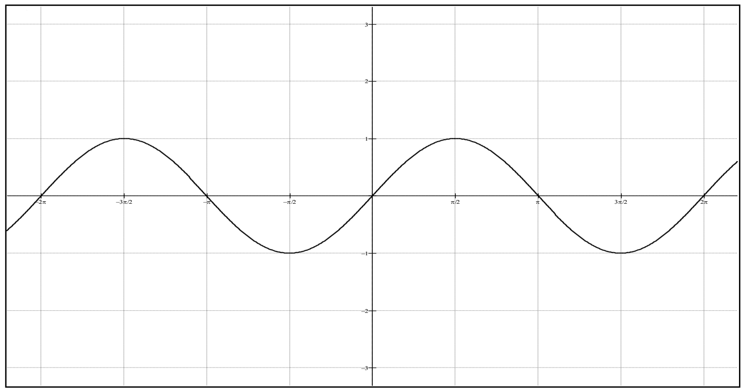
5.3 Modeling with Sine and Cosine

Name ________________

Date ________________

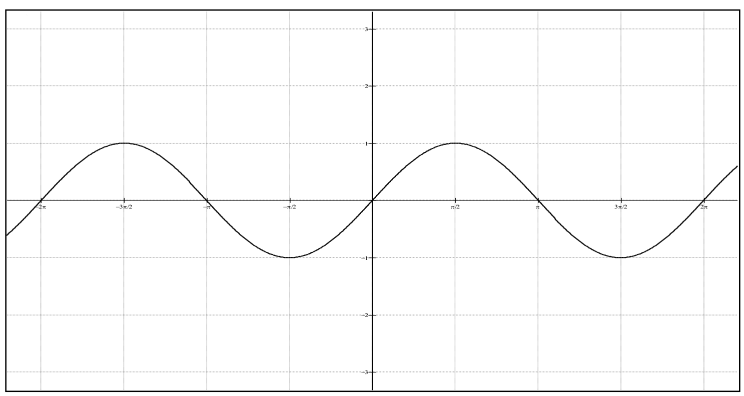
1. The graph of $y = \sin(x)$, x in radians, is shown in each of the grids below.

a. Graph the function $y = 3\sin(x)$.



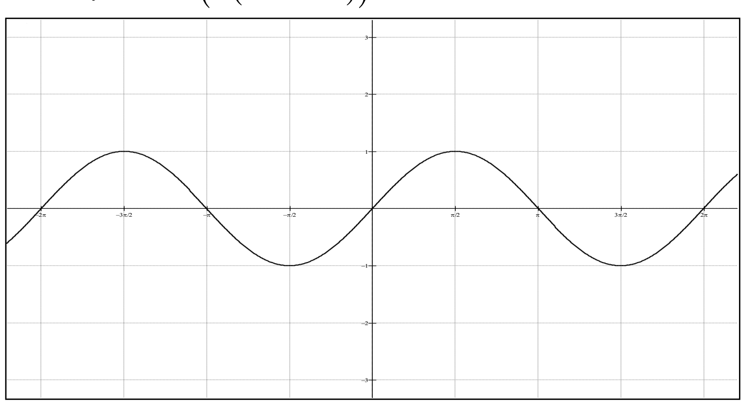
Check your graph with your graphing calculator.

b. Graph the function $y = 3\sin(2x)$.



Check your graph with your graphing calculator.

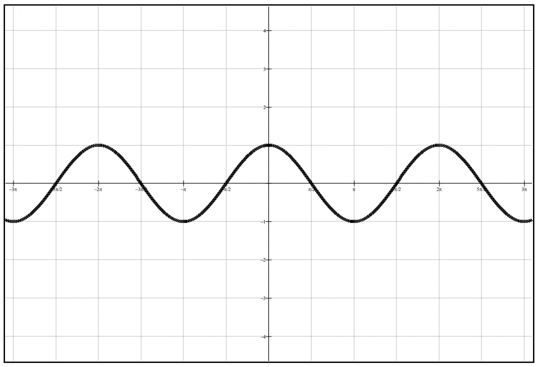
c. Graph the function $y = 3\sin\left(2\left(x - \pi/2\right)\right)$.



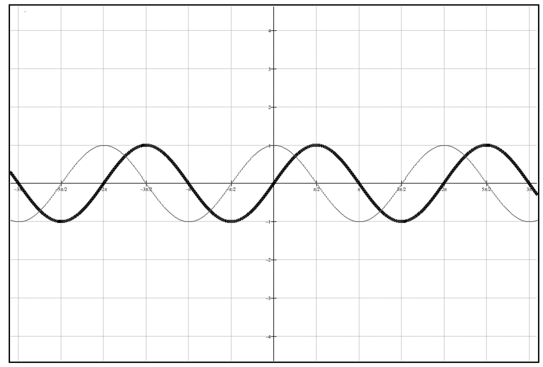
Check your graph with your graphing calculator.

2. Write an equation involving cosine for each of the **bold** graphs below. The graph of cosine has been included on each graph as a visual aid.

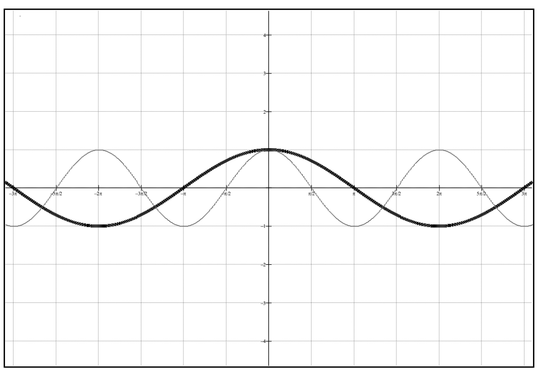
a. This is the function $y = \cos(x)$.



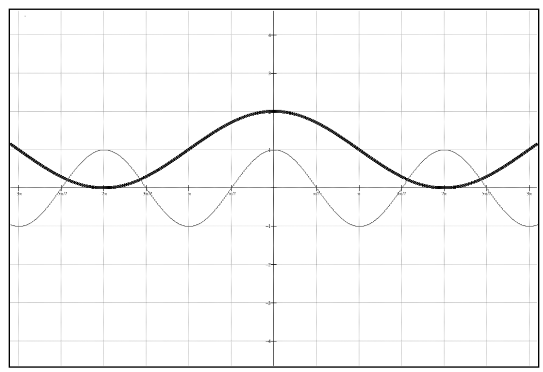
b. This is $y =$ ____________.



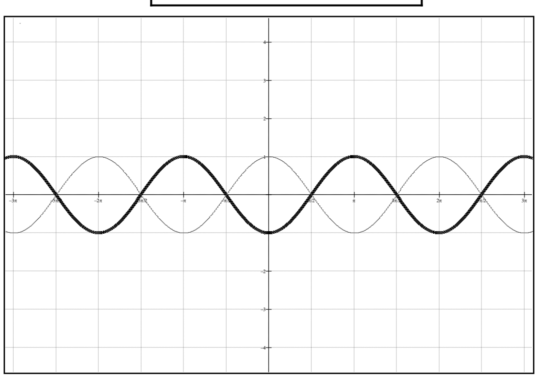
c. This is $y =$ ____________.



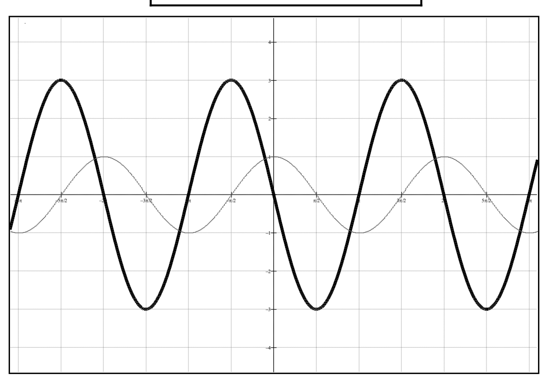
d. This is $y =$ ____________.



e. This is $y =$ ____________.

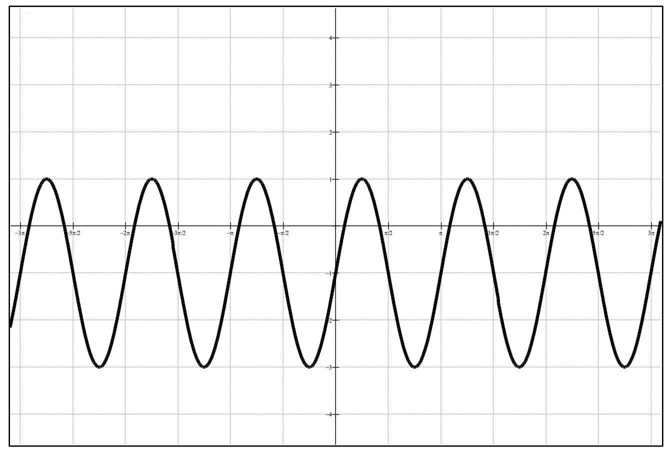


f. This is $y =$ ____________.



3. The familiar shape of the sine wave is called a *sinusoidal* function. Functions of this type can be written in terms of cosine or sine, with a standard form of $y = A\cos\left(B\left(x-C\right)\right)+D$ or $y = A\sin\left(B\left(x-C\right)\right)+D$.

Write two equations for the graph below, one involving sine and the other involving cosine. The vertical scale is 1 unit per tick and the horizontal scale is $\frac{\pi}{2}$ per tick.



4. *Terminology*. The *period T* is the horizontal distance over which the graph repeats itself. The *amplitude A* is the vertical distance from the mid-line of the graph to the peak or valley.

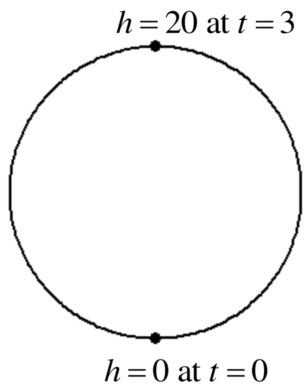
a. Which of the following equations are true (T is the period)? *Explain why.*

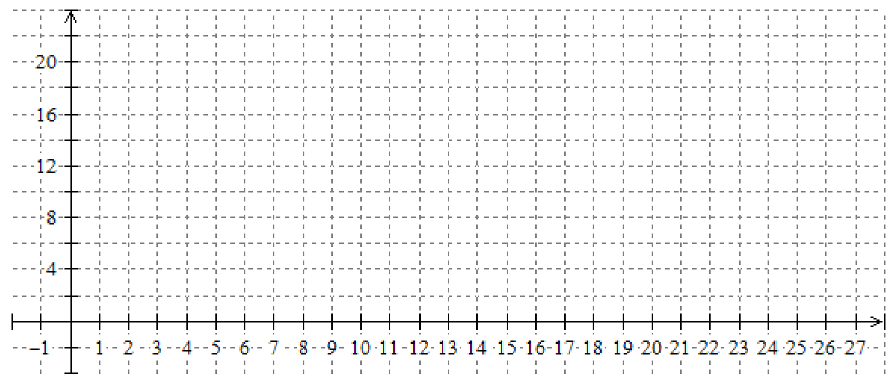
$$\cos\left(Bt\right)=\cos\left(B\left(t+T\right)\right) \qquad \cos\left(Bt\right)=\cos\left(Bt+T\right)$$
$$\cos\left(Bt\right)=\cos\left(Bt+2\pi\right) \qquad \cos\left(Bt\right)=\cos\left(B\left(t+2\pi\right)\right)$$

b. Use your answer to **a** to derive a relationship between B, T, and 2π.

5. Consider a Ferris wheel with a diameter of 20 meters that revolves once every 6 minutes. A ride on the Ferris wheel lasts 4 full revolutions (yeah, a kind of slow boring ride). Let h be the height of a rider on the wheel t minutes after getting on the ride ($t = 0$).

a. On the grid below, graph $h(t)$ for the duration of the ride.





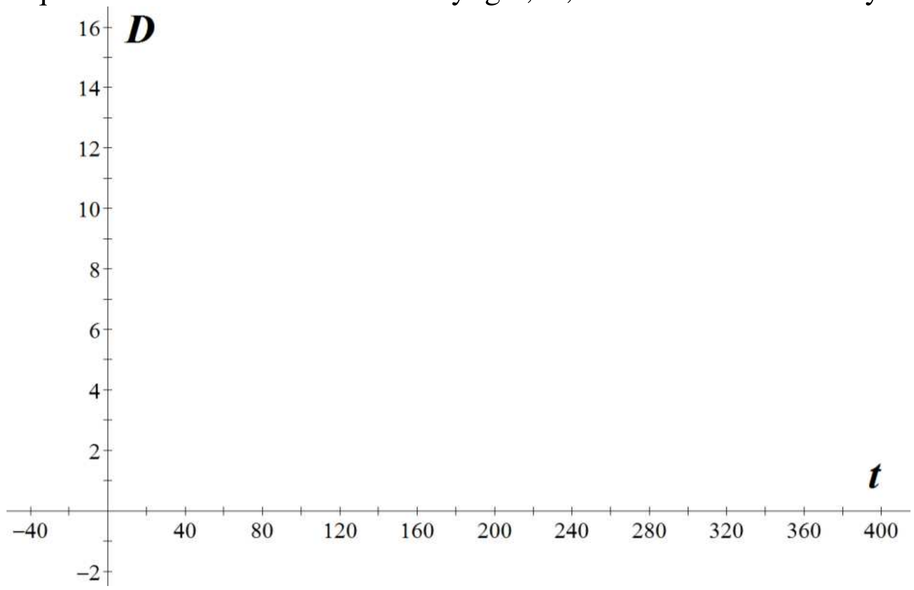
b. Find an equation, involving sine, that models $h(t)$.

c. At what times will the rider be exactly 12 meters above the ground?

d. What is the angular speed of the Ferris wheel, in degrees/minute? Though an arc of how many meters does the rider move every minute?

6. The amount of daylight (amount of time between sunrise and sunset) at a given location varies sinusoidally during the year. Seattle's longest day (June 21st, the 172nd day of the year) and shortest day (December 21st, the 355th day of the year) receive around 16 hours and 8.5 hours of daylight, respectively.

a. Make a quick sketch of the amount of daylight, *D*, as a function of the day of the year, *t*.



b. Write a model, in terms of either sine or cosine, for $D(t)$.

c. Use your model to estimate the amount of daylight today. On what other day this year will we have the same amount of daylight as today?