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Unemployment and hysteresis: a nonlinear unobserved components approach

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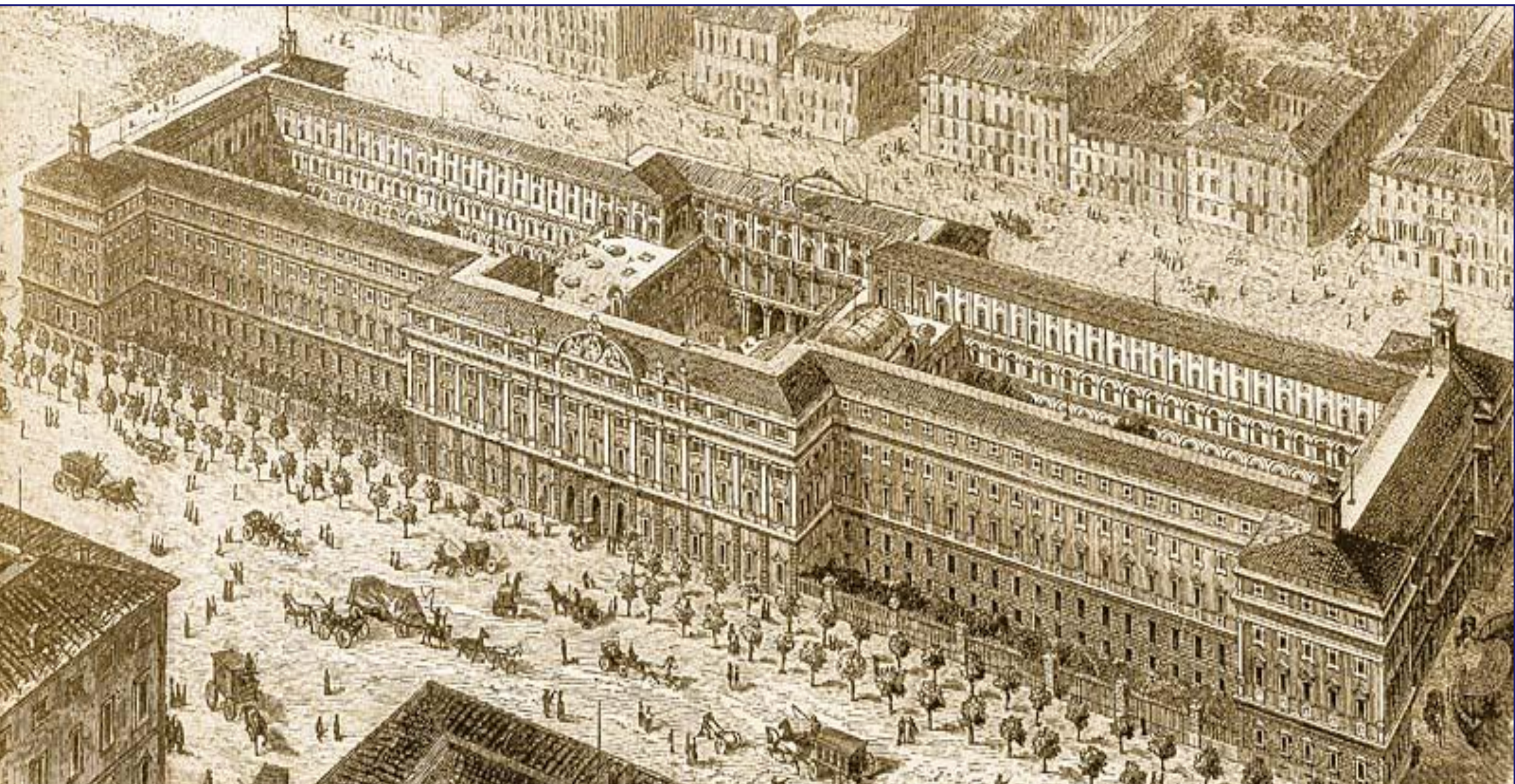

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Unemployment and Hysteresis: A Nonlinear Unobserved Components Approach[1]

Silvestro DI SANZO*,Alicia PEREZ-ALONSO**

Abstract

A new test for hysteresis based on a nonlinear unobserved components model is proposed. Observed unemployment rates are decomposed into a natural rate component and a cyclical component. Threshold type nonlinearities are introduced by allowing past cyclical unemployment to have a different impact on the natural rate depending onthe regime of the economy. The impact of lagged cyclical shocks on thecurrent natural component is the measure of hysteresis. To derive anappropriate p-value for a test for hysteresis two alternative bootstrapalgorithms are proposed: the first is valid under homoskedastic errorsand the second allows for heteroskedasticity of unknown form. A MonteCarlo simulation study shows the good performance of both bootstrapalgorithms. The bootstrap testing procedure is applied to data fromItaly, France and the United States. We find evidence of hysteresis forall countries under study.

JEL Classification: C12, C13, C15, C32, E24
Keywords: Hysteresis; Unobserved Components Model; Threshold Autoregressive Models; Nuisance parameters; Bootstrap

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CONTENTS

1 INTRODUCTION

In the mid 1970's European unemployment started a transition from rates in the order of 1-2% to rates in the order of 10-15% in the 1990's. More recently, according to Eurostat, the euro area seasonally-adjusted unemployment rate stood at 7.5% in September 2008. This experience reveals a slow tendency of actual unemployment to revert to a stable underlying unemployment rate, if any. Many theories have emerged to provide an economic explanation which could account for this observed unemployment persistence. Most of the work in the relevant literature assumes that it can be attributed to changes in the natural rate of unemployment and/or changes in the cyclical rate of unemployment. Based on this framework, two main approaches are the natural rate theory and the unemployment hysteresis theory.

The first approach assumes that output fluctuations generate cyclical movements in the unemployment rate, which in the long run, will tend to revert to its equilibrium. The crux of the natural rate hypothesis is that the cyclical unemployment rate and the natural unemployment rate evolve independently. Hence, the tendency of the natural rate to remain at a high level is the result of permanent shocks on the structure of the labour market, whereas transitory shocks only cause a temporary deviation from a unique equilibrium, see Friedman (1968), Bean et al. (1987) and Layard et al. (1991).

The second approach assumes that the cyclical unemployment rate and the natural unemployment rate do not evolve independently. The basic idea is that a change in the cyclical component of the unemployment rate may be permanently propagated to the natural rate (see Amable et al. 1995 and Roed 1997 for a survey). Therefore, a direct implication of the hysteresis hypothesis is that short-run adjustments of the economy can take place over a very long period. Consequently, aggregate demand policy, traditionally considered as ineffective in changing the natural rate of unemployment, can have a permanent effect on it.

In this paper, we focus on the second approach. The concept of hysteresis is brought to the forefront of labour market theory through a paper by Blanchard and Summers (1986). They consider an insider-outsider model of wage bargaining between insiders and the firm with outsiders playing no role[2].Given the presence of labour turnover costs, a shock that reduces the number of insiders one period raises the optimal insider-wage in subsequent periods, which prevents unemployed workers from being hired. In the particular casewhere insider status always coincides with current employment, employment follows a random walk. Based on this framework, a great number of empirical studies have investigated whether unemployment series, which is mainly modelled as a linear ARMA-type process, exhibits a unit root (see Roed 1997 and references therein). However, this practice of checking for the presence of hysteresis using linear ARMA-type processes has an important shortcoming: natural and cyclical shocks are summarized in the innovation with no distinction. Given that hysteresis in unemployment arises when a change in cyclical unemployment induces a permanent change in the natural rate, the presence of a unit root in the unemployment rate, modelled as a linear dynamic system, could be generated by accumulation of natural shocks and be completely independent of

[2]For further details on the insider-outsider theory of employment see Lindbeck and Snower (1988).

whether there is hysteresis. Hence, separating the respective effects of transitory and permanent shocks on the natural rate of unemployment is the only way to assess if changes in it are due to cyclical (this is the case of hysteresis) or natural shocks or both. So, we need an econometric model that discriminates between natural and cyclical sources of influence on the unemployment rate.

Jaeger and Parkinson (1994, henceforth JP) put this idea into perspective and adopt an unobserved components model to test the validity of the hysteresis hypothesis[3]. They generate a pure statistical decomposition of the actual unemployment rate into a natural rate component and a cyclical component, which are both treated as latent variables. They also assume a particular structure to describe the variation over time of these latent variables. The hysteresis effect is introduced by allowing cyclical unemployment to have a lagged effect on the natural rate, which is assumed to contain a unit root. They only consider symmetric responses of the natural rate as regards cyclical unemployment fluctuations. Thus, they implicitly assume hysteresis is a linear phenomenon, and this assumption may be too restrictive.

We propose an extended version of JP's model introducing nonlinearities. There is a wide range of theoretical and empirical evidence that shows that the unemployment rate displays asymmetries in adjustment dynamics, and thus hysteresis may be characterized by nonlinear dynamics when it exists. Following, we look at some of the various explanations for the nonlinearity of the unemployment rate. Firstly, there are asymmetric adjustment labour costs, such as hiring and firing costs (see Johansen 1982, Bentolila and Bertola 1990, and Hamermesh and Pfann 1996). Secondly, there is asymmetry in job creation and destruction, for instance, Mortensen and Pissarides (1993) emphasize the time it takes for a firm to find a good match to explain why jobcreation takes longer than job destruction. Similarly, Caballero and Hammour (1994) develop a model in which jobs are destroyed at a higher rate during recessions than expansions. Finally, Bean (1989) stresses asymmetry in capital destruction. The theoretical arguments stressing the nonlinearity of unemployment have been matched by plenty of empirical evidence. Using nonparametric techniques, the seminal paper of Neftci (1984) finds unemployment rises to be sudden, and falls to be gradual; see also Sichel (1989) and Rothman (1991). Various parametric nonlinear time series models of unemployment have also been estimated in the literature by Hansen (1997), Bianchi and Zoega (1998), Koop and Potter (1999), Papell et al. (2000), Caner and Hansen (2001), Skalin and Teräsvirta (2002), Coakley and Fuertes (2006), Caporale and Gil-Alana (2007), among others. All these studies assume Markov-switching, threshold or smooth transition specifications.

This theoretical and empirical evidence suggests that any satisfactory model for the unemployment rate has to be able to account for nonlinearity. The contribution of this paper is to extend JP's model by introducing nonlinearities using a threshold autoregressive (TAR) model[4]. In particular, we allow past cyclical unemployment to have a different effect on the natural rate, which depends on the regime of the economy. We consider two regimes reflecting favorable and unfavorable times, which have been defined based on previous changes in unemployment. We choose this particular form of nonlinearity because TAR models are the most widely used class of models in the nonlinear time series literature on the dynamics of unemployment, given

[3]See Harvey (1989) for a detailed description of the Unobserved Component models.

[4]For an extensive discussion of TAR models we refer to Tong (1990).

that they can exhibit the type of dynamic asymmetries that theoretical models suggest, and are computationally easy to work with (see references above). Furthermore, Petruccelli (1992) shows that threshold specifications may be viewed as an approximation to a more general class of nonlinear models. We propose a test for assessing the presence of regime specific nonlinearity within the phenomenon of hysteresis when it exists. The relevant null hypothesis is a one-regime model against the alternative of two regimes, i.e. the null hypothesis of linearity is tested against a threshold alternative. Testing for threshold type nonlinearities raises a particular problem known in the statistics literature as hypothesis testing when a nuisance parameter is not identified under the null hypothesis (see, Davies 1977 and 1987, Andrews and Ploberger 1994, Chan 1990, and Hansen 1996). If the model is not identified under the null, the asymptotic distribution of classical tests is unknown, so tabulated critical values are unavailable. To circumvent this problem, we use bootstrap methods to approximate the null distribution of the test statistic. More precisely, we use the resamplingalgorithm proposed by Stoffer and Wall (1991) for linear state-space models. Finally, we use this bootstrap testing procedure to check for the presence of hysteresis in Italy, France and the United States.

The rest of this paper is organized as follows. Section 2 briefly describes JP's model and proposes an extended version that introduces hysteresis allowing for threshold type nonlinearity. Section 3 proposes two alternative bootstrap procedures to compute the p-value for a linearity test under our hysteresis model. Empirical results for Italy, France and the United States are presented in Section 4. The conclusion is provided in the last section. Appendix A discusses the design of the Monte Carlo experiments that are used to investigate the small sample performance of the bootstrap version of the test statistic, and presents the results of some limited simulations. Estimation methods are relegated to Appendix B. Appendix C contains all the tables and figures.

2 AN EXTENSION OF JAEGER AND PARKINSON'S MODEL

JP propose a pure statistical decomposition of the unemployment rate to evaluate the data for evidence on hysteresis effects. They assume the actual unemployment rate to be the sum of two unobservable components: a non-stationary natural rate component,U^N, and a stationary cyclical component, U^C,

$$U_t = U_t^N + U_t^C. \tag{2.1}$$

The natural rate component is defined as a random walk plus a term capturingpossible hysteresis effects,

$$U_t^N = U_{t-1}^N + \alpha U_{t-1}^C + \epsilon_t^N. \tag{2.2}$$

Coefficient αmeasures, in percentage points, how much the natural rate increases if the economy experiences a cyclical unemployment rate of 1.0 percent. The size of this coefficient is their measure of hysteresis.

The cyclical component of the unemployment rate is defined as a stationary second-order autoregressive process[5],

$$(2.3) \qquad U_t^C = \phi_1 U_{t-1}^C + \phi_2 U_{t-2}^C + \epsilon_t^C.$$

To identify the model, the system is completed by augmenting it with a version of Okun's law, which relates cyclical unemployment and output growth,

$$(2.4) \qquad D_t = \beta D_{t-1} + \delta U_t^C + \epsilon_t^D,$$

where D_t stands for the output growth rate at date t[6]. Equation (2.4) defines the output growth rate as an autoregressive process of order one plus a term capturing the influence of the cyclical rate of unemployment. Since the cyclical component is assumed to be stationary, we consider U_t^C instead of ΔU_t^C as in JP's model in order to avoid a problem of over-differentiation.

The disturbances ϵ_t^N, ϵ_t^C and ϵ_t^D are assumed to be mutually uncorrelated shocks, which are normally distributed with variances σ_N^2, σ_C^2 and σ_D^2, respectively.

To test the hysteresis hypothesis, i.e. past cyclical movements on unemployment have a permanent impact on the natural rate, JP perform a significance test on parameter α,

$$H_0^{JP} : \alpha = 0 \quad \text{vs.} \quad H_1^{JP} : \alpha \neq 0.$$

If parameter α is significantly different from zero, they argue there existsa hysteresis effect on the unemployment rate. Note that JP's model is lineargiven that past cyclical unemployment changes have the same impact, in absoluteterms, on the natural unemployment rate. For example, a variation inthe cyclical component of 1% or (-1)% causes a variation in the natural rateof α% or $(-\alpha)$%, respectively.

Relaxing the linearity assumption may allow a better estimation of hysteresis if it exists. It is widely acknowledged that the unemployment rate displays asymmetries in adjustment dynamics. In particular, fast-up, slow-down dynamics. As pointed out in the introduction, among the multitude of alternative nonlinear models available, we choose the class of models with TAR dynamics. Hence, to relax the assumption of linearity, we introduce threshold type nonlinearities into JP's model. These are introduced by allowing past cyclical unemployment to have a different impact on the natural rate, which depends on the regime of the economy. To that end, equation (2.2) becomes

$$(2.2') \qquad U_t^N = U_{t-1}^N + \alpha_1 U_{t-1}^C I(q_{t-1} \geq \gamma) + \alpha_2 U_{t-1}^C I(q_{t-1} < \gamma) + \epsilon_t^N,$$

where q_{t-1} is the threshold variable assumed to be stationary, γ stands for the threshold parameter and $I(B)$ is the usual indicator function taking the valueone when B holds and zero otherwise. Equations (2.1), (2.3) and (2.4) remain the same together with assumptions about shocks.

This model is estimated via maximum likelihood (ML) in the frameworkof the Kalmanfilter[7]. More precisely, we employ a modified Kalman filter to incorporate a deterministic cut-off of the sample that corresponds to araw indicator for *favorable* and *unfavorable* periods, which is based on themethodology implemented for the estimation of TAR models. We choose thelong

[5]To select the order of the autoregressive process, the Akaike and Schwarz information criteria and the diagnostic checking tests proposed by Harvey (1985) are employed. As in JP, we find that an AR(2) process for the cyclical component fits the data well for all the countries under study.

[6]Our results regarding the nature of the hysteresis phenomenon are rather stable even when the model is estimated using different identification equations such as the Phillips curve and the Beveridge curve.

[7]See Appendix A for a detailed description of this estimation methodology.

difference$U_{t-1} - U_{t-d}$ with $d \epsilon \{2, 3\}$as our threshold variableq_{t-1}. This variable is an indicator of the state of the economy to identify the regimes. The integer d is called the threshold delay lag. Whether the threshold variable is lower or higher than the threshold parameter γ determines whether an observation belongs to one regime or the other. We consider an economy with two regimes, one related to high long differences (regime 1), i.e. an *unfavorable* regime, and the other with low long differences (regime 2), i.e. a *favorable*regime. Parameters d and γ are unknown so they are estimated along with the other parameters of the model. The maximization is best solved through agrid search over the two-dimensional space(γ, d). To execute a grid search weneed to fix a region over which to search. It is important to restrict the set ofthreshold candidates *a priori* so that each regime contains a minimal numberof observations. For each value of$d$, we restrict the search to values ofγlying on $\left[\underline{\gamma}, \overline{\gamma}\right]$, where $\underline{\gamma}$ is the τth quantile of q_{t-1}, and $\overline{\gamma}$ is the $(1-\tau)$th quantile. In our applications we choose $\tau = 0.30$. Then we estimate the model for each pair (γ, d) belonging to the grid $\Delta = (\left[\underline{\gamma}, \overline{\gamma}\right] \times \{2, 3\})$ and retain the one that provides the highest log-likelihood value.

In this framework, we want to test the null hypothesis of a linear modelversus the alternative of a nonlinear one, that is:

$$H_0 : \alpha_1 = \alpha_2 \quad \text{vs.} \quad H_1 : \alpha_1 \neq \alpha_2.$$

If we reject H_0 (the null of linearity), there is evidence for the presence of hysteresis in the unemployment rate, which displays a nonlinear behaviour. This finding is consistent with cyclical shocks being propagated asymmetrically to the natural rate. In this case, JP's model is misspecified and any inference based on the parameters of their model may lead us to wrong conclusions. If it is not rejected, the next step is to estimate the linear model proposed by JP and test for hysteresis following the strategy they propose. If we reject H_0^{JP}, the natural rate component is affected by movements in the cyclical componentand thus hysteresis in unemployment occurs. If it is not rejected, there is noplace for hysteresis.

Here we propose a Wald type test statistic for testingH_0. Note that under this null hypothesis the threshold parameter given byγ and the delay d remain unidentified. As a result, the asymptotic distribution of conventional test statistics is not χ^2. This is a well-known problem in the literature on testingfor regime switching type of nonlinearities; here we test for a single regimeagainst the alternative of two regimes. This problem is usually handled byviewing the test statistic as a random function of the nuisance parametersand basing inference on a particular functional of the test statistic such as,for instance, its supremum over (γ, d)(see, Davies 1977 and 1987, Andrews and Ploberger 1994, Chan 1990, and Hansen 1996). Letting W(γ, d)denote the Wald type test statistic obtained for each (γ, d), we base our inferences on $SupW = sup_{(\gamma,d)\epsilon\Delta} W(\gamma, d)$. To our knowledge the null asymptotic distribution of SupW is unknown under the above framework. To circumvent this problem,we suggest using bootstrap methods to approximate the sampling distributionof SupW under H_0..

3 TESTING FOR LINEARITY

As the asymptotic distribution of the SupW test statistic is unknown in thepresent framework, in this section we discuss two bootstrap methods to calculate*p*-values. As a general rule, resampling should always reflect the nullhypothesis, according to Hall and Wilson (1991). Under the null hypothesis oflinearity we have JP's model, and Stoffer and Wall (1991) establish the validityof a resampling scheme for the innovations sequence of linear state-spacemodels. Other work using the bootstrap to study the problem of testing forlinearity includes Hansen (1999), Caner and Hansen (2001) and Hansen andSeo (2002).

To approximate the sampling distribution of the SupW test statistic, wesuggest using either a parametric residual bootstrap, or alternatively a wildbootstrap. The parametric residual bootstrap requires a complete specificationof the model under H_0.This is JP's model but relaxing the strong assumptionthat the error terms are normally distributed. While the assumptions of themodel also include homoskedasticity, we do not think that it is prudent toimpose this condition when constructing test statistics. It is therefore desirableto calculate a bootstrap distribution of SupW allowing for the possibility oferror terms with an unknown pattern of heteroskedasticity. The disadvantageof the parametric residual bootstrap is that if the pattern is unknown, it cannotbe imitated in the bootstrap data-generating process under H_0.A techniqueused to overcome this difficulty is the so-called wild bootstrap proposed byWu (1986) and developed by Liu (1988).

The finite sample performance of the test statistic obtained from the twobootstrap algorithms is investigated with Monte Carlo experiments in AppendixA. The simulation results suggest that the bootstrap test statistic worksquite well concerning size and power in our framework. Of course, we have noguarantee that it works in general.

3.1 The state-space model

The state-space model is defined by the equations

$$s_t = F(q_{t-1})s_{t-1} + w_t, \tag{3.1}$$

$$y_t = Hs_t + Dy_{t-1} + v_t, \tag{3.2}$$

where $s_t = (U_t^N, U_t^C, U_{t-1}^C)'$ is a vector of unobserved state variables and $y_t = (U_t, D_t)'$is a vector of observed variables. Equation (3.1) is known as the transition equation and equation (3.2) is known as the measurement equation.The coefficients of the model are stored in the constant matrices

$$F(q_{t-1}) = F_1 I(q_{t-1} \geq \gamma) + F_2 I(q_{t-1} < \gamma), \quad q_{t-1} = U_{t-1} - U_{t-d}, \quad d \in \{2, 3\};$$

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \delta & 0 \end{bmatrix}; \quad F_i = \begin{bmatrix} 1 & \alpha_i & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}, \; i = 1, 2; \quad D = \begin{bmatrix} 0 & 0 \\ 0 & \beta \end{bmatrix}.$$

The vectors $w_t = (\epsilon_t^N, \epsilon_t^C, 0)'$ and $v_t = (0, \epsilon_t^D)'$ represent white noise processes with $E(w_t w_t') = Q, E(v_t v_t') = R$ and $E(w_t v_t') = 0$, where

$$Q = \begin{bmatrix} \sigma_N^2 & 0 & 0 \\ 0 & \sigma_C^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_D^2 \end{bmatrix}.$$

Note that under the null hypothesis of linearity $F(q_{t-1}) = F$. To simplify the notation, let $\Theta_0 = (\sigma_N, \sigma_C, \sigma_D, \alpha, \phi_1, \phi_2, \delta, \beta)'$ be the vector with the model coefficients and the correlation structure under H_0, and $\Theta_1 = (\sigma_N, \sigma_C, \sigma_D, \alpha_1, \alpha_2, \phi_1, \phi_2, \delta, \beta)'$be the vector of parameters under the alternative of nonlinearity.

3.2 Two bootstrap algorithms

The first algorithm we propose is the parametric residual bootstrap (RB). It consists of the following steps:

Bootstrap I (RB)

Step 1

We compute the SupW test statistic. To compute it we need only to estimate the model under H_1. For each given value of $(\gamma, d) \epsilon \Delta$, let $\widehat{\Theta}_1(\gamma, \mathrm{d})$ denote the ML estimate of Θ_1. We compute the pointwise Wald test statistic as $W(\gamma, d) = R\widehat{\Theta}_1(\gamma, \mathrm{d}) \left(\mathrm{R}\widehat{\mathrm{Var}}\left(\widehat{\Theta}_1(\gamma, \mathrm{d})\right)\mathrm{R}'\right)^{-1} (R\widehat{\Theta}_1(\gamma, \mathrm{d}))'$, where R is the selector matrix $R = (0\ 0\ 0\ 1 - 1\ 0\ 0\ 0\ 0)$ and $\widehat{\mathrm{Var}}\left(\widehat{\Theta}_1(\gamma, \mathrm{d})\right)$ is the robust variance-covariance matrix estimator proposed by White (1982). Davies (1977, 1987) suggest testing H_0 by $SupW = sup_{(\gamma,d)\epsilon\Delta} W(\gamma, d)$.

Step 2

We use the Kalmanfilter to construct the standardized residualsunder H_0. We first obtain linear forecasts of the state vector at time$t$based on all the available information up to time$t-1$, say $s_{t|t-1}$,and the mean square error matrix associated with each of these forecasts, say$P_{t|t-1}$. We also obtain from the Kalmanfilter the *innovations*$\epsilon_t = y_t - Hs_{t|t-1} - Dy_{t-1}$, the *innovations covariance matrix* $\sum_t = HP_{t|t-1}H' + R$, the *Kalman gain matrix* $K_t = P_{t|t-1}H' \sum_t^{-1}$, and the *updating of the state variable* $s_{t|t} = s_{t|t-1} + K_t\epsilon_t$. We also derive the *innovations form representation* of the observations as

$$s_{t+1|t} = Fs_{t|t-1} + FK_t\epsilon_t, \tag{3.3}$$
$$y_t = Hs_{t|t-1} + Dy_{t-1} + \epsilon_t. \tag{3.4}$$

Let $\widehat{\Theta}_0$ denote the ML estimate of Θ_0. Evaluating $\epsilon_t, \Sigma_t, \mathrm{K}_t$ and $s_{t|t}$ at $\widehat{\Theta}_0$, we obtain $\hat{\epsilon}_t, \hat{\Sigma}_t, \widehat{\mathrm{K}}_t$ and $\hat{s}_{t|t}$. We construct the standardized residuals by setting $e_t = \hat{\Sigma}_t^{-1/2}\hat{\epsilon}_t$. By using standardized residuals, we guarantee that all model residuals have, at least, the same first two moments.

Step 3

The bootstrap errors $\{e_t^*, t = 1, \dots, T\}$ are independent values obtainedby resampling, with replacement, from the set of standardized residuals$\{e_t\ , t = 1, \dots, T\}$.

Step 4

To construct the bootstrap data set underH_0, say $\{y_t^*, t = 1, \dots, T\}$ we use equations (3.3) and (3.4). Let $\hat{F}, \hat{H}$ and $\hat{D}$ be the matrices of coefficients evaluated at$\widehat{\Theta}_0$, and $s_{1|0} = 0$contains the first 3 values of the state variables (thus, these are prespecified and set equal to the initial conditionsfor the Kalmanfilter). The remaining elements of the vector$s_{t+1|t}$ are constructed by computing a first-order autoregressive process given by (3.3):

$$s^*_{t+1|t} = \widehat{F}s^*_{t|t-1} + \widehat{F}\widehat{K}_t\widehat{\Sigma}_t^{1/2}e^*_t.$$

The vector $y_t$is constructed by computing a first-order autoregressive process,with initial conditions fixed at the observed values, and then by adding theresults to the corresponding elements of$s_{t+1|t}$. That is, the row t*th* of *y* isgiven by (3.4):

$$y^*_t = \widehat{H}s^*_{t|t-1} + \widehat{D}y^*_{t-1} + \widehat{\Sigma}_t^{1/2}e^*_t.$$

All initial conditions are kept fixed throughout the bootstrap replications.

Step 5

The bootstrap sample $\{y^*_t, t = 1, \dots, T\}$ is then used to calculate the statistic SupW* using the same procedure as to calculate SupW on theactual series.

Step 6

Repeating steps 3 through 5 for $b = 1$, …, *B*, gives a sample$\{SupW^*: b = 1, \dots, B\}$ of SupW values. This sample mimics a random sample of draws of SupW underH_0. We compute the bootstrap *p*-value as $p_B = card(SupW^* \geq SupW)/B$, that is the fraction of $SupW^*$ values that are greater than the observed value SupW. We carry out B = 1000 bootstrap replications.

The wild bootstrap (WB) is an alternative way of obtaining the bootstrapdistribution of the SupW test statistic allowing for the possibility ofheteroskedasticity of unknown form. This bootstrap algorithm differs fromthe former in the resampling scheme of the residuals and in the use of a (conditionally) fixed design on the regressors to obtain the bootstrap data set.

Bootstrap II (WB)

Step 3'

To construct the wild bootstrap errors$\{\tilde{e}^*_t, t = 1, \dots, T\}$, we firstgenerate $\eta_t$independent and identically distributed random variables from a fixed distribution, such that $E(\eta_t) = 0$ and $E(\eta_t^2) = E(\eta_t^3) = 1$.[8]We next define $\tilde{e}^*_t = \hat{\epsilon}_t\eta_t$, where $\hat{\epsilon}_t$ is the t*th*non-standardized residual calculated in step 2. Thus, the errors$\tilde{e}^*_t$ satisfy $E^*(\tilde{e}^*_t) = 0$, $E^*(\tilde{e}^{*2}_t) = \hat{e}^2_t$ and $E^*(\tilde{e}^{*3}_t) = \hat{e}^3_t$, where $E^*(.)$ denotes the expectation under the bootstrap distribution.

Step 4'

To construct the bootstrap data set$\{\tilde{y}^*_t, t = 1, \dots, T\}$:

I. Set the initial condition $s_{1|0} = 0$ and, for $t = 2, \dots, T$, set $\tilde{s}^*_{t|t-1} = \hat{s}_{t|t-1}$, that is, unobserved bootstrap components are generated with conditionally setdesign on the estimated unobserved components in step 2:$\tilde{s}^*_{t+1|t} = \widehat{F}\hat{s}_{t|t-1} + \widehat{F}\widehat{K}_t\tilde{e}^*_t$.

II. Using a conditional resampling on $(y_0, y_1, \dots, y_{T-1})$, derive $\tilde{y}^*_t = \widehat{H}\tilde{s}^*_{t|t-1} + \widehat{D}y_{t-1} + \tilde{e}^*_t, t = 1, \dots, T$.

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[8]In particular, the variable η_t was sampled from Mammen's (1993, p.257) two-pointdistribution attaching masses $(5 + \sqrt{5})/10$ and $(5 - \sqrt{5})/10$at the points$-(\sqrt{5} - 1)/2$ and $(\sqrt{5} + 1)/2$, respectively.

4 EMPIRICAL RESULTS

Our study concerns Italy, France and the United States. The economic seriesemployed are the quarterly unemployment rate (U) and real gross domesticproduct (GDP). Data for Italy (1970:1-2007:2) come from *Prometeia*, and data for France (1978:1-2007:2) and U.S. (1965:1-2007:2) come from *OECD Main Economic Indicators*. All data are obtained as seasonally adjusted and all the variables exept the unemployment rate are in natural logs.

We have decomposed the unemployment rate assuming that the natural rate contains a unit root. This assumption must be tested. We employ the methodology proposed by Caner and Hansen (2001) to test for a unit root in a single-equation two-regime TAR model. They restrict their analysis to univariate time series. Therefore, we adapt their method to our framework of state-space models by mimicking the method. We obtain that the unemployment rate series displays a non-stationary behaviour for all countries. We perform an augmented Dickey-Fuller (ADF) unit root test for the GDP series, which also displays a non-stationary behaviour for all countries. Results are presented in Table 1.

Tests for hysteresis are reported in Table 2. The p-values presented in Table 2 are calculated following the bootstrap technique described in Section 3. For comparison reasons, we also report the *p*-values obtained with the linear model of JP. Diagnosis checking of the residuals of the linear model leads us to implement a wild bootstrap for the U.S. and a parametric residual bootstrap for France and Italy. According to bootstrap *p*-values, the hysteresis effect is significant at the 1% level for all countries. As argued in Section 2, under the presence of nonlinearity, JP's model may lead to obtain spurious inference results. In fact, note that JP's methodology fails to detect hysteresis for the case of Italy, France and U.S..

Results concerning the estimated models for Italy, France and U.S. are available in Table 3. For the case of Italy, the ML estimate of the threshold parameter is$\hat{\gamma} = 0.1$ with a 90% bootstrap confidence interval [0:023; 0:247].Our estimate of the delay parameter is$\hat{d} = 2$. Hence, the threshold model splits the regression into two regimes depending on whether or not the threshold variable is higher than this threshold parameter. That is, we consider we arein regime 1 when$U_{t-1} - U_{t-2} \geq 0.1$and in regime 2 when$U_{t-1} - U_{t-2} < 0.1$.

ForItaly, there are less observations in regime 1 (41%) than in regime 2 (59%), which means that this country spent more periods of time in the *favorable*regime. This is also the case for U.S. and France. Analyzing the estimated hysteresis parameter, we observe a point of great interest. Both parameters are positive and the one associated with Regime 1 is greater than that of Regime 2. This points to asymmetric responses of the natural rate as regards cyclical unemployment movements in the following direction: the natural rate does not decrease in *favorable* cyclical periods as much as it increases in *unfavorable*cyclical periods. The size of the coefficients suggests that this mechanism ismore pronounced in France than in U.S. and Italy. In fact, for Italy, the natural rate decreases (2.512%) in *unfavorable* periods, while cyclical shockshave an impact of (1.476%) in *favorable* periods. In the U.S., these values are (1.343%) and (0.562%), respectively. On the other hand, for France, we find(3.540%) and (1.570%), respectively.

In Figures 1-3, the estimate of the natural rate is depicted against the recessionary periods for each country. Apart from the U.S., for which the *NBER Business Cycle Dating Committee* has been dating expansion and recessions, which have been generally recognized as the official U.S. business cycle dates, there is no widely accepted reference chronology of the classical business cycle for other countries. To overcome this problem, we date the turning points by using the dating algorithm of Harding and Pagan (2002) that isolates the local minima and maxima in a quarterly series, subject to reasonable constraints on both the length and amplitude of expansions and contractions. For the U.S., periods of increasing natural rate correspond to, but generally lag, the NBER recession periods. This is consistent with the classification of the unemployment rate as a lagging indicator at troughs.

Our findings have three important implications. Firstly, our empirical evidence supports theoretical models of hysteresis that describe it as a nonlinear phenomenon (see Bentolila and Bertola 1990, and Caballero and Hammour 1994 among others). As we pointed out in the introduction, in these models, hysteresis arises when cyclical shocks are propagated asymmetrically to the natural rate. Secondly, since statistical linear models are not able to describe the dynamic asymmetries of the unemployment rate, nonlinear models are needed to correctly represent and test hysteresis phenomena. Here, JP's hysteresis test may lead to obtain misleading inference results. Thirdly, our results are important for policy-makers. When hysteresis is present in the labour market, monetary policies, traditionally considered as ineffective, can be used to combat unemployment without immediate inflationary consequences. This evidence is in contrast with non-accelerating inflation rate of unemployment (NAIRU) models where shocks are not long-lived, and thus the unemployment rate reverts back to its underlying equilibrium level (see Friedman 1968).

5 CONCLUSIONS

In this paper we propose a new test for hysteresis based on a nonlinear unobserved components model. We extend the model of Jaeger and Parkinson (1994) by introducing threshold type nonlinearities into the specification of the natural rate component. We do this by allowing past cyclical unemployment to have a different effect on the current natural rate depending on the regime ofthe economy. Under this framework, a test on the hysteresis parameter implies to perform a test for linearity. In particular, the null hypothesis of interest is that of a one-regime model versus the alternative of two regimes. Testing for the presence of a threshold effect involves nuisance parameters which are not identified under the null hypothesis of linearity, so the asymptotic distribution of standard tests is unknown under the null, precluding tabulation of critical values. We rely on bootstrapping techniques to calculate an appropriate *p*-value for the test statistic. In particular, we propose two bootstrap procedures: the first is valid if the errors are homoskedastic and the second allows for general forms of heteroskedasticity. To assess the usefulness of the bootstrap test for linearity, finite sample results are reported in a simple Monte Carlo study. Our study concerns Italy, France and the United States. The empirical results show that the presence of hysteresis cannot be rejected for all the countries under study.

A. MONTE CARLO EVIDENCE

In this section we report on a Monte Carlo simulation study designed to evaluatethe small sample performance of both bootstrap procedures in the problemof testing for linearity. We start with a brief description of the design of theexperiment, then proceed with the discussion of the results.

Design of the experiment

The time series considered in our analysis are generated according to the state-spacemodel given by equations (3.1) and (3.2), under the null and the alternativehypotheses. Let M_0 and M_1 denote the class of linear and nonlinearstate-space models, respectively. Thus, in our experiments we use M_0 and M_1 as data-generating processes (DGPs) with$(\epsilon_t^N, \epsilon_t^C, \epsilon_t^D)' iidN(0, \Omega)$, where

$$\Omega = \begin{bmatrix} \sigma_N^2 & 0 & 0 \\ 0 & \sigma_C^2 & 0 \\ 0 & 0 & \sigma_D^2 \end{bmatrix}.$$

We aim at testing the null hypothesis of linearity. As discussed at the endof Section 2, the null hypothesis is true if and only if$\alpha_1 = \alpha_2$. Hence, M_0 is nested in M_1. We use the statistic SupW based on an estimatedM_1 setting $d = 2$, and compute the *p*-value using both the residual bootstrap and the wild bootstrap. The size of the test is investigated when the data are generatedaccording to M_0, while turning to the power properties of the test underM_1.

To ensure the relevance of the simulations, the parameter values are chosento correspond to models that have been fitted successfully to real-worldtime series. More specifically, we choose the estimated parameters for theU.S. obtained by JP as the DGP under the null hypothesis (DGP_0). AsDGP under the alternative hypothesis, we use the estimated model for U.S.$(DGP_{\alpha 1 - \alpha 2 = 0.8})$. That is, respectively,

$$\begin{aligned} \Theta_0 &= (0.0863, 0.2542, 0.4156, 0.023, 1.613, -0.678, -1.439, 0.031)'; \\ \Theta_1 &= (0.4, 0.12, 0.69, 1.343, 0.562, 1.237, -0.65, -1.626, 0.611)'; \quad \gamma = 0.3. \end{aligned}$$

To study the effect of the size of the difference $\alpha_1 - \alpha_2$on the performance of the test, we vary$\alpha_1$between (1:062; 1:562); while$\alpha_2$remains constant at its fixed value. Each of these values gives rise to$DGP_{\alpha 1 - \alpha 2 = 0.5}$ and $DGP_{\alpha 1 - \alpha 2 = 1}$, respectively.

The experiments proceed by generating artificial series of length$T + 50$ according to M_0 or M_1 with $T = 150$, and initial values set to zero.

Wethen discard the first 50 pseudo-data points in order to attenuate the effectof initial conditions and the remaining T points are used to compute the teststatistic. We simulate the proportion of rejections of the test at the 5%; 10%and 20% significance levels. The estimation of the rejection probabilities iscalculated from $B = 100$ bootstrap replications and $R = 500$ simulation runs.The processing time becomes excessive when greater values of B or R are used.

Simulation results

In Table 4 we present simulation evidence concerning the empirical size andpower of the test under both RB and WB. We observe a reasonable approximationof the nominal level at all significance levels considered. Deviationsfrom the null hypothesis are detected with high probability across the variousparameterizations. We observe that in all cases under consideration the testbased on the wild bootstrap approach yields slightly lower rejection probabilitiesthan the residual bootstrap test. Thus, with homoskedastic errors, thepenalty attached to using the wild bootstrap is very small. As expected, theperformance of both bootstrap procedures improves as the difference betweenthe values of parameters in the two regimes increases.

B. ESTIMATION PROCEDURES

In this appendix we present different filters that have been proposed in the relevant literature for estimating the sort of model described in Section 2.Firstly, we examine the Kalmanfilter, which allows us to estimate JP's model.Secondly, we introduce the threshold Kalmanfilter, which is a Kalmanfiltermodified to include a threshold state equation.

The Kalman filter

In 1960, R.E. Kalman published a famous paper describing a recursive solutionto the discrete data linear filtering problem. Since that time, greatly dueto advances in digital computing, the Kalmanfilter has been the subject ofextensive research and applications, particularly in the area of autonomous orassisted navigation.

The Kalmanfilter is a set of mathematical equations that provides an efficient recursive computational procedure for estimating the state of a process,in a way that minimizes the mean squared error (MSE)[9]. The filter is verypowerful in several aspects: it supports estimations of past, present, and evenfuture states, and it can do so even when the precise nature of the systemmodelled is unknown.

To start with, consider an $(n \times 1)$ vector of observed variables at date t, y_t.

These observable variables are related to a possibly unobserved $(r \times 1)$ vectorh_t, known as the state vector, via a *measurement equation*,

$$y_t = H'h_t + A'x_t + w_t, \tag{B.1}$$

where H' and A' are matrices of parameters of dimensions $(n \times r)$ and $(n \times k)$,respectively;x_t is a $(k \times 1)$ vector containing exogenous or lagged dependentvariables, and $w_t$is an $(n \times 1)$white noise disturbance vector with $E(w_t w_t') = R$for$t = \tau$, and 0 otherwise. Despite the fact that the variables of h_t are, ingeneral, not observable, they are known to be generated by a first-order Markovprocess,

$$h_t = Fh_{t-1} + \Pi' x_t + v_t, \tag{B.2}$$

where F and Π' are matrices of parameters of dimensions$(r \times r)$ and $(r \times k)$, respectively.

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The $(r \times 1)$ vector v_t is a white noise disturbance vector with $E(v_t v_t') = Q$ for $t = \tau$, and 0 otherwise. Equation (B.2) is known as the*transition equation*.

The disturbancesv_t and $w_t$are assumed to be uncorrelated at all lags, i.e.$E(v_t w_t') = 0$ for all t and τ.

Further assumptions on measurement and transitiondisturbances are as follows: i) they are uncorrelated with the exogenousvariables; ii) they are assumed to be normally distributed in order to calculatethe likelihood function.

The state-space form that represents the dynamics of the univariate timeseries y_t is composed of equations (B.1) and (B.2). There are two sets ofunknowns: the parameters of the modelH', A', R, F, Π' and Q (these matrices will be referred as the system matrices), and the elements of thestate vectorh_t. We will assume for now that the particular numerical valuesof the system matrices are known. The goal of the Kalmanfilter procedureis to form a forecast of the unobserved state vector at time t based on theinformation at date $t-1$. The information set at time$t-1$ is given by matrix $\Psi_{t-1} \equiv (y_{t-1}', y_{t-2}', \ldots, y_1', x_{t-1}', x_{t-2}', \ldots, x_1')'$. Let $\hat{h}_{t|t-1}$ denote the linear forecast of the state vectorh_t based on (x_t, Ψ_{t-1}), and $P_{t|t-1}$ denote the MSE matrix associated with this forecast.

Given that the filter is a recursion, it is started assuming initial valuesfor the mean and variance of the state variables, $\hat{h}_{1|0}$ and $P_{1|0}$, respectively. We can therefore conduct the Kalmanfilter in four major steps. Firstly, wecalculate the one-period-ahead forecast of the unobserved state vector and itsassociated MSE at $t-1$:

$$\begin{aligned} \widehat{h}_{t|t-1} &= E\left[h_t|x_t, \Psi_{t-1}\right] = F\widehat{h}_{t-1|t-1} + \Pi' x_t, \\ P_{t|t-1} &= E\left[(h_t - \widehat{h}_{t|t-1})(h_t - \widehat{h}_{t|t-1})'|\Psi_{t-1}\right] = FP_{t-1|t-1}F' + Q. \end{aligned}$$

Secondly, we calculate the one-period-ahead forecast of the measurementvariable at$t-1$:

$$\widehat{y}_{t|t-1} = E(y_t|x_t, \Psi_{t-1}) = H'\widehat{h}_{t|t-1} + A'x_t. \tag{B.3}$$

Thirdly, once the new observationy_t becomes available at date t, we calculatethe *innovation* and the *innovation covariance matrix*:

$$\begin{aligned} \lambda_t &= y_t - \hat{y}_{t|t-1}, \\ \Lambda_t &= E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'|\Psi_t] = H'P_{t|t-1}H + R. \end{aligned} \tag{B.4}$$

Finally, we update the state estimate and the estimate MSE:

$$\begin{aligned} \widehat{h}_{t|t} &= E\left[h_t|\Psi_t\right] = \widehat{h}_{t|t-1} + \Phi_t \lambda_t, \\ P_{t|t} &= (I - \Phi_t H')P_{t|t-1}, \end{aligned}$$

where $\Phi_t = P_{t|t-1} H \Lambda_t^{-1}$ is known as the *Kalman gain matrix*since a certainfraction of the difference between the observed and the predicted measurementvariable is added to the previous prediction of the state vector.$\hat{h}_{t|t}$ and $P_{t|t}$ are inputs of the next filter iteration.

Hence, if the system matrices are known the Kalmanfilter will yield asoutcome the sequences$\{\hat{h}_{t|t-1}\}_{t=1}^{T}$ and $\{P_{t|t-1}\}_{t=1}^{T}$.

We can view the Kalman filter as a sequential updating procedure that consists of forming a prior guessabout the state of nature and then adding a correction to that guess, thiscorrection being determined by how well the guess has performed in predictingthe next observation. However, the state-space model is not entirely estimatedsince we do not usually know the parameters of the system matrices. Assuming that $\{v_t, w_t\}_{t=1}^{T}$ are normally distributed, then the distribution of y_t conditional on (x_t, Ψ_{t-1}) is Normal with mean given by (B.3) and variance given by (B.4).We use the *prediction error decomposition* to construct the logarithm of thedistribution function as follows:

$$\ln f(y_t|x_t, \Psi_{t-1}) = -\frac{T}{2}\ln 2\pi - \frac{1}{2}\ln|\Lambda_t| - \frac{1}{2}\lambda_t'\Lambda_t^{-1}\lambda_t.$$

To estimate the parameters of the system matrices, we maximize the log-likelihoodfunction$\ln L = \Sigma_{t=1}^{T} \ln f(y_t|x_t, \Psi_{t-1})$ with respect to the underlying unknown parameters using nonlinear optimization techniques.

The threshold Kalman filter

Nonlinearities can be introduced into state-space models in a variety of ways.One of the most important classes of models has Gaussian (or Normal) disturbancesbut allows the system matrices to depend on past observations availableat time$t-1$. This class of models is known in time series literatureas *conditionally Gaussian*[10]. These models have the attractive property ofstill being tractable by the Kalmanfilter. In our model, we only introduceregime-switching in the state equation. The state-space representation is thefollowing:

$$\begin{aligned} y_t &= H'h_t + A'x_t + w_t \\ h_t &= F(q_{t-1})h_{t-1} + \Pi'x_t + v_t, \end{aligned}$$

where q_{t-1}stands for a stationary threshold variable. Despite the fact thatthe coefficient matrix associated withh_{t-1}

depends on observations up to andincluding $t-1$, it may be regarded as being fixed once we are at time $t-1$. The same hypotheses about the disturbance vectors v_t and w_t are retained.

Hence the derivation of the Kalmanfilter proceeds as in the previous sectionbut a simple modification is introduced.As mentioned above, the goal of the Kalmanfilter procedure is to derive aforecast of the unobserved state vectorh_t based on the information set Ψ_{t-1}.

Here the goal is to form a forecast of$h_t$conditional not only on(x_t, Ψ_{t-1}) but also on the regime of the economy. Let$j$ be a dummy variable that refers tothe regime of the economy, i.e.$j = 1$ if $q_{t-1} \geq \gamma$, and $j = 2$ if $q_{t-1} < \gamma$.

We calculate the conditional forecast of the state variables and its conditionalerror covariance, or MSE, matrix as follows:

$$\begin{aligned} \hat{h}^j_{t|t-1} &= F_j \widehat{h}^j_{t-1|t-1} + \Pi'x_t \\ P^j_{t|t-1} &= F_j P^j_{t-1|t-1} F_j' + Q, \end{aligned}$$

where F_j refers to the transition matrix in each regime.

The conditional forecast of observed variables is given by:

$$\widehat{y}^j_{t|t-1} = H'\widehat{h}^j_{t|t-1} + A'x_t.$$

[10]See Harvey (1989, Section 3.7.) for a more detailed description of this class of models.

Once observable variables are realized at date t, we can calculate the conditionalerror forecast and its conditional variance:

$$\lambda_t^j = y_t - \hat{y}_{t|t-1}^j$$
$$\Lambda_t^j = H' P_{t|t-1}^j H + R.$$

Finally, we update the previous conditional forecast of unobserved variablesand its conditional variance as follows:

$$\hat{h}_{t|t}^j = \hat{h}_{t|t-1}^j + \Phi_t^j \lambda_t^j$$
$$P_{t|t}^j = (I - \Phi_t^j H') P_{t|t-1}^j,$$

with $\Phi_t^j = P_{t|t-1}^j H (\Lambda_t^j)^{-1}$. These last two terms correspond to the inputs of the next filter iteration.

In our particular case,$q_{t-1} = U_{t-1} - U_{t-d}$. To estimate parameters γ and d we first construct a grid $\Delta = \Theta \otimes \Pi$ over the two-dimensional space(γ, d), where Θ and Π are the grids for γ and d, respectively.

We proceed in twosteps. Firstly, we estimate the model for each candidate(γ, d) belonging to the selected grid. That is, conditionally on (γ, d) we maximize the log-likelihoodfunction $\ln L(\gamma, d) = \Sigma_{t=1}^{T} \ln f(y_t | x_t, \Psi_{t-1}, \gamma, d)$ with respect to the underlyingunknown parameters using nonlinear optimization techniques. Secondly, weretain the values of the threshold parameter and the delay lag that providethe highest log-likelihood. That is,$\hat{\gamma}$ and $\hat{d}$ are given by:

$$(\widehat{\gamma}, \widehat{d}) = \underset{(\gamma,d)\in\Delta}{\arg\max} \ln L(\gamma, d).$$

C. TABLES AND FIGURES

Table 1: **Unit Root Tests**[a]

p-value of ADF test on GDP series	
Italy	0.9970
France	0.2622
U.S.	0.9459
p-value of Caner and Hansen's test on Unemployment series	
Italy	0.002
France	0.001
U.S.	0.000

[a] For the ADF test, we use Mackinnon (1996) one-sided *p*-values for the null hypothesis of a unit root. For the test of Caner and Hansen (2001), the relevant null hypothesis is that there is not a unit root.

Table 2 : **Tests for the Hysteresis Assumption**

	Nonlinear Model $H_0 : \alpha_1 = \alpha_2$	Linear Model $H_0^{JP} : \alpha = 0$
Italy	Bootstrap p-value=0.0021	p-value=0.269
France	Bootstrap p-value=0.0015	p-value=0.285
U.S.	Bootstrap p-value=0.0013	p-value=0.405

Table 3 : **Estimation Results for the Nonlinear Model**[a]

	ITALY		**FRANCE**		**U.S.**	
Natural Rate Eq.						
Regime	$i=1$	$i=2$	$i=1$	$i=2$	$i=1$	$i=2$
α_i	2.512 (0.101)	1.476 (0.042)	3.540 (0.913)	1.570 (0.022)	1.343 (0.121)	0.562 (0.022)
σ_N	0.431 (0.021)		0.416 (0.035)		0.400 (0.018)	
Cyclical Rate Eq.						
ϕ_1	1.521 (0.187)		1.615 (0.041)		1.237 (0.082)	
ϕ_2	-0.638 (0.050)		-0.733 (0.056)		-0.650 (0.089)	
σ_C	0.020 (0.004)		0.139 (0.074)		0.120 (0.023)	
Identification Eq.						
β	0.463 (0.023)		0.600 (0.069)		0.611 (0.042)	
δ	-5.401 (1.211)		-0.390 (0.052)		-1.626 (0.0159)	
σ_D	0.640 (0.035)		0.540 (0.033)		0.690 (0.031)	
Threshold	$\gamma = 0.1$		$\gamma = 0.16$		$\gamma = 0.3$	
90% - CI[b]	$[0.023, 0.247]$		$[0.123, 0.532]$		$[0.143, 0.632]$	
Delay lag	$d = 2$		$d = 3$		$d = 2$	
% observations	41%	59%	44%	56%	42%	58%

[a] Following Stoffer and Wall (1991), standard errors are calculated from $B = 1000$ runs of the bootstrap and provided into brackets. These standard errors are the square root of $\sum_{b=1}^{B}(\widehat{\theta}^*_{ib} - \widehat{\theta}_i)^2/(B-1)$, where θ_i represents the ith parameter of the vector Θ_1, $i = 1, ..., 9$, and $\widehat{\theta}_i$ is the ML estimate of θ_i.

[b] We compute the confidence interval based on the bootstrap percentiles described by Hall (1992).

Table 4: Monte Carlo Results

Nominal size		5%	10%	20%
Simulated size				
DGP_0	**RB**	0.051	0.096	0.196
	WB	0.042	0.093	0.203
Simulated power				
$DGP_{\alpha_1-\alpha_2=0.5}$:	**RB**	0.566	0.587	0.590
	WB	0.455	0.520	0.549
$DGP_{\alpha_1-\alpha_2=0.8}$:	**RB**	0.710	0.723	0.741
	WB	0.649	0.690	0.703
$DGP_{\alpha_1-\alpha_2=1}$:	**RB**	0.899	0.930	0.976
	WB	0.795	0.887	0.900

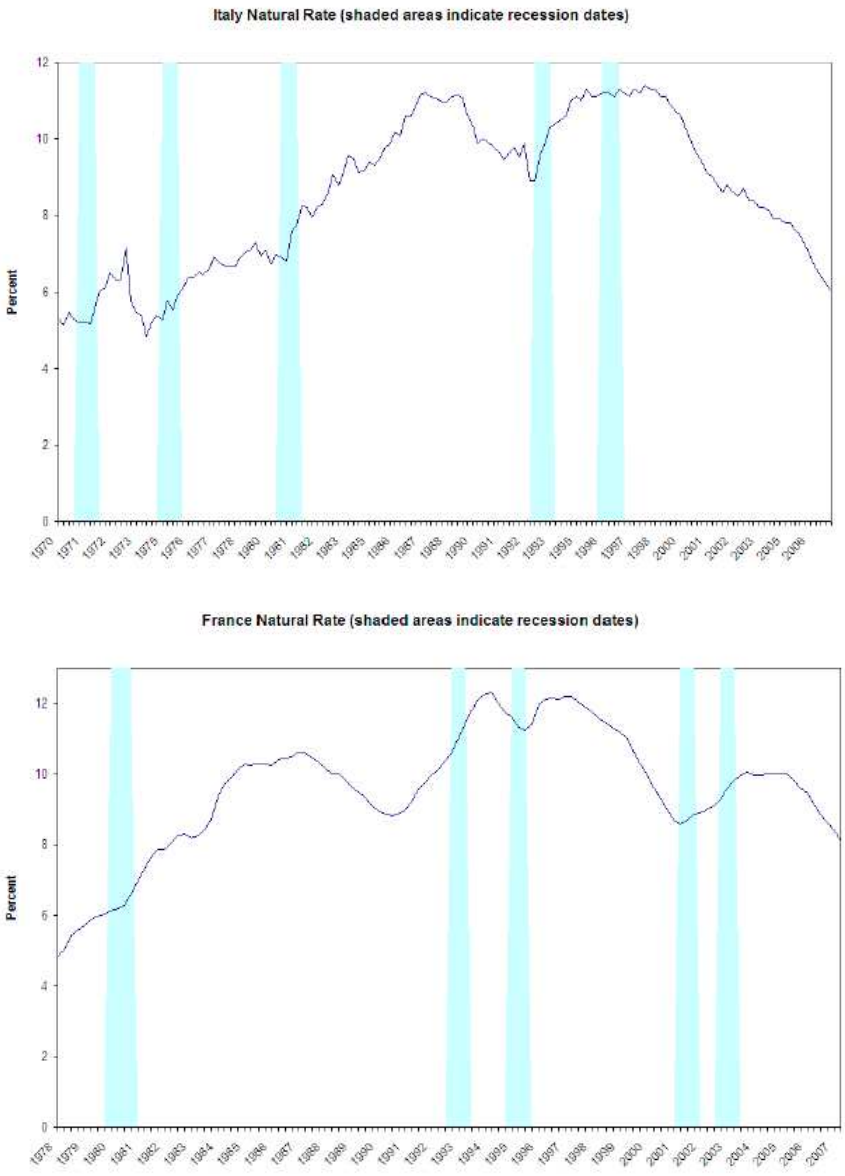

Figure 2: France

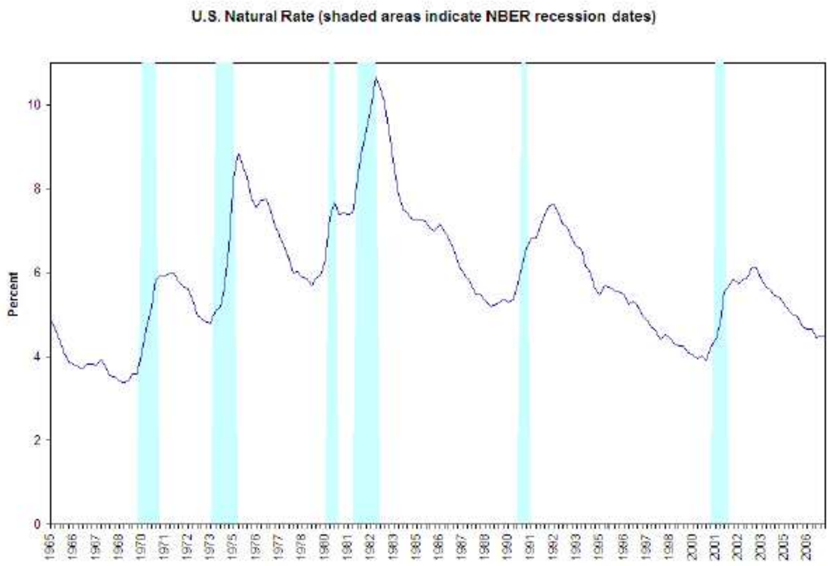
U.S. Natural Rate (shaded areas indicate NBER recession dates)
Percent
10
8
6
4
2
0
1965
1966
1967
1968
1970
1971
1972
1973
1975
1976
1977
1978
1980
1981
1982
1983
1985
1986
1987
1988
1990
1991
1992
1993
1995
1996
1997
1998
2000
2001
2002
2003
2005
2006

Figure 3: U.S.

REFERENCES

Amable, B., J. Henry, F. Lordon and R. Topol (1995): “Hysteresis revisited: a methodological approach,” in R. Cross, ed., The Natural Rate of Unemployment: Reections on 25 Years of the Hypothesis, Cambridge: Cambridge University Press, 153-180.

Andrews, D.W.K., and W. Ploberger (1994): “Optimal tests when a nuisanceparameter is present only under the alternative," Econometrica, 62, 1383-1414.

Bean, C.R., P.R.G. Layard and S.J. Nickell (1986): “The rise in unemployment:a multi-country study,”Economica, 53 Supplement, S1-S22.

Bean, C.R. (1989): “Capital shortage,” Economic Policy, 8, 11-53.

Bentolila, S., and G. Bertola (1990): “Firing costs and labour demand: howbad is eurosclerosis?,” Review of Economic Studies, 57, 381-402.

Bianchi, M., and G. Zoega (1998): “Unemployment persistence: does the sizeof the shock matter?,” Journal of Applied Econometrics, 13, 283-304.

Blanchard, O.J., and L.H. Summers (1986): “Hysteresis and the Europeanunemployment problem,” in S. Fischer, ed., NBER Macroeconomics Annual 1986, Cambridge MA: MIT Press, 15-78.

Caballero, R.J., and M.L. Hammour (1994): “The cleansing effect of recessions,” American Economic Review, 84, 1350-68.

Caner, M., and B.E. Hansen (2001): “Threshold autoregression with a unit root,” Econometrica, 69, 1555-1596.

Caporale, G.M., and L.A. Gil-Alana (2007): “Nonlinearities and fractionalintegration in the US unemployment rate,” Oxford Bulletin of Economicsand Statistics, 69, 521-544.

Coakley, J., and A.-M. Fuertes (2006): “Testing for sign and amplitude asymmetriesusing threshold autoregressions,” Journal of Economic Dynamicsand Control, 30, 623-654.

Chan, K.S. (1990): “Testing for threshold autoregression,” The Annals ofStatistics, 18, 1886-1894.

Davies, R.B. (1977): “Hypothesis testing when a nuisance parameter ispresent only under the alternative,”Biometrika, 64, 247-254.

Davies, R.B. (1987): “Hypothesis testing when a nuisance parameter ispresent only under the alternative,”Biometrika, 74, 33-43.

Friedman, M. (1968): “The role of monetary policy,” American EconomicReview, 58, 1-17.

Hall, P., and S.R. Wilson (1991): “Two guidelines for bootstrap hypothesistesting,” Biometrics, 47, 757-762.

Hall, P. (1992): The Bootstrap and Edgeworth Expansion. New York:Springer.

Hamermesh, D.S., and G.A. Pfann (1996): “Adjustment costs in factor demand,” Journal of Economic Literature, 34, 1264-1292.

Hamilton, J.D. (1994). Time Series Analysis. Princeton: Princeton UniversityPress.

Hansen, B.E. (1996): “Inference when a nuisance parameter is not identifiedunder the null hypothesis,”Econometrica, 64, 413-30.

Hansen, B.E. (1999): “Testing for linearity,” Journal of Economic Surveys,13, 551-576.

Hansen, B.E., and B. Seo (2002): “Testing for two-regime threshold cointegrationin vector error correction models,” Journal of Econometrics, 110,293-318.

Harding, D., and A. Pagan (2002): “Dissecting the cycle: a methodologicalinvestigation,” Journal of Monetary economics, 49, 365-381.

Harvey, A.C. (1985): “Trends and cycles in macroeconomic time series,” Journal of Business & Economic Statistics, 3, 216-227.

Harvey, A.C. (1989): Forecasting, Structural Time Series Models and theKalman Filter. Cambridge: Cambridge University Press.

Jaeger, A., and M. Parkinson (1994): “Some evidence on hysteresis in unemploymentrates,” European Economic Review, 38, 329-342.

Johansen, L. (1982): “Some notes on employment and unemployment withheterogeneous labour,” National økonomiskTidsskrift, 102-117.

Kalman, R.E. (1960): “A new approach to linear filtering and predictionproblems,” Transactions of the ASME - Journal of Basic Engineering,Series D 82, 35-45.

Koop, G., and S.M. Potter (1999): “Dynamic asymmetries in US unemployment,” Journal of Business & Economic Statistics, 17, 298-313.

Layard, R., S. Nickell and R. Jackman (1991): Unemployment: Macroeconomic Performance and the Labour Market. Oxford: Oxford UniversityPress.

Lindbeck, A., and D. Snower (1988): Then Insider-Outsider Theory of Employment and Unemployment. Cambridge MA: MIT Press.

Liu, R.Y. (1988): “Bootstrap procedure under some non-i.i.d. models,” Annals of Statistics, 16, 1696-1708.

MacKinnon, J.G. (1996): “Numerical distribution functions for unit root andcointegration tests,” Journal of Applied Econometrics, 11, 601-618.

Mammen, E. (1993): “Bootstrap and wild bootstrap for high-dimensionallinear models,” The Annals of Statistics, 21, 255-285.

Mortensen, D.T., and C.A. Pissarides (1993): “The cyclical behavior of jobcreation and job destruction,” in J.C. van Ours, G. Pfann, and G. Ridder,eds., Labor Demand and Equilibrium Wage Formation, Amsterdam:North-Holland.

Neftci, S. (1984): “Are economic time series asymmetric over the businesscycle?,” Journal of Political Economy, 92, 307-328.

Papell, D.H., C.J. Murray and H. Ghiblawi (2000): “The structure of unemployment,” The Review of Economics and Statistics, 82, 309-315.

Petruccelli, J.D. (1992): “On the approximation of time series by thresholdautoregressive models,”Sankhya, Series B 54, 106-113.

Rothman, P. (1991): “Further evidence on the asymmetric behaviour of unemploymentrates over the business cycle,” Journal of Macroeconomics,13, 291-298.

Røed, K. (1997): “Hysteresis in unemployment,”Journal of Economic Surveys, 11, 389-418.

Sichel, D.E. (1989): “Are business cycles asymmetric? A correction,” Journalof Political

Economy, 97, 1255-1260.

Skalin, J., and T. Teräsvirta (2002): “Modelling asymmetries and movingequilibria in unemployment rates,” Journal of Econometrics, 110, 135-165.

Stoffer, D.S., and K.D. Wall (1991): “Bootstrapping state-space models:gaussian maximum likelihood estimation and the Kalmanfilter,” Journalof the American Statistical Association, 86, 1024-1033.

Tong, H. (1990): Nonlinear Time Series: a Dynamical System Approach.Oxford: Oxford University Press.

White, H. (1982): “Maximum likelihood estimation of misspecified models,” Econometrica, 50, 1-25.

Wu, C.F.J. (1986): “Jackknife, bootstrap and other resampling methods inregression analysis,” The Annals of Statistics, 14, 1261-1295.

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