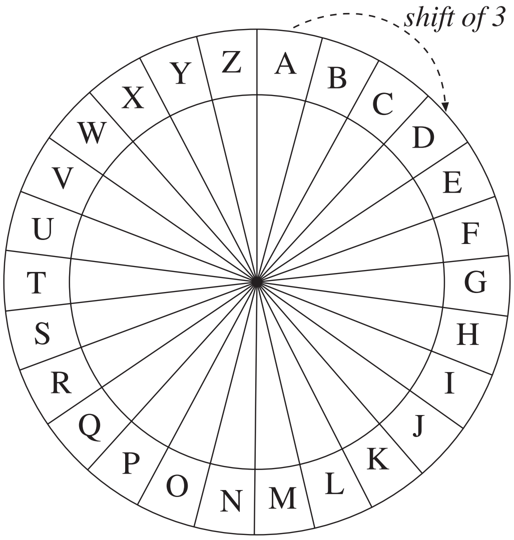
# UNIT 16 *Algebra: Linear Equations* Activities

## Activities

# ACTIVITY 16.1 *Codebreakers*




This codewheel can be used to code messages. Using a shift of 3, A becomes D, B becomes E, etc.

1. Code this message with a shift of 3:

   M E E T  M E  A T  T H E  S T A T I O N

   (You can use the codewheel rings on Activity 16.1, Sheet 2, to help.)

2. Decode this message, which has an *unknown* shift:

   B N J D  H S R  S G D  Q D Z K  S G H M F

A key, for example, [1, 2, 3], can be used to code a message. Using this, each letter is shifted by the next number in the key. For example:

| M | A | T | H | S |
|---|---|---|---|---|
| ↓ (1) | ↓ (2) | ↓ (3) | ↓ (1) | ↓ (2) |
| N | C | W | I | U |

3. Code with the key [1, 2, 3]: D O N T  L O O K  B A C K

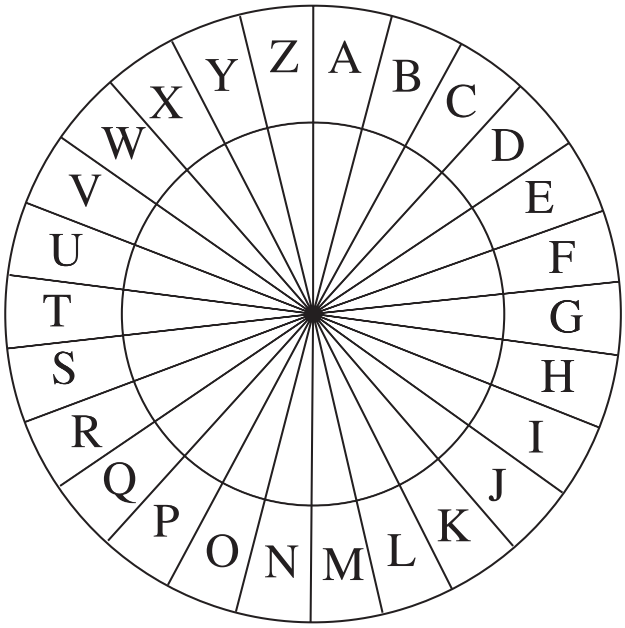
4. Decode this message, with the key [0, 2, 4]: L G R D  O I  A  H M V G V

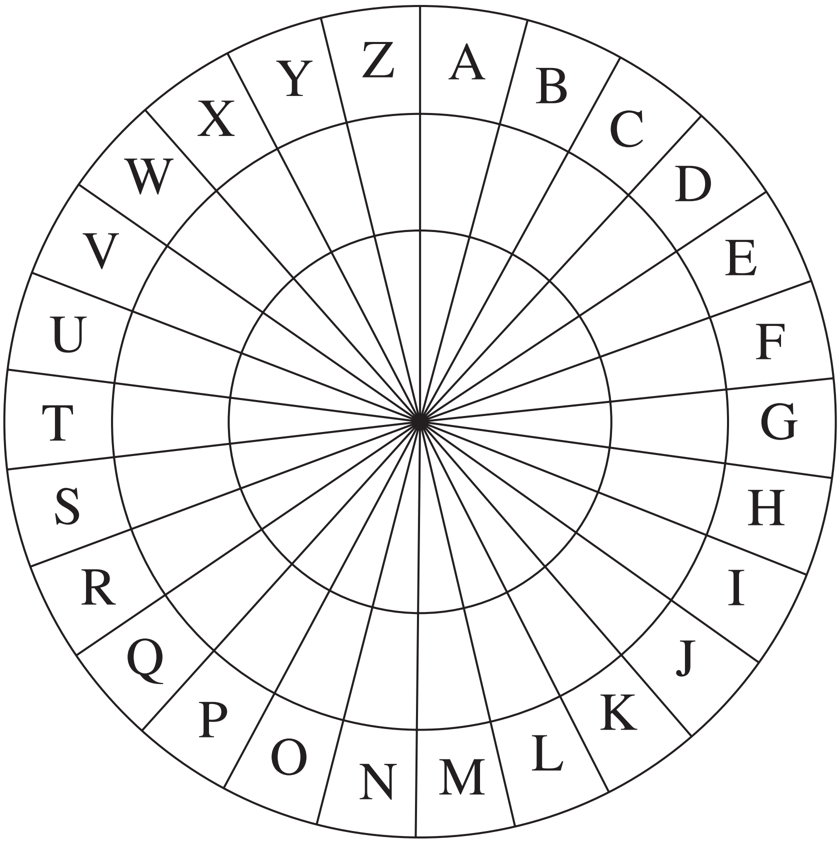
5. Decode with an *unknown* key! A  H O P D  U R Z

6. Try to decode this message: R G K C D M P Y Z P C Y I

# ACTIVITY 16.1 Sheet 2

**CODEWHEEL – INNER AND OUTER RINGS**

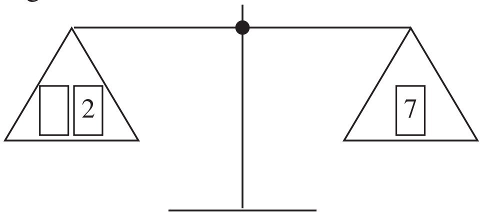


# ACTIVITY 16.2 *Balancing Equations*

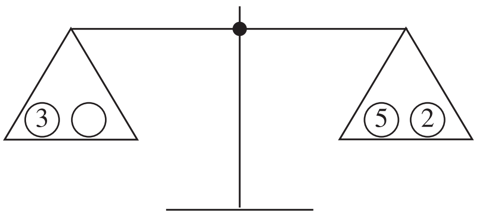
For each balance shown below, calculate the unknown weights; where there are two identical symbols, their weights are the same. Also, write down and solve an equation for each situation.

Here is an example:

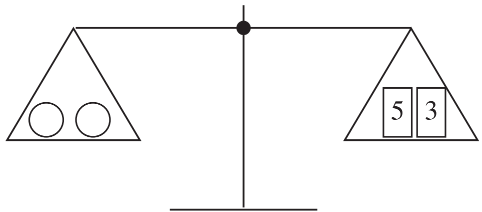



$$x + 2 = 7$$
$$x = 5$$

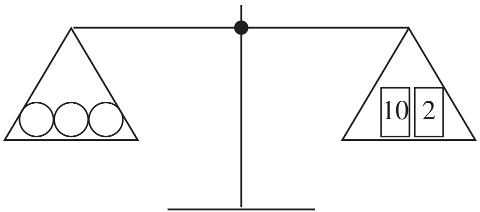
1.



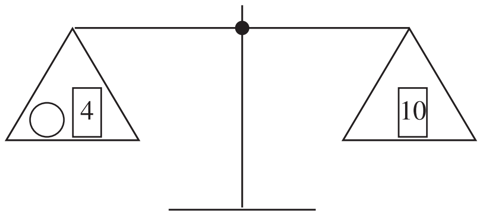

2.



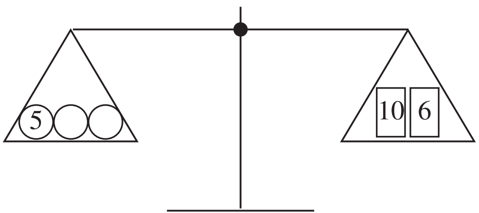

3.



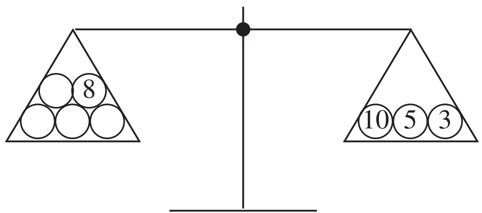

4.



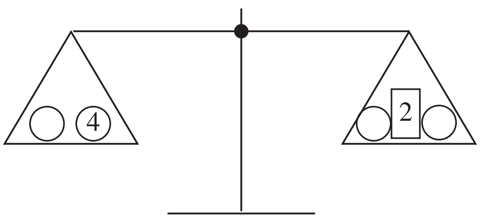

5.



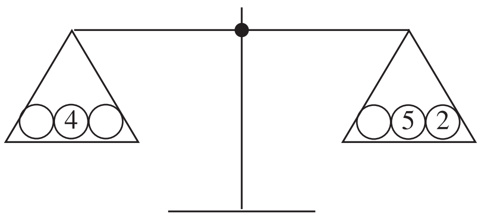

6.




7.




8.



# ACTIVITY 16.3 *Number Trick*

Try this trick out on a friend:

1. Choose three different digits from:

   1, 2, 3, 4, 5, 6, 7, 8 and 9.

2. Write down all the possible 2-digit numbers you can form from these numbers.
3. Add all these 2-digit numbers together.
4. Work out the sum of the three chosen digits.
5. Divide the total of the numbers by the total of the digits.

Now amaze your friend by predicting this answer to be 22.

We will now see how and why this method works:

Let the three chosen digits be $a$, $b$ and $c$. The number formed by using $a$ in the tens and $b$ in the units can be written as $10a + b$.

1. Write down the algebraic expression for each of the six numbers formed by taking 2-digit numbers with $a$, $b$ and $c$.
2. Find an expression for $T$, the total of these six numbers.
3. Simplify your expression for $T$.
4. Write down the sum, $S$, of $a$, $b$ and $c$.
5. What is $\dfrac{T}{S}$ ?

**Extension**

Design a similar trick, using, for example, all 3-digit numbers that can be found using 3 numbers.

# ACTIVITY 16.4 *Solving Equations*

Solving equations is a fundamental part of algebra. At first, it might seem that there are different rules for different types of equations, and you might be confused over which rule to apply. In fact, all algebraic manipulations are based on the concept of

*balancing equations.*

Whichever process is applied to one side of the equation *must* also be applied to the other side. To solve a general linear equation of the form

$$\boxed{ax + b = c}$$

where $x$ is the unknown and $a$, $b$ and $c$ are given constants, we must make $x$ the subject.

The procedure is shown below for both the *general* case and, as an example, $2x + 5 = 9$.

| | *General formula* | *Example* |
|---|---|---|
| | $\boxed{ax + b = c}$ | $\boxed{2x + 5 = 9}$ |
| *Step 1* Subtract $b$ from both sides: | $ax + b - b = c - b$ | $2x + 5 - 5 = 9 - 5$ |
| | $ax = c - b$ | $2x = 4$ |
| *Step 2* Divide both sides by $a$: | $\dfrac{ax}{a} = \dfrac{c-b}{a}$ | $\dfrac{2x}{2} = \dfrac{4}{2}$ |
| | $x = \dfrac{c-b}{a}$ | $x = 2$ |

Using this method, calculate the value of $x$ in the following equations:

1. (a) $3x + 7 = 10$ (b) $5x - 4 = 26$ (c) $6x + 7 = 22$

   (d) $4 - x = -3$ (e) $10 - \dfrac{3x}{2} = 16$ (f) $11x - 52 = 3$

The special cases where either $a = 1$ or $b = 0$ can be solved easily.

Find the solution to the following equations:

2. (a) $x + 3 = 7$ (b) $x - 5 = -4$ (c) $7 - x = -4$

3. (a) $2x = 6$ (b) $3x = -15$ (c) $\dfrac{x}{2} = 10$ (d) $\dfrac{5x}{3} = 15$

**Extension**

1. Find the *general* solution of $\boxed{ax + b = cx + d}$ where $a$, $b$, $c$ and $d$ are constants.

2. Solve:

   (a) $2x + 1 = x + 6$ (b) $3x + 2 = 5 - x$ (c) $4 - 7x = 3x + 14$

# ACTIVITY 16.5 *Magic Squares*

In *magic squares,* in each row, column and diagonal, the sum of the numbers is always equal to the *magic number* for that square.

1. Here is an example which happens to use 9 consecutive numbers.

   Check that the sum of the numbers in each row, column and diagonal is equal to the magic number, 12.

| | | |
|---|---|---|
| 3 | 2 | 7 |
| 8 | 4 | 0 |
| 1 | 6 | 5 |

*Solving magic squares*

| | | |
|---|---|---|
| | 11 | 7 |
| 9 | | |
| | 5 | 10 |

| | | |
|---|---|---|
| $x$ | 11 | 7 |
| 9 | | |
| $y$ | 5 | 10 |

This magic square is more challenging! The answer may be found by trial and error but, ideally, a more systematic method is required.

Let $x$ be the unknown number in *Column 1, Row 1,*
$y$ be the unknown number in *Column 1, Row 3,*
$n$ be the *magic* number.

Then, from *Row 1,* $n = x + 11 + 7$
$= x + 18$

and from *Column 1,* $n = x + 9 + y$

So, $x + 18 = x + 9 + y$ (Subtract $x$ from both sides.)
$18 = 9 + y$ (Subtract 9 from both sides.)
$y = 9$

From *Row 3,* $n = y + 5 + 10$, so $n = 24$. From *Row 1,* $x + 18 = n = 24$, so $x = 6$.
The other two missing numbers can then be found to be 8 (*Column 2*) and 7 (*Column 3*).

2. Use an algebraic approach to solve the following magic squares:

(a)

| | | |
|---|---|---|
| 9 | 2 | |
| 12 | 8 | |
| | | |

(b)

| | | |
|---|---|---|
| 10 | 3 | |
| 5 | | 9 |
| | 11 | 4 |

(c)

| | | |
|---|---|---|
| 14 | | 12 |
| | | |
| 10 | | 8 |

**Extension**

| | | |
|---|---|---|
| $a$ | $b$ | |
| $c$ | $d$ | |
| | | |

For the general magic square opposite:

1. Find an expression for the missing entries in terms of $a$, $b$, $c$, $d$ and $n$.
2. Form equations for the sums in the two diagonals.
3. Hence solve for the unknowns, $c$ and $d$, in terms of $a$, $b$ and $n$ and find the form of a general magic square.
4. Use this general form to solve the magic squares in question 2.

# ACTIVITIES 16.1 - 16.3 *Notes and Solutions*

*Notes and solutions are given only where appropriate.*

**16.1** Questions 5 and 6 are difficult and pupils may need some hints.

1. P H H W P H D W W K H V W D W L R Q
2. C O K E I T S T H E R E A L T H I N G
3. E Q Q U N R P M E B E N
4. L E N D M E A F I V E R
5. A G O O D T R Y [ 0, 1]
6. T I M E F O R A B R E A K (Codewheel shift of 2)

**16.2**

1. $3 + x = 7$
   $x = 4$
2. $2x = 8$
   $x = 4$
3. $3x = 12$
   $x = 4$
4. $x + 4 = 10$
   $x = 6$
5. $5 + 2x = 16$
   $2x = 11$
   $x = 5\frac{1}{2}$
6. $4x + 8 = 18$
   $4x = 10$
   $x = 2\frac{1}{2}$
7. $x + 4 = 2x + 2$
   $4 = x + 2$
   $x = 2$
8. $2x + 4 = x + 7$
   $x + 4 = 7$
   $x = 3$

**16.3**

1. $10a + b,\ 10b + a,\ 10a + c,\ 10b + c,\ 10c + b,\ 10c + a$
2. $T = 22a + 22b + 22c$
3. $T = 22(a + b + c)$
4. $S = a + b + c$

**Extension** In the example, suggested, $T \div S = 222$

# ACTIVITIES 16.4 - 16.5 *Notes and Solutions*

**16.4**

1. (a) 1 (b) 6 (c) 2.5 (d) 7 (e) – 4 (f) 5
2. (a) 4 (b) 1 (c) 11
3. (a) 3 (b) – 5 (c) 20 (d) 9

**Extension**

1. $\dfrac{d-b}{a-c} \quad (a \neq c)$
2. (a) 5 (b) $\dfrac{3}{4}$ (c) – 1

**16.5**

2. (a)

| | | |
|---|---|---|
| 9 | 2 | 13 |
| 12 | 8 | 4 |
| 3 | 14 | 7 |

$n = 24$

(b)

| | | |
|---|---|---|
| 10 | 3 | 8 |
| 5 | 7 | 9 |
| 6 | 11 | 4 |

$n = 21$

(c)

| | | |
|---|---|---|
| 14 | 7 | 12 |
| 9 | 11 | 13 |
| 10 | 15 | 8 |

$n = 33$

**Extension**

1.

| | | |
|---|---|---|
| $a$ | $b$ | $n-a-b$ |
| $c$ | $d$ | $n-c-d$ |
| $n-a-c$ | $n-b-d$ | $a+b+c+d-n$ |

2. $n-2a-b-c+d=0, \quad -2n+2a+b+c+2d=0$

3. $d=\dfrac{n}{3}, \quad c=\dfrac{4n}{3}-2a-b$

4.

| | | |
|---|---|---|
| $a$ | $b$ | $n-a-b$ |
| $\dfrac{4n}{3}-2a-b$ | $\dfrac{n}{3}$ | $-\dfrac{2n}{3}+2a+b$ |
| $-\dfrac{n}{3}+a+b$ | $\dfrac{2n}{3}-b$ | $\dfrac{2n}{3}-a$ |