

# Answer Explanations for: PSAT Wednesday 2012 (Form W)

## Section 2: Mathematics

- 1) C) To calculate how many lawns can be mowed in 8 hours, set up a proportion. To make sure you set it up right, make sure the units line up on both sides. In other words, this/that = this/that. Here, you have  $(\text{lawns})/(\text{hours}) = (\text{lawns})/(\text{hours})$ , so  $45 \text{ lawns}/12 \text{ hours} = x \text{ lawns}/8 \text{ hours}$   $x = 8 \cdot 45/12 = 30$ . See [ratios and proportions](#).
- 2) C) If PQ is twice as long as QR, which is 6 units long, PQ is 12 units long. Therefore, PR =  $12 + 6 = 18$ .
- 3) E) Since all of the numerators are even, they all must yield integers when divided by 2.  $6/2 = 3$ ,  $12/2 = 6$ , and  $4/2 = 2$ .
- 4) B) Remember that probability can be calculated as  $(\# \text{ of desired outcomes})/(\# \text{ of possible outcomes})$ . After the green marble is chosen, there are three possible outcomes since there are three marbles left in the jar. Only one of these three, the blue marble, is a desired outcome, so the probability of drawing a blue marble is  $1/3$ . If you chose A, you might have missed the sentence stating that the green marble is already removed. See [counting and probability](#).
- 5) A) Based on parallel line properties (specifically alternating interior angles), the missing angle of the bottom triangle must be  $30^\circ$ . Because the angles of a triangle add up to  $180^\circ$ ,  $20 + 30 + z = 180$ , so  $z = 130$ . See [plane geometry](#).
- 6) B) The slow way to do this problem is to solve for  $x$  using the first equation.  $4x = 16$   $x = 4$ . Now substitute 4 for  $x$  in the second expression.  $2(4) - 1 = 7$ . The fast way to do the problem is to recognize that  $2x - 1$  is equal to half the expression on the left side of the first equation, so it must be equal to half of 14, which is 7.
- 7) D) Model the price of both plans using linear equations. When modeling a linear situation, the  $y$ -intercept ( $b$  in  $y = mx + b$ ) is equal to the fixed amount and the slope ( $m$  in  $y = mx + b$ ) is equal to the variable amount. In this case, the  $y$ -intercept is equal to the per day cost (since the car is only being rented for one day), and the slope is equal to the cost per mile. Therefore, for plan 1,  $c = .3m + 20$  and for plan 2,  $c = .35m + 10$ , where  $c$  is the cost and  $m$  is the mileage driven. Set the right sides of these equations

equal to find the value of  $m$  for which the cost is the same.  $.3m + 20 = .35m + 10$   $10 = .05m$   $m = 200$  miles. See [equation building](#) and [linear systems](#).

- 8) D) Since point O is located at the origin (0, 0) and point A is located at (1, 2), point A is up 2 units and to the right 1 unit from point O. Because the slope of a line (the ratio of its rise to its run) is constant, the point that is 1 up and 2 over from A would also be on this line, and this point is (2, 4). Alternatively, you could have used the slope formula (rise/run = (change in y)/(change in x) =  $(y_2 - y_1)/(x_2 - x_1)$ ) to calculate that the slope of the line is 2. Then you could check each answer using this same formula to see if it creates a slope of 2 with either point A or point O. However, this method is unnecessarily time consuming. See [coordinate geometry](#).
- 9) A) Because 40% of cable subscribers subscribe to Invent Cable, 60% of cable subscribers subscribe to other services. Therefore, the answer can be found by finding 60% of the 5,000,000 cable subscribers, so  $.6 \cdot 5,000,000 = 3,000,000$ . See [percentages](#).
- 10) A) Do guess and check by plugging the answers in for  $b$  and seeing if the expression yields the corresponding value of  $g(b)$  from the table. Remember that the bars around the  $2b$  are absolute value bars, which make the value positive whether  $2b$  is positive or negative to begin with. Based on the method above, A is the only answer that works, since  $|2(-3)| + 1 = |-6| + 1 = 6 + 1 = 7$ , which is the value of  $g(b)$  in the table when  $b = -3$ . See [absolute values](#).
- 11) B) Simplify the equation in the question to get  $3x = -m$ . Then plug in 3 for  $m$  to get  $3x = -3$ , so  $x = -3/3 = -1$ . If you answered A, you may have miscalculated the right side of the equation as  $-3m$  instead of  $-m$ .
- 12) A) "Twice" means  $2 \cdot$ , "sum" means addition, "decreased" means minus, and "product" means multiplication. Use these definitions to slowly translate the English in the statement into a mathematical expression, and you should get A. The most tempting incorrect answer is C, which is incorrect because the language in the statement makes it clear that  $x$  and  $y$  must be added together and then multiplied by 2. See [equation building](#).
- 13) B) There are six faces of a cube, and the surface area of a cube is the sum of the areas of all six of these faces, so the area of 1 face is equal to the surface area of the cube divided by 6.  $48/6 = 8$ . If you answered E, you may have mistakenly thought a cube had four faces.
- 14) E) To minimize the value of the fraction in the question, you must maximize the denominator,  $\sqrt{x}$ .  $\sqrt{x}$  will be largest when  $x$  itself is its largest, so E, the largest answer, is correct. By no means is it necessary to calculate the actual value of the fraction for each of the answer choices. Instead, simply choose the largest answer choice offered based on the above reasoning.

- 15) C)** Since  $CE = CD$ , triangle  $CED$  is isosceles. The base angles of an isosceles triangle (the angles opposite the two equal sides) are always equal, so because  $x = 70$ ,  $\angle D = 70^\circ$ . Therefore  $\angle ECD = 180^\circ - 70^\circ - 70^\circ = 40^\circ$ . Because vertical angles are equal,  $\angle BCA = 40^\circ$ . Since  $AB = BC$ , triangle  $ABC$  is isosceles. Therefore, its base angles,  $\angle BCA$  and  $\angle A$  must be equal, so  $\angle A = 40^\circ$ . Because the angles of a triangle add up to  $180^\circ$ ,  $\angle ABC = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ . See [plane geometry](#).
- 16) C)** A parabola is symmetric about a vertical line drawn through its vertex, which for this parabola is the line  $x = -2$ . Therefore, since  $(-5, 3)$  is on the parabola, its reflection across the line  $x = -2$  must also be on the parabola. Since  $-5$  is 3 less than  $-2$ , the point of the reflection must have an x-value 3 more than  $-2$ , so its x-value must be 1. Its y-value must be the same as the y-value of  $(-5, 3)$ , so its y-value must be 3. Therefore, the point  $(1, 3)$  must also be on this parabola. See [coordinate geometry](#).
- 17) D)** Because there are a total of 91 numbers, you know that  $x + y = 91 - 15 - 19 - 11$ , so  $x + y = 46$ . The mode of a set is the most frequently occurring number in the set, so if 80 is the mode of the set, its frequency must be greater than that of the next greatest frequency, so  $x > 19$ . You also know that  $y < x$ , or 88 would be the mode of the set. Since  $y < x$  and since  $x + y = 46$ , which is an even number, the largest possible value of  $y$  will be found when it is 2 less than  $x$ . (If  $y$  were only 1 less than  $x$ ,  $x + y$  would have to be odd and could therefore not be equal to 46). So let  $y = x - 2$ . Now substitute  $x - 2$  for  $y$  in the equation  $x + y = 46$  to get  $x + (x - 2) = 46$   $2x - 2 = 46$   $2x = 48$   $x = 24$ . If  $x = 24$ , then  $24 + y = 46$ , so  $y = 22$ . It turned out to be irrelevant that the median (the middle number) of the set is 88, because when  $x = 24$  and  $y = 22$ , the median of the set is indeed 88. See [linear systems](#) and [averages](#).
- 18) D)** The distance from 1 to  $x$  is equal to their difference,  $x - 1$ . Because this distance is one tick mark and there are two tick marks between  $x$  and  $y$ , the distance between  $x$  and  $y$  is equal to  $2(x - 1) = 2x - 2$ . Since the distance between  $x$  and  $y$  is also equal to their difference,  $y - x$ , you can set  $y - x$  equal to  $2x - 2$  and solve for  $y$ .  $y - x = 2x - 2$   $y = 3x - 2$ .
- 19) E)** In counting problems, you should consider how many events are occurring and how many ways each event can occur. Here you have 4 events since the dog goes through four doors. It might be wise at this point to write out slots for each event with multiplication symbols in between them:  $\_ \bullet \_ \bullet \_ \bullet \_$ . Then, populate these slots with how many ways each event can occur. Here, the dog has 2 door options from Room T to Room S and 3 options from Room S to Room R. Because the dog cannot reuse the same door, he then has 2 door options from Room R to Room S and 1 door option from Room S to Room T. Populate the slots drawn above with these numbers to get  $2 \bullet 3 \bullet 2 \bullet 1 = 12$ . See [counting and probability](#).

**20) D)** I is not true for all values because it is false for any negative value of  $x$ , since negative numbers squared are positive. II is true for all values because if  $(x - 2)$  is positive or negative,  $(x - 2)^2$  is positive, and if  $(x - 2)$  is equal to zero, then  $(x - 2)^2$  is also equal to zero. III is true for all values because the two expressions are equal when  $x = 1$ , and the left side is greater than the right side for all other values of  $x$ . One easy way to figure out which inequalities are true for all values is to graph the left side as  $y_1$  and the right side as  $y_2$  on your graphing calculator. If the graph of  $y_1$  is always above (has a greater  $y$ -value) or equal to the graph of  $y_2$  at every  $x$ -value, then the inequality is true.