20] Boldrin and Montes (2005) note that it is difficult to mimic the market outcome without lump-sum taxation, since income taxation leads to distortions. In this model, this is not a problem due to inelastic labor supply. Moreover, it should be possible to avoid such distortions if applying participation constraints in the pension system.

clear that if the pension scheme is a pure FB scheme, then $R^H_{t+1} < R$, irrespective of the education design. This implies a diversification effect, from which all generations benefit from, given that they are risk-averse.

A pure FC pension scheme together with a pure FC education system implies that R^H_{t+1} moves together with R_{t+1}. The investment in human capital will thus in this case not offer any diversification effect.

6 Discussion

The underlying question in this paper was if an unfunded pension system with a *fixed contribution rate* is desirable when considering changes in dependency ratios. When considering fertility fluctuations from an *ex ante* perspective, the optimal pension design was shown to crucially depend on the degree of openness. For a closed economy a *fixed benefit rate* is preferred, which confirms findings in other studies. It was further shown that this result holds, even with a considerable degree of openness, and irrespective of how the education system might be financed.

For a fully open economy, the pure FC pension scheme is desirable if the education system operates according to the pure FB scheme. This specific design combination for the education system and the pension system implies that individuals in the small open economy are unaffected by fertility fluctuations, even with large intergenerational transfer systems in place. The move towards FC could thus be motivated for small open economies, but it requires that the corresponding education design is in place. Switching to FC pension design, however, implies that unlucky generations, i.e. the large ones, will bear a huge cost in economies that are not small and open.

It is never possible to avoid the effects of fertility fluctuations without interacting the pension system with the education system. What has been proposed, and implemented in Sweden for instance, is to use buffer funds to counter changes in dependency ratios. This will only work in a very specific situation; when a shock to the old age dependency ratio is followed by a shock of equal size and opposite sign in the next period. If this specific pattern does not occur, then the gain/burden pension in the pension system created by a decrease/increase in the old age dependency ratio will only be distributed to an unknown future generation.

The paper also investigated what kind of allocations would arise in the market outcome when children and adults can enter into contractual agreements. The finding is that the resulting transactions in the closed economy resemble a pure FC pension scheme and a pure FC education scheme. For the small open economy, the arising allocations are the same as for the pure FC pension scheme combined with a pure FB education scheme.

For the closed economy, the market outcome does not yield the highest expected welfare which depends on the fact that the market outcome leads to the same rate of return on both human and physical capital. From an *ex ante* perspective, one would want these two form of capital to be negatively correlated which is accomplished by a pure FB pension scheme.

The opposing results about the pension scheme when viewed *ex ante* (maximizing expected welfare) and viewed *ex post* (market outcome when the uncertainty has been realized) leads to the question of sustainability. Is it possible to impose a pension system that deviates from the market outcome? This is left to future research, but it is interesting that countries that have re-designed their pension system have moved from a *fixed replacement rate* to a *fixed contribution rate* (e.g. Sweden, Italy, Germany). This is noteworthy, since the *fixed replacement rate* can never replicate the market outcome.

Linking the education design with the pension design is crucial for minimizing the effects of fertility fluctuations. This does not imply that the two should have a unified budget constraint. What is needed is to specify how the systems should react to changes in dependency ratios. For this reason, the long-term intergenerational contract in the education system should be made explicit. The education system is based on intergenerational transfer and it is important to start considering its design. If we want to give equal opportunities of education, across generations, introducing an explicit educational long-term contract according to the FB scheme might be warranted. If this were to be the case, then the FC pension scheme is desirable for a small open economy.

There are many important aspects that have been omitted from the analysis, e.g. uncertainties about the mortality rate, productivity, and the like. The stylized model did not capture the effects that a varying tax rate might have on the labor supply. Has this made the analysis biased towards the FB scheme? This depends on whether the income or the substitution effect dominates. Another possibility is to make fertility endogenous. This would probably lead to the inclusion of altruism in the model. Including altruism would probably not dismantle the qualitative prescriptions since the results regarding preferred designs mainly rest on how different designs minimize the effects of fertility fluctuations. Moreover, assuming parental altruism towards children would probably strengthen the results since a large cohort is worse off than the parent generation, even without altruism.

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Essay 3

Pension design and longevity

1 Introduction

During the past century life expectancy has increased dramatically. In the U.S., for instance, it has increased by about 30 years from 48 in 1900 to 77 years today. In the beginning this increase was mainly caused by mortality improvements at pre-retirement ages. Now, however, 70 percent of the gain in life expectancy comes from mortality reductions after age 65 (e.g. Lee and Tuljapurkar 1997). This has led to changes in the old age dependency ratio. It is well known that unfunded pension systems are sensitive to changes in old age dependency ratios. For this reason several countries have re-designed their pension systems. While still mainly unfunded, they now have a *fixed contribution rate* instead of a *fixed replacement rate*. Is a *fixed contribution rate* the preferred design when considering old age dependency ratios?

From the pension literature it is known that alternative unfunded pension schemes have very different distributional properties (Hassler and Lindbeck 1997, Thøgersen 1998, Wagener 2003). Changes in the old age dependency ratio alter the contributions from the workers, the benefits to the retirees, or both. Which of these alternatives that occur depends on how the pension system is designed. In this paper I analyze convex combinations between a pure *fixed contribution rate* (FC scheme) and a pure *fixed benefit rate* (FB scheme).[1] In the pure FC scheme the workers always pay a certain fraction of their income to the system, irrespective of the dependency ratio. In a pure FB scheme retirees are guaranteed a certain fraction of current workers' income. Convex combinations between these two extremes imply that both workers and retirees are affected by changes in dependency ratios.

[1]Note that a *fixed benefit rate* is not the same as a *fixed replacement rate*.

Longevity changes are bound to affect other aspects of human life, including the economic life-cycle. Lee and Goldstein (2003) discuss, among other things, how longevity changes might alter the length of childhood, work and retirement. How different life periods respond to longevity changes is what they call rescaling the life-cycle. The question is how different pension designs will affect this rescaling.

This paper will investigate how the pension design affects the rescaling of the life-cycle. The paper will also investigate which unfunded pension design that yields the highest expected *ex ante* welfare when old age mortality is uncertain. Welfare is measured according to a standard utilitarian welfare function. This corresponds to finding the pension scheme that the individuals would choose *behind the veil of ignorance*, i.e. before knowing the realization of the old age mortality.

Previous studies that have investigated preferred pension designs, deal with uncertainty in either mortality, fertility, or factor prices. Studies by Smith (1982), Blomquist and Wijkander (1994), and Bohn (2001) focus on fertility fluctuations in a closed economy setup, while Thøgersen (1998) and Wagener (2003) focus on factor price uncertainty in a small open economy setup. None of these studies find justification for the fixed contribution rate design.[2] In essay 2, I showed that the FC design could be motivated *ex ante* for a small open economy that has a specific design for the education system. The question is then if the FC design might be preferred when considering mortality fluctuations.

The studies by Bohn (2001) and Andersen (2005) investigate intergenerational risk-sharing for different pension designs under uncertainty about mortality. The difference is that they do not include human capital formation. The study by Echevarra and Iza (2005) includes human capital accumulation but does not compare different pension designs and does not analyze the transition after a shock. This study includes the transition, and human capital accumulation using numerical methods, in the spirit of Auerbach and Kotlikoff (1987).

The computable general equilibrium model used in this paper consist of three overlapping generations. The individual problem consists of choosing the optimal amount of human and physical capital, and the optimal retirement time. When individuals exit the labor force, i.e. retire, they will receive pension benefits according to the benefit formula which is determined by the pension design. Once chosen *behind the veil of ignorance* the pension design remains unaltered. The optimal pension design maximizes the expected *ex ante* life-time utility for all generations, taking account of individuals behavior. The analysis is conducted for a small open economy.

[2]Thøgersen (1998) finds motivation for the FC design but as Wagener (2003) shows this is not valid from an *ex ante* perspective.

One finding is that the ability for the old to adjust their retirement period is very important. If they are able to do this then there is little difference between the pension designs. The pension design will in this case have negligible impact on the rescaling of the life-cycle. The resulting *ex ante* welfare difference is also negligible in this case. Further, the difference in labor distortions induced by the different pension designs is quite small.

If the old cannot adjust their labor supply then the preferred pension design is the FB design. This design implies that the generation that cannot adjust is sheltered from longevity shocks and that future generations that are more able to adjust are involved in the risk-sharing. The rescaling of the life-cycle will also differ between the designs. A fixed benefit rate will lead to a variable tax rate which in turn will affect the entry into and the exit from the labor force.

2 The model

The model depicts a small open economy with 3 overlapping generations. In every period t there is a new generation born, which will be called the t generation.The size of generation t increases over time:

$$N_t = n_t N_{t-1}, \tag{1}$$

where n_t is the gross population growth between period $t-1$ and t. Since the focus in this paper is mortality and not fertility it is assumed that $n_t = n \ \forall t$. Agents are homogenous within generations and their objective is to maximize life-time utility. There is an exogenous unfunded pension system, that can operate under different designs. The individuals know which pension system that is in place. The only shock that can occur in the economy is shocks to mortality in the final period of life. The first two periods of life have a fixed length normalized to unity while the length of the final stage of life is ε_t and can vary.

2.1 Individuals

Individuals live through three phases: the young phase, the working phase, and the old phase. They can work in all three phases, but during the working phase they devote all their time to work. During the young phase, agents devote a fraction of their one unit time to human capital accumulation while the remaining time is spent on work. The fraction, e_t, that the young generation in period t devotes to human capital accumulation is chosen endogenously. The first fraction of young time, e_t, is thus spent on education which produces human capital according to:

$$h_{y,t} = \varphi e_t^{\sigma},$$

where $h_{y,t}$ is the human capital of generation t during the young phase, φ is a scale parameter and $\sigma \in (0,1]$ is a measure of the returns scale. The human capital accumulated when young determines the stock of human capital during the last two phases of life, according to:

$$h_{w,t+1} = \eta_w h_{y,t}, \tag{2}$$
$$h_{o,t+2} = \eta_o h_{y,t}, \tag{3}$$

where $h_{w,t+1}$ and $h_{o,t+2}$ is the human capital of generation t during working phase and old phase, respectively. The parameters η_w and η_o allow for varying efficiency at different stages of the life-cycle. The individuals also choose how much to save when young.

In the second stage of life the individuals combine their one unit of time with their human capital to receive wage income. The only choice during this phase is to decide on the amount of savings. During the last phase, when old, the generation t has a time endowment of ε_{t+2}. During this phase they choose to work a fraction $z_{o,t+2}$, implying that $p_{t+2} \equiv \varepsilon_{t+2} - z_{o,t+2}$ is spent in retirement. It is further assumed that ε_{t+2} is realized before the time of death as to enable the generation to die with zero assets holdings.

The objective of the individuals is to maximize their life-time utility. I assume an additively separable utility function:

$$U_t = u(c_{y,t}) + \beta u(c_{w,t+1}) + \beta^2 \varepsilon_{t+2} \left(u\left(\frac{c_{o,t+2}}{\varepsilon_{t+2}}\right) + \upsilon\left(\frac{p_{t+2}}{\varepsilon_{t+2}}\right)\right), \tag{4}$$

where β is the subjective discount factor, and c_i where $i = \{y, w, o\}$ is consumption during the different phases. The period utility during young and working phase is solely based on consumption. The utility when old comes both from consumption and retirement, and is scaled by the period length. The utility from retirement is similar to the specification in Andersen (2005) and implies that the individuals value longer lives but that this at the same time creates consumption and retirement needs. Retirement is viewed as a consumption good. It is assumed that $\upsilon' > 0$, $\upsilon'' < 0$, $\lim_{p \to 0} \upsilon' = \pm\infty$, which ensures that the individual always choose some retirement before they die. This captures the fact that in most cases it will become increasingly difficult to work when approaching the time of death. A simple functional form that satisfies the conditions above is:

$$\upsilon\left(p/\varepsilon\right)=\kappa\ln\left(\frac{p}{\varepsilon}\right), \tag{5}$$

where κ is a scale parameter and will determine the marginal rate of substitution between consumption and retirement in the last period. The utility from consumption is specified according to:

$$u\left(c\right)=\ln c. \tag{6}$$

The chosen specification implies that the marginal rate of substitution between retirement and consumption in last period is unaffected by longevity. The choice is made to capture the fact that morbidity has reduced in the same extent, or more, as mortality (e.g. Fogel 2004). This implies that the ability to work increases together with longevity. Moreover, the aim is to keep all aspects as neutral as possible to mortality changes, and thus isolate the effect of different pension designs.

The objective of the individuals is to choose e, c_i, and p as to maximize U_t under the constraints:

$$c_{y,t} = \left(1-\tau_t\right)z_{y,t}w_t h_{y,t}-s_{y,t}, \tag{7}$$

$$c_{w,t+1} = \left(1-\tau_{t+1}\right)w_{t+1}h_{w,t+1}+R_{t+1}s_{y,t}-s_{w,t+1}, \tag{8}$$

$$c_{o,t+2} = R_{t+2}s_{w,t+1}+\left(1-\tau_{t+2}\right)w_{t+2}h_{o,t+2}z_{o,t+2}+b_{t+2}p_{t+2}, \tag{9}$$

where τ_t is the tax in period t devoted to finance the pension system, $s_{y,t}$ and $s_{w,t+1}$ are the savings of generation t during young and working phase, w_t is the wage rate per efficient labor unit, R_t is the gross interest rate on savings between period $t-1$ and t, and b_t are the benefits per retirement unit from the pension system. Given the small open economy assumption we have that $w_t = w$ and $R_t = R\ \forall\ t$. The first order conditions with respect to consumption gives the intertemporal Euler conditions:

$$c_{w,t+1} = \beta R c_{y,t}, \tag{10}$$

$$c_{o,t+2} = \varepsilon_{t+2}\beta R c_{w,t+1}. \tag{11}$$

The first order condition with respect to education can, after some rearrangement, be stated as:

$$e_t=\frac{\sigma}{1+\sigma}\left(1+\frac{\left(1-\tau_{t+1}\right)\eta_w}{\left(1-\tau_t\right)R}+\frac{\left(1-\tau_{t+2}\right)\eta_o}{\left(1-\tau_{t+1}\right)R^2}z_{o,t+2}\right), \tag{12}$$

and comes from the equalization between the marginal return on investment and the marginal cost (in terms of opportunity cost of forgone labor income). The final first order condition with respect the to retirement period links the marginal rate of substitution between consumption and retirement to the marginal product of labor, according to:

$$\frac{p_{t+2}}{c_{o,t+2}\kappa} = \frac{1}{(wh_{o,t+2}(1-\tau_{t+2}) - b_{t+2})}. \tag{13}$$

The equations (10), (11), (12), and (13) characterize the behavior of individuals in the economy.

2.2 Pension system

For the pension system to be unfunded it is necessary that the budget is balanced in each period, which implies that it is possible to state the transfers in period t as:

$$b_t N_{t-2}(\varepsilon_t - z_{o,t}) = d_t(N_t z_{y,t} + N_{t-1} + N_{t-2} z_{o,t}), \tag{14}$$

where d_t is the mean contribution per worker, while b_t is the benefit per retired. Further let m_t denote the worker retiree ratio in period t, i.e.:

$$m_t \equiv \frac{n^2 z_{y,t} + n + z_{o,t}}{\varepsilon_t - z_{o,t}}, \tag{15}$$

which implies that $b_t = d_t m_t$. We see that when m_t varies either benefits per retired, contributions per worker, or both need to adjust. This is the direct effect that changing old age dependency will have on the pension system.

Since the contributions are collected through wage taxes it is reasonable to relate the mean contribution per worker to the income of the workers.[3] Letting $\bar{w}_t$ be the mean wage of the work force, implies that:

$$d_t = \tau_t \bar{w}_t. \tag{16}$$

The benefits can also be related to the income, but it is not as obvious to what income. Should it be to the mean income of current workers, mean income of one's own income over the life-cycle, or perhaps the mean income during x years of the working period? All these three approaches are equivalent in steady state (incentive motives put aside). During a disturbance it, however, matters which system that is in place. Since this paper focuses on disturbances to the worker retiree ratio, caused

[3] Regarding the contributions there seems to be more or less consensus that these should be related to the mean wage of the work force. There are however proposals such as to finance pension system by consumption taxes and the like.

by mortality fluctuations, I will choose the first approach and base the benefits on the mean income of current workers. By doing so it is possible to abstract from direct effects from changes in wages, and thus focus on the worker retiree ratio.

Relating the benefits to $\bar{w}_t$ according to:

$$b_t = \gamma_t \bar{w}_t, \tag{17}$$

makes it possible to rewrite the budget restriction on the following form:

$$\gamma_t = \tau_t m_t, \tag{18}$$

where γ_t represents the benefit rate. The benefit rate is the fraction of the current mean wage that is given to each retiree. In steady state when $m_t = m$ it is possible to have both γ and τ fixed. During a disturbance both cannot be fixed at the same time. When facing a shock there are thus two extreme ways that the system can adjust: either keeping $\tau_t = \tau$, or keeping $\gamma_t = \gamma$. The first extreme will be referred to as *fixed contribution rate*, FC, while the latter will be referred to as *fixed benefit rate*, FB. The FB system implies that the retirees will not bear any direct risk from fluctuations in m. If on the other hand the system operates according to FC then the retirees bear the whole direct effect while the workers are entirely sheltered. There is, however, a possibility for an indirect effect on benefits and contributions through changes to the mean wage. Changes to the mean wage are always shared between retirees and workers.

With only the extreme cases it is not possible to make the workers and retirees share the direct effect from changes in m. To allow for this consider the following benefit formula:

$$b_t = \bar{w}_t \gamma \left(\phi + (1-\phi)\, m_t/m\right), \tag{19}$$

where ϕ indicates the mix between FC and FB, and thus how the risk is shared between the workers and retires. When $\phi = 1$ we have a pure FB system and when $\phi = 0$ we have a pure FC system. Choosing the design for the pension system amounts to choose the value for ϕ, while γ indicates the size of the system.

2.3 The intergenerational welfare function

To obtain a compact measure of how all generations are affected by a mortality shock, welfare is defined as:

$$W = E\left[\sum_{t=1}^{\infty} \psi_t U_t\right], \tag{20}$$

This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.

There are different views on how the per capita lifetime utility of generation t should be weighted. The question is if the utility should be weighted by the generation size, and whether the utility of future generations should be discounted. It seems more or less necessary to account for the generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will be included and the weighting factor will be the following:

$$\psi_t/\psi_{t-1} = \beta_s n_t, \tag{21}$$

where β_s is the social discount rate. In the simulation the social discount rate will be set equal to the individuals discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying the social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth then the discount rate should not exceed the inverse of the population growth; if it does, then the future generations would get an ever increasing impact on the welfare function, due to their larger number.[4]

3 Simulation and calibration

What will be simulated are shocks to ε_t. Of interest is first to investigate how different pension designs rescale the life-cycle. After that the objective is to maximize the intergenerational welfare function in eq. (20) by choosing the pension design, i.e. ϕ. This corresponds to finding the optimal pension design in the Rawlsian sense, *behind the veil of ignorance*. This is an *ex ante* analysis similar to the one applied in Ball and Mankiw (2001). To be able to do this one first needs to calibrate the model.

[4]See, for instance, Blanchet and Kessler (1991) and Boadway, Marchand, and Pestieau (1991), for a short comment concerning the weighting problem.

3.1 Demography

There is no doubt that longevity is increasing in the real life. The question of interest is however not the predictable part of of the increase but the unpredictable. For this reason I abstract from the trend in longevity and focus on the uncertain part. This means that $\varepsilon_t = \varepsilon$ in steady state. With respect to demographics what needs to be calibrated is thus ε and n.

I start by choosing ε as to obtain reasonable lengths between the different stages of the life-cycle. The first two phases are normalized to unity and should be of equal size in number of years. The old phase has a different amount of years by the factor ε. Assuming that children under 16 years of age cannot choose labor over education, while the ones above can, gives that the young phase starts at age 16. The young phase and the working phase should be of equal size. The number of years in each of these phases is thus half the period between the age 16 and the age at which the old phase starts. The age when the old phase starts is marked by the fact that labor market work is not the only activity. To find this age I use the labor participation rate (LPR) at different ages, this is presented in table (1). During no age is the LPR 100 percent, which makes it necessary to choose a threshold value for LPR which will be considered as full time work. I choose this threshold value to 85 percent. This makes the old phase start at the age 55, while the young and the working phase will correspond to 20 years each. Note that this does not imply that the agents retire at 55 or that they spend their time in education until 35. What it implies is that individuals do not spend time on education or retirement between the ages 35 and 55.

To obtain the value for ε we also need to know how long people live. Life expectancy at 55 will be used as a proxy for the number of years in the old phase. Using the 2001 period life-table yields that remaining number of years at the age 55 are 23 for men and 27 for women.[5] Since the calibration has been done according to the male labor participation rates, I will choose the male life table and use 23 as the remaining years at 55. This implies that the last period comprises of 23 years while the first two phases have 20 years. Normalizing the first two to unity implies that $\varepsilon = 1.15$.

What remains to determine with respect to demographics is the growth of each new generation, n. The steady state gross population growth, n, will be set to 1.22, based on the annual average for the U.S. between 1910-2001.[6]

[5]See: Annual Statistical Supplement, 2004, to the Social Security Bulletin. Ideally one would want to base this on cohort data. However this is not available and instead the period life-tables is used instead.

[6]The annual average, from Vital Statistics of the United States, Volume I, Natality, is approxi-

Table 1: Male labor participation rates at different ages in 2005 for the U.S.

Age	16-19	20-24	25-29	30-34	35-39	40-44	45-49
LPR	43	79	91	93	93	92	90
Age	50-54	55-59	60-64	65-70	70-74	75 +	
LPR	86	78	58	34	21	9	

Source: Bureau of labor statistics, Current population survey, Annual averages - Household data, 2005.

3.2 Preferences, wages, and the interest rate

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one year estimate from (Auerbach and Kotlikoff 1987) of 0.98 translates to $\beta = 0.7$, since the normalized period length represents 20 years. The parameter κ will be chosen so that the share of retirement in steady state is half the time of the old phase, i.e. $z_o = 0.5\varepsilon$. This corresponds to the working share of the 55+ in table (1). This implies that the model retirement age is 66.5. This is somewhat higher then average real life retirement age, which is 63, but still within reason. What is important is that the resulting worker retiree ratio is reasonable.

Regarding the wage and the interest rate these will be set as to equalize the autarky prices with the world market prices in steady state. To obtain the autarky prices a standard Cob-Douglas production function is used with efficient labor and capital as factor inputs.[7]

3.3 The benefit rate and human capital

Choosing the size of the pension system amounts to calibrating the benefit rate in steady state, γ. According to Pecchenino and Utendorf (1999) the benefit ratio, i.e. the benefit over the average wage ratio in the same period, is 0.42. I will use the same value and set $\gamma = 0.42$.

Regarding the human capital process I start by calibrating the relative efficiency during the three phases, i.e. η_w and η_o. Using the same efficiency profile over the life-cycle as in Auerbach and Kotlikoff (1987) leads to $\eta_w = 1.17$ and $\eta_o = 0.89$. Which means that the individuals are 17 percent more efficient during the working phase compared to the young phase, while they are 11 percent less efficient during the old phase compared to the young phase. The scale parameter φ will not affect the outcome and will be set to unity.

mately 1.01. This implies that per period $n = 1.01^{20}$, since one period corresponds to 20 years.

[7] The exact specification and calibration can be found in essay 2.

The rate of return on human capital, is determined by σ. Unfortunately it is very hard to get an accurate measure for σ. Card and Krueger (1992) investigate how the pupil teacher ratio affects future productivity. Translating their results, via assumptions on how the spending per pupil is related to the pupil teacher ratio, would yield $\sigma = 0.17$. The other approach how σ could be determined is by the use of table (1). Assuming that most of the individuals that do not work between the ages 16 and 35 spend time on education makes it possible get an estimate of z_y.[8] This value for z_y corresponds to 0.77, and implies that the model age at which individuals enter the labor market is 20.6. If the education length is fixed (as in this case, $e = 0.23$) then it is possible to back out the value for σ according to equation (12). Doing so leads to $\sigma = 0.17$, which is what was obtained from the estimate by Card and Krueger (1992). In the simulation I use $\sigma = 0.17$, and this is close to what was used in the two previous essays.

Table 2: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.7
Efficiency during working phase	η_w	1.17
Efficiency during old phase	η_o	0.89
Elasticity of scale in hum. cap. prod.	σ	0.17
Steady state benefit rate	γ	0.42
Population gross growth rate	n	1.22
$\text{MRS}_{c,p}$ parameter	κ	0.12
Longevity in steady state	ε	1.15

3.4 Steady state

To see how this stylized model performs it is possible to compare some variables in steady state with data. The steady state results, according to the calibration in table 2, are presented in table 3.

The model was calibrated so as to obtain the first two values, regarding e and z_o. This was done by adjusting the free parameters κ and σ. More interesting is that the resulting worker retiree ratio is reasonable. The comparable value according to data in table 1 is 5.5. The resulting pension tax is 8 percent which is lower than the comparable OASDI pay-roll tax, which is 12.4 percent. The difference comes from the fact that I have calibrated the model to male data, which gives a higher worker

[8] What is meant by most individuals, is the excess out of labor force share during the young phase compared to the working phase.

Table 3: Steady state based according to calibration in table 2.

Working share of in old phase	z_o/ε	0.5
Education share of young phase	e	0.23
Worker retiree ratio	m	5.12
Pension tax	τ	0.08
Gross interest rate	R	2.41

retiree ratio compared to the ratio used for the OASDI pay-roll tax. Looking at the gross interest rate it might seem high but when adjusting for the models period length it implies a yearly rate of 4.5 percent. This is somewhat higher but within reason.

Considering that it is a highly stylized model, it seems as if the comparable variables match data.

4 Temporary longevity increase

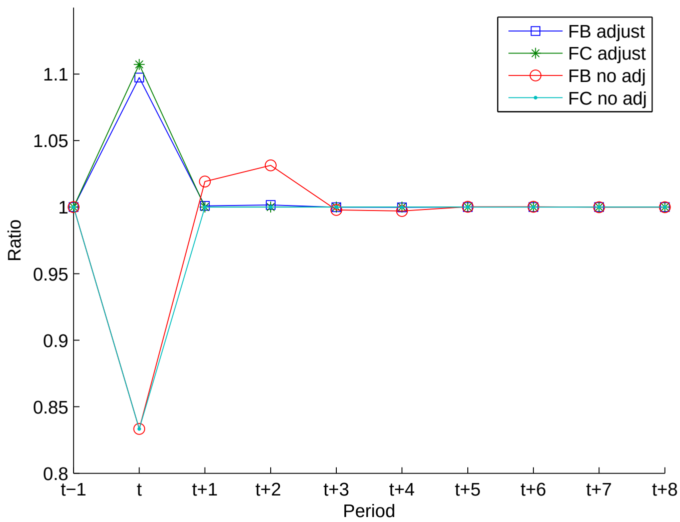
Before presenting the optimal pension design I first consider the extreme cases. They will be compared under the experiment that there is a positive longevity shock in one period. The experiment is the following $\varepsilon_{t+i} = \varepsilon \ \forall \ i \neq 0$ and $\varepsilon_t > \varepsilon$. This will give some insight about the importance of the design when it comes to rescaling the life-cycle and also some understanding of what effects that are at work. I will analyze how four variables evolve after a positive longevity shock, namely: retirement length, education length, worker retiree ratio, and the utilities for the generations.

It will be important to distinguish between how quick individuals might react to a shock, or put differently, how long head notice they have about the shock. To make this distinction two cases are considered. One, is to allow all individuals in period t to re-optimize fully when facing a longevity shock in period t. This case will be referred to as full adjustment. The other alternative is not to allow the old individuals in period t to alter their working length in response to the shock in period t. This corresponds to a scenario where the shock about time of death is not revealed until after the individuals have stopped working. This case will be referred to as no adjustment.

4.1 Labor length in last period

Let us first see how the old divide the longevity increase between retirement and work. Figure 1 presents the work share during old phase relative to steady state for the pure FB and the pure FC design. Given the shock at hand the maximum increase in the reported variable is up to 1.17 while the maximum decrease is down to 0.83.

Figure 1: Working share of old phase time relative to steady state.



When the old can adjust then they increase their labor supply relatively more than they increase their retirement length. The interesting result is that the pension design has a small impact on the increase in labor. One would expect a larger difference since with an FB design creates incentive to earlier retirement due to the tax increase. We see, however, that this is effect is quite small.

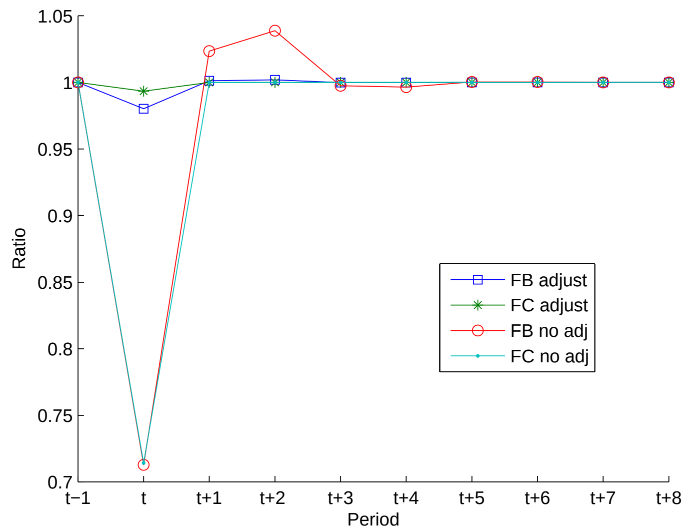
If the old cannot adjust then the entire increase in longevity will be in terms of retirement. In this case there will be a mechanical decrease in the share of old age workers, in period t. With an FB pension system the share of old age workers will increase in the two following periods after the shock. This happens since the increased tax rate during period t implies less savings and thus the generations prolong their working period to compensate for this.

The main result is that the pension design has a marginal impact on the rescaling in the last period, and that when it has an impact it is mainly after the shock.

4.2 Worker retiree ratio

The worker retiree ratio will always go down, but the magnitude differs a lot. From a negligible decrease when the old can adjust to almost 30 percent reduction when there is no adjustment. This can be seen from figure 2, where the worker retiree ratio is presented as deviations from steady state. Once again we see that it is much more important if the old generation can adjust or not, than which pension design that is in place. We also see the increase in worker retiree ratio that is created under

Figure 2: Worker retiree ratio, relative to steady state.



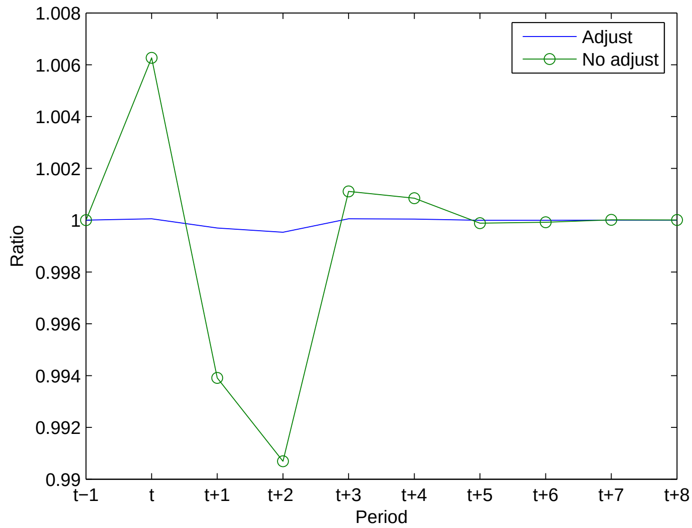
the FB design when there are no adjustment possibilities. This occurs for the same reason as before, to compensate for the lower life-time income that a tax increase implies.

Further, if combining figure 1 with 2 we see that almost all of the change in worker retiree ratio is caused by changes in the old age working share. This means that there is no substantial difference between pension designs regarding the education length.

4.3 Life-time utility

Figure 3 shows how life-time utility differs between the pension designs. Here the ratio between the life-time utility for the FB design over the life-time utility for the FC design is presented. This is done to highlight the differences in design. Once again the difference between full and limited adjustment is important. With full adjustment the design of the pension system does not matter. With limited adjustment the generation that is old during the longevity shock prefers the FB design while the following generations prefer the FC design. The generations prefer

Figure 3: Ratio between FB design over the FC design for life-time utility, with and without adjustment.



opposite designs since it is a matter of redistribution between them. This is, however, their preferred design *ex-post* when the uncertainty has been revealed. The shock could well have been negative instead of positive in which case the old would prefer the FC design while the young and working generations would prefer the FB design. The question is what design is the preferred *ex-ante*. This is analyzed in the next section.

5 The *ex-ante* approach

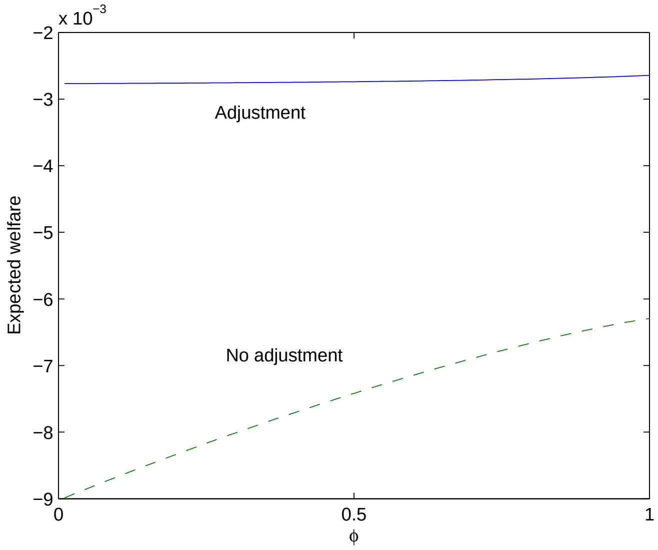
The preferred *ex-ante* design will be identified by evaluating the welfare function specified in equation (20). The idea is that the generations have to decide on a pension design before being born, or at least before they know the longevity that will affect them. This corresponds to the choice *behind the veil of ignorance* a'la (Rawls

1971). Once chosen, the pension design will be unaltered and it will respond to any longevity shock in the pre-determined manner according to the benefit formula.

The longevity in period t will either be smaller or higher, by equal magnitude, and there is equal probability for both outcomes. Stated differently, the longevity sequence analyzed is: $\varepsilon_{t+j} = \varepsilon \ \forall j \neq 0$ and $\varepsilon_t = \varepsilon(1+x)$, where $x = \{-0.2, 0.2\}$ with equal probability. Here the disturbance is set to 20 percent but this could be changed without affecting the results.

Evaluating the welfare function at different designs for the two cases yields results according to figure 4. The preferred design is the pure FB design, i.e. $\phi = 1$. It also emerges that the difference between the designs when the old can adjust is negligible.

Figure 4: Expected welfare with and without adjustment.



The reason for why the FB design is preferred is that this design leads to least deviation from steady state. In the no adjustment case it so that the FB design makes the generations share the longevity risk. With a pure FC design only one generation bears the longevity risk. Moreover, this generation is not able to adjust in any manner except to increase the retirement period. With the FB design the generation that cannot adjust is compensated, while future generations that can adjust their education and working period are involved in the risk-sharing.

6 Conclusion

The preferred pension design with respect to mortality fluctuations is highly dependent on if the old can adjust to the shock or not. To assess the distribution of the old age mortality and how far into the future it is predictable is thus of great concern.

When the old cannot adjust it is desirable that future generations that can adjust are involved in the risk-sharing. Such risk-sharing will occur with the FB pension design. The FB pension design shelters the old generation that cannot adjust by offering a pension that is not dependent on the longevity realization.

The pension designs imply similar rescaling of the life-cycle, in particular if the old can adjust to the shock. Previous studies that have not accounted for the labor distortion effect of different systems can thus be viewed as a good approximations. Further, since the labor distortion effect is negligible the preferred design will for the most parts be on the extremes.

Pension design affects the rescaling when the old cannot adjust. The difference between design occurs during the periods after the longevity shock. This difference is brought out by the fact the FB design makes future generations rescale their life-cycle while the FC does not. It is precisely this rescaling that makes the generations share the risk in a more efficient manner.

Finally I will comment on the shift from the fixed replacement rate towards a fixed contribution rate in many countries. This paper has evaluated the FC design against the FB design. The difference between the FB and the FR design lies in how they respond to productivity shocks. The disadvantage of the FR design to expose the life-time income to one productivity realization is not shared by the FB design. The FB design, and the FC design, implies that life-time income depends on the productivity realization both during the working period and the retirement period. The FB design, however, shares the advantage of not exposing the retired to longevity uncertainty that the FR scheme entails. For this reason it is not easy to value the transition from the FR scheme to the FC scheme. What can be said is that with respect to old age mortality the FB design seems to be a better alternative then the FC design. In reality, however, one needs to consider more than just the longevity uncertainty. Considering fertility uncertainty it was demonstrated in essay 2 that the FB pension design is preferred if there are capital mobility restrictions.

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