Honors Precalculus Summer Review Packet

2013

This packet can also be downloaded from the B.M.C. Durfee High School Mathematics Department website at http://www.fallriverschools.org/math.cfm.

Honors Precalculus Summer Assignment Information

Welcome to Honors Precalculus! This summer review assignment is designed to refresh your Algebra 2 skills. It includes information that was taught in Honors Algebra 2 and will be used daily in Precalculus. Please understand that the material in this assignment is the information that provides the foundation for learning Precalculus. You must understand this information in order to be successful in the Precalculus class.

A GRAPHING CALCULATOR IS REQUIRED FOR THIS COURSE AND WILL NOT BE PROVIDED TO STUDENTS BY THE SCHOOL.

ASSIGNMENT REQUIREMENTS: You **MUST** show all work in order to receive credit! This includes the multiple choice problems. All work must be done on the attached answer sheets in a neat and organized manner. No work, no credit! Please write your multiple choice answers on the answer sheet that has been provided in this packet.

3 Big Shifts in Instruction: In keeping with Durfee's 3 Big Shifts in Instruction involving literacy where you are expected to get "regular practice with complex text and its academic language", you are given two sections from the Precalculus textbook that you are required to read, take notes on, and answer a set of questions about the material.

Due Date: This packet must be completed **by the first day of school**. Twenty-five points will be deducted for each day that this packet is late.

Grading: This assignment will be collected and graded based upon completion and correctness. It will count as your first **test grade** for Term I. The information in this assignment will be used regularly in the Precalculus classroom, therefore, you will also be tested on this same material throughout the year.

About Honors Precalculus: Honors Precalculus is a **rigorous and fast-paced** course. This standards based year-long course emphasizes the use and application of polynomial, logarithmic, and trigonometric functions and their applications, the extension of conic sections and the concept of theory of limits. There will be extensive use of the graphing calculator. ***A TI-84 Plus calculator is recommended*** and will be used by the instructor during lessons throughout the year. Instructors are only familiar with this type of calculator in teaching Precalculus, therefore if you buy a different type of calculator, you will need to learn how to perform the operations being done in class with your individual calculator. Be prepared for ***at least a half hour to an hour of homework each night*** with weekly quizzes and/or tests. You will also be assigned an extensive project each term.

3 BIG SHIFTS in MATHEMATICS INSTRUCTION: These shifts in mathematics instruction will be evident throughout the Precalculus course.

1. Greater focus on fewer topics
2. Linking Topics & Thinking across Grades
3. Rigorous Pursuit of Conceptual Understanding, Procedural Skill, & Application

Need Help With Something? The following links can be used to help you complete this assignment.

http://www.coolmath.com/algebra/Algebra2/

https://www.khanacademy.org/

http://www.webmath.com/

http://www.math.com/

Name: ____

Answer Sheet

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
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11. ________
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13. ________
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38. ________
39. ________
40. ________
41. ________
42. ________
43. ________
44. ________
45. ________
46. ________
47. ________
48. ________
49. ________
50. ________

1. **Given the function , find .**

   a. 3
   b. 195
   c. 99
   d. -5

2. **Given the function , find .**

   a.
   b.
   c.
   d.

3. **Simplify:**

   a.
   b.
   c.
   d.

4. **Simplify:**

   a.
   b.
   c.
   d.

5. **Simplify:**

   a.
   b.
   c.
   d.

6. **Simplify**

a.

b.

c.

d.

7. **Simplify**

a. $\frac{-54\sqrt{3} - 18\sqrt{2}}{75}$

b. $\frac{-54\sqrt{3} + 18\sqrt{2}}{\left(-9 - \sqrt{6}\right)}$

c. $\frac{\left(-9 - \sqrt{6}\right)}{75}$

d. $\frac{6\sqrt{3}}{75}$

8. **Solve the radical equation:** $\sqrt{9x - 9} + 5 = 10$

a.

b.

c.

d.

9. **Solve the system of equations:**

a. (-1, 3)

b. (1, -3)

c. (-3, 1)

d. (3, -1)

10. Solve the system of equations:

a. (-4, 6)

b. (4, -6)

c. (6, -4)

d. (-6, 4)

11. Solve the system of equations:

a. (-1, -2)

b. (1, 2)

c. (2, -1)

d. (1, -2)

12. Factor completely:

a.

b.

c.

d.

13. Factor completely:

a.

b.

c.

d.

14. Solve the equation by factoring:

a. –

b. –

c. –

d. –

15. Solve the equation by factoring:

a.

b.

c.

d.

16. Solve the equation by factoring:

a. − −

b. − −

c. − −

d. − −

17. Solve the equation by factoring:

a. −

b. −

c. −

d. −

18. Find the product:

a.

b.

c.

d.

19. **Find the product:**

a.

c.

b.

d.

20. **Simplify the expression:**

a.

c.

b.

d.

21. **Simplify the expression:**

a.

c.

b.

d.

22. **Multiply the polynomials:**

a.

c.

b.

d.

23. Multiply the polynomials:

a. c.

b. d.

24. Write a quadratic equation in standard form with the given roots: –5 and 2

a. c.

b. d.

25. Determine whether the given function has a maximum or a minimum value. Then find the maximum of minimum value of the function:

$$f(x) = x^2 - 2x + 2$$

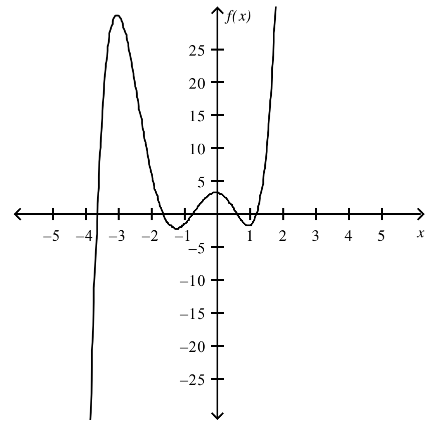
a. The function has a maximum value. The maximum value of the function is 1.

b. The function has a maximum value. The maximum value of the function is 5.

c. The function has a minimum value. The minimum value of the function is 1.

d. The function has a minimum value. The minimum value of the function is 5.

26. For the given graph,



a. *describe the end behavior,*
b. *determine whether it represents an odd-degree or even-degree polynomial function, and*
c. *state the number of real zeros.*

a. The end behavior of the graph is $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to +\infty$ as $x \to -\infty$.
It is an odd-degree polynomial function.
The function has five real zeros.

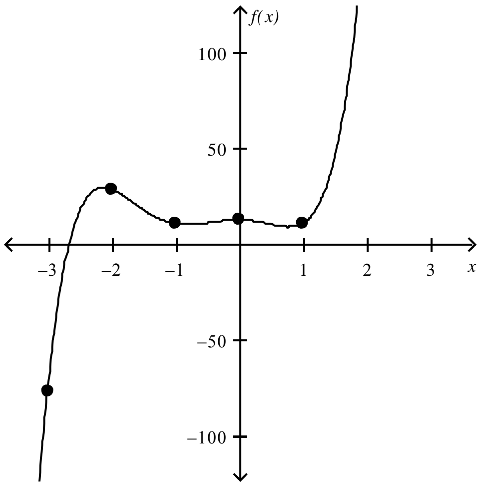
b. The end behavior of the graph is $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to -\infty$ as $x \to -\infty$.
It is an odd-degree polynomial function.
The function has five real zeros.

c. The end behavior of the graph is $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to -\infty$ as $x \to -\infty$.
It is an odd-degree polynomial function.
The function has four real zeros.

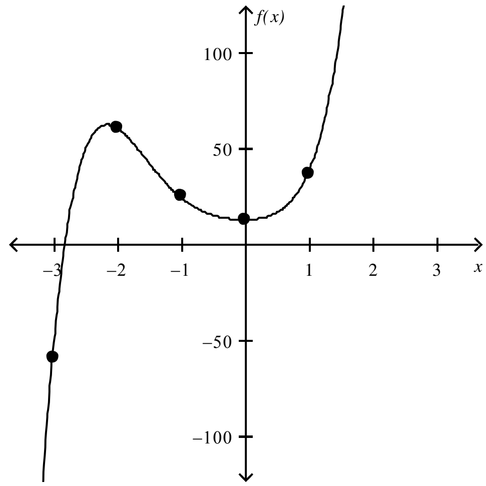
d. The end behavior of the graph is $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to -\infty$ as $x \to -\infty$.
It is an even-degree polynomial function.
The function has five real zeros.

27. Graph the function $f(x) = 3x^5 + 8x^4 - 3x^3 - 10x^2 + 12$ by making a table of values or using a graphing calculator if you have one. If using a graphing calculator, please use the table feature to copy a portion of the table of values in the work area of this assignment.

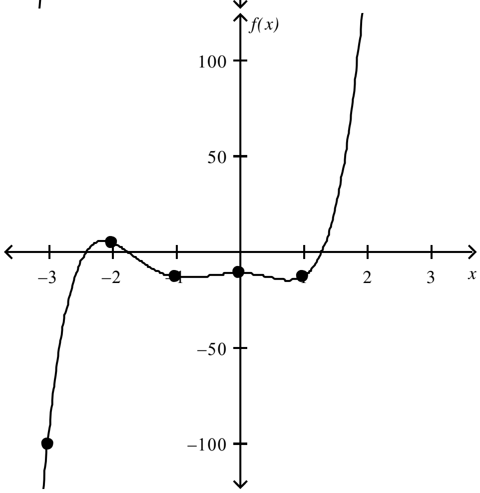
a.



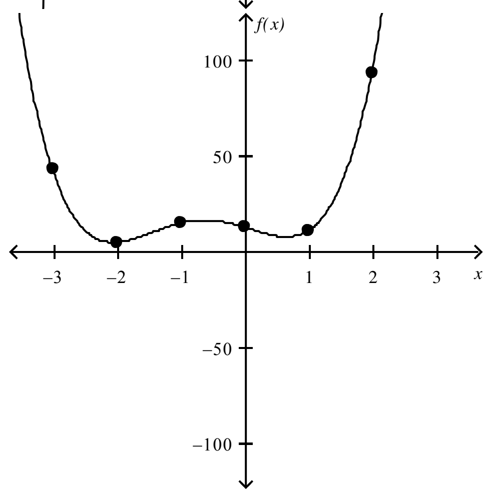
c.



b.



d.



28. Simplify the rational expression. Then state the excluded values: ______

a. —;

c. —;

b. —;

d. —;

29. Multiply the rational expression:

a.

b.

c.

d.

30. Divide the rational expressions and write your answer in simplest terms:

a.

b.

c.

d.

31. Divide the rational expression and write your answer in simplest terms:

a.

b.

c.

d.

32. Add the rational expressions:

a.

b.

c.

d.

33. Subtract the rational expressions:

a.

b.

c.

d.

34. Solve the rational equation:

a.

b.

c.

d.

35. Solve the rational equation:

a. —

b. —

c. 24

d. -24

36. Write – in logarithm form.

a. –

b. –

c. –

d. –

37. Simplify:

a

b.

c.

d.

38. Solve for n: —

a. –

b. –

c. –

d.

39. Evaluate the logarithm expression: —

a.

b.

c. —

d.

40. Solve for n: –

a. –

c.

b. —

d.

41. Solve the logarithmic equation:

a.

c. —

b. —

d.

42. Solve the logarithmic equation:

a.

c.

b.

d.

43. Solve the exponential equation:

a. –

c. –

b. –

d. ——

44. Solve the exponential equation:

a. —

c. – –

b. —

d. –

45. **Solve the logarithmic equation:** –

a.

c. –

b.

d.

46. **Solve the exponential equation:**

a. –

c. –

b.

d.

47. **Condense:**

a.

c.

b.

d.

48. **Find the inverse of the given function:** $f(x) = \frac{7x - 3}{16}$

a. $f^{-1}(x) = \frac{16x - 3}{7}$

c. $f^{-1}(x) = \frac{7x + 16}{3}$

b. $f^{-1}(x) = \frac{16x + 3}{7}$

d. $f^{-1}(x) = \frac{7x - 16}{3}$

49. **Find the vertical asymptote(s), if any, for** ———

a. $x = 7,\ x = 2$

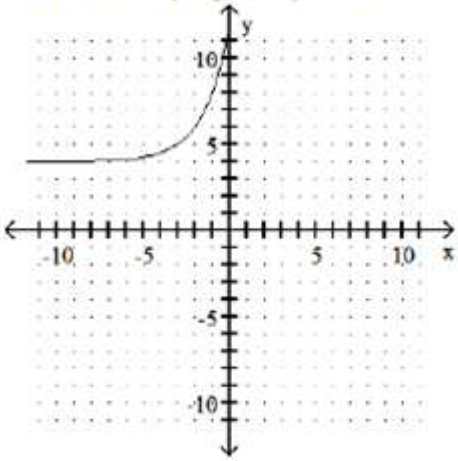
c. $x = 2,\ x = 3$

b. $x = 2,\ x = 3,\ x = 7$

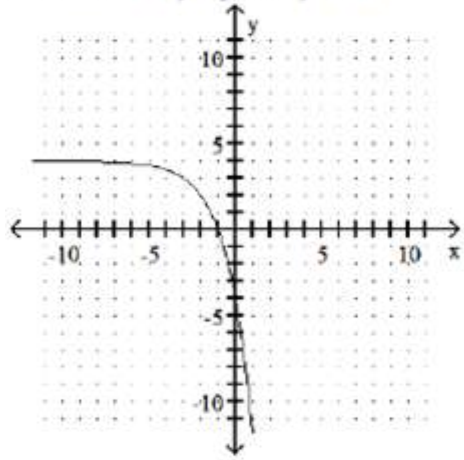
d. No vertical asymptotes

50. **Use transformation to identify the correct graph of . Then determine the domain, range, and horizontal asymptote of the function.**

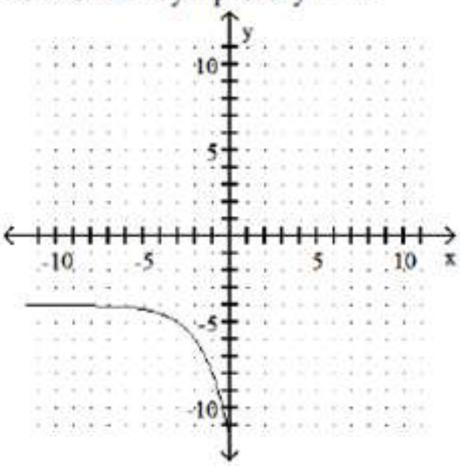
a. domain of f: (- ∞, ∞); range of f: (-4, ∞); horizontal asymptote: y = 4



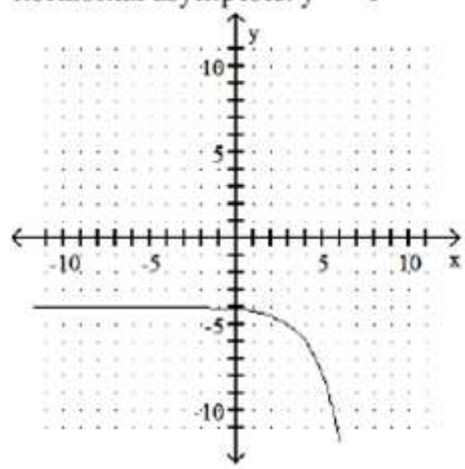
c. domain of f: (- ∞, ∞); range of f: (- ∞, 4); horizontal asymptote: y = 4



b. domain of f: (- ∞, ∞); range of f: (- ∞, -4); horizontal asymptote: y = -4



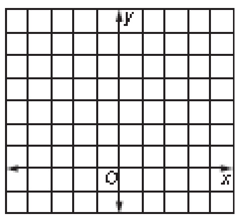
d. domain of f: (- ∞, ∞); range of f: (- ∞, -4); horizontal asymptote: y = -4



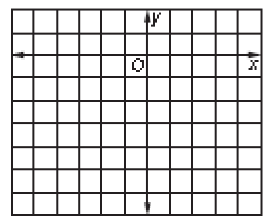
Short Answer: *You must show all your work on the student work sheet that has been provided to you. If you need more room, please attach a separate sheet of paper.*

Graph the functions:

51. $h(x) = |2x + 1|$



52. $h(x) = \begin{cases} \frac{x}{3} \text{ if } x \leq 0 \\ 2x - 6 \text{ if } 0 < x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$



Use synthetic division to divide:

53. 2)

54.

55. For ,

a. How many zeros should this polynomial function have?

b. How many turns could the graph of the equation make?

c. What is the end behavior of the graph of the function?

d. State the number of positive, negative, and imaginary zeros using Descartes Rule of Signs.

e. Use the Rational Zero Theorem to find the possible rational zeros of this polynomial function.

f. Find all the zeros of the polynomial function (real and imaginary).

55. Given the following quadratic equation , find

a. the direction of opening

b. the axis of symmetry

c. the vertex

d. the maximum/minimum value

e. the y-intercept

f. the x-intercepts/roots/zeros

g. graph the parabola, finding at least 3 additional points

Write the equation on vertex form.

Mathematics Literacy Portion

3 Big Shifts in Instruction: In keeping with Durfee's 3 Big Shifts in Instruction involving literacy where you are expected to get "regular practice with complex text and its academic language", you are given two sections from the Precalculus textbook that you are required to read, take notes on, and answer a set of questions about the material.

Directions: Read Lessons 0-1 and 1-1 and use the attached note-taking guides to take notes on these sections. Then complete the following problems and record your answers below:

Lesson 0-1: pg. 5 #1-3, 7-8, 9-15 odd, 18-24 even, & 26-30 all

Lesson 1-1: p. 9-10 #2-14 even, 19-27 odd, 30, 32, 35, 36, & 40-48 even

Literacy Portion Answers:

Lesson 0-1 Sets

1. ______________________________ 18. ______________________________

2. ______________________________ 20. ______________________________

3. ______________________________ 22. ______________________________

7. ______________________________ 24. ______________________________

8. ______________________________ 26. ______________________________

9. ______________________________ 27. ______________________________

11. ______________________________ 28. ______________________________

13. ______________________________ 29. ______________________________

15. ______________________________ 30. ______________________________

Lesson 1-1: Functions

2. ______________________________

4. ______________________________

6. ______________________________

8. ______________________________

10. ______________________________

12. ______________________________

14. ______________________________

19. ______________________________

21. ______________________________

23. ______________________________

25. ______________________________

27. ______________________________

30. ______________________________

32. ______________________________

35. ______________________________

36. ______________________________

40. ______________________________

42. ______________________________

44. ______________________________

46. ______________________________

48. ______________________________

LESSON 0-1 Sets

Objective

1. Use set notation to denote elements, subsets, and complements.
2. Find intersections and unions of sets.

NewVocabulary

set
element
subset
universal set
complement
union
intersection
empty set

1 Set Notation

A **set** is a collection of objects. Each object in a set is called an **element**. A set is named using a capital letter and is written with its elements listed within braces { }.

Set Name	Description of Set	Set Notation
C	pages in a chapter of a book	$C = \{35, 36, 37, 38, 39, 40\}$
A	students who made an A on the test	$A = \{\text{Olinda, Mario, Karen}\}$
L	the letters from A to H	$L = \{A, B, C, D, E, F, G, H\}$
N	positive odd numbers	$N = \{1, 3, 5, 7, 9, 11, 13, \ldots\}$

To write that Olinda is *an element* of set A, write Olinda $\in A$.

Example 1 Use Set Notation

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*.

a. ***N* is the set of whole numbers greater than 12 and less than 16. $15 \in N$**

The elements in this set are 13, 14, and 15, so $N = \{13, 14, 15\}$. Because 15 is an element of N, $15 \in N$ is a true statement.

b. ***V* is the set of vowels. $g \in V$**

The elements in this set are the letters a, e, i, o, and u, so $V = \{a, e, i, o, u\}$. Because the letter g is not an element of V, a correct statement is $g \notin V$. Therefore, $g \in V$ is a false statement.

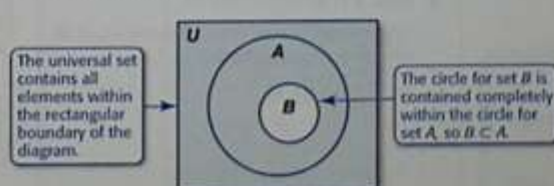
c. ***M* is the set of months that begin with J. April $\in M$**

The elements in this set are the months January, June, and July, so $M = \{\text{January, June, July}\}$. Because the month of April is not an element of this set, a correct statement is April $\notin M$. Therefore, April $\in M$ is a false statement.

d. ***X* is the set of numbers on a die. $4 \in X$**

The elements in this set are 1, 2, 3, 4, 5, and 6, so $X = \{1, 2, 3, 4, 5, 6\}$. Because 4 is an element of X, $4 \in X$ is a true statement.

If every element of set B is also contained in set A, then B is called a **subset** of A, and is written as $B \subset A$. The **universal set** U is the set of all possible elements for a situation. All other sets in this situation are subsets of the universal set.



Suppose $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 2, 3\}$. Because $1 \in A$, $2 \in A$, and $3 \in A$, $B \subset A$.

The set of elements in U that are not elements of set B is called the **complement** of B, and is written as B'. In the Venn diagram, the complement of B is all of the shaded regions.

StudyTip

Math Symbols

- $\in$ is an element of
- $\notin$ is not an element of
- $\subset$ is a subset of
- A' the complement of set A
- $A \cap B$ the intersection of set A and set B
- $A \cup B$ the union of set A and set B

Example 2 Identify Subsets and Complements of Sets

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 4, 5, 7, 8, 9\}$, $B = \{5, 7\}$, $C = \{1, 5, 7, 8\}$, $D = \{2, 3\}$, and $E = \{6, 3\}$.

a. State whether $B \subset A$ is *true* or *false*.

$B = \{5, 7\}$ $\quad$ $A = \{1, 4, 5, 7, 8, 9\}$

True; $5 \in A$ and $7 \in A$, so all of the elements of B are also elements of A. Therefore, B is a subset of A.

b. State whether $E \subset D$ is *true* or *false*.

$E = \{6, 3\}$ $\quad$ $D = \{2, 3\}$

False; $6 \notin D$, so not all of the elements of E are in D. Therefore, E is not a subset of D.

c. Find A'.

Identify the elements of U that are not in A.

$A = \{1, 4, 5, 7, 8, 9\}$ $\quad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $A' = \{0, 2, 3, 6\}$.

d. Find D'.

Identify the elements of U that are not in D.

$D = \{2, 3\}$ $\quad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $D' = \{0, 1, 4, 5, 6, 7, 8, 9\}$.

2 Unions and Intersections

The **union** of sets A and B, written $A \cup B$, is a new set consisting of all of the elements that are in either A or B. The **intersection** of sets A and B, written $A \cap B$, is a new set consisting of elements found in A and B. If two sets have no elements in common, their intersection is called the **empty set**, and is written as $\varnothing$ or $\{\ \}$.

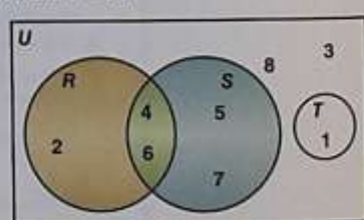
Example 3 Find the Union and Intersection of Two Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $R = \{2, 4, 6\}$, $S = \{4, 5, 6, 7\}$, and $T = \{1\}$.

a. Find $R \cup S$.

The union of R and S is the set of all elements that belong to R, S, or to both sets.

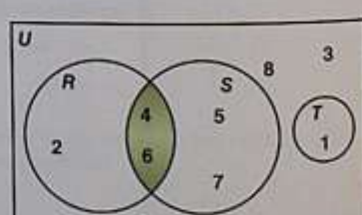
So, $R \cup S = \{2, 4, 5, 6, 7\}$.



b. Find $R \cap S$.

The intersection of R and S is the set of all elements found in both R and S.

So, $R \cap S = \{4, 6\}$.



c. Find $T \cap S$.

Because there are no elements that belong to both T and S, the intersection of T and S is the empty set. So, $T \cap S = \varnothing$.

WatchOut!

Disjoint Sets Sets A and B are said to be *disjoint* if they have no elements in common. The intersection of two disjoint sets is the empty set, while the union of two disjoint sets includes all of the elements from each set.

Exercises

● = Step-by-Step Solutions begin on page R29.

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*. (Example 1)

1. J is the set of whole number multiples of 3 that are less than 15. $15 \in J$
2. K is the set of consonant letters in the English alphabet. $h \in K$
3. L is the set of the first six prime numbers. $9 \in L$
4. V is the set of states in the U.S. that border Georgia. Alabama $\notin V$
5. N is the set of natural numbers less than 12. $0 \in N$
6. D is the set of days that start with S. Sunday $\in D$
7. A is the set of girls names that start with A. Ashley $\in A$
8. S is the set of the 48 continental states in the U.S. Hawaii $\notin S$

For Exercises 9–24, use the following information.
Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, $A = \{1, 2, 6, 9, 10, 12\}$, $B = \{2, 9, 10\}$, $C = \{0, 1, 6, 9, 11\}$, $D = \{4, 5, 10\}$, $E = \{2, 3, 6\}$, and $F = \{2, 9\}$.

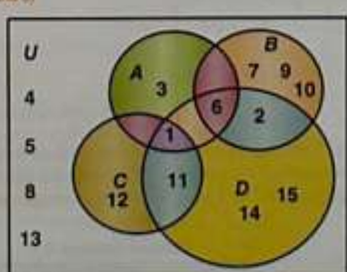
Determine whether each statement is *true* or *false*. Explain your reasoning. (Examples 1 and 2)

9. $3 \in D$
10. $8 \notin A$
11. $B \subset A$
12. $U \subset A$
13. $5 \notin D$
14. $2 \in E$
15. $0 \in F$
16. $6 \notin F$

Find each of the following. (Examples 2 and 3)

17. C'
18. U'
19. A'
20. $D \cap E$
21. $C \cap E$
22. $E \cup F$
23. $A \cup B$
24. $A \cap B$

Use the Venn diagram to find each of the following. (Examples 2 and 3)



25. $A \cup B$
26. $A \cap D$
27. $C \cup D$
28. A'
29. $A \cap B \cap D$
30. $(A \cup B) \cup C$

31. **SPORTS** Sixteen students in Mr. Frank's gym class each participate in one or more sports as shown in the table. (Examples 2 and 3)

Mr. Frank's Gym Class		
Basketball	**Soccer**	**Volleyball**
Ayanna	Lisa	Pam
Pam	Ayanna	Lisa
Sue	Ron	Shiv
Lisa	Tyron	Max
Ron	Max	Aida
Max	Aida	Juan
Ito	Evita	Tino
Juan	Nelia	Kai
Nelia	Percy	Percy

a. Let B represent the set of basketball players, S represent the set of soccer players, and V represent the set of volleyball players. Draw a Venn diagram of this situation.

b. Find $S \cap V$. What does this set represent?

c. Find S'. What does this set represent?

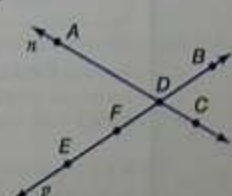
d. Find $B \cup V$. What does this set represent?

32. **ACADEMICS** There are 26 students at West High School who take either calculus or physics or both. Each student is represented by a letter of the alphabet below. Draw a Venn diagram of this situation. (Examples 2 and 3)

Calculus	A, D, F, I, J, K, L, M, P, R, T, V, X, Y, Z
Physics	B, C, D, E, F, G, H, I, J, K, L, N, O, Q, S, U, W

33. **BEVERAGES** Suppose you can select a juice from three possible kinds: apple, orange, or grape, or you can select a soda from two possible kinds, Brand A or Brand B. If you can choose a juice *or* a soda to drink, according to the Addition Principle, you have $3 + 2$ or 5 possible choices. Using notation that you have learned in this lesson, justify this result. In what situation could this principle not be applied?

GEOMETRY Use the figure to find the simplest name for each of the following.



34. $\overline{DE} \cap \overline{BF}$
35. $\overline{AD} \cup \overline{DC}$
36. $\overrightarrow{DE} \cup \overrightarrow{DC}$
37. line $n \cap$ line p
38. $\overleftrightarrow{AC} \cap \overline{EF}$
39. $\overrightarrow{FB} \cup \overrightarrow{EB}$

LESSON

1-1 Functions

:: Then	:: Now	:: Why?
• You used set notation to denote elements, subsets, and complements. (Lesson 0-1)	• **1** Describe subsets of real numbers. **2** Identify and evaluate functions and state their domains.	• Many events that occur in everyday life involve two related quantities. For example, to operate a vending machine, you insert money and make a selection. The machine gives you the selected item and any change due. Once your selection is made, the amount of change you receive *depends* on the amount of money you put into the machine.

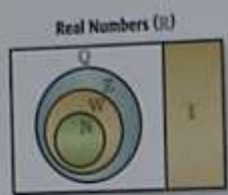
NewVocabulary
- set-builder notation
- interval notation
- function
- function notation
- independent variable
- dependent variable
- implied domain
- piecewise-defined function
- relevant domain

1 Describe Subsets of Real Numbers

Real numbers are used to describe quantities such as money and distance. The set of real numbers $\mathbb{R}$ includes the following subsets of numbers.

KeyConcept Real Numbers

Real Numbers ($\mathbb{R}$)



Letter	Set	Examples
Q	rationals	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.666\ldots$
I	irrationals	$\sqrt{3} = 1.73205\ldots$
Z	integers	$-5, 17, -23, 8$
W	wholes	$0, 1, 2, 3\ldots$
N	naturals	$1, 2, 3, 4\ldots$

These and other sets of real numbers can be described using set-builder notation. **Set-builder notation** uses the properties of the numbers in the set to define the set.

$$\{x \mid -3 \leq x \leq 16, x \in \mathbb{Z}\}$$

- The set of numbers x such that…
- x has the given properties…
- and x is an element of the given set of numbers.

Example 1 Use Set Builder Notation

Describe the set of numbers using set-builder notation.

a. $\{8, 9, 10, 11, \ldots\}$

The set includes all whole numbers greater than or equal to 8.

$\{x \mid x \geq 8, x \in \mathbb{W}\}$ *Read as the set of all x such that x is greater than or equal to 8 and x is an element of the set of whole numbers.*

b. $x < 7$

Unless otherwise stated, you should assume that a given set consists of real numbers. Therefore, the set includes all real numbers less than 7. $\{x \mid x < 7, x \in \mathbb{R}\}$

c. all multiples of three

The set includes all integers that are multiples of three. $\{x \mid x = 3n, n \in \mathbb{Z}\}$

GuidedPractice

1A. $\{1, 2, 3, 4, 5, \ldots\}$ **1B.** $x \leq -3$ **1C.** all multiples of π

Stu… Look… set n… and i… Less…

Stu… Dom… the n… will b…

StudyTip
Look Back You can review set notation, including unions and intersections of sets, in Lesson 0-1.

Interval notation uses inequalities to describe subsets of real numbers. The symbols [or] are used to indicate that an endpoint is included in the interval, while the symbols (or) are used to indicate that an endpoint is not included in the interval. The symbols ∞, positive infinity, and $-\infty$, negative infinity, are used to describe the unboundedness of an interval. An interval is *unbounded* if it goes on indefinitely.

Bounded Intervals		Unbounded Intervals	
Inequality	Interval Notation	Inequality	Interval Notation
$a \le x \le b$	$[a, b]$	$x \ge a$	$[a, \infty)$
$a < x < b$	(a, b)	$x \le a$	$(-\infty, a]$
$a \le x < b$	$[a, b)$	$x > a$	(a, ∞)
$a < x \le b$	$(a, b]$	$x < a$	$(-\infty, a)$
		$-\infty < x < \infty$	$(-\infty, \infty)$

Example 2 Use Interval Notation

Write each set of numbers using interval notation.

a. $-8 < x \le 16$ $(-8, 16]$

b. $x < 11$ $(-\infty, 11)$

c. $x \le -16$ or $x > 5$ $(-\infty, -16] \cup (5, \infty)$ $\cup$ read as union

GuidedPractice

2A. $-4 \le y < -1$ **2B.** $a \ge -3$ **2C.** $x > 9$ or $x < -2$

2 Identify Functions

Recall that a *relation* is a rule that relates two quantities. Such a rule pairs the elements in a set A with elements in a set B. The set A of all inputs is the *domain* of the relation, and set B contains all outputs or the *range*.

Relations are commonly represented in four ways.

1. **Verbally** A sentence describes how the inputs and outputs are related.
   The output value is 2 more than the input value.
2. **Numerically** A table of values or a set of ordered pairs relates each input (x-value) with an output value (y-value).
   $\{(0, 2), (1, 3), (2, 4), (3, 5)\}$
3. **Graphically** Points on a graph in the coordinate plane represent the ordered pairs.

4. **Algebraically** An equation relates the x- and y-coordinates of each ordered pair.
   $y = x + 2$

A **function** is a special type of relation.

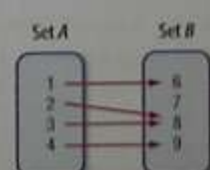
KeyConcept Function

Words A function f from set A to set B is a relation that assigns to each element x in set A *exactly one* element y in set B.

Symbols The relation from set A to set B is a function.

Set A is the domain. $D = \{1, 2, 3, 4\}$

Set B contains the range. $R = \{6, 8, 9\}$



StudyTip
Domain and Range In this text, the notation for domain and range will be D = and R =, respectively.

StudyTip

Tabular Method When a relation fails the vertical line test, an x-value has more than one corresponding y-value, as shown below.

x	y
−2	−4
3	−1
3	4
5	6
7	9

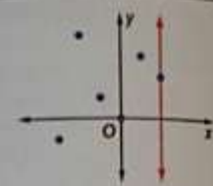
An alternate definition of a function is a set of ordered pairs in which no two different pairs have the same x-value. Interpreted graphically, this means that no two points on the graph of a function in the coordinate plane can lie on the same vertical line.

KeyConcept Vertical Line Test

Words

A set of points in the coordinate plane is the graph of a function if each possible vertical line intersects the graph in at most one point.

Model



Example 3 Identify Relations that are Functions

Determine whether each relation represents y as a function of x.

a. The input value x is a student's ID number, and the output value y is that student's score on a physics exam.

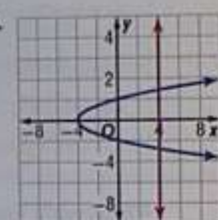
Each value of x cannot be assigned to more than one y-value. A student cannot receive two different scores on an exam. Therefore, the sentence describes y as a function of x.

b.

x	y
−8	−5
−5	−4
0	−3
3	−2
6	−3

Each x-value is assigned to exactly one y-value. Therefore, the table represents y as a function of x.

c.



A vertical line at $x = 4$ intersects the graph at more than one point. Therefore, the graph does not represent y as a function of x.

d. $y^2 - 2x = 5$

To determine whether this equation represents y as a function of x, solve the equation for y.

$y^2 - 2x = 5$ — Original equation

$y^2 = 5 + 2x$ — Add $2x$ to each side.

$y = \pm\sqrt{5 + 2x}$ — Take the square root of each side.

This equation does not represent y as a function of x because there will be two corresponding y-values, one positive and one negative, for any x-value greater than −2.5.

StudyTip

Functions with Repeated y-Values While a function cannot have more than one y-value paired with each x-value, a function can have one y-value paired with more than one x-value, as shown in Example 3b.

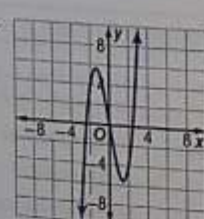
GuidedPractice

3A. The input value x is the area code, and the output value y is a phone number in that area code.

3B.

x	y
−6	−7
2	3
5	8
5	9
9	22

3C.



3D. $3y + 6x = 18$

In **function notation**, the symbol $f(x)$ is read f of x and interpreted as *the value of the function f at x.* Because $f(x)$ corresponds to the y-value of f for a given x-value, you can write $y = f(x)$.

Equation	Related Function
$y = -6x$	$f(x) = -6x$

Because it can represent any value in the function's domain, x is called the **independent variable**. A value in the range of f is represented by the **dependent variable**, y.

Example 4 Find Function Values

If $g(x) = x^2 + 8x - 24$, find each function value.

a. $g(6)$

To find $g(6)$, replace x with 6 in $g(x) = x^2 + 8x - 24$.

$g(x) = x^2 + 8x - 24$	Original function
$g(6) = (6)^2 + 8(6) - 24$	Substitute 6 for x.
$= 36 + 48 - 24$	Simplify.
$= 60$	Simplify.

b. $g(-4x)$

$g(x) = x^2 + 8x - 24$	Original function
$g(-4x) = (-4x)^2 + 8(-4x) - 24$	Substitute $-4x$ for x.
$= 16x^2 - 32x - 24$	Simplify.

c. $g(5c + 4)$

$g(x) = x^2 + 8x - 24$	Original function
$g(5c + 4) = (5c + 4)^2 + 8(5c + 4) - 24$	Substitute $5c + 4$ for x.
$= 25c^2 + 40c + 16 + 40c + 32 - 24$	Expand $(5c + 4)^2$ and $8(5c + 4)$.
$= 25c^2 + 80c + 24$	Simplify.

GuidedPractice

If $f(x) = \frac{2x + 3}{x^2 - 2x + 1}$, find each function value.

4A. $f(12)$ **4B.** $f(6x)$ **4C.** $f(-3x + 8)$

Math HistoryLink

Leonhard Euler (1707–1783) A Swiss mathematician, Euler was a prolific mathematical writer, publishing over 800 papers in his lifetime. He also introduced much of our modern mathematical notation, including the use of $f(x)$ for the function f.

When you are given a function with an unspecified domain, the **implied domain** is the set of all real numbers for which the expression used to define the function is real. In general, you must exclude values from the domain of a function that would result in division by zero or taking the even root of a negative number.

Example 5 Find Domains Algebraically

State the domain of each function.

a. $f(x) = \frac{2 + x}{x^2 - 7x}$

When the denominator of $\frac{2 + x}{x^2 - 7x}$ is zero, the expression is undefined. Solving $x^2 - 7x = 0$, the excluded values for the domain of this function are $x = 0$ and $x = 7$. The domain of this function is all real numbers except $x = 0$ and $x = 7$, or $\{x \mid x \neq 0, x \neq 7, x \in \mathbb{R}\}$.

b. $g(t) = \sqrt{t - 5}$

Because the square root of a negative number cannot be real, $t - 5 \geq 0$. Therefore, the domain of $g(t)$ is all real numbers t such that $t \geq 5$ or $[5, \infty)$.

StudyTip

Naming Functions You can use other letters to name a function and its independent variable. For example, $f(x) = \sqrt{x - 5}$ and $g(t) = \sqrt{t - 5}$ name the same function.

c. $h(x) = \frac{1}{\sqrt{x^2 - 9}}$

This function is defined only when $x^2 - 9 > 0$. Therefore, the domain of $h(x)$ is $(-\infty, -3) \cup (3, \infty)$.

GuidedPractice

State the domain of each function.

5A. $f(x) = \frac{5x - 2}{x^2 + 7x + 12}$ **5B.** $h(a) = \sqrt{a^2 - 4}$ **5C.** $g(x) = \frac{8x}{\sqrt{2x + 6}}$

A function that is defined using two or more equations for different intervals of the domain is called a **piecewise-defined function**.

Real-WorldLink

Robert Pershing Wadlow of Alton, Illinois, was the tallest man recorded in medical history at 8 feet 11.1 inches. Wadlow weighed almost 440 pounds.

Source: *Guinness Book of World Records*

Real-World Example 6 Evaluate a Piecewise-Defined Function

HEIGHT The average maximum height of children in inches as a function of their parents' maximum heights in inches can be modeled by the following piecewise function. Find the average maximum heights of children whose parents have the given maximum heights. Use $h(x)$, where x is the independent variable representing the parents' height and $h(x)$ is the dependent variable representing the child's height.

$$h(x) = \begin{cases} 1.6x - 41.6 & \text{if } 63 < x < 66 \\ 3x - 132 & \text{if } 66 \le x \le 68 \\ 2x - 66 & \text{if } x > 68 \end{cases}$$

a. $h(67)$

Because 67 is between 66 and 68, use $h(x) = 3x - 132$ to find $h(67)$.

$h(67) = 3x - 132$		Function for $66 \le x \le 68$
	$= 3(67) - 132$	Substitute 67 for x.
	$= 201 - 132$ or 69	Simplify.

According to this model, children whose parents have a maximum height of 67 inches will attain an average maximum height of 69 inches.

b. $h(72)$

Because 72 is greater than 68, use $h(x) = 2x - 66$.

$h(72) = 2x - 66$		Function for $x > 68$
	$= 2(72) - 66$	Substitute 72 for x.
	$= 144 - 66$ or 78	Simplify.

According to this model, children whose parents have a maximum height of 72 inches will attain an average maximum height of 78 inches.

StudyTip

Relevant Domain A **relevant domain** is the part of a domain that is relevant to a model. Consider a function in which the output is a function of length. It is unreasonable to have a negative length, so the relevant domain is the set of numbers greater than or equal to 0.

GuidedPractice

6. SPEED The speed v of a vehicle in miles per hour can be represented by the following piecewise function when t is the time in seconds. Find the speed of the vehicle at each indicated time.

$$v(t) = \begin{cases} 4t & \text{if } 0 \le t \le 15 \\ 60 & \text{if } 15 < t < 240 \\ -6t + 1500 & \text{if } 240 \le t \le 250 \end{cases}$$

A. $v(5)$ **B.** $v(15)$ **C.** $v(245)$

Exercises

= Step-by-Step Solutions begin on page R29.

Write each set of numbers in set-builder and interval notation, if possible. (Examples 1 and 2)

1. $x > 50$
2. $x < -13$
3. $x \leq -4$
4. $\{-4, -3, -2, -1, \ldots\}$
5. $8 < x < 99$
6. $-31 < x \leq 64$
7. $x < -19$ or $x > 21$
8. $x < 0$ or $x \geq 100$
9. $\{-0.25, 0, 0.25, 0.50, \ldots\}$
10. $x \leq 61$ or $x \geq 67$
11. $x \leq -45$ or $x > 86$
12. all multiples of 8
13. all multiples of 5
14. $x \geq 32$

Determine whether each relation represents y as a function of x. (Example 3)

15. The input value x is a bank account number and the output value y is the account balance.

16. The input value x is the year and the output value y is the day of the week.

17.

x	y
−50	2.11
−40	2.14
−30	2.16
−20	2.17
−10	2.17
0	2.18

18.

x	y
0.01	423
0.04	449
0.04	451
0.07	466
0.08	478
0.09	482

19. $\frac{1}{x} = y$
20. $x^2 = y + 2$
21. $3y + 4x = 11$
22. $4y^2 + 18 = 96x$
23. $\sqrt{48y} = x$
24. $\frac{x}{y} = y - 6$

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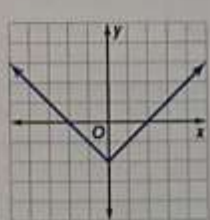

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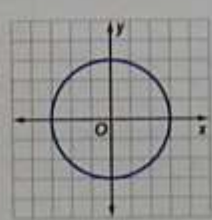

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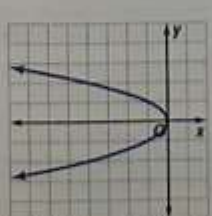

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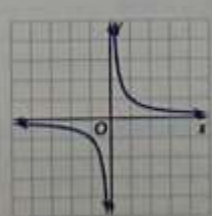


29. **METEOROLOGY** The five-day forecast for a city is shown. (Example 3)

a. Represent the relation between the day of the week and the estimated high temperature as a set of ordered pairs.

b. Is the estimated high temperature a function of the day of the week? the low temperature? Explain your reasoning.

Find each function value. (Example 4)

30. $g(x) = 2x^2 + 18x - 14$
   a. $g(9)$
   b. $g(3x)$
   c. $g(1 + 5m)$

31. $h(y) = -3y^3 - 6y + 9$
   a. $h(4)$
   b. $h(-2y)$
   c. $h(5b + 3)$

32. $f(t) = \frac{4t + 11}{3t^2 + 5t + 1}$
   a. $f(-6)$
   b. $f(4t)$
   c. $f(3 - 2a)$

33. $g(x) = \frac{3x^3}{x^2 + x - 4}$
   a. $g(-2)$
   b. $g(5x)$
   c. $g(8 - 4b)$

34. $h(x) = 16 - \frac{12}{2x + 3}$
   a. $h(-3)$
   b. $h(6x)$
   c. $h(10 - 2c)$

35. $f(x) = -7 + \frac{6x + 1}{x}$
   a. $f(5)$
   b. $f(-8x)$
   c. $f(6y + 4)$

36. $g(m) = 3 + \sqrt{m^2 - 4}$
   a. $g(-2)$
   b. $g(3m)$
   c. $g(4m - 2)$

37. $t(x) = 5\sqrt{6x^2}$
   a. $t(-4)$
   b. $t(2x)$
   c. $t(7 + n)$

38. **DIGITAL AUDIO PLAYERS** The sales of digital audio players in millions of dollars for a five-year period can be modeled using $f(t) = 24t^2 - 93t + 78$, where t is the year. The actual sales data are shown in the table. (Example 4)

Year	Sales ($)
1	1 million
2	3 million
3	14 million
4	74 million
5	219 million

a. Find $f(1)$ and $f(5)$.

b. Do you think that the model is more accurate for the earlier years or the later years? Explain your reasoning.

State the domain of each function. (Example 5)

39. $f(x) = \frac{8x+12}{x^2+5x+4}$

40. $g(x) = \frac{x+1}{x^2-3x-40}$

41. $g(a) = \sqrt{1+a^2}$

42. $h(x) = \sqrt{6-x^2}$

43. $f(a) = \frac{5a}{\sqrt{4a-1}}$

44. $g(x) = \frac{3}{\sqrt{x^2-16}}$

45. $f(x) = \frac{2}{x} + \frac{4}{x+1}$

46. $g(x) = \frac{6}{x+3} + \frac{2}{x-4}$

47. **PHYSICS** The period T of a pendulum is the time for one cycle and can be calculated using the formula $T = 2\pi\sqrt{\frac{\ell}{9.8}}$, where ℓ is the length of the pendulum and 9.8 is the gravitational acceleration due to gravity in meters per second squared. Is this formula a function of ℓ? If so, determine the domain. If not, explain why not. (Example 5)

Find $f(-5)$ and $f(12)$ for each piecewise function. (Example 6)

48. $f(x) = \begin{cases} -4x+3 & \text{if } x < 3 \\ -x^3 & \text{if } 3 \le x \le 8 \\ 3x^2+1 & \text{if } x > 8 \end{cases}$

49. $f(x) = \begin{cases} -5x^2 & \text{if } x < -6 \\ x^2+x+1 & \text{if } -6 \le x \le 12 \\ 0.5x^3-4 & \text{if } x > 12 \end{cases}$

50. $f(x) = \begin{cases} 2x^2+6x+4 & \text{if } x < -4 \\ 6-x^2 & \text{if } -4 \le x < 12 \\ 14 & \text{if } x \ge 12 \end{cases}$

51. $f(x) = \begin{cases} -15 & \text{if } x < -5 \\ \sqrt{x+6} & \text{if } -5 \le x \le 10 \\ \frac{2}{x}+8 & \text{if } x > 10 \end{cases}$

52. **INCOME TAX** Federal income tax for a person filing single in the United States in a recent year can be modeled using the following function, where x represents income and $T(x)$ represents total tax. (Example 6)

$$T(x) = \begin{cases} 0.10x & \text{if } 0 \le x \le 7285 \\ 782.5 + 0.15x & \text{if } 7285 < x \le 31{,}850 \\ 4386.25 + 0.25x & \text{if } 31{,}850 < x \le 77{,}100 \end{cases}$$

a. Find $T(7000)$, $T(10{,}000)$, and $T(50{,}000)$.

b. If a person's annual income were \$7285, what would his or her income tax be?

53. **PUBLIC TRANSPORTATION** The nationwide use of public transportation can be modeled using the following function. The year 1996 is represented by $t = 0$, and $P(t)$ represents passenger trips in millions. (Example 6)

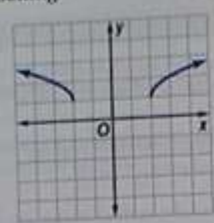
$$P(t) = \begin{cases} 0.35t + 7.6 & \text{if } 0 \le t \le 5 \\ 0.04t^2 - 0.6t + 11.6 & \text{if } 5 < t \le 10 \end{cases}$$

a. Approximately how many passenger trips were there in 1999? in 2004?

b. State the domain of the function.

Use the vertical line test to determine whether each graph represents a function. Write *yes* or *no*. Explain your reasoning.

54.



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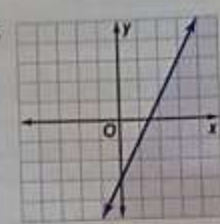

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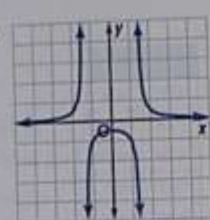

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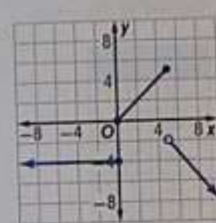


58. **TRIATHLON** In a triathlon, athletes swim 2.4 miles, then bike 112 miles, and finally run 26.2 miles. Jesse's average rates for each leg of a triathlon are shown in the table.

Leg	Rate
swim	4 mph
bike	20 mph
run	6 mph

a. Write a piecewise function to describe the distance D that Jesse has traveled in terms of time t. Round t to the nearest tenth, if necessary.

b. State the domain of the function.

59. **ELECTIONS** Describe the set of presidential election years beginning in 1792 in interval notation or in set-builder notation. Explain your reasoning.

60. **CONCESSIONS** The number of students working the concession stands at a football game can be represented by $f(x) = \frac{x}{50}$, where x is the number of tickets sold. Describe the relevant domain of the function.

61. **ATTENDANCE** The Chicago Cubs franchise has been in existence since 1874. The total season attendance for its home games can be modeled by $f(x) = 70{,}050x - 137{,}400{,}000$, where x represents the year. Describe the relevant domain of the function.

62. **ACCOUNTING** A business' assets, such as equipment, wear out or depreciate over time. One way to calculate depreciation is the straight-line method, using the value of the estimated life of the asset. Suppose $v(t) = 10{,}440 - 290t$ describes the value $v(t)$ of a copy machine after t months. Describe the relevant domain of the function.

Find $f(a)$, $f(a + h)$, and $\frac{f(a+h) - f(a)}{h}$ if $h \neq 0$.

63. $f(x) = -5$

64. $f(x) = \sqrt{x}$

65. $f(x) = \frac{1}{x+4}$

66. $f(x) = \frac{2}{5-x}$

67. $f(x) = x^2 - 6x + 8$

68. $f(x) = -\frac{1}{4}x + 6$

69. $f(x) = -x^5$

70. $f(x) = x^3 + 9$

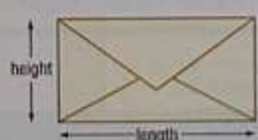
71. $f(x) = 7x - 3$

72. $f(x) = 5x^2$

73. $f(x) = x^3$

74. $f(x) = 11$

75. **MAIL** The U.S. Postal Service requires that envelopes have an aspect ratio (length divided by height) of 1.3 to 2.5, inclusive. The minimum allowable length is 5 inches and the maximum allowable length is $11\frac{1}{2}$ inches.



a. Write the area of the envelope A as a function of length ℓ if the aspect ratio is 1.8. State the domain of the function.

b. Write the area of the envelope A as a function of height h if the aspect ratio is 2.1. State the domain of the function.

c. Find the area of an envelope with the maximum height at the maximum aspect ratio.

76. **GEOMETRY** Consider the circle below with area A and circumference C.

a. Represent the area of the circle as a function of its circumference.

b. Find $A(0.5)$ and $A(4)$.

c. What do you notice about the area as the circumference increases?

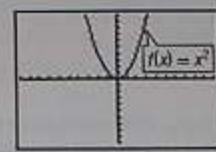
Determine whether each equation is a function of x. Explain.

77. $x = |y|$

78. $x = y^3$

79. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the range of a function.

a. **GRAPHICAL** Use a graphing calculator to graph $f(x) = x^n$ for whole-number values of n from 1 to 6, inclusive.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

b. **TABULAR** Predict the range of each function based on the graph, and tabulate each value of n and the corresponding range.

c. **VERBAL** Make a conjecture about the range of $f(x)$ when n is even.

d. **VERBAL** Make a conjecture about the range of $f(x)$ when n is odd.

H.O.T. Problems Use Higher-Order Thinking Skills

80. **ERROR ANALYSIS** Ana and Mason are evaluating $f(x) = \frac{2}{x^2 - 4}$. Ana thinks that the domain of the function is $(-\infty, -2)$ or $(1, 1)$ or $(2, \infty)$. Mason thinks that the domain is $\{x \mid x \neq -2, x \neq 2, x \in \mathbb{R}\}$. Is either of them correct? Explain.

81. **WRITING IN MATH** Write the domain of $f(x) = \frac{1}{(x+3)(x+1)(x-5)}$ in interval notation and in set-builder notation. Which notation do you prefer? Explain.

82. **CHALLENGE** $G(x)$ is a function for which $G(1) = 1$, $G(2) = 2$, $G(3) = 3$, and $G(x+1) = \frac{G(x-2)\,G(x-1) + 1}{G(x)}$ for $x \geq 3$. Find $G(6)$.

REASONING Determine whether each statement is *true* or *false* given a function from set X to set Y. If a statement is false, rewrite it to make a true statement.

83. Every element in X must be matched with only one element in Y.

84. Every element in Y must be matched with an element in X.

85. Two or more elements in X may not be matched with the same element in Y.

86. Two or more elements in Y may not be matched with the same element in X.

WRITING IN MATH Explain how you can identify a function described as each of the following.

87. a verbal description of inputs and outputs

88. a set of ordered pairs

89. a table of values

90. a graph

91. an equation

Spiral Review

Find the standard deviation of each population of data. (Lesson 0-8)

92. {200, 476, 721, 579, 152, 158}

93. {5.7, 5.7, 5.6, 5.5, 5.3, 4.9, 4.4, 4.0, 4.0, 3.8}

94. {369, 398, 381, 392, 406, 413, 376, 454, 420, 385, 402, 446}

95. **BASEBALL** How many different 9-player teams can be made if there are 3 players who can only play catcher, 4 players who can only play first base, 6 players who can only pitch, and 14 players who can play in any of the remaining 6 positions? (Lesson 0-7)

Find the values for x and y that make each matrix equation true. (Lesson 0-6)

96. $\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 4x - 3 \\ y - 2 \end{bmatrix}$

97. $\begin{bmatrix} 3y \\ 10 \end{bmatrix} = \begin{bmatrix} 27 + 6x \\ 5y \end{bmatrix}$

98. $[9 \quad 11] = [3x + 3y \quad 2x + 1]$

Use any method to solve the system of equations. State whether the system is *consistent, dependent, independent,* or *inconsistent*. (Lesson 0-5)

99. $2x + 3y = 36$
$4x + 2y = 48$

100. $5x + y = 25$
$10x + 2y = 50$

101. $7x + 8y = 30$
$7x + 16y = 46$

102. **BUSINESS** A used book store sells 1400 paperback books per week at \$2.25 per book. The owner estimates that he will sell 100 fewer books for each \$0.25 increase in price. What price will maximize the income of the store? (Lesson 0-3)

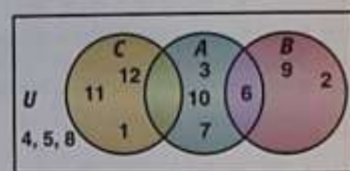
Use the Venn diagram to find each of the following. (Lesson 0-1)

103. A'

104. $A \cup B$

105. $B \cap C$

106. $A \cap B$



Skills Review for Standardized Tests

107. **SAT/ACT** A circular cone with a base of radius 5 has been cut as shown in the figure.

What is the height of the smaller top cone?

A $\frac{8}{13}$
B $\frac{96}{13}$
C $\frac{96}{12}$
D $\frac{96}{5}$
E $\frac{104}{5}$

108. **REVIEW** Which function is linear?

F $f(x) = x^2$
G $g(x) = 2.7$
H $f(x) = \sqrt{9 - x^2}$
J $g(x) = \sqrt{x - 1}$

109. Louis is flying from Denver to Dallas for a convention. He can park his car in the Denver airport long-term lot or in the nearby shuttle parking facility. The long-term lot costs \$1 per hour or any fraction thereof with a maximum charge of \$6 per day. In the shuttle facility, he has to pay \$4 for each day or part of a day. Which parking lot is less expensive if Louis returns after 2 days and 3 hours?

A shuttle facility
B airport lot
C They will both cost the same.
D cannot be determined with the information given

110. **REVIEW** Given $y = 2.24x + 16.45$, which statement best describes the effect of moving the graph down two units?

F The y-intercept increases.
G The x-intercept remains the same.
H The x-intercept increases.
J The y-intercept remains the same.

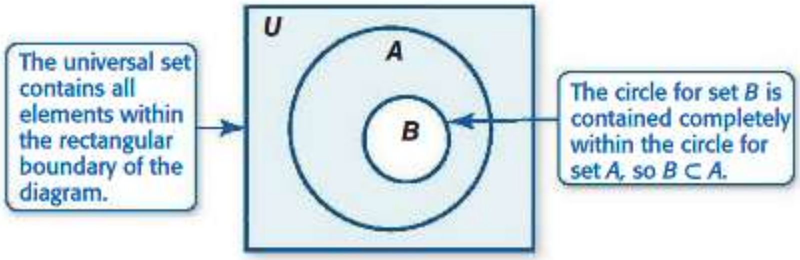
Honors Precalculus Notes
Lesson 0-1: Sets

Date: ______________________________

Objectives:

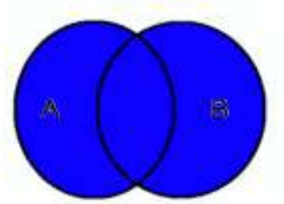
- Use set notation to denote elements, subsets, and complements
- Find intersections and unions of sets

Topic	Notes
What is a **set** and an **element**?	
How to Write Sets and Elements of Sets	"B is the set of natural numbers less than 10." "8 is an element of set B"
Examples	Directions: Use set notation to write the elements of each set. Then determine whether the statement about the set is true or false. a. N is the set of natural numbers less than 3; b. S is the set of states that border California; c. D is the set of days of the week that begin with T; d. C is the set of consonants before h in the alphabet;

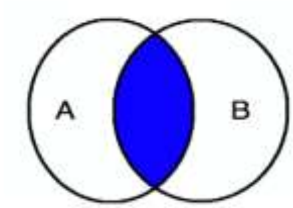
Subset, Universal Set & Complement	 The **Subset**: The **Universal Set**: The **Complement**:
Example	Let U={0, 1, 2, 3, 4, 5, 6}, A={1,3}, B={0, 2, 4, 6}, C={4, 5, 6}, D={2, 4, 6} and E={0}. a. State whether is true or false. b. State whether is true or false. c. Find D'. d. Find B'.

<table>
<tr>
<td>Unions, Intersections & The Empty Set</td>
<td>

Union:



Intersection:



The Empty Set:

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<tr>
<td>Examples</td>
<td>

Let U={0, 1, 2, 3, 4, 8, 10, 12}, R={3, 4, 8, 10}, S={0, 2, 4}, and T={1, 3, 10}.
a. Draw a Venn Diagram illustrating the problem.
b. Find .
c. Find .
d. Find .

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</tr>
</table>

Math Symbol Summary	You must know these math symbols: : : : A’: : : :

Precalculus Notes
Lesson 1-1: Functions

Date: ___________________________

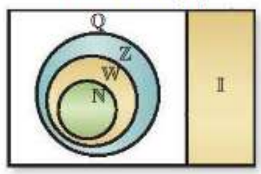
Objectives:

- Describe subsets of real numbers
- Identify and evaluate functions and state their domains

Main Idea	Notes
Real Numbers	The set of real numbers, , includes the following subsets of numbers:
Reminder of Symbols	: : : A’: : : :

KeyConcept Real Numbers

Real Numbers ($\mathbb{R}$)



Letter	Set	Examples
$\mathbb{Q}$	rationals	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.666\ldots$
$\mathbb{I}$	irrationals	$\sqrt{3} = 1.73205\ldots$
$\mathbb{Z}$	integers	$-5, 17, -23, 8$
$\mathbb{W}$	wholes	$0, 1, 2, 3\ldots$
$\mathbb{N}$	naturals	$1, 2, 3, 4\ldots$

Set Builder Notation

Set Builder Notation uses the properties of the numbers in the set to define the set.

Example #1 Use Set Builder Notation

Describe the set of numbers using set builder notation:

A. {2, 3, 4, 5, 6, 7}

B.

C. all multiples of seven

Guided Practice for Example #1

1A. {1, 2, 3, 4, 5, ….}

1B. $x \leq -3$

1C. all multiples of π

Interval Notation

Interval Notation uses inequalities to describe subsets of real numbers.

- The symbols [or] are used to indicate that an endpoint is included in the interval.
- The symbols (or) are used to indicate that an endpoint is not included in the interval.
- The symbols ° and -° are used to describe the unboundedness of an interval. (An interval is unbounded if it goes on indefinitely.)

Bounded Intervals		Unbounded Intervals	
Inequality	Interval Notation	Inequality	Interval Notation
$a \leq x \leq b$	$[a, b]$	$x \geq a$	$[a, \infty)$
$a < x < b$	(a, b)	$x \leq a$	$(-\infty, a]$
$a \leq x < b$	$[a, b)$	$x > a$	(a, ∞)
$a < x \leq b$	$(a, b]$	$x < a$	$(-\infty, a)$
		$-\infty < x < \infty$	$(-\infty, \infty)$

<table>
<tr>
<td>Example #2: Use Interval Notation</td>
<td>Write each set of numbers using interval notation:

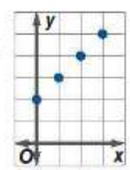
A.

B.

C.</td>
</tr>
<tr>
<td>Guided Practice for Example #2</td>
<td>2A. $-4 \leq y < -1$ 2B. $a \geq -3$ 2C. $x > 9$ or $x < -2$</td>
</tr>
<tr>
<td>Identifying Functions</td>
<td>
A <u>Relation</u> is a rule that relates two quantities. The rule pairs the elements of set A with elements that are in a set B.

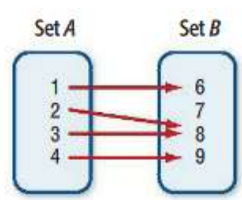
The set A of all the inputs is the <u>Domain</u>.
The set B of all the outputs is the <u>Range</u>.

Relations are commonly represented in 4 ways:

1. Verbally A sentence describes how the inputs and outputs are related.
The output value is 2 more than the input value.
2. Numerically A table of values or a set of ordered pairs relates each input (x-value) with an output value (y-value).
$\{(0, 2), (1, 3), (2, 4), (3, 5)\}$
3. Graphically Points on a graph in the coordinate plane represent the ordered pairs.


4. Algebraically An equation relates the x- and y-coordinates of each ordered pair.
$y = x + 2$

A <u>Function</u> is a special type of relation. It is a set of ordered pairs in which no two different pairs have the same x-value.

KeyConcept Function
Words A function f from set A to set B is a relation that assigns to each element x in set A exactly one element y in set B.
Symbols The relation from set A to set B is a function.
Set A is the domain. $D = \{1, 2, 3, 4\}$
Set B contains the range. $R = \{6, 8, 9\}$


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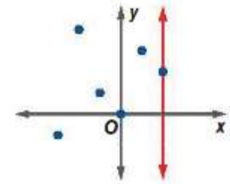
Vertical Line Test

KeyConcept **Vertical Line Test**

Words

A set of points in the coordinate plane is the graph of a function if each possible vertical line intersects the graph in at most one point.

Model



Example #3: Identifying Relations that are Functions

StudyTip

Tabular Method When a relation fails the vertical line test, an x-value has more than one corresponding y-value, as shown below.

x	y
−2	−4
3	−1
3	4
5	6
7	9

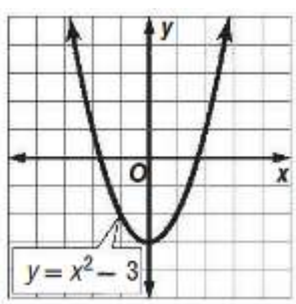
Determine whether each relation represents y as a function of x:

A. The input value x is the height of each student in inches, and the output value y is the number of books that the student owns.

B.

x	y
1	−1
1	1
4	−2
4	2
9	−3

C.



D.

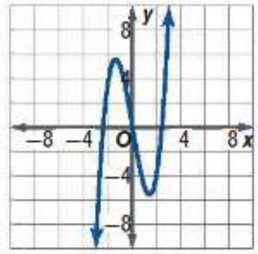
Guided Practice for Example #3

3A. The input value x is the area code, and the output value y is a phone number in that area code.

3B.

x	y
−6	−7
2	3
5	8
5	9
9	22

3C.



3D. $3y + 6x = 18$

Function Notation

In **Function Notation**, the symbol is read f of x and interpreted as the value of the function f at x.

- stands for the y-value of a function for a given x value.
- x is the **Independent Variable**
- y is the **Dependent Variable**

Example #4: Finding Function Values

If , find each function value:

A. f(3)

B. f(-3d)

C. f(2a – 1)

Guided Practice for Example #4

If $f(x) = \frac{2x+3}{x^2 - 2x + 1}$, find each function value.

4A. $f(12)$

4B. $f(6x)$

4C. $f(-3a + 8)$

Domain

The **Implied Domain** is set of all real numbers for which the expression used to define the function is real.

- Exclude values from the domain of a function that:
  - Result in division by zero
  - Result in taking an even root of a negative number

A **Relevant Domain** is the part of the domain that is relevant to a model (example: a real-world example where you may be looking for only whole numbers)

Example #5: Finding Domains Algebraically	State the domain of each function: A. B. C.
Guided Practice for Example #5	**State the domain of each function.** **5A.** $f(x) = \frac{5x - 2}{x^2 + 7x + 12}$ **5B.** $h(a) = \sqrt{a^2 - 4}$ **5C.** $g(x) = \frac{8x}{\sqrt{2x + 6}}$
Relevant Domain Example	**CONCESSIONS** The number of students working the concession stands at a football game can be represented by $f(x) = \frac{x}{50}$, where x is the number of tickets sold. Describe the relevant domain of the function.
Piecewise Functions	A **piecewise-defined function** is a function that is defined using two or more equations for different intervals of the domain.

Example #6: Evaluate a Piecewise-Defined Function	Finance: Realtors in metropolitan area studied the average home price per square foot as a function of total square footage. Their evaluation yielded the following piecewise-defined function. Find the average price per square foot for a home with the given square footage. $p(a) = \begin{cases} \frac{a-1000}{40} + 75 \\ \quad \text{if } 1000 \leq a < 2600 \\ \frac{-(a-2600)}{100} + 110 \\ \quad \text{if } 2600 \leq a < 4000 \\ \frac{a-4000}{25} + 98 \\ \quad \text{if } a \geq 4000 \end{cases}$ a. 1400 square feet b. 3200 square feet
Guided Practice for Example #6	**6. SPEED** The speed v of a vehicle in miles per hour can be represented by the following piecewise function when t is the time in seconds. Find the speed of the vehicle at each indicated time. $v(t) = \begin{cases} 4t & \text{if } 0 \leq t \leq 15 \\ 60 & \text{if } 15 < t < 240 \\ -6t + 1500 & \text{if } 240 \leq t \leq 250 \end{cases}$ **A.** $v(5)$ **B.** $v(15)$ **C.** $v(245)$

<table>
<tr><td>Writing a Piecewise Function</td><td>

58. **TRIATHLON** In a triathlon, athletes swim 2.4 miles, then bike 112 miles, and finally run 26.2 miles. Jesse's average rates for each leg of a triathlon are shown in the table.

Leg	Rate
swim	4 mph
bike	20 mph
run	6 mph

a. Write a piecewise function to describe the distance D that Jesse has traveled in terms of time t. Round t to the nearest tenth, if necessary.

b. State the domain of the function.

</td></tr>
<tr><td>A Look Ahead to Calculus</td><td>

Find $f(a)$, $f(a+h)$, and $\frac{f(a+h)-f(a)}{h}$ if $h \neq 0$.

A.

B.

</td></tr>
<tr><td>Guided Practice for</td><td>

Find $f(a)$, $f(a+h)$, and $\frac{f(a+h)-f(a)}{h}$ if $h \neq 0$.

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</table>

Writing in Precalculus	81 **WRITING IN MATH** Write the domain of $f(x) = \frac{1}{(x+3)(x+1)(x-5)}$ in interval notation and in set-builder notation. Which notation do you prefer? Explain.

Name: ____

Precalculus Summer Assignment Student Work

1.	2.
3.	4.
5.	6.

7.	8.
9.	10.
11.	12.

13.	14.
15.	16.
17.	18.

19.	20.
21.	22.
23.	24.

25.	26.
27.	28.
29.	30.

31.	32.
33.	34.
35.	36.

37.	38.
39.	40.
41.	42.

43.	44.
45.	46.
47.	48.

49.	50.

51.
52.

53.
54.
55.