Name: ________________________ **Date:** ________________________________

Honors Physics: Worksheet on Position, Velocity, and Acceleration Graphs

Most of the questions on your next quiz will be taken from the questions and answers on this worksheet! Feel free to come in to check your answers against my solutions before next class.

Attached you will find three sheets of graph paper, each subdivided into position vs. time, velocity vs. time, and acceleration vs. time motion graphs. On each sheet you see a piecewise function has been drawn on one of the three sub-graphs. For each sheet your objective is to:

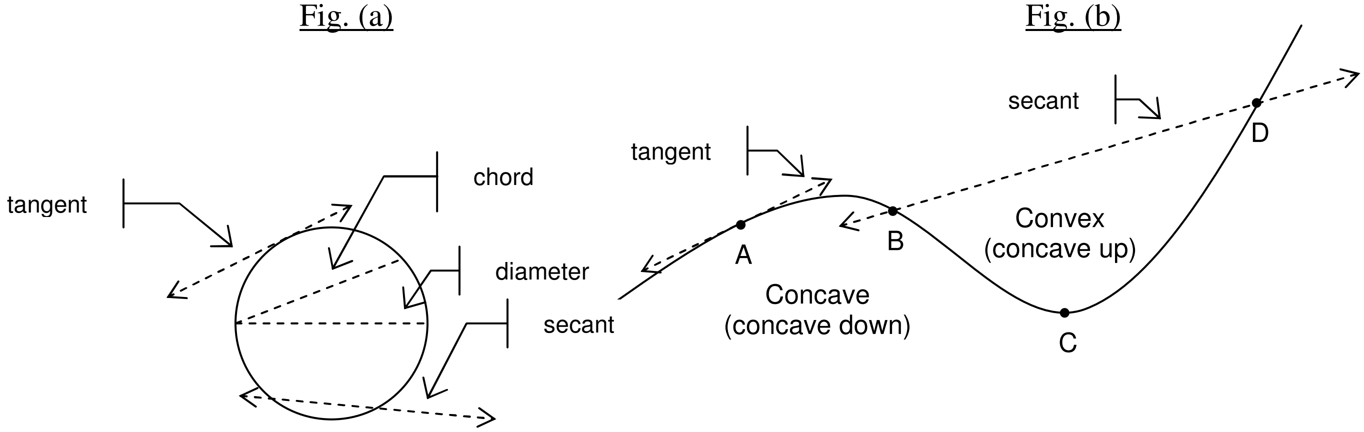
(a) Accurately describe the motion (position, velocity, acceleration) of an object based on the given sub-graph.

(b) Correctly identify the graphical and mathematical relationships between one sub-graph and the two others (e.g., meaning of the slope and area)

(b) Find the piecewise function that describes the given motion graph (to make it easier for you, the graphs on all three sheets are identical, but are drawn on a different sub-graph).

1. First label the axes on each graph and draw in your axes. I did most of this for you, but you must fill in the appropriate variable for 1-D motion and the units. Use **feet** and seconds for the basic units (1 square = 1 foot or 1 second; similar for velocity and acceleration). Mark the major (dark) grid lines with the unit values. *In the future if you do not fully label a graph, you will lose half credit for the problem.*
2. Fill out Table 1: ***Relationship between Graphs – Physical Meaning of Slope and Area***. We've discussed the meaning of every cell in the top section of the table (note that you should label two of the physical meaning cells as "none"). In class you've already seen examples and interpreted the meaning of all of the shapes in the bottom part of the table. If you're not sure about an answer in this section, check your notes or wait until you complete the next two steps, then go back and fill in the blanks.
3. Here's a diagram from geometry class to refresh your memory:

Fig. (a) Fig. (b)



a) In Figure (b), the tangent line touches the curve at only one point (Point A). The slope of the tangent line is the slope of the curve at a single point (Point A).

b) In Figure (b), the secant line passes through two points (B and D) on the curve (although it would pass through a third point if you extended it to the left). The slope of the tangent line could be considered as the average slope of between B and D.

c) Point C is a relative minimum on the curve in Figure (b). Draw a tangent to point C. What is the slope of the curve at Point C? ________________ .

d) The slope at relative minima or maxima is __________ . Is this always true? _____ . You may want to look up the definition for relative min/max from Alg 2 or Calc.

4. Now look at the graphs and answer the following questions:

a) Do the graphs meet the mathematical definition of a <u>function</u>? (explain using the <u>definition of a function</u>): ____

____

b) What pieces of the function are <u>increasing</u>? ___________ . Just above those pieces, pencil in "+" signs. Increasing functions have a __________ slope.

c) What pieces of the function are <u>decreasing</u>? ___________ . Just above those pieces, pencil in "–" signs. Decreasing functions have a __________ slope.

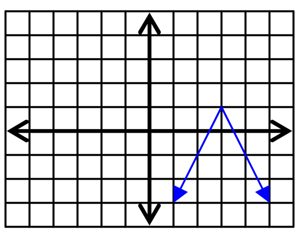
d) What pieces of the function are <u>constant</u>? ____________ . Just above those pieces, pencil in "0" signs. Decreasing functions have a __________ slope.

e) Which piece is increasing, decreasing, <u>and</u> constant? ______ Just above Piece F pencil in "+", "0", or "–" signs in the appropriate locations.

f) Are these piecewise functions? If so, how many pieces make up this piecewise function? _____________

g) Write the equation for the **<u>Position vs. Time</u>** piecewise function below using the appropriate variables and format. Note the Velocity and Acceleration functions are identical (just change the dependent variable name)

For example, the piecewise equation and graph for the absolute value function f(x) = –2|x – 3| + 1 is shown below. Basically you extend each line to find its "y-intercept" and provide a domain for each line, listed in order from left to right.



$$f(x)=\begin{cases}2x-5, & x<3\\ -2x+7, & x\geq 3\end{cases}$$

<u>Hint for the curved section (F)</u>: it is a parabola passing through points (12, -1), (14, -2), (16, -1), and (18, 2). Please show how you derived the equation for the parabola (use one of several methods practiced in Alg II and PreCalc).

<u>Piece name</u>	<u>Equation</u>	<u>Comma</u>	<u>Domain</u>
A			
B			
C			
D			
E			
F			
G			

___________ = {

5. Now work through each graph one at a time to describe what is happening to the object in each case (Table 2). For your explanation, let's pretend that the object in question is Stuart Little in his car (he is a talking mouse who drives a tiny car—it's a good book if you are looking for an easy read).

Don't try to work on all 3 graphs at once! Do each separately--although the 3 graphs are identical, they will describe completely different motions!

6. Start with Graph 1 (function for **Position vs. Time** is given):

a) When you look at Piece A, can you determine Stuart's initial position (x_0), velocity (v_{x0}), and acceleration (a_{xo})? Remember to always provide +/- signs for displacement, velocity, and acceleration (they are vectors). Write your answers in the first row of Table 2(a).

b) Now look at the shape and values for each piece of the function and carefully describe what is happening to Stuart's displacement, velocity, and acceleration. Write your answers in Table 2(a). You should look back at your answers on Table 1 to guide you through your explanations!

c) From the graph, find Stuart's displacement and show/explain how you did it:

d) From the graph, find Stuart's total distance travelled and show/explain how you did it:

e) From the graph, find Stuart's avg velocity over his entire trip and show/explain how you did it. Also, what is Stuart's highest speed?

f) Is there any point on Piece F for which Stuart's instantaneous velocity is zero? Explain!

g) Using your answers from above and from Table 1 and 2(a), carefully create a Velocity vs. Time graph for Stuart's trip. After you create the graph, you might want to review your answers in Table 1 and 2(a) to make sure they are consistent. Briefly explain how you developed this graph (hint: slope!)

The only "tricky" part of this task is finding the slope for Piece F because it is not constant. What you need is to find the instantaneous slope at every single point, then graph them all (an infinite number of points!). How is this possible?

- PreCalc students: You learned some key initial knowledge from section 1.1 of your PreCalc textbook. Complete the next page, and then use the final answer to complete the blank below.
- Calc students: Calc students may skip the next page (but it would be a good review for the AP Calc Test). Instead, you should use the equation you found for Piece F and use calculus to an equation for the velocity of Stuarts's car for Piece F. Show your work below.

v(t) (piece F) = _______________

h) Repeat step 6(g) to create the Acceleration vs. Time graph (briefly explain how you developed it). Hint: use slope of 6(g)! Calc students: is there a discrepancy between your v(t) and a(t) graphs?

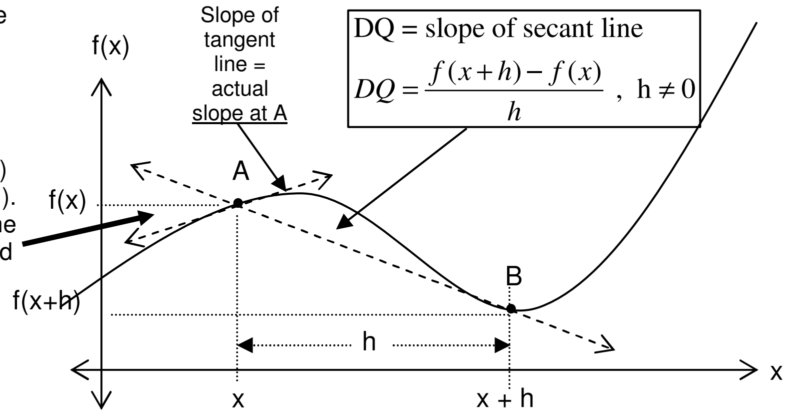
i) Are there any motions on the X vs. T graph that would be physically impossible to achieve? Why?

Special Exercise for Pre-Calculus Students: Finding the slope of EVERY point on a curve

First review the concept of the Difference Quotient (DQ) in Section 1.1, Example 9 on page 90 of your PreCalc textbook. The DQ is essentially calculating the slope of a secant line on the function f(x) using two generic points [x, f(x)] and [(x+h), f(x+h)].

For example, suppose we eventually want to find the slope of the curve at point A. If you look at Point A, you can see that the slope at that point is slightly positive.

As an approximation you could begin by finding the slope of the secant line drawn from Point A = (x, f(x)) to a point h units to the right, Point C = ((x+h), f(x+h)). Obviously the slope of this secant line, also called the difference quotient, is negative—it is not a very good estimate for the positive slope at Point A!



Difference Quotient (DQ) = slope of secant line

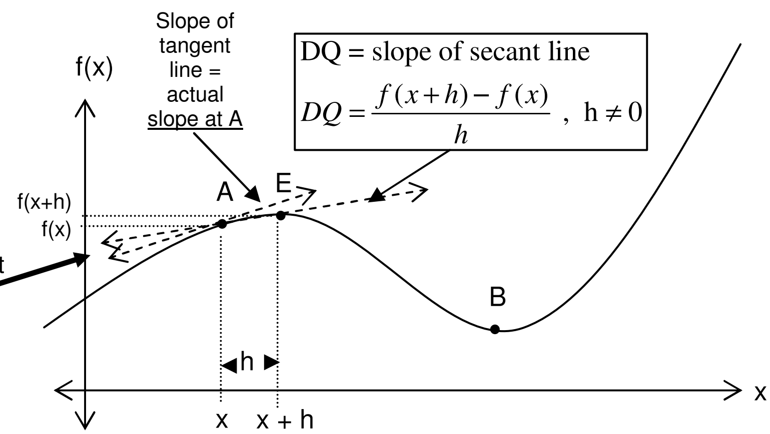
$$DQ = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $(x_1, y_1) = [x, f(x)]$ and $(x_2, y_2) = [(x+h), f(x+h)]$

$$\therefore DQ = \frac{f(x+h)-f(x)}{(x+h)-(x)}$$

$$= \frac{f(x+h)-f(x)}{h}, \ h \neq 0 \text{ (domain restricton)}$$

However, if you "shrink" h (the distance between the two points) by moving to Point E, the two-point secant approximation for the slope (the DQ), becomes a much better approximation for the actual slope at point A!



Now if you shrink h down to an infinitesmally small size (h approaches zero, h→0), then the slope of the secant (the DQ) should converge with the slope of the tangent line to point A! This process is called "taking the limit" as h→0. ⟶

$$\text{Actual Slope} = \text{Lim}_{h\to 0} \text{ DQ} = \text{Lim}_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

<u>So here is what you need to do: Show all you math neatly on a separate piece of paper</u>

a. Write down the equation for Piece F of your Position vs. Time piecewise function (from question 4g).

x(t) =

b. Now find the location of a point h units to the right of t (substitute (t+h) for (t) in the equation above, then simplify (restate in standard polynomial form, FOIL squared terms, distribute, combine like terms). Be careful to use parentheses when evaluating the functions and keep track of your negative signs. This will take a few lines of careful algebra…

x(t + h) =

Now take your answers from part (a) and (b) above and create the difference quotient, DQ. Then simplify the numerator (combine like terms, then factor out any common term factors). Then cancel any common terms in the numerator and denominator. Hint: something should cancel!

$$\text{Slope of Secant Line} = DQ = \frac{f(x+h)-f(x)}{h} = \frac{x(t+h)-x(t)}{h} =$$

c. After you have found and simplified the DQ, what happens as h approaches zero? Do any terms "disappear"? Take this limit by substituting h = 0 (this works for this simple case, but not generally).

$$\text{Actual Slope at (t, x(t)) (slope of tangent line)} = \text{Lim}_{h\to 0} \frac{x(t+h)-x(t)}{h} =$$

d. The equation you just derived gives you the actual slope at every point on Piece F of the X vs. T curve – i.e., you just found the equation for the Stuart's instantaneous velocity as a funtion of time, v(t), for Piece F. To check your result, substitute the time you found for 6(f) to see if you come up with v = 0!

7. Now work on Graph 2 (function for **Velocity vs. Time** is given):

   a. When you look at Piece A, can you determine Stuart's initial position (x_o), velocity (v_{xo}), and acceleration (a_{xo})? Remember to always provide +/- signs for displacement, velocity, and acceleration (they are vectors). Write your answers in the first row of Table 2(a).

      Explain why you cannot determine the initial position (x_o) using only the information provided on the graph. For that unknown initial value, you may use a value of $x_o = +3$ ft.

   b. Now look at the shape and values for each piece of the function and carefully describe what is happening to Stuart's displacement, velocity, and acceleration. Write your answers in Table 2(a). You should look back at your answers on Table 1 to guide you through your explanations!

   c. From the graph, find Stuart's displacement and show/explain how you did it (hint: Area!). Use this information to create a graph of Position vs. Time for Stuart.

      The only "tricky" part of this task is finding the area "under" Piece F because it is not linear:

      - Pre-calc students: Shade in the area "captured" between the function and the t-axis and estimate the area by counting full and partial blocks (remember that area below the t-axis is negative, above is positive). Round your estimate to the nearest whole block.
      - Calc Students: Find the area using calculus. Show all of your work below. Do NOT use a calculator (use your head, the graph, a pencil, and fractions).

   d. From the graphs, find Stuart's total distance travelled and show/explain how you did it:

   e. From the graph, find Stuart's avg. velocity on his trip and show/explain how you did it:

   f. Is there any point on Piece F for which Stuart's instantaneous acceleration is zero? Explain!

   g. Using your answers from above and from Table 1 and 2(a), carefully create an Acceleration vs. Time graph for Stuart's trip. Hint: Shouldn't it look identical to the graph you made for 6(g)??

   h. Are there any motions on the V vs. T graph that would be physically impossible to achieve? Why?

8. Now work on Graph 3 (function for **<u>Acceleration vs. Time</u>** is given):

   a. When you look at Piece A, can you determine Stuart's <u>initial</u> position (x_o), velocity (v_{x0}), and acceleration (a_{xo})? Remember to always provide +/- signs for displacement, velocity, and acceleration (they are vectors). Write your answers in the first row of Table 2(a).

      <u>Explain</u> why you <u>cannot</u> determine the <u>initial</u> position (x_o) or velocity (v_{xo}) using only the information provided on the graph.

   b. If the initial acceleration was <u>zero</u> for the first two seconds instead of – 2 ft/s^2, what would this tell you about Stuart's…

      i. Velocity during the first two seconds?

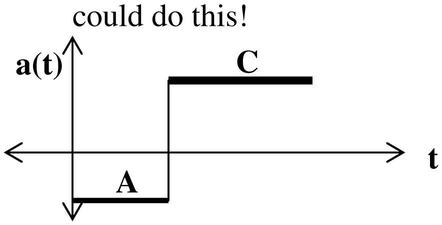
      ii. Position during the first two seconds?

   c. Now look at the shape and values for <u>each piece</u> of the function and carefully describe what is happening to Stuart's <u>displacement</u>, <u>velocity</u>, and <u>acceleration</u>. Write your answers in Table 2(a). You should look back at your answers on Table 1 to guide you through your explanations! You only need to discuss Pieces A, B and C. If you would like to do the rest for extra credit, please make your own table on separate paper.

   d. From the Pieces A, B, and C of the acceleration graph, find Stuart's <u>velocity</u> and <u>displacement</u> and <u>show</u>/<u>explain</u> how you did it (hint: <u>Area</u>!). Use this information to create a graph of Position vs. Time for Stuart.

   e. Are there any motions on the Acceleration vs. Time that would be physically impossible to achieve? Explain.

   f. Suppose Stuart's Acceleration vs. Time graph started using Piece A for the first two seconds, then Piece C for the next three seconds. Is this scenario physically possible? Explain how he could do this!



9. Try using the University of Colorado's PhET Moving Man simulation to recreate two or more consecutive pieces from X vs. T in Graph 1, V vs. T in graph 2 and A vs. T in Graph 3. You'll want to use the "Charts" tab on the simulation. Take a printscreen picture of your results and cut and paste into a blank document, print, and hand in.

   For example, I ran an approximate simulation for #8(f) by doing the following…

   a. Type "– 2" into the "Acceleration" box
   b. Press the play ▸ button just below the "Acceleration" box
   c. **Quickly** type "3" into the "Acceleration" box, then **quickly** hit enter before Abe Lincoln runs into the back wall!
   d. Hit the "pause" ⏸ button afer a couple of seconds
   e. Adjusted the scale of the vertical horizontal axes of the acceleration graph using the zoom buttons on the right side (magnifying glass with a + sign)
   f. Use the shift-printscreen keys on your keyboard to capture an image of your screen
   g. Cut and paste the image to your document as shown below…

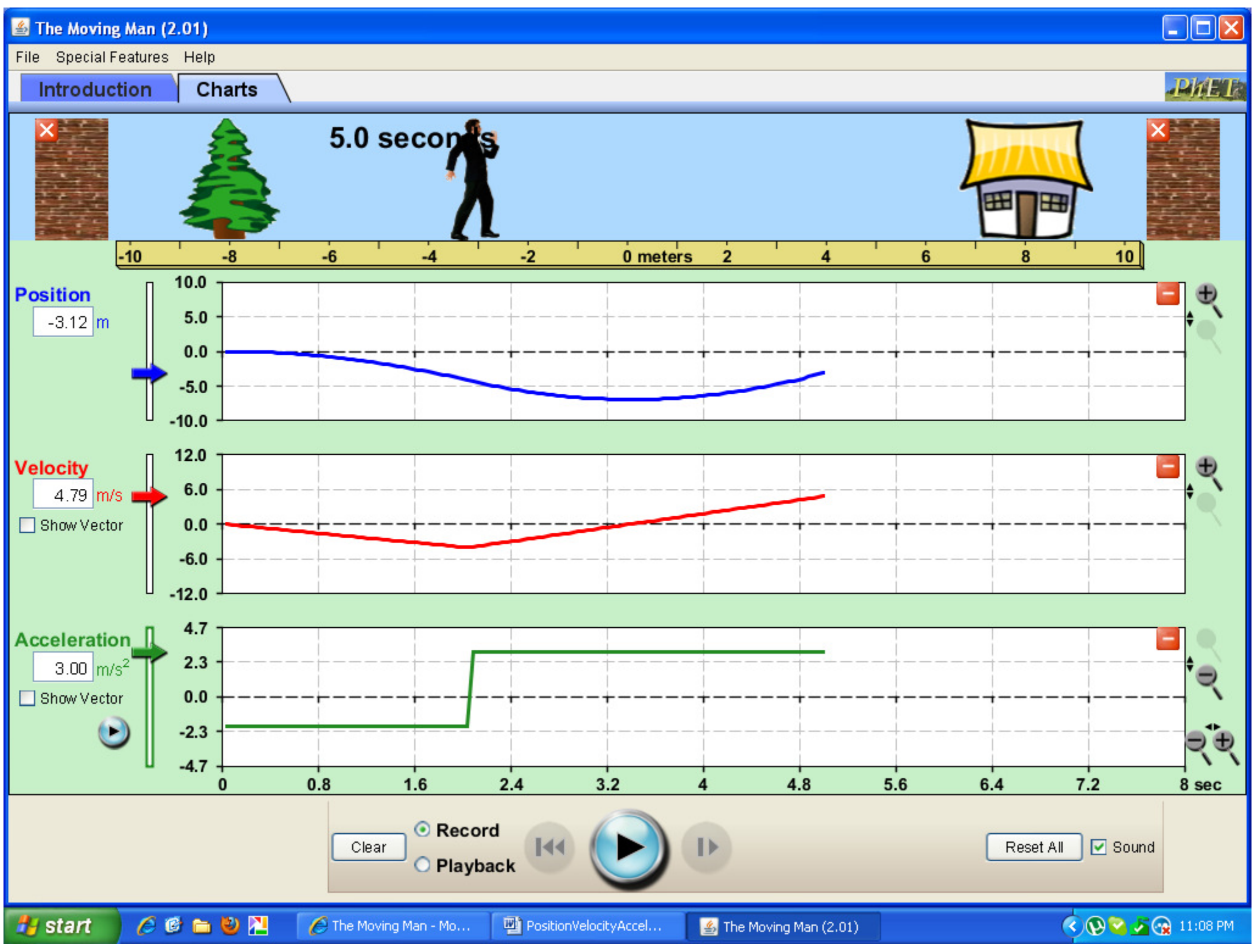


Table 1: Relationship between Graphs – Physical Meaning of Slope and Area

	Position vs. Time Graph		Velocity vs. Time Graph		Acceleration vs. Time Graph	
	Units	Physical Meaning	Units	Physical Meaning	Units	Physical Meaning
Horizontal Axis (abscissa)						
Vertical Axis (ordinate)						
Slope of a line connecting two points (secant line)						
Slope of a line tangent to one point (tangent line)						
Area below (or above) a line or curve segment and the horizontal axis						
A horizontal line segment	n/a		n/a		n/a	
A line segment with a + slope	n/a		n/a		n/a	
A line segment with a – slope	n/a		n/a		n/a	
A convex (concave up) curve segment	n/a		n/a		n/a	
A concave (down) curve segment	n/a		n/a		n/a	

Table 2(a): Given <u>Position vs. Time</u> Piecewise Function – Use plain English + physics terminology to accurately describe…

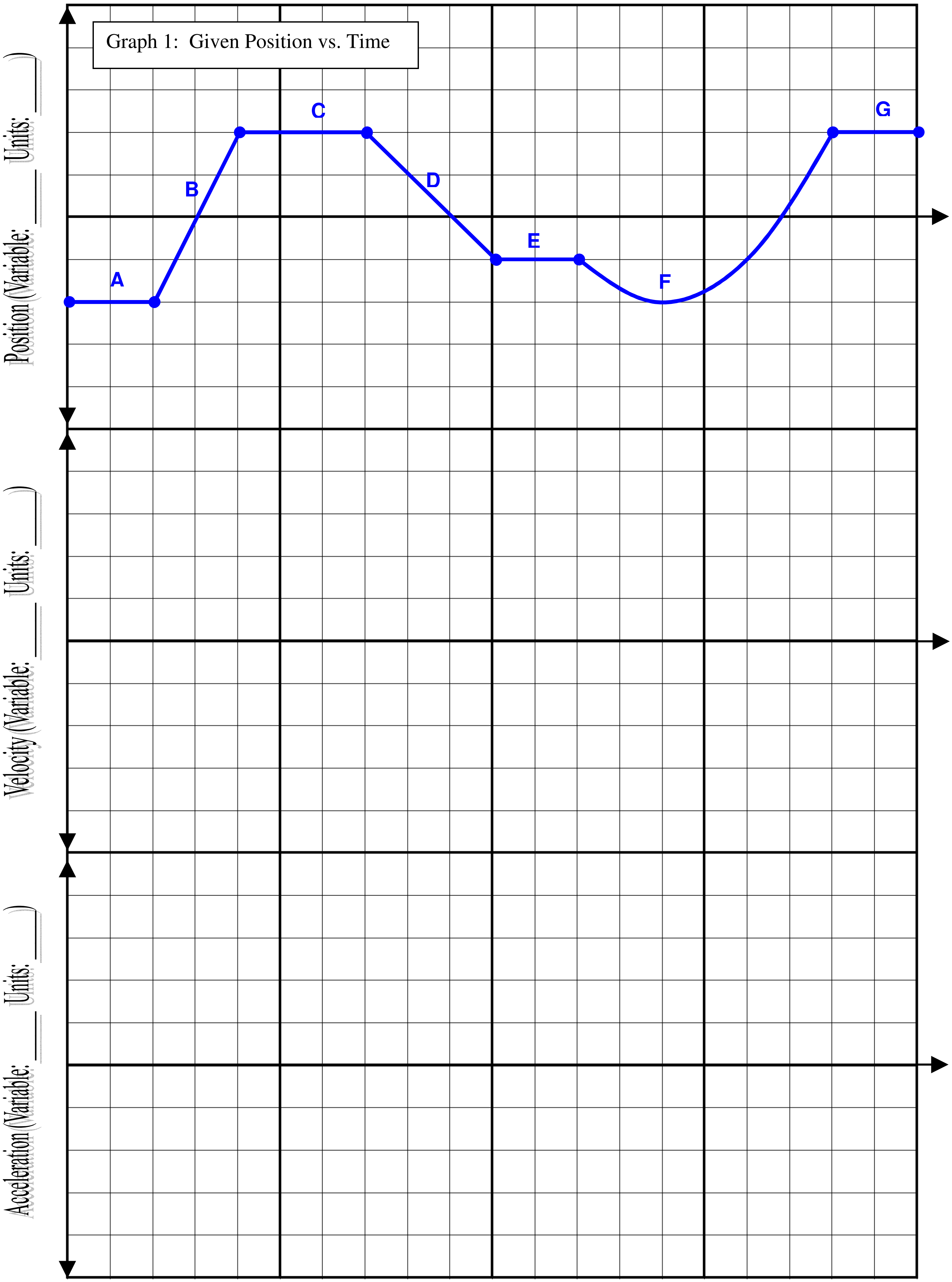
From the graph, determine, if possible…	x_0 =	v_{x0} =	a_{x0} =
	Position	**Velocity**	**Acceleration**
A			
B			
C			
D			
E			
F			
G			

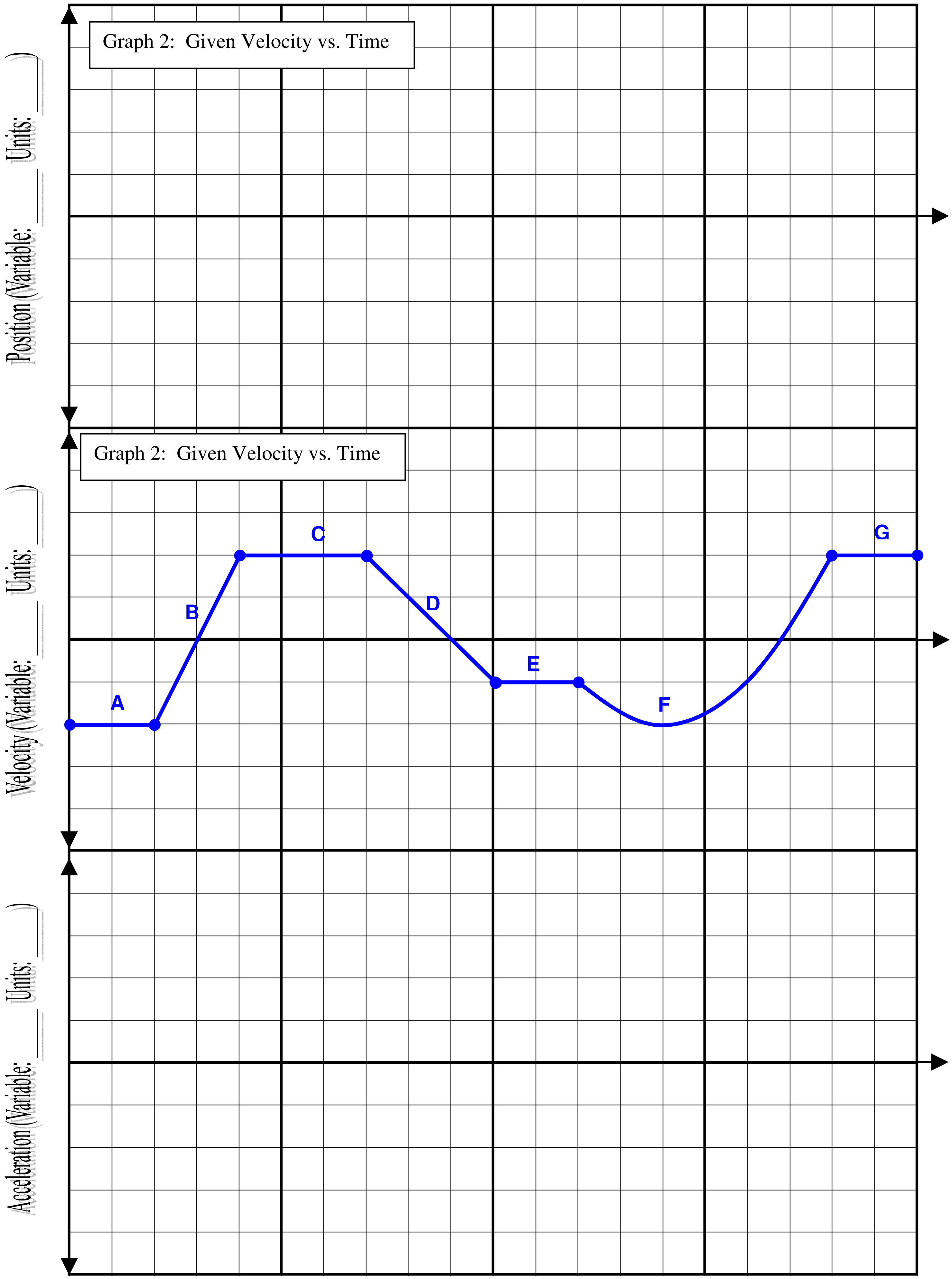
Table 2(b): Given <u>Velocity vs. Time</u> Piecewise Function – Use plain English + physics terminology to accurately describe…

From the graph, determine, if possible…	x_0 =	v_{x0} =	a_{x0} =
	Position	**Velocity**	**Acceleration**
A			
B			
C			
D			
E			
F			
G			

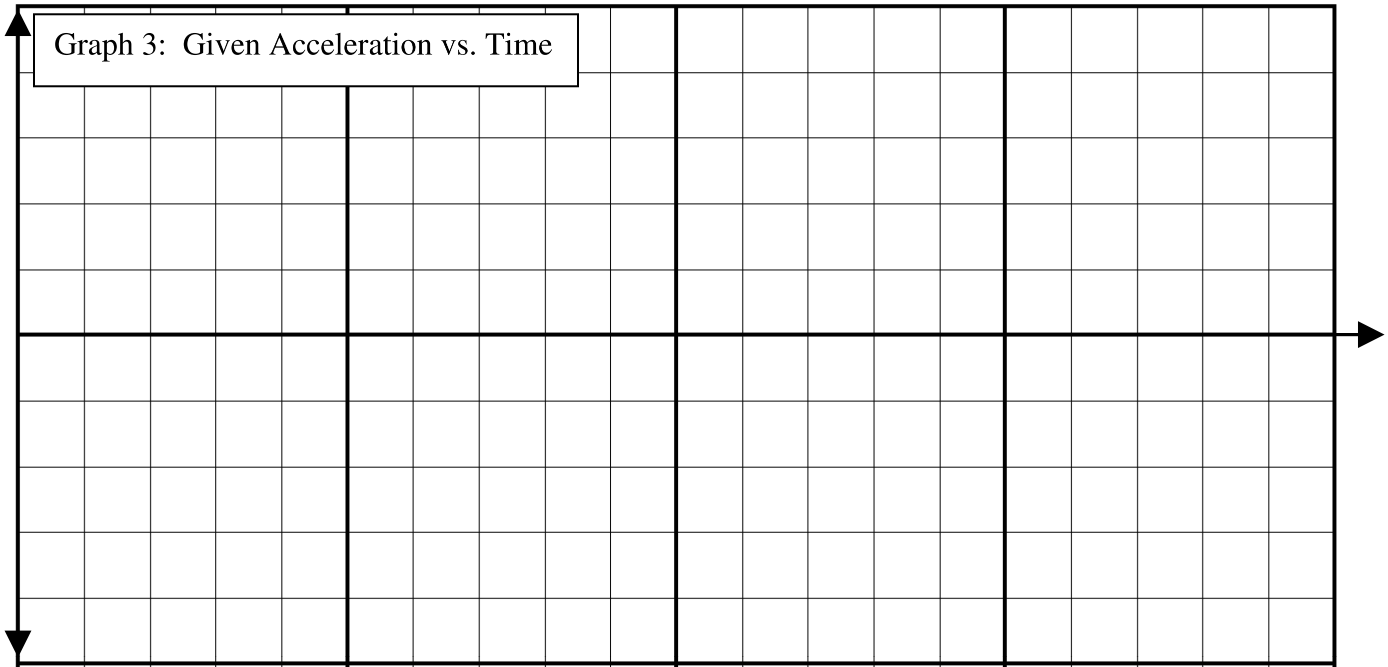
Table 2(c): Given Acceleration vs. Time Piecewise Function – Use plain English + physics terminology to accurately describe…

From the graph, determine, if possible…	x_0 =	v_{x0} =	a_{x0} =
A			
B			
C			

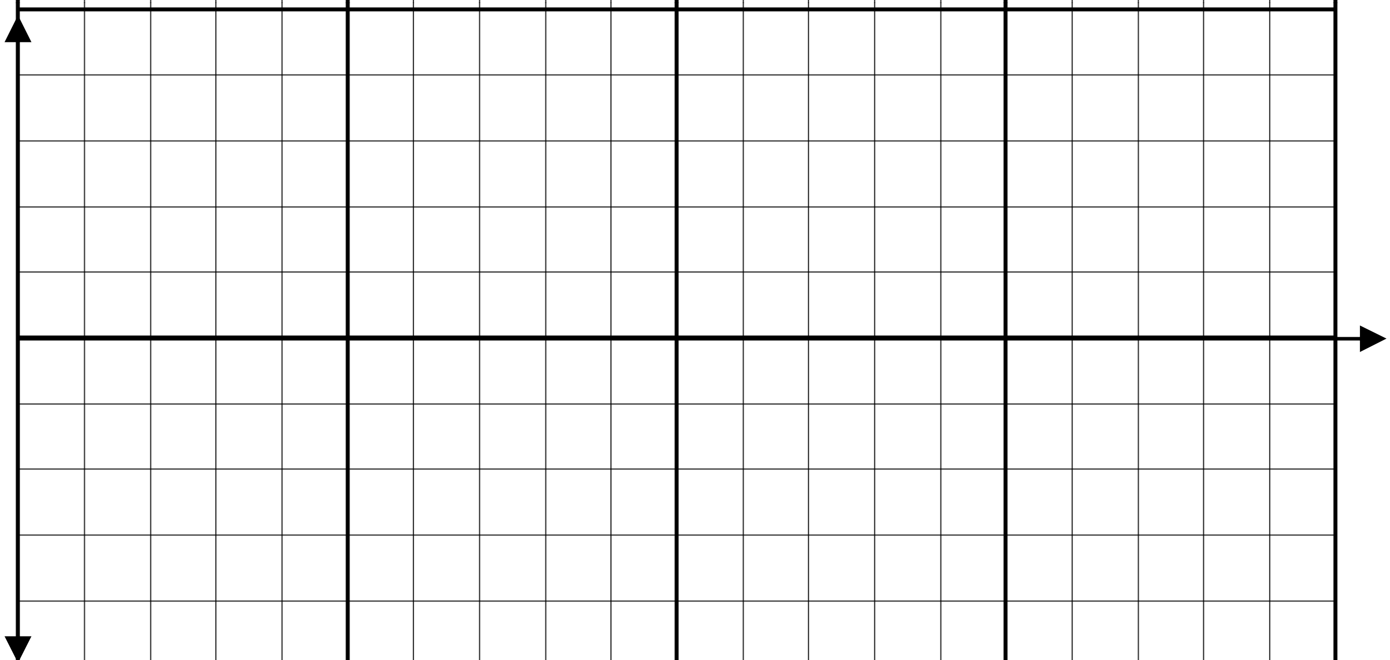

Graph 1: Given Position vs. Time
A
B
C
D
E
F
G
Position (Variable: ____ Units: ____)
Velocity (Variable: ____ Units: ____)
Acceleration (Variable: ____ Units: ____)


Graph 2: Given Velocity vs. Time
Position (Variable: ____ Units: ____)
Graph 2: Given Velocity vs. Time
Velocity (Variable: ____ Units: ____)
A
B
C
D
E
F
G
Acceleration (Variable: ____ Units: ____)

Position (Variable: ________ Units: ________)



Velocity (Variable: ________ Units: ________)



Acceleration (Variable: ________ Units: ________)