Looking Ahead to Chapter 9

Focus

In Chapter 9, you will learn how to identify prime and composite numbers, factor composite numbers by using prime factorizations, and write numbers in scientific notation. You will also work with powers and learn properties of powers, as well as different forms of *n*th roots of numbers.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 9.

Use mental math to find the value of *x*.

1. $3x = 81$ **2.** $5x = 105$ **3.** $17x = 85$

Write each expression as a power.

4. (8)(8)(8)(8) **5.** 11 • 11 • 11 • 11 • 11 • 11 **6.** (–13)(–13)(–13)

9

Evaluate each power.

7. 2^6 **8.** 5^2 **9.** 10^4

Read the problem scenario below.

You work for a cooking supply manufacturer and you are in charge of making circular pizza pans. Your boss would like to know the area of each circular pan to get an idea of the amount of material that will be needed for each of the pizza pans. Note that the area of a circle is $A = \pi r^2$, where r is the radius of the circle. Use 3.14 for π.

10. What is the area of a pizza pan that has a radius of 3 inches?

11. What is the area of a pizza pan that has a radius of 6 inches?

12. One pizza pan has a radius of 7 inches. Another pizza pan has a radius of 5 inches. How much more area does the first pizza pan have than the second pizza pan?

Key Terms

CHAPTER

9

Properties of Exponents

In 2004, Steve Fossett set a world record for the fastest trip around the world in a sailboat. He and his crew of thirteen made the journey in 58 days and 9 hours. In Lesson 9.6, you will examine the capsize factor of a boat by using its weight and width.

9

9

9.1 The Museum of Natural History

Powers and Prime Factorization

Objectives

In this lesson, you will:

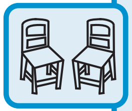
- List the factors of numbers.
- Identify prime and composite numbers.
- Write the prime factorizations of numbers.

Key Terms

- factor
- prime number
- composite number
- prime factorization

SCENARIO Some of the students at an elementary school are going on a field trip to a museum of natural history. Some adults are also going on the trip to serve as guides. The school principal wants to divide the students and adults into groups so that each group has the same number of students and the same number of adults.

Problem 1 Splitting Up



Suppose that there are 12 students and 4 adults going on the trip.

A. Can there be two groups of students and adults? Use complete sentences to explain your reasoning.

B. Can there be three groups of students and adults? Use complete sentences to explain your reasoning.

C. Can there be four groups of students and adults? Use complete sentences to explain your reasoning.

D. Can there be five groups of students and adults? Use complete sentences to explain your reasoning.

E. What are the different-sized groups that are possible for this number of students and adults? Describe the number of groups, and the numbers of students and adults in each group. Use complete sentences in your answer.

Investigate Problem 1

1. Now, suppose that there are 30 students and 6 adults going on the trip. What are the different-sized groups that are possible for this number of students and adults? Describe the number of groups, and the numbers of students and adults in each group. Use complete sentences to explain your reasoning.

2. Now suppose that there are 29 students and 5 adults going on the trip. What are the different-sized groups that are possible for this number of students and adults? Describe the number of groups, and the number of students and adults in each group. Use complete sentences to explain your reasoning.

3. How does the number of groups relate to the number of students and the number of adults going on the trip? Use complete sentences in your answer.

4. How did you determine the answers to part (E) and Questions 1 and 2? Use complete sentences to explain your method.

5. **Just the Math: Factors of a Number** The **factors** of a given number are the numbers that evenly divide the given number. For each pair of numbers, list all the factors.

4 and 12 | 6 and 30 | 5 and 29

Investigate Problem 1

In part (E) and in Questions 1 and 2, you were finding all the *common* factors of the numbers of students and adults. List the common factors of each pair of numbers in Question 5.

In part (E) and in Questions 1 and 2, did you find all the different-sized groups? How do you know? Use complete sentences in your answer.

6. Just the Math: Prime Numbers What do you notice about the factors of 5 and 29?

The numbers 5 and 29 are called *prime numbers*. A **prime number** is a number greater than 1 that has exactly two whole number factors, 1 and itself. The numbers 4, 12, 6, and 30 are **composite numbers,** or numbers greater than 1 that have more than two whole number factors. Make a list of the first 10 prime numbers after the prime number 2.

7. Just the Math: Prime Factorization Every composite number can be written as a product of prime numbers. This is called the **prime factorization** of a number. For instance, write 36 as the product of prime factors.

36 = ______ • ______ • ______ • ______

You can use your knowledge of powers from Lesson 1.2 to write this prime factorization as products of powers: $2^2 \cdot 3^2$. Write the prime factorization of each of the numbers in Question 5. Write your answers as products of powers.

8. Use complete sentences to explain how you can use the prime factorization of a number to help you determine the common factors of a pair of numbers.

Investigate Problem 1

9. Use prime factorizations to find the common factors of each pair of numbers. Show all your work and use a complete sentence in your answer.

9 and 12

21 and 42

36 and 40

54 and 24

31 and 50

10. Suppose that 126 students and 48 adults are going on the trip. What are the different-sized groups that are possible for this number of students and adults? Describe the number of groups, and the numbers of students and adults in each group. Show all your work and use a complete sentence in your answer.

11. Which of the groupings would you choose from Question 10? Use complete sentences to explain your answer.

Take Note

Whenever you see the share with the class icon, your group should prepare a short presentation to share with the class that describes how you solved the problem. Be prepared to ask questions during other groups' presentations and to answer questions during your presentation.

9.2 Bits and Bytes

Multiplying and Dividing Powers

Objectives

In this lesson, you will:

- Write numbers as powers.
- Multiply powers.
- Divide powers.

Key Terms

- power
- exponent
- product
- quotient

SCENARIO The amount of memory available on CDs, hard drives, zip drives, and so on are measured in bytes. A byte is a basic unit of measurement of size for computer information, but the most basic unit is a bit. There are 8 bits contained in one byte.

Problem 1 Kilobytes, Megabytes, and Gigabytes

The most common units of measurement for computer data storage are kilobytes, megabytes, and gigabytes.

A. One kilobyte is the same as 1024 bytes. Write the prime factorization of the number of bytes in one kilobyte.

B. One megabyte is the same as 1,048,576 bytes. Write the prime factorization of the number of bytes in one megabyte.

C. One gigabyte is the same as 1,073,741,824 bytes. Write the prime factorization of the number of bytes in one gigabyte.

D. What do the prime factorizations in parts (A) through (C) have in common? Use a complete sentence in your answer.

Investigate Problem 1

1. Divide the number of bytes in one megabyte by the number of bytes in one kilobyte. Show all your work.

What does this number represent? Use a complete sentence in your answer.

2. Write the numerator, denominator, and quotient from Question 1 as powers of 2.

Investigate Problem 1

3. Divide the number of bytes in one gigabyte by the number of bytes in one kilobyte. Show all your work and use a complete sentence in your answer.

What does this number represent? Use a complete sentence in your answer.

4. Write the numerator, denominator, and quotient from Question 3 as powers of 2.

9

5. In Question 2, how do the bases of all the powers compare? In Question 4, how do the bases of all the powers compare? In each division problem, how does the exponent in the quotient relate to the exponents in the numerator and the denominator? Use complete sentences in your answer.

6. Write a rule that you can use to find the quotient of two powers that have the same base. Use a complete sentence in your answer.

7. Use your rule from Question 6 to simplify each expression. Show all your work and write your answer as a power.

$\frac{4^9}{4^8}$ $\frac{5^6}{5^3}$ $\frac{10^{12}}{10^8}$

Problem 2 Storage Options

External (outside) storage for a computer comes in different forms: CD, DVD, external hard drive, flash drive, and so on. The sizes for these different options vary and get larger as new technologies develop.

A. One model of hard drive can store up to 4 gigabytes of data. Find the number of bytes that this hard drive can store. Remember that 1 gigabyte = 1,073,741,824 bytes. Show all your work and use a complete sentence in your answer.

B. One model of flash drive can store up to 128 megabytes of data. Find the number of bytes that this flash drive can store. Remember that 1 megabyte = 1,048,576 bytes. Show all your work and use a complete sentence in your answer.

C. Use complete sentences to explain how you found your answers to parts (A) and (B).

Investigate Problem 2

1. Write the multiplication problem that you used in part (A) to find the number of bytes of storage for the hard drive.

Write each factor in the product as a power of 2. Then write the result as a power of 2. What do you notice about the exponents? Use a complete sentence in your answer.

2. Write the multiplication problem that you used in part (B) to find the number of bytes of storage for the hard drive.

Investigate Problem 2

Write each factor in the product as a power of 2. Then write the result as a power of 2. What do you notice about the exponents? Use a complete sentence in your answer.

3. How do the bases of the powers in Question 1 compare? How do the bases of the powers in Question 2 compare? Use a complete sentence in your answer.

4. Write a rule that you can use to find the product of two powers that have the same base. Use a complete sentence in your answer.

5. Use your rule from Question 4 to simplify each expression. Show all your work and write your answer as a power.

$5^6 \cdot 5^8$ $\qquad$ $2^3 \cdot 2^5$ $\qquad$ $1^9 \cdot 1^7$

6. Simplify each expression, if possible. Show all your work and write your answer as a power.

$4^5 \cdot 4^5$ $\qquad$ $\dfrac{8^{10}}{8^3}$ $\qquad$ $2^3 \cdot 3^2$

$\dfrac{6^4}{5^4}$ $\qquad$ $10^5 \cdot 10^1$ $\qquad$ $8^4 \cdot 3^4$

7. Describe the situations in which you would use powers to multiply and divide. Use complete sentences in your answer.

9.3 As Time Goes By

Zero and Negative Exponents

Objectives

In this lesson, you will:

- Write a number as a power.
- Evaluate powers with positive, negative, and zero exponents.

Key Terms

- positive exponent
- negative exponent
- zero exponent

SCENARIO Computer operations and communications between electronic equipment happen so fast that they need to be measured in units of time that are smaller than seconds. Three units of time that are smaller than seconds are milliseconds, microseconds, and nanoseconds.

Problem 1 Faster and Faster

A. One millisecond is the same as $\frac{1}{1000}$ second. Complete the statement below to determine the number of seconds that there are in 10,000 milliseconds.

$$10{,}000\ \cancel{\text{milliseconds}}\left(\frac{1\text{ second}}{1000\ \cancel{\text{milliseconds}}}\right) = \frac{\square\ \text{seconds}}{\square}$$

$$= \square\ \text{seconds}$$

There are ______ seconds in 10,000 milliseconds.

B. Complete the statement below to determine the number of seconds there are in 1000 milliseconds.

$$1000\ \cancel{\text{milliseconds}}\left(\frac{1\text{ second}}{1000\ \cancel{\text{milliseconds}}}\right) = \frac{\square\ \text{seconds}}{\square}$$

$$= \square\ \text{second}$$

There is ______ second in 1000 milliseconds.

C. Complete the statement below to determine the number of seconds there are in 100 milliseconds.

$$100\ \cancel{\text{milliseconds}}\left(\frac{1\text{ second}}{1000\ \cancel{\text{milliseconds}}}\right) = \frac{\square\ \text{seconds}}{\square}$$

$$= \square\ \text{second}$$

There is ______ second in 1000 milliseconds.

D. How does the numerator compare to the denominator in each problem in parts (A) through (C)? Use complete sentences in your answer.

9

Investigate Problem 1

1. Show how you could use powers to find the answer to part (A). Show all your work.

2. Write the numerator and denominator from the statement in part (B) as a quotient of powers.

According to your answer in part (B), what is the value of this quotient?

Use the rule for finding a quotient of powers to simplify the quotient of powers above. Show your work and write your answer as a power.

3. Consider the quotient $\frac{2^4}{2^4}$. What is the value of this quotient? Use a complete sentence to explain your reasoning.

Write your answer as a power of 2. Show your work.

4. What is the value of any number raised to the power of zero? Use a complete sentence in your answer.

5. Write the numerator and denominator from the statement in part (C) as a quotient of powers.

Use the rule for finding a quotient of powers to write the quotient as a power of 10.

According to your answer to part (C), what is the value of this power of 10?

So, $10^{-1} = \frac{1}{10^1}$.

Investigate Problem 1

6. Complete the table below that shows small units of time.

Unit	Number of seconds	Number of seconds as a power with a positive exponent	Number of seconds as a power with a negative exponent
Millisecond	$\frac{1}{1000}$	$\frac{1}{10^{\square}}$	$10^{\square}$
Microsecond	$\frac{1}{1,000,000}$	$\frac{1}{10^{\square}}$	$10^{\square}$
Nanosecond	$\frac{1}{1,000,000,000}$	$\frac{1}{10^{\square}}$	$10^{\square}$

7. Use complete sentences to explain how you can write a power with a negative exponent as a power with a positive exponent.

8. Rewrite the power so that the exponent is positive.

5^{-3} $\quad$ 4^{-6} $\quad$ 3^{-10}

9. Suppose that you are given a fraction that has a power in the denominator. How can you rewrite the value so that the power is no longer in the denominator? Use complete sentences in your answer.

10. Rewrite the fraction so that there is no power in the denominator.

$\frac{1}{8^3}$ $\quad$ $\frac{1}{1^5}$ $\quad$ $\frac{1}{6^7}$

Investigate Problem 1

11. What can you conclude about the value of a power when the exponent is greater than zero? Use a complete sentence in your answer.

What can you conclude about the value of a power when the exponent is less than zero? Use a complete sentence in your answer.

What can you conclude about the value of a power when the exponent is zero? Use a complete sentence in your answer.

12. Use what you have learned so far in this chapter to simplify each expression completely. Show all your work.

$\frac{8^3}{8^5}$

$6^{-3} \cdot 6^2$

$10^0 \cdot 10^{-3}$

$\frac{2^0}{2^4}$

$\frac{2^{-3}}{2^2}$

$5^4 \cdot 5^{-2}$

9.4 Large and Small Measurements

Scientific Notation

Objectives

In this lesson, you will:

- Write numbers in scientific notation.
- Write numbers in standard form.

Key Terms

- standard form
- scientific notation

Take Note

To write a number in *standard form* means to write the number as a numeral.

SCENARIO In our universe, we encounter very large numbers, such as the diameter of Earth, the distances to Mars, other planets and stars, and so on. We also encounter very small numbers, such as the weight of a butterfly, the length of a blood cell, the width of a grain of sand, and so on. **Scientific notation** is a shorthand notation that is used to write these numbers so that they can be more easily used in computations.

Problem 1 The Big Stuff

Scientific notation uses powers of ten to represent large and small numbers. A number is written in scientific notation if it is the product of a number greater than or equal to one and less than 10 and a power of 10. You can use scientific notation to describe the distances from the sun to planets in the solar system.

A. Earth's average distance from the sun is 1.496×10^8 kilometers. Complete the statement below to write this distance in standard form.

1.496×10^8 kilometers = ☐ kilometers

B. Mercury's average distance from the sun is 5.8×10^7 kilometers. Complete the statement below to write this distance in standard form.

5.8×10^7 kilometers = ☐ kilometers

C. What happens to the value of a power of ten as the exponent gets larger? Use a complete sentence in your answer.

D. Mars' average distance from the sun is 227,900,000 kilometers. Complete the statement below to write this distance in scientific notation.

227,900,000 kilometers = 2.279 × ☐ kilometers

E. Saturn's average distance from the sun is 1,433,000,000 kilometers. Complete the statement below to write the diameter in scientific notation.

1,433,000,000 kilometers = ☐ $\times 10^9$ kilometers

9

Investigate Problem 1

1. The following numbers represent the closest distance of a comet to the sun during the comet's orbit. Which distances are *not* written in scientific notation? How do you know? Use complete sentences in your explanation.

   Comet Halley: 8.78×10^7 kilometers

   Comet Encke: 50×10^6 kilometers

   Comet Wild 2: 2.368×10^8 kilometers

   Comet Hale-Bopp: 136×10^6 kilometers

2. Write the distances that you identified in Question 1 in scientific notation.

3. Without writing the distances in standard form, list the comets in order from shortest distance from the sun to longest distance from the sun.

4. Use a complete sentence to explain how to compare two large numbers that are in scientific notation.

5. Write each of the distances from Question 1 in standard form.

   Comet Halley: ______________________

   Comet Encke: ______________________

   Comet Wild 2: ______________________

   Comet Hale-Bopp: ______________________

Take Note

Note that when you compare two numbers written in scientific notation that have the same exponent, the number that is larger is the number with the larger value between 1 and 10. For instance, the number 3.4×10^5 is greater than the number 3.2×10^5.

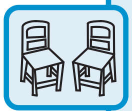
Problem 2 The Small Stuff

You can also use scientific notation to write very small numbers.

A. The diameter of a large grain of sand is 2.0×10^{-1} centimeter. Complete the statement below to write the diameter in standard form.

2.0×10^{-1} centimeter = ☐ centimeter

Problem 2 The Small Stuff



B. The diameter of a red blood cell is 8.4×10^{-4} centimeter. Complete the statement below to write this diameter in standard form.

8.4×10^{-4} centimeter = $\square$ centimeter

C. What happens to the value of a number as the absolute value of the exponent of a power of ten gets larger? Use a complete sentence in your answer.

D. The diameter of a bacterium is 0.00002 centimeter. Complete the statement below to write this diameter in scientific notation.

0.00002 centimeter $= 2.0 \times \square$ centimeter

E. The diameter of a human hair is 0.0025 centimeter. Complete the statement below to write the diameter in scientific notation.

0.0025 centimeter $= \square \times 10^{-3}$ centimeter

Investigate Problem 2

1. The following numbers represent the diameters of the nucleus (inside) of different kinds of atoms. Which diameters are *not* written in scientific notation? How do you know? Use complete sentences in your explanation.

   Aluminum atom nucleus: 7.2×10^{-13} centimeter

   Cobalt atom nucleus: 0.93×10^{-15} centimeter

   Gold atom nucleus: 14×10^{-11} centimeter

   Sodium atom nucleus: 6.8×10^{-13} centimeter

2. Write the diameters that you identified in Question 1 in scientific notation.

3. Without writing the diameters in standard form, list the atoms by diameter from least to greatest.

Investigate Problem 2

4. Use a complete sentence to explain how to compare two small numbers that are in scientific notation.

5. Write each of the diameters from Question 1 in standard form.

Aluminum atom nucleus: ______________________________

Cobalt atom nucleus: ______________________________

Gold atom nucleus: ______________________________

Sodium atom nucleus: ______________________________

6. Complete the table that shows measurements for different objects.

Object	Measurement	Measurement in standard form	Measurement in scientific notation
Jupiter	average distance from sun (kilometers)	778,600,000	
Water molecule	diameter (centimeters)		2.82×10^{-8}
Comet Hyakutake	nearest distance to the sun (kilometers)		3.4×10^{7}
Atom	diameter (centimeters)	0.00000005	
Dust speck	diameter (centimeters)		3.0×10^{-5}
Saturn's E ring	inner radius (kilometers)	300,000	

7. What happens to the decimal point in a number after you multiply the number by a power of ten with an exponent that is an integer greater than zero?

What happens to the decimal point in a number after you multiply the number by a power of ten with an exponent that is an integer less than zero?

9.5 The Beat Goes On

Properties of Powers

Objectives

In this lesson, you will:

- Use the power of a power property.
- Use the power of a product property.
- Use the power of a quotient property.

Key Terms

- power
- product
- quotient

Take Note

In a circle, the radius is half the length of the diameter.

Take Note

Recall that π is an irrational number whose value is approximately 3.14.

SCENARIO A basic drum is made of a cylindrical frame and one or two circular drumheads that are stretched over the frame. The sound that a drum makes depends on the amount that the drumhead is stretched, as well as the size of the drumhead.

Problem 1 Area of a Drumhead

A. A snare drum's drumhead is 8 inches. This measurement describes the diameter of the drumhead. Find the area of the drumhead by using the formula for the area of a circle: $A = \pi r^2$, where r is the radius and A is the area. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π.

B. A bass drum's drumhead is 18 inches. Find the area of the drumhead. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π.

C. A tom-tom's drumhead is 16 inches. Find the area of the drumhead. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π.

Investigate Problem 1

1. For each answer in parts (A) through (C), write the prime factorization of the number that is multiplied by π.

Area of 8-inch drumhead: $A = \square\pi$

Area of 18-inch drumhead: $A = \square\pi$

Area of 16-inch drumhead: $A = \square\pi$

9

Take Note

The plural of radius is *radii.*

Investigate Problem 1

2. What do you notice about the radii of the drumheads in parts (A) through (C)? Use a complete sentence in your answer.

3. Write each radius from the problem situation as a power.

Radius of 8-inch drumhead: $r =$ ☐

Radius of 18-inch drumhead: $r =$ ☐

Radius of 16-inch drumhead: $r =$ ☐

4. In finding each drumhead area, you squared the radius of the drumhead. Review your work in Questions 1 and 3. What is the result of squaring a power? Use a complete sentence in your answer.

Take Note

Recall that a *conjecture* is a possible explanation that can be tested by further investigation.

9

5. What do you think is the result of cubing a power? Write an example that demonstrates your conjecture. Use a complete sentence in your answer.

6. Write a rule using a complete sentence that explains how to simplify a power that has a base that is a power, such as:

$(2^4)^5$. ← exponent

↖ base is a power

Problem 2 More Bass Drum Drumheads

A. Bass drum drumheads come in a multitude of sizes. Another size of a bass drum drumhead is 24 inches. Find the area of the drumhead. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π.

B. Write the prime factorization of the radius of the drumhead in part (A).

$r =$ ☐

Now write the prime factorization of the area.

$A =$ ☐ π

Problem 2 More Bass Drumheads

How are the prime factorizations related? Use a complete sentence in your answer.

C. Another bass drum drumhead size is 36 inches. Find the area of the drumhead. Show all your work and use a complete sentence in your answer. Leave your answer in terms of π.

D. Write the prime factorization of the radius of the drumhead in part (C).

$r = \boxed{}$

Now write the prime factorization of the area.

$A = \boxed{}\pi$

How are the prime factorizations related? Use a complete sentence in your answer.

9

Investigate Problem 2

1. Use a complete sentence to explain how you can simplify a power that has a product as its base without first simplifying the base.

2. Now consider a power that has a quotient as its base, such as: $\left(\frac{3}{4}\right)^{10}$.

Write a conjecture for a method of how to simplify this kind of power. Write an example that demonstrates your conjecture. Use a complete sentence in your answer.

Investigate Problem 2

3. Use your properties of powers to simplify each expression. Show all your work.

$(3^2)^3$

$(15 \cdot 12)^2$

$\left(\frac{2}{3}\right)^4$

$(4^5)^0$

4. A summary of the rules for powers that you have learned so far in this chapter is shown below. Complete each property.

Product rule of powers: $a^b a^c = a^{\square}$

Quotient rule of powers: $\frac{a^b}{a^c} = a^{\square}$

Negative exponents: $\frac{1}{a^b} = a^{\square}$

Zero exponents: $a^0 = \square$

Power of a power rule: $(a^b)^c = a^{\square}$

Power of a product rule: $(ab)^c = a^{\square}b^{\square}$

Power of a quotient rule: $\left(\frac{a}{b}\right)^c = \frac{a^{\square}}{b^{\square}}$

5. All the properties that you have used so far apply to algebraic expressions too. Simplify each expression. Show all your work.

$(x^3)^3$

$(2p)^2$

y^3y^7

$\left(\frac{x}{4}\right)^3$

9

Investigate Problem 2

6. For each problem below, identify the property that is used in each step to simplify the expression. The first problem is done as an example.

$$\left(\frac{x^3}{y^2z}\right)^4 = \frac{(x^3)^4}{(y^2z)^4}$$ Power of a quotient rule

$$= \frac{(x^3)^4}{(y^2)^4z^4}$$ Power of a product rule

$$= \frac{x^{12}}{y^8z^4}$$ Power of a power rule

$(2ab^2)^3(-3b)^2 = 2^3a^3(b^2)^3(-3)^2b^2$ ______________________

$= 8a^3(b^2)^3(9)b^2$ ______________________

$= 8a^3b^6(9)b^2$ ______________________

$= 8(9)a^3b^6b^2$ ______________________

$= 72a^3b^6b^2$ ______________________

$= 72a^3b^8$ ______________________

Take Note

Recall that in Lesson 4.5 you learned other properties of real numbers.

$\left(\frac{x^2}{2y^3}\right)^4\left(\frac{xy^2}{3}\right)^3 = \frac{(x^2)^4}{(2y^3)^4} \cdot \frac{(xy^2)^3}{3^3}$ ______________________

$= \frac{(x^2)^4}{2^4(y^3)^4} \cdot \frac{x^3(y^2)^3}{3^3}$ ______________________

$= \frac{x^8}{2^4y^{12}} \cdot \frac{x^3y^6}{3^3}$ ______________________

$= \frac{x^8x^3y^6}{2^4y^{12}(3^3)}$ ______________________

$= \frac{x^{11}y^6}{2^4y^{12}(3^3)}$ ______________________

$= \frac{x^{11}}{2^4y^6(3^3)}$ ______________________

$= \frac{x^{11}}{16y^6(27)}$ ______________________

$= \frac{x^{11}}{432y^6}$ ______________________

9

9

9.6 Sailing Away

Radicals and Rational Exponents

Objectives

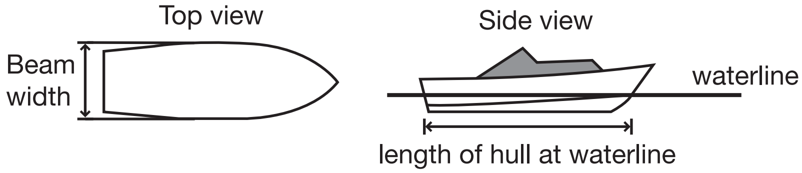
In this lesson, you will:

- Find the *n*th root of a number.
- Write an expression in radical form.
- Write an expression in rational exponent form.

Key Terms

- cube root
- index
- *n*th root
- radicand
- rational exponent
- radical

SCENARIO Some boats move by skimming across the surface of the water, while other boats move by pushing through the water. The boats that push through the water have a body, or hull, that is called a displacement hull. Sailboats are boats with displacement hulls. Important measurements of a boat that can be used to determine the boat's speed and stability are labeled on the figure below.



Problem 1 Boat Stability

The weight of a boat depends on its beam width. An equation that relates a boat's weight to its beam width is $w = 64\left(\frac{b}{c}\right)^3$, where w is the boat's weight in pounds, b is the beam width in feet, and c is the capsize factor. When the capsize factor c is less than 2, the boat is less likely to capsize, or turn over.

A. Suppose that a boat has a capsize factor of three and a beam width of six feet. What is the weight of the boat? Show your work and use a complete sentence in your answer.

B. Suppose that a boat has a capsize factor of three and a beam width of nine feet. What is the weight of the boat? Show your work and use a complete sentence in your answer.

Take Note

When a power has an exponent of 3, it is called the third power. When you evaluate a third power, you are *cubing* the base.

Problem 1 Boat Stability

C. Use complete sentences to explain how you found your answers to parts (A) and (B).

D. Suppose that a boat has a capsize factor of 2 and weighs 1000 pounds. Write an equation that you can use to find the beam width of the boat.

In the equation that you wrote, get the power by itself on one side of the equation. Show your work.

E. To find the value of *b*, you need to find the number that when multiplied by itself three times is equal to 125. What is this number? Use a complete sentence in your answer.

F. What is the boat's beam width? Use a complete sentence in your answer.

Investigate Problem 1

1. Just the Math: Cube Root Your answer to part (E) is the *cube root* of 125. You can say that a number b is a **cube root** of a number a if $b^3 = a$.

So, 5 is a cube root of 125 because $5^3 = 125$.

Is there another number whose cube is 125?

A cube root is designated by using the symbol $\sqrt[3]{\ }$. The number 3 is called the **index** of the radical. So, $\sqrt[3]{125} = 5$.

Complete each statement below.

$\sqrt[3]{8} = \square$ $\sqrt[3]{-27} = \square$

Take Note

Finding the cube root of a number is the *inverse operation* of finding the cube of a number.

Investigate Problem 1

2. Just the Math: *n*th Roots You can extend the idea of square roots and cube roots. Use n to represent a positive number. Then a number b is the ***n*th root of *a*** if $b^n = a$. For instance, 2 is the fourth root of 16 because $2^4 = 16$.

Complete each statement below.

3 is the fifth root of 243 because $\square = 243$.

–2 is the cube root of –8 because $(-2)^3 = \square$.

4 is the $\square$ root of 4096 because $4^6 = 4096$.

The *n*th root of a number a is designated as $\sqrt[n]{a}$, where n is the index of the radical and a is the *radicand*.

Take Note

It is understood that $\sqrt{}$ and $\sqrt[2]{}$ indicate a square root.

3. Complete each statement below.

$\sqrt{100} = \square$ because $\square^2 = 100$.

$\sqrt[3]{216} = \square$ because $\square^3 = 216$.

$\sqrt[4]{81} = \square$ because $\square^4 = 81$.

$\sqrt[5]{-32} = \square$ because $(\square)^5 = -32$.

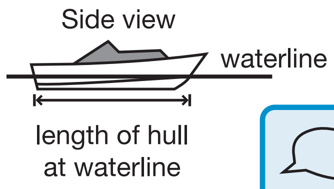
4. Notice that a power can be positive or negative, depending on the base and the exponent. When the exponent of a power is an even number and the base is a positive number, is the value of the power a positive number or a negative number? How do you know? Use a complete sentence to explain.

When the exponent of a power is an even number and the base is negative, is the value of the power a positive number or a negative number? Use a complete sentence to explain.

5. When the exponent of a power is an odd number and the base is a positive number, is the value of the power a positive number or negative number? Use a complete sentence to explain.

When the exponent of a power is an odd number and the base is negative, is the value of the power a positive number or negative number? Use a complete sentence to explain.

9



Problem 2 Boat Speed

The speed of a boat depends on the length of the hull at the waterline. An equation that relates the speed s in knots and the length ℓ in feet of the hull at the waterline is $s = 1.34\sqrt{\ell}$.

A. What is the speed of a boat that has a length of 16 feet at the waterline? Show your work and use a complete sentence in your answer.

B. Write an equation that you can use to find the length of a boat at its waterline if the boat's speed is 6.7 knots.

C. In the equation that you wrote, get the radical by itself on one side of the equation. Show your work.

D. What is the length of the boat at the waterline? How do you know? Use complete sentences in your answer.

Your answer is the value of which variable? Use a complete sentence in your answer.

Investigate Problem 2

1. In the problem above, you found that the solution of the equation $\sqrt{\ell} = 5$ is $\ell = 25$. Complete the steps below.

$\ell = 25$	Given
$\ell = \square$	Write 25 as a power.
$\ell = \square$	Replace 5 by $\sqrt{\ell}$.

Investigate Problem 2

2. Because the definition of the *n*th root involves powers, it would be nice if we could write an *n*th root of a number as a power of the number. Consider your equation from Question 1:

$\ell^1 = (\sqrt{\ell})^2$.

If we could write the square root of ℓ as a power of ℓ, we would have $\ell^1 = (\ell^?)^2$. You know that when you find the power of a power, you multiply the exponents: $(a^b)^c = a^{\square}$.

Represent the square root as an exponent so the exponent times 2 is equal to 1. Use complete sentences to explain.

3. Just the Math: Radicals and Rational Exponents
A **rational exponent** is an exponent that is a rational number. We can write each *n*th root as a rational exponent in the following way: If *n* is an integer greater than 1, then $\sqrt[n]{a} = a^{1/n}$. Write each radical as a power.

$\sqrt[3]{7}$ $\qquad$ $\sqrt[5]{x}$ $\qquad$ $\sqrt{y}$

Write each power as a *radical*.

$8^{1/4}$ $\qquad$ $z^{1/5}$ $\qquad$ $m^{1/3}$

Take Note

Remember that a *rational number* is a number of the form $\frac{a}{b}$ where $b \neq 0$.

4. Write the equation in Problem 2 in rational exponent form.

5. Another nautical equation can be used to determine how fast a sailboat can travel in light wind. This equation is $s = \frac{a}{d^{2/3}}$ where *a* is the sail area and *d* is the boat's displacement. You can write this equation in radical form by using the properties you know about powers. Use the properties of powers to write the equation in radical form.

$s = \frac{a}{d^{2/3}}$ Given equation

$s = \frac{a}{\square}$ Write power as a product.

$s = \frac{a}{\square}$ Power of a power property

$s = \frac{a}{\square}$ Definition of rational exponent

Take Note

A boat's *displacement* is the volume of water that is moved by the weight of the boat in the water.

Investigate Problem 2

6. The equation in Question 5 gives you s, the *sail area to displacement ratio.* Use the equation to find s for a boat that has a sail area of 600 square feet and a displacement of 125 cubic feet. Show your work and use a complete sentence in your answer.

7. Write each expression in radical form. Show your work and simplify your answer, if possible.

$4^{3/2}$

$5^{3/4}$

$x^{4/5}$

$y^{2/3}$

8. Write each expression in rational exponent form. Show your work and simplify your answer, if possible.

$(\sqrt[4]{2})^3$

$(\sqrt{5})^4$

$(\sqrt[5]{x})^8$

$(\sqrt[5]{y})^{10}$