Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Skills Intervention

8-1 Factors and Greatest Common Factors

Find these vocabulary words in Lesson 8-1 and the Multi-Language Visual Glossary.

Vocabulary	
prime factorization	greatest common factor

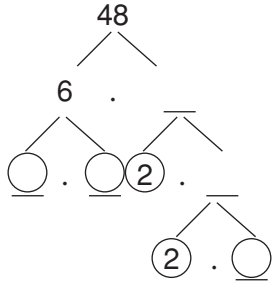
Writing Prime Factorizations

Write the prime factorization of 48.

What is a prime number? _____

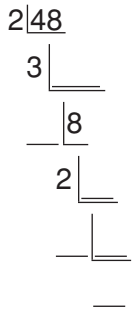
Method 1 Factor tree

Choose any two factors of 48 to begin.
Keep finding factors until each branch ends in a prime factor.
Circle the prime factors.



Method 2 Ladder diagram

Choose a prime factor of 48 to begin.
Keep dividing by prime factors until the quotient is 1.



The prime factorization of 48 is $2 \cdot 2 \cdot \_\_ \cdot \_\_ \cdot \_\_$ or $2\_\_ \cdot \_\_$.

Finding the GCF of Monomials

Find the GCF of each pair of monomials.

A. $9a^3$ and $3a$

$9a^3 = 3 \cdot \_\_ \cdot a \cdot \_\_ \cdot \_\_$ Write the prime factorization of each coefficient and write the powers as products.

$3a = 3 \cdot \_\_$ Align the common factors.

What are the factors common to both monomials? 3 and __

Multiply the common factors. The GCF of $9a^3$ and $3a$ is ___.

B. $24x^3$, $30x^5$

$24x^3 = 2 \cdot 2 \cdot \_\_ \cdot \_\_ \cdot x \cdot \_\_ \cdot \_\_$ Write the prime factorization of each coefficient and write the powers as products.

$30x^5 = 2 \cdot \_\_ \cdot \_\_ \cdot x \cdot x \cdot \_\_ \cdot \_\_ \cdot \_\_$ Align the common factors.

What are the factors common to both monomials? 2, __ and ___

Multiply the common factors. The GCF is ____.

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-1 Factors and Greatest Common Factors

Factors that are shared by two or more numbers are called common factors. The largest common factor is called the GCF, or greatest common factor.

A softball league bought new equipment for the teams. The league bought 40 balls and 24 bats. How many teams are there if all the equipment is distributed evenly between the teams? How many bats and balls does each team receive?

Understand the Problem

1. How many bats and balls were purchased? ____ bats and ____ balls
2. How is the equipment divided? _____
3. What are you being asked to find? _____

_____

Make a Plan

4. If the equipment is to be divided evenly between teams then 40 and 24 must be _____ by the same number.
5. What are the factors of 24: 1, 2, _____

40: 1, 2, 4, _____

Solve

6. Which factors are common to 24 and 40? 1, 2, ____, ____
7. What is the largest common factor? ____
8. So, there are ____ softball teams.
9. Since there are 40 balls and ____ teams, how many balls does each team receive? ____
10. Since there are 24 bats and ____ teams, how many bats does each team receive? ____

Look Back

11. How many balls does each team receive? ____ How many teams are there? ____
    What is the product of number of teams and number of balls? ____
12. How many bats does each team receive? ____ How many teams are there? ____
    What is the product of number of teams and numbers of bats? ____
13. Do your answers to Exercises 11 and 12 match the information in the problem statement? _____

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Skills Intervention

8-2 Factoring by GCF

Factoring by Using the GCF

Factor each polynomial. Check your answer.

A. $8y^3 + 24y^2$

Find the GCF of each term.

$8y^3 = 2 \cdot \_\_ \cdot \_\_ \cdot y \cdot \_\_ \cdot \_\_$

$24y^2 = 2 \cdot \_\_ \cdot \_\_ \cdot \_\_ \cdot y \cdot \_\_$

The GCF is $2 \cdot \_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_ = 8\_\_\_\_$.

Write terms as products using the GCF as a factor. $y(8y^2) + 3\_\_\_\_\_$

Use the Distributive Property to factor out the GCF. $8y^2(\_\_ + \_\_)$

Check: Multiply to check your answer. $8y^2(\_\_ + \_\_) = 8y^3 + 24y^2$

B. $4x^2 + 20x + 28$

Find the GCF of each term.

$4x^2 = 2 \cdot \_\_ \cdot \_\_ \cdot \_\_$

$20x = 2 \cdot \_\_ \cdot \_\_ \cdot \_\_$

$28 = 2 \cdot \_\_ \cdot \_\_$

The GCF is $2 \cdot \_\_ = \_\_$.

Write terms as products using the GCF as a factor. $x^2(4) + 4\_\_\_\_\_ + \_\_(7)$

Use the Distributive Property to factor out the GCF. $4(\_\_\_\_ + 5x + \_\_)$

Check: Multiply to check your answer. $4(\_\_\_\_ + 5x + \_\_) = 4x^2 + 20x + 28$

Factoring by Grouping

Factor the polynomial, $4a^3 + 8a^2 - 3a - 6$, by grouping. Check your answer.

Group terms that have a common number of variables as a factor. $(4a^3 + 8a^2) + (-3a - 6)$

What is the GCF of $4a^3$ and $8a^2$? _____ What is the GCF of $-3a$ and -6? ____

Write each group with the GCF. $\_\_\_\_\_(a + \_\_) + (-3)(a + \_\_)$

Factor out the common factor of $a + 2$. $(a + 2)(\_\_\_\_\_ - \_\_)$

Multiply to check your solution.

$(a + 2)(\_\_\_\_\_ - \_\_) = \_\_(4a^2) + \_\_(-3) + \_\_(4a^2) + \_\_(-3)$

$= \_\_\_\_\_ - 3a + \_\_\_\_\_ - 6$ Multiply.

$= \_\_\_\_\_ + 8a^2 - \_\_\_ - 6$ Rewrite in descending order.

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Skills Intervention

8-3 Factoring $x^2 + bx + c$

Factoring $x^2 + bx + c$ When c is Positive.

Factor each trinomial. Check your answer.

A. $x^2 + 9x + 14$

When the constant term is positive, its factors have the _____ sign.

What is the value of the c term? ____ What is the value of the b term? __

Factors of 14	Sum
1 and ___	15
2 and __	__

What are the factors of c whose sum is b? __ and __

Complete: $(x + \_\_)(x + \_\_)$

Check using FOIL $(x + \_\_)(x + \_\_) = x^2 + \_\_\_ + 2x + \_\_\_$

$x^2 + \_\_\_ + \_\_\_$

B. $x^2 + 8x + 15$

What is the value of the c term? ___ What is the value of the b term? __

Factors of 15	Sum
1 and ___	16
3 and __	__

What are the factors of c whose sum is b? __ and __

Complete: $(x + \_\_)(x + \_\_)$

Check using FOIL $(x + \_\_)(x + \_\_) = x^2 + 5x + \_\_\_ + \_\_\_$

$x^2 + \_\_\_ + \_\_\_$

Factoring $x^2 + bx + c$ When c is Negative

Factor the trinomial $x^2 + 3x - 18$. Check your answer.

When the constant term of a trinomial is negative, its factors have _____ signs.

What is the value of the c term? _____ What is the value of the b term? __

Factors of −18	Sum
−1 and ___	17
−2 and __	__
−3 and __	__

What are the factors of c whose sum is b? ____ and __

Complete: $(x - \_\_)(x + \_\_)$

Check using FOIL $(x - \_\_)(x + \_\_) = x^2 + \_\_\_ - 3x - \_\_\_$

$x^2 + \_\_\_ - \_\_\_$

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-4 Factoring $ax^2 + bx + c$

Factoring $ax^2 + bx + c$ when c is Positive.

Factor the trinomial $2x^2 + 9x + 10$. Check your answer.

What is the value of a? __ What is the value of c? ___

What is the sum of the inner and outer products? __

Factors of 2	Factors of 10	Outer + Inner
$1 \cdot 2$	$1 \cdot 10$	$1 \cdot 10 + 2 \cdot 1 = 12$
$1 \cdot 2$	$10 \cdot 1$	$1 \cdot 1 + 2 \cdot$ ___ $= 21$
$1 \cdot 2$	$5 \cdot$ __	$1 \cdot 2 + 2 \cdot$ __ $=$ ___
$1 \cdot 2$	$2 \cdot$ __	$1 \cdot 5 + 2 \cdot$ __ $=$ __

$(x +$ __$)(2x +$ __$)$ Complete the factoring.

Check $(x +$ __$)(2x +$ __$) = 2x^2 +$ ___ $+$ ___ $+$ ___

$= 2x^2 +$ ___ $+ 10$

Factoring $ax^2 + bx + c$ when c is Negative

Factor the trinomial $4x^2 + 21x - 18$. Check your answer.

What is the value of a? __ What is the value of c? _____

What is the sum of the inner and outer products? ____

Factors of 4	Factors of -18	Outer + Inner
$1 \cdot 4$	$1(-18)$	$1 \cdot -18 + 4 \cdot 1 = -14$
$1 \cdot 4$	$-18(1)$	$1 \cdot 1 + 4 \cdot -18 =$ _____
$1 \cdot 4$	$2 \cdot$ ____	$1 \cdot -9 + 4 \cdot$ __ $=$ ____
$1 \cdot 4$	$-9 \cdot$ __	$1 \cdot$ __ $+ 4 \cdot$ ____ $=$ _____
$1 \cdot 4$	$3 \cdot$ ____	$1 \cdot$ ____ $+ 4 \cdot$ __ $=$ ____
$1 \cdot 4$	$-6 \cdot$ __	$1 \cdot$ __ $+ 4 \cdot$ ____ $=$ _____
$1 \cdot 4$	$-3 \cdot$ __	$1 \cdot$ __ $+ 4 \cdot$ ____ $=$ ____
$1 \cdot 4$	$6 \cdot$ ____	$1 \cdot$ ____ $+ 4 \cdot$ __ $=$ ___

$(x +$ __$)(4x -$ __$)$ Complete the factoring.

Check $(x +$ __$)(4x -$ __$) = 4x^2 -$ ___ $+$ ____ $-$ ___

$= 4x^2 +$ ____ $- 18$

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Quiz

8-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 84 _____

2. 60 _____

3. 150 _____

4. 66 _____

5. 72 _____

6. 156 _____

Find the GCF of each pair of monomials.

7. $6x^4$ and $9x^2$ _____

8. $25x^3$ and $20x^4$ _____

9. -18 and $27c^5$ _____

10. $5a$ and $7c$ _____

11. Nichole is designing a quilt. She has cut out 60 red squares and 48 green squares. She plans for the quilt to have the same number of colored squares in each row, but, green and red squares will not be in the same row. How many rows will the quilt have if Nichole puts the greatest number of squares in each row? _____

8-2 Factoring by GCF

Factor each polynomial. Check your answer.

12. $30x^2 + 12x$ _____

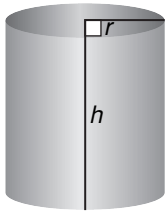
13. $2a^2 - 10a^3$ _____

14. $6x^4 - 15x^3 - 9x^2$ _____

15. $4x^2 + 12x + 16$ _____

16. The surface area of a cylinder can be found using the expression $2\pi r^2 + 2\pi rh$, where r represents the radius of the cylinder and h represents the height.

Factor this expression. _____



Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Quiz continued

Factor each polynomial by grouping. Check your answer.

17. $x^3 - 3x^2 + 2x - 6$

18. $5x^3 + 10x^2 + x + 2$

19. $2r^3 - 8r^2 - 3r + 12$

20. $3s^3 - 12s^2 - s + 4$

21. $4y^3 + 16y^2 - 2y - 8$

22. $8b^3 + 16b^2 - 2b - 4$

8-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

23. $a^2 + 12a + 35$

24. $x^2 - 3x - 10$

25. $x^2 - 8x + 7$

26. $x^2 + 13x - 30$

27. $c^2 - 13c + 36$

28. $y^2 - 12y + 32$

29. Simplify and factor the polynomial $n(n + 7) + 12$. Show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3,$ and 4.

8-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

30. $2a^2 + 5a + 2$

31. $6x^2 + 11x + 4$

32. $3x^2 - x - 10$

33. $4x^2 - 14x + 12$

34. $6c^2 - 12c - 18$

35. $12y^2 - y - 20$

36. The area of a rectangle is $(10x^2 + 21x + 9)$ cm^2. The length is $(5x + 3)$ cm. What is the width of the rectangle?

Name _____ Date _____ Class _____

SECTION 8A

Ready To Go On? Enrichment

Sum and Differences of Cubes

While it is possible to factor the sum and differences of two squares, it is also possible to factor the sum and differences of two cubes.

The sum of two cubes can be factored in the following way:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

The differences of two cubes can be factored in the following way:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor each of the following.

1. $r^3 - s^3$

_____

2. $x^3 + y^3$

_____

3. $x^3 + 8$

_____

4. $n^3 - 64$

_____

5. $8y^3 + 27$

_____

6. $pq^3 - 64p$

_____

Express each of the following as the sum or difference of two cubes.

7. $(m - 1)(m^2 + m + 1)$

_____

8. $(2 + 3t)(4 - 6t + 9t^2)$

_____

9. $(b - 64)(b^2 + 4b + 16)$

_____

10. $(x + 7)(x^2 - 7x + 49)$

_____

11. $(2y - 1)(4y^2 + 2y + 1)$

_____

12. $(3 + 2t)(9 - 6t + 4t^2)$

_____

13. $(s + 10)(s^2 - 10s + 100)$

_____

14. $2(x - 4)(x^2 + 4x + 16)$

_____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Skills Intervention

8-5 Factoring Special Products

Recognizing and Factoring Perfect-Square Trinomials

A trinomial is a perfect square if the first and the _____ terms are perfect squares. The _____ term is two times one factor from the first term and one factor from the last term.

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

A. $x^2 + 14x + 42$

Method 1 Factor.

Factors of 42	Sum
1 and 42	43
2 and 21	___
__ and 14	___
6 and __	___

Is $x^2 + 14x + 42$ a perfect square trinomial? _____ Why? _____

_____

B. $9x^2 - 24x + 16$

Method 2 Use the pattern.

$9x^2 - 24x + 16$

$3x \cdot 3x - 2(3x \cdot \_\_) + 4 \cdot 4$

The trinomial is a perfect _____.

$a = 3x$, $b =$ __

$(3x)^2 - 2(\_\_\_)(4) + 4^2$ Write the trinomial as $a^2 - 2ab + b^2$.

$(\_\_\_ - 4)^2$ Write as $(a - b)^2$.

Determine whether each binomial is the difference of two squares. If so, factor it. If not explain why.

A. $x^2 - 16$

$(x \cdot x) - (4 \cdot \_\_)$ The polynomial is the difference of two _____.

$a = x$, $b = 4$

$(x + 4)(x - \_\_)$ Write the polynomial as $(a + b)(a - b)$.

So $x^2 - 16 = (x + \_\_)(x - \_\_)$.

B. $9x^6 - 17y^2$

Is $9x^6$ a perfect square? _____

Is $-17y^2$ a perfect square? _____

Is $9x^6 - 17y^2$ the difference of two squares? _____

Why? _____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-5 Factoring Special Products

If a trinomial is a perfect square, it can be factored to determine measurements of real-world objects.

A community garden, rectangular in shape, has an area of $(25x^2 + 20x + 4)$ ft^2. The dimensions of the garden are approximately $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the garden. Then find the perimeter where $x = 12$.

Understand the Problem

1. What is the shape of the garden? _____

2. What is the area of the garden? _____

3. What kind of numbers are a and b? _____

4. What are you being asked to find? _____

_____ and _____

Make a Plan

5. The formula for the area of a rectangle is: Area = length $\times$ _____

6. What must be done to the trinomial to find the length and width? _____

Solve

7. Does the trinomial $25x^2 + 20x + 4$ have a common factor? _____

8. Is $25x^2 + 20x + 4$ a perfect square trinomial? _____ So, a = _____ and $b = 2$.

9. What are the factors of the perfect square trinomial? _____

10. Since the length and width of the garden are equal, what is the actual shape of the garden? _____

11. The formula for the perimeter of a square is $P = 4s$.

a. What should you substitute in the perimeter formula for s? _____

b. Simplify $4(5x + 2)$ to find an expression for the perimeter. _____

12. Evaluate the expression when $x = 12$. What is the perimeter? _____

Look Back

13. Using the perimeter from Exercise 12, what is the length of one side of the square? _____ Use this side length to find the area. _____

14. Evaluate $25x^2 + 20x + 4$ for $x = 12$: $25(12)^2 + 20(12) + 4$ = _____

15. Are the areas in Exercises 13 and 14 equal? _____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Skills Intervention

8-6 Choosing a Factoring Method

Determining Whether a Polynomial Is Completely Factored

Tell whether each polynomial is completely factored. If not, factor it.

A. $4x^2 + 2x - 6 = 2(2x^2 + x - 3)$

The greatest common factor is ___.

Is $(2x^2 + x - 3)$ factorable? _____

Is $2(2x^2 + x - 3)$ completely factored? _____

Factor: $2(2x^2 + x - 3) \Rightarrow 2(2x + \_\_\_)(x - \_\_\_)$

B. $42x - 14x^3 = 14(3x - x^3)$

What is the greatest common factor? _____

Is the polynomial factored completely? _____

Factor: $42x - 14x^3 \Rightarrow 14x(\_\_ - \_\_^{\,2})$

Can the polynomial be factored any further? _____

Factoring by Multiple Methods

Factor the polynomial $6x^3 + 21x^2 + 15x$ completely. Check your answer.

Consider $6x^3 + 21x^2 + 15x$.

What is the GCF? _____

Factor out the GCF: $3x(\_\_\_\_\_ + \_\_\_\_\_ + 5)$

What is the value of a? __ What is the value of c? __,

The Outer + Inner terms = __.

Factors of 2	Factors of 5	Outer + Inner
$1 \cdot 2$	$5 \cdot 1$	$1 \cdot 1 + 2 \cdot 5 =$ ___
$1 \cdot 2$	$1 \cdot$ __	$1 \cdot$ __ $+ 2 \cdot$ __ $=$ __

Complete the binomial: $(x + \_\_)(2x + \_\_)$

Write as a factored polynomial: $3x(x + 1)(2x + \_\_)$

Check: $3x(x + 1)(2x + 5)$

$= 3x(2x^2 + \_\_\_\_\_ + \_\_\_\_\_ + 5)$ FOIL

$= 3x(2x^2 + \_\_\_\_\_ + 5)$ Combine like terms.

$= 6x^{-} + \_\_\_x^2 + \_\_\_x$ Distribute the $3x$.

Is this the original polynomial? _____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Quiz

8-5 Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $a^2 + 6a + 9$ _____

2. $16x^2 - 40x + 25$ _____

3. $w^2 - 12w + 4$ _____

4. $5y^2 - 14y + 16$ _____

5. $x^2 + 4x + 4$ _____

6. $25h^2 - 70h + 49$ _____

7. An architect is designing rectangular windows with an area of $(x^2 + 22x + 121)$ ft^2. The dimensions of the windows are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the windows. Find the perimeter of a window when $x = 3$ ft.

_____

Determine whether each trinomial is the difference of two squares. If so, factor it. If not, explain why.

8. $r^2 - 144$ _____

9. $4a^2 - 30$ _____

10. $1 - 25a^4$ _____

11. $36k^2 - 9k^6$ _____

12. $49a^2 + 64$ _____

13. $w^4 - a^2$ _____

14. The area of a square is $(49x^2 - 28x + 4)$ in^2.

a. What is the length of a side of the square? _____

b. What is the perimeter of the square? _____

c. What are the length of a side, the perimeter, and the area of the square when $x = 4$ in.? _____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Quiz continued

8-6 Choosing a Factoring Method

Tell whether each polynomial is completely factored. If not, factor it.

15. $7x^2 + 35x + 7 = 7(x^2 + 5x + 1)$

_____

16. $16x^3 - 24x^2 = 8x(2x^2 - 3x)$

_____

17. $4x^5 - 16x = 4x(x^4 - 4)$

_____

18. $3a^2 - 42a + 147 = 3(a^2 - 14a + 49)$

_____

19. $8y^3 - 8y^2 - 12y + 12 = 4(2y^2 - 3)(y - 1)$

_____

20. $3y^2 + 17y + 10 = (3y + 2)(y + 5)$

_____

Factor each polynomial completely. Check your answer.

21. $4x^3 - 24x^2 + 36x$

_____

22. $3x^2y - 12xy^2$

_____

23. $3a^2 + a + 1$

_____

24. $12x^3 - 3x$

_____

25. $4x^2 + 12x - 112$

_____

26. $x^5 - 16x$

_____

Write an expression for each situation. Factor your expression.

27. The difference of the square of a pipe's length and 49.

_____

28. The square of Catherine's age plus 18 times Catherine's age plus 81.

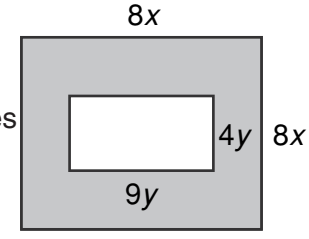
_____

29. Three times the square of a truck's speed minus three times the trucks speed plus 18.

_____

30. Four times the square of cherries on a tree minus 11 times cherries minus 21.

_____



31. Write an expression for the area of the shaded region. Then factor the expression.

_____

_____

Name _____ Date _____ Class _____

SECTION 8B

Ready To Go On? Enrichment

Fourth Degree Trinomials

Sometimes it is possible to write a trinomial of the fourth degree, $a^4 + a^2b^2 + b^4$, as a difference of two squares and then factor.

Example: Factor $4a^4 - 21a^2b^2 + 9b^4$.

Step I Find the square roots of the first and last terms.

$\sqrt{4a^4} = 2a^2$ $\qquad$ $\sqrt{9b^4} = 3b^2$

Step II Find twice the product of the square roots from the terms in Step 1.

$2(2a^2)(3b^2) = 12a^2b^2$

Step III Split the middle term of the trinomial into two parts. One part is either the answer from the Step II or its opposite. The other part should be the opposite of a perfect square.

$-21a^2b^2 = -12a^2b^2 - 9a^2b^2$

Step IV Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned} 4a^4 - 21a^2b^2 + 9b^4 &= (4a^4 - 12a^2b^2 + 9b^4) - 9a^2b^2 \\ &= (2a^2 - 3b^2)^2 - 9a^2b^2 \\ &= [(2a^2 - 3b^2) - 3ab]\,[(2a^2 - 3b^2) - 3ab] \\ &= (2a^2 + 3ab - 3b^2)(2a^2 - 3ab - 3b^2) \end{aligned}$$

Factor each trinomial.

1. $16d^4 + 7d^2 + 1$

_____

2. $p^4 + p^2 + 1$

_____

3. $4x^4 - 13x^2 + 1$

_____

4. $4x^4 - 9x^2y^2 + 16y^4$

_____

5. $9r^4 + 26r^2s^2 + 25s^4$

_____

6. $4a^4 - 5a^2c^2 + 25c^4$

_____

SECTION 7B Ready To Go On? Skills Intervention
7-9 Special Product of Binomials

Find these vocabulary words in Lesson 7-9 and the Multil-Language Visual Glossary.

Vocabulary	
perfect-square trinomial	difference of two squares

Finding Products in the Form $(a + b)^2$

A. Multiply $(x + 9)^2$.

What is the rule for $(a + b)^2$? $a^2 + 2$__ab__ $+ b^2$

Let $a = x$ and $b =$ __9__.

$(x + 9)^2 = (x)^2 + 2($__x__$)(9) + ($__9__$)^2$

$= x^2 +$ __18__$x +$ __81__

B. Multiply $(3x + 5y)^2$.

Let $a = 3x$ and $b =$ __5y__.

$(3x + 5y)^2 = (3x)^2 + 2($__3x__$)(5y) + ($__5y__$)^2$

$=$ __9__$x^2 +$ __30__$xy +$ __25__y^2

Finding Products in the Form $(a - b)^2$

A. Multiply $(x - 5)^2$.

What is the rule for $(a - b)^2$? $a^2 - 2$__ab__ $+$ __b__ ^{2}

Let $a = x$ and $b =$ __5__.

$(x - 5)^2 = (x)^2 - 2($__x__$)(5) + ($__5__$)^2$

$= x^2 -$ __10__$x +$ __25__

B. Multiply $(4x - 3)^2$.

Let $a =$ __4x__ and $b =$ __3__.

$(4x - 3)^2 = (4x)^2 - 2($__4x__$)($__3__$) + ($__3__$)^2$

$=$ __16__$x^2 -$ __24__$x +$ __9__

Finding Products in the Form (a + b)(a − b)

A. Multiply $(x + 8)(x - 8)$.

What is the rule for a difference of two squares $(a + b)(a - b)$? $a^2 -$ __b^2__

Let $a = x$ and $b =$ __8__.

$(x + 8)(x - 8) = (x)^2 -$ __8__ ^{2}

$=$ __x^2__ $-$ __64__ Simplify.

B. Multiply $(3x + 4)(3x - 4)$.

Let $a = 3x$ and $b =$ __4__.

$(3x + 4)(3x - 4) = ($__3x__$)^2 - (4)^2$

$=$ __9__$x^2 -$ __16__ Simplify.

SECTION 7B Ready to Go On? Problem Solving Intervention
7-9 Special Products of Binomials

A circular fish pond has a radius of $(x - 12)$ feet. Write a polynomial that represents the area of the fish pond. The formula for the area of a circle is $A = \pi r^2$, where r represents the radius of the circle. Leave the symbol π in your answer.

Understand the Problem

1. What are you being asked to find? __The area of the pond__
2. What piece of information are you given regarding the pond? __The radius is $(x - 12)$ feet.__
3. How do you find the area of a circle? __$A = \pi r^2$__

Make a Plan

4. Write an expression representing the area of the fish pond. __$\pi(x - 12)^2$__
5. What are two possible ways that $(x - 12)^2$ can be simplified? __Using the Distributive Property more than once, FOIL or a perfect-square trinomial__

Solve

6. What is $(x - 12)^2$? $x^2 -$ __24x__ $+$ __144__
7. Write an expression representing the area of the pond. $\pi x^2 -$ __$24\pi x$__ $+$ __144π__

Look Back

8. Suppose that $x = 20$. What would the length of the radius be $(x - 12)$? __8__
9. What would the area of the pond equal using $A = \pi r^2$?

$A = \pi$__8__ ^{2}

$A =$ __64__ π

10. What is the value of the expression in Exercise 7, using $x = 20$?

$A = \pi x^2 - 24\pi x + 144\pi$

$= \pi(20)^2 - 24\pi($__20__$) + 144\pi$

$=$ __400__$\pi -$ __480__$\pi + 144\pi$

$=$ __64__ π

11. Are your answers to Exercises 9 and 10 the same? __Yes__

SECTION 8A Ready To Go On? Skills Intervention
8-1 Factors and Greatest Common Factors

Find these vocabulary words in Lesson 8-1 and the Multi-Language Visual Glossary.

Vocabulary	
prime factorization	greatest common factor

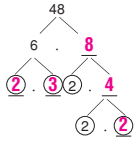
Writing Prime Factorizations

Write the prime factorization of 48.

What is a prime number? __A number with only two factors, itself and 1.__

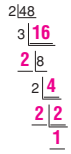
Method 1 Factor tree

Choose any two factors of 48 to begin. Keep finding factors until each branch ends in a prime factor. Circle the prime factors.



Method 2 Ladder diagram

Choose a prime factor of 48 to begin. Keep dividing by prime factors until the quotient is 1.



The prime factorization of 48 is $2 \cdot$ __2__ $\cdot$ __2__ $\cdot$ __3__ or $2^{\underline{4}} \cdot$ __3__.

Finding the GCF of Monomials

Find the GCF of each pair of monomials.

A. $9a^3$ and $3a$

$9a^3 = 3 \cdot$ __3__ $\cdot a \cdot$ __a__ $\cdot$ __a__ Write the prime factorization of each coefficient and write the powers as products.

$3a = 3 \cdot$ __a__ Align the common factors.

What are the factors common to both monomials? 3 and __a__

Multiply the common factors. The GCF of $9a^3$ and $3a$ is __$3a$__.

B. $24x^3$, $30x^5$

$24x^3 = 2 \cdot 2 \cdot$ __2__ $\cdot$ __3__ $\cdot \quad x \cdot$ __x__ $\cdot$ __x__

$30x^5 = \quad 2 \cdot$ __3__ $\cdot$ __5__ $\cdot x \cdot x \cdot$ __x__ $\cdot$ __x__ $\cdot$ __x__ Write the prime factorization of each coefficient and write the powers as products. Align the common factors.

What are the factors common to both monomials? 2, __3__ and __x^3__

Multiply the common factors. The GCF is __$6x^3$__.

SECTION 8A Ready To Go On? Problem Solving Intervention
8-1 Factors and Greatest Common Factors

Factors that are shared by two or more numbers are called common factors. The largest common factor is called the GCF, or greatest common factor.

A softball league bought new equipment for the teams. The league bought 40 balls and 24 bats. How many teams are there if all the equipment is distributed evenly between the teams? How many bats and balls does each team receive?

Understand the Problem

1. How many bats and balls were purchased? __24__ bats and __40__ balls
2. How is the equipment divided? __Evenly between the teams__
3. What are you being asked to find? __How many teams there are and how many bats and balls each team receives.__

Make a Plan

4. If the equipment is to be divided evenly between teams then 40 and 24 must be __divisible__ by the same number.
5. What are the factors of 24: 1, 2, __3, 4, 6, 8, 12, 24__

40: 1, 2, 4, __5, 8, 10, 20, 40__

Solve

6. Which factors are common to 24 and 40? 1, 2, __4__, __8__
7. What is the largest common factor? __8__
8. So, there are __8__ softball teams.
9. Since there are 40 balls and __8__ teams, how many balls does each team receive? __5__
10. Since there are 24 bats and __8__ teams, how many bats does each team receive? __3__

Look Back

11. How many balls does each team receive? __5__ How many teams are there? __8__

What is the product of number of teams and number of balls? __40__

12. How many bats does each team receive? __3__ How many teams are there? __8__

What is the product of number of teams and numbers of bats? __24__

13. Do your answers to Exercises 11 and 12 match the information in the problem statement? __Yes__

SECTION 8A

Ready To Go On? Skills Intervention

8-2 Factoring by GCF

Factoring by Using the GCF
Factor each polynomial. Check your answer.

A. $8y^3 + 24y^2$

Find the GCF of each term.

$8y^3 = 2 \cdot$ 2 $\cdot$ 2 $\cdot y \cdot$ y $\cdot$ y

$24y^2 = 2 \cdot$ 2 $\cdot$ 2 $\cdot$ 3 $\cdot y \cdot$ y

The GCF is $2 \cdot$ 2 $\cdot$ 2 $\cdot$ y $\cdot$ y $= 8$ y^2.

Write terms as products using the GCF as a factor. $y(8y^2) + 3$ $(8y^2)$

Use the Distributive Property to factor out the GCF. $8y^2($ y $+$ 3 $)$

Check: Multiply to check your answer. $8y^2($ y $+$ 3 $) = 8y^3 + 24y^2$

B. $4x^2 + 20x + 28$

Find the GCF of each term.

$4x^2 = 2 \cdot$ 2 $\cdot x \cdot$ x

$20x = 2 \cdot$ 2 $\cdot$ 5 $\cdot x$

$28 = 2 \cdot$ 2 $\cdot$ 7

The GCF is $2 \cdot$ 2 $= 4$.

Write terms as products using the GCF as a factor. $x^2(4) + 4$ $(5x)$ $+ 4(7)$

Use the Distributive Property to factor out the GCF. $4($ x^2 $+ 5x +$ 7 $)$

Check: Multiply to check your answer. $4($ x^2 $+ 5x +$ 7 $) = 4x^2 + 20x + 28$

Factoring by Grouping
Factor the polynomial, $4a^3 + 8a^2 - 3a - 6$, by grouping. Check your answer.

Group terms that have a common number of variables as a factor. $(4a^3 + 8a^2) + (-3a - 6)$

What is the GCF of $4a^3$ and $8a^2$? $4a^2$ What is the GCF of $-3a$ and -6? -3

Write each group with the GCF. $4a^2(a + 2) + (-3)(a + 2)$

Factor out the common factor of $a + 2$. $(a + 2)($ $4a^2$ $-$ 3 $)$

Multiply to check your solution.

$(a + 2)($ $4a^2$ $-$ 3 $) =$ $a(4a^2) +$ $a(-3) +$ 2 $(4a^2) +$ 2 (-3)

$=$ $4a^3$ $- 3a +$ $8a^2$ $- 6$ Multiply.

$=$ $4a^3$ $+ 8a^2 -$ $3a$ $- 6$ Rewrite in descending order.

SECTION 8A

Ready To Go On? Skills Intervention

8-3 Factoring $x^2 + bx + c$

Factoring $x^2 + bx + c$ When c is Positive.
Factor each trinomial. Check your answer.

A. $x^2 + 9x + 14$

When the constant term is positive, its factors have the same sign.

What is the value of the c term? 14 What is the value of the b term? 9

Factors of 14	Sum
1 and 14	15
2 and 7	9

What are the factors of c whose sum is b? 2 and 7

Complete: $(x +$ 2 $)(x +$ 7 $)$

Check using FOIL $(x +$ 2 $)(x +$ 7 $) = x^2 +$ $7x$ $+ 2x +$ 14

$x^2 +$ $9x$ $+$ 14

B. $x^2 + 8x + 15$

What is the value of the c term? 15 What is the value of the b term? 8

Factors of 15	Sum
1 and 15	16
3 and 5	8

What are the factors of c whose sum is b? 3 and 5

Complete: $(x +$ 3 $)(x +$ 5 $)$

Check using FOIL $(x +$ 3 $)(x +$ 5 $) = x^2 + 5x +$ $3x$ $+$ 15

$x^2 +$ $8x$ $+$ 15

Factoring $x^2 + bx + c$ When c is Negative
Factor the trinomial $x^2 + 3x - 18$. Check your answer.

When the constant term of a trinomial is negative, its factors have opposite signs.

What is the value of the c term? -18 What is the value of the b term? 3

Factors of -18	Sum
-1 and 18	17
-2 and 9	7
-3 and 6	3

What are the factors of c whose sum is b? -3 and 6

Complete: $(x -$ 3 $)(x +$ 6 $)$

Check using FOIL $(x -$ 3 $)(x +$ 6 $) = x^2 +$ $6x$ $- 3x -$ 18

$x^2 +$ $3x$ $-$ 18

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-4 Factoring $ax^2 + bx + c$

Factoring $ax^2 + bx + c$ when c is Positive.
Factor the trinomial $2x^2 + 9x + 10$. Check your answer.

What is the value of a? 2 What is the value of c? 10

What is the sum of the inner and outer products? 9

Factors of 2	Factors of 10	Outer + Inner
$1 \cdot 2$	$1 \cdot 10$	$1 \cdot 10 + 2 \cdot 1 = 12$
$1 \cdot 2$	$10 \cdot 1$	$1 \cdot 1 + 2 \cdot$ 10 $= 21$
$1 \cdot 2$	$5 \cdot$ 2	$1 \cdot 2 + 2 \cdot$ 5 $=$ 12
$1 \cdot 2$	$2 \cdot$ 5	$1 \cdot 5 + 2 \cdot$ 2 $=$ 9

$(x +$ 2 $)(2x +$ 5 $)$ Complete the factoring.

Check $(x +$ 2 $)(2x +$ 5 $) = 2x^2 +$ $5x$ $+$ $4x$ $+$ 10

$= 2x^2 +$ $9x$ $+ 10$

Factoring $ax^2 + bx + c$ when c is Negative
Factor the trinomial $4x^2 + 21x - 18$. Check your answer.

What is the value of a? 4 What is the value of c? -18

What is the sum of the inner and outer products? 21

Factors of 4	Factors of -18	Outer + Inner
$1 \cdot 4$	$1(-18)$	$1 \cdot -18 + 4 \cdot 1 = -14$
$1 \cdot 4$	$-18(1)$	$1 \cdot 1 + 4 \cdot -18 =$ -71
$1 \cdot 4$	$2 \cdot$ -9	$1 \cdot -9 + 4 \cdot$ 2 $=$ -1
$1 \cdot 4$	$-9 \cdot$ 2	$1 \cdot$ 2 $+ 4 \cdot$ -9 $=$ -34
$1 \cdot 4$	$3 \cdot$ -6	$1 \cdot$ -6 $+ 4 \cdot$ 3 $=$ 6
$1 \cdot 4$	$-6 \cdot$ 3	$1 \cdot$ 3 $+ 4 \cdot$ -6 $=$ -21
$1 \cdot 4$	$-3 \cdot$ 6	$1 \cdot$ 6 $+ 4 \cdot$ -3 $=$ -6
$1 \cdot 4$	$6 \cdot$ -3	$1 \cdot$ -3 $+ 4 \cdot$ 6 $=$ 21

$(x +$ 6 $)(4x -$ 3 $)$ Complete the factoring.

Check $(x +$ 6 $)(4x -$ 3 $) = 4x^2 -$ $3x$ $+$ $24x$ $-$ 18

$= 4x^2 +$ $21x$ $-$ 18

SECTION 8B

Ready To Go On? Skills Intervention

8-5 Factoring Special Products

Recognizing and Factoring Perfect-Square Trinomials

A trinomial is a perfect square if the first and the last terms are perfect squares. The middle term is two times one factor from the first term and one factor from the last term.

Determine whether each trinomial is a perfect square.
If so, factor it. If not, explain why.

A. $x^2 + 14x + 42$

Method 1 Factor.

Factors of 42	Sum
1 and 42	43
2 and 21	23
3 and 14	17
6 and 7	13

Is $x^2 + 14x + 42$ a perfect square trinomial? No Why? No factors give the sum of 14.

B. $9x^2 - 24x + 16$

Method 2 Use the pattern.

$9x^2 - 24x + 16$

$3x \cdot 3x - 2(3x \cdot$ 4 $) + 4 \cdot 4$

The trinomial is a perfect square.

$a = 3x$, $b =$ 4

$(3x)^2 - 2($ $3x$ $)(4) + 4^2$ Write the trinomial as $a^2 - 2ab + b^2$.

$($ $3x$ $- 4)^2$ Write as $(a - b)^2$.

Determine whether each binomial is the difference of two squares. If so, factor it. If not explain why.

A. $x^2 - 16$

$(x \cdot x) - (4 \cdot$ 4 $)$ The polynomial is the difference of two squares.

$a = x$, $b = 4$

$(x + 4)(x -$ 4 $)$ Write the polynomial as $(a + b)(a - b)$.

So $x^2 - 16 = (x +$ 4 $)(x -$ 4 $)$.

B. $9x^6 - 17y^2$

Is $9x^6$ a perfect square? Yes

Is $-17y^2$ a perfect square? No

Is $9x^6 - 17y^2$ the difference of two squares? No

Why? Because $-17y^2$ is not a perfect square.

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-5 Factoring Special Products

If a trinomial is a perfect square, it can be factored to determine measurements of real-world objects.

A community garden, rectangular in shape, has an area of $(25x^2 + 20x + 4)$ ft^2. The dimensions of the garden are approximately $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the garden. Then find the perimeter where $x = 12$.

Understand the Problem

1. What is the shape of the garden? Rectangle
2. What is the area of the garden? $(25x^2 + 20x + 4)$ ft^2
3. What kind of numbers are a and b? Whole numbers
4. What are you being asked to find? The expression for the perimeter of the garden and the perimeter when $x = 12$.

Make a Plan

5. The formula for the area of a rectangle is: Area = length × width
6. What must be done to the trinomial to find the length and width? Factor it

Solve

7. Does the trinomial $25x^2 + 20x + 4$ have a common factor? No
8. Is $25x^2 + 20x + 4$ a perfect square trinomial? Yes So, $a =$ $5x$ and $b = 2$.
9. What are the factors of the perfect square trinomial? $(5x + 2)^2$
10. Since the length and width of the garden are equal, what is the actual shape of the garden? A square
11. The formula for the perimeter of a square is $P = 4s$.
    a. What should you substitute in the perimeter formula for s? $5x + 2$
    b. Simplify $4(5x + 2)$ to find an expression for the perimeter. $20x + 8$
12. Evaluate the expression when $x = 12$. What is the perimeter? 248 ft

Look Back

13. Using the perimeter from Exercise 12, what is the length of one side of the square? 62 ft Use this side length to find the area. 3844 ft^2
14. Evaluate $25x^2 + 20x + 4$ for $x = 12$: $25(12)^2 + 20(12) + 4 =$ 3844
15. Are the areas in Exercises 13 and 14 equal? Yes

SECTION 8B

Ready To Go On? Skills Intervention

8-6 Choosing a Factoring Method

Determining Whether a Polynomial Is Completely Factored
Tell whether each polynomial is completely factored. If not, factor it.

A. $4x^2 + 2x - 6 = 2(2x^2 + x - 3)$

The greatest common factor is 2.

Is $(2x^2 + x - 3)$ factorable? Yes

Is $2(2x^2 + x - 3)$ completely factored? No

Factor: $2(2x^2 + x - 3) \Rightarrow 2(2x +$ 3 $)(x -$ 1 $)$

B. $42x - 14x^3 = 14(3x - x^3)$

What is the greatest common factor? $14x$

Is the polynomial factored completely? No

Factor: $42x - 14x^3 \Rightarrow 14x($ 3 $-$ x^2 $)$

Can the polynomial be factored any further? No

Factoring by Multiple Methods
Factor the polynomial $6x^3 + 21x^2 + 15x$ completely. Check your answer.

Consider $6x^3 + 21x^2 + 15x$.

What is the GCF? $3x$

Factor out the GCF: $3x($ $2x^2$ + $7x$ + 5 $)$

What is the value of a? 2 What is the value of c? 5,

The Outer + Inner terms = 7.

Factors of 2	Factors of 5	Outer + Inner
1 · 2	5 · 1	1 · 1 + 2 · 5 = 11
1 · 2	1 · 5	1 · 5 + 2 · 1 = 7

Complete the binomial: $(x +$ 1 $)(2x +$ 5 $)$

Write as a factored polynomial: $3x(x + 1)(2x +$ 5 $)$

Check: $3x(x + 1)(2x + 5)$

$= 3x(2x^2 +$ $5x$ + $2x$ + 5 $)$ FOIL

$= 3x(2x^2 +$ $7x$ + 5 $)$ Combine like terms.

$= 6x^3 +$ 21 $x^2 +$ 15 x Distribute the $3x$.

Is this the original polynomial? Yes

SECTION 9A

Ready To Go On? Skills Intervention

9-1 Identifying Quadratic Functions

Find these vocabulary words in Lesson 9-1 and the Multi-Language Visual Glossary.

Vocabulary				
quadratic function	parabola	vertex	minimum	maximum

Identifying Quadratic Functions
The quadratic function $y = x^2$ does not have constant first differences. It has constant second differences. This is true for all quadratic functions.

A. Tell whether the function $y + 6x^2 = 5x$ is quadratic. Explain.

Try writing the function in the form $y = ax^2 + bx + c$ by solving for y.

$y + 6x^2 = 5x$

$-6x^2$ $-6x^2$ Subtract $6x^2$ from each side.

$y = -6x^2 +$ $5x$ Write in $ax^2 + bx + c$ form.

So, $a =$ -6, $b = 5$, and $c =$ 0.

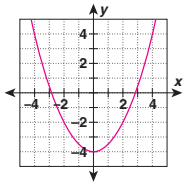
Is the function quadratic? Yes How do you know?

It can be written in the form $ax^2 + bx + c$.

B. Graph the function $y = \frac{1}{2}x^2 - 4$ and give the domain and range.

Make a table of values. Choose values of x and use them to find values of y. Graph the points and connect with a smooth curve.

x	$y = \frac{1}{2}x^2 - 4$
−4	4
-2	−2
0	-4
2	-2
4	4



Is the value of a positive or negative? Positive
Therefore, the graph opens upward.
The vertex is located at (0, -4).
Is the vertex a maximum or a minimum? Minimum

The domain is all real numbers. All the y-values of the function are greater than or equal to -4. So the range is $y \geq -4$.

SECTION 9A

Ready To Go On? Skills Intervention

9-2 Characteristics of Quadratic Functions

Find these vocabulary words in Lesson 9-2 and the Multi-Language Visual Glossary.

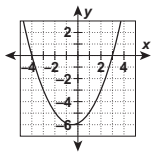
Vocabulary	
zero of a function	axis of symmetry

Finding Zeros of Quadratic Functions From Graphs
Find the zeros of the quadratic function from its graph. Then find its axis of symmetry.
The zero of a function is an x-value that makes the function equal to zero.
The zero of a function is the same as an x-intercept.

A quadratic function may have one, two, or no zeros.

The axis of symmetry always passes through the vertex of the parabola.

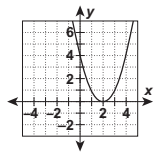
A.



Where does the graph cross the x-axis?
-4 and 3
To determine the axis of symmetry, find the average of the zeros.

$\frac{-4 + 3}{2} = -\frac{1}{2}$ $x = -\frac{1}{2}$

B.



Where does the graph cross the x-axis?
2
In this case, the x-coordinate is the axis of symmetry.
$x =$ 2

Finding the Vertex of a Parabola
Find the vertex of the parabola $y = -2x^2 + 4x - 3$.

Step 1 Find the x-coordinate using the formula $x = -\frac{b}{2a}$.

What does a equal? -2 What does b equal? 4

$x = \frac{-b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1$

Step 2 Find the corresponding y-coordinate.

$y = -2x^2 + 4x - 3$

$y = -2($ 1 $)^2 + 4($ 1 $) - 3$

$y = -2 +$ 4 $- 3 =$ -1

Step 3 Write the coordinates as an ordered pair. The vertex is (1, -1).

SECTION 8A Ready To Go On? Quiz

8-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 84 — $2^2 \cdot 3 \cdot 7$
2. 60 — $2^2 \cdot 3 \cdot 5$
3. 150 — $2 \cdot 3 \cdot 5^2$
4. 66 — $2 \cdot 3 \cdot 11$
5. 72 — $2^3 \cdot 3^2$
6. 156 — $2^2 \cdot 3 \cdot 13$

Find the GCF of each pair of monomials.

7. $6x^4$ and $9x^2$ — $3x^2$
8. $25x^3$ and $20x^4$ — $5x^3$
9. -18 and $27c^5$ — 9
10. $5a$ and $7c$ — 1

11. Nichole is designing a quilt. She has cut out 60 red squares and 48 green squares. She plans for the quilt to have the same number of colored squares in each row, but, green and red squares will not be in the same row. How many rows will the quilt have if Nichole puts the greatest number of squares in each row? — 9 rows

8-2 Factoring by GCF

Factor each polynomial. Check your answer.

12. $30x^2 + 12x$ — $6x(5x + 2)$
13. $2a^2 - 10a^3$ — $2a^2(1 - 5a)$
14. $6x^4 - 15x^3 - 9x^2$ — $3x^2(2x + 1)(x - 3)$
15. $4x^2 + 12x + 16$ — $4(x^2 + 3x + 4)$

16. The surface area of a cylinder can be found using the expression $2\pi r^2 + 2\pi rh$, where r represents the radius of the cylinder and h represents the height. Factor this expression. — $2\pi r(r + h)$

SECTION 8A Ready To Go On? Quiz continued

Factor each polynomial by grouping. Check your answer.

17. $x^3 - 3x^2 + 2x - 6$ — $(x^2 + 2)(x - 3)$
18. $5x^3 + 10x^2 + x + 2$ — $(5x^2 + 1)(x + 2)$
19. $2r^3 - 8r^2 - 3r + 12$ — $(2r^2 - 3)(r - 4)$
20. $3s^3 - 12s^2 - s + 4$ — $(3s^2 - 1)(s - 4)$
21. $4y^3 + 16y^2 - 2y - 8$ — $(4y^2 - 2)(y + 4)$
22. $8b^3 + 16b^2 - 2b - 4$ — $(8b^2 - 2)(b + 2)$

8-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

23. $a^2 + 12a + 35$ — $(a + 5)(a + 7)$
24. $x^2 - 3x - 10$ — $(x - 5)(x + 2)$
25. $x^2 - 8x + 7$ — $(x - 1)(x - 7)$
26. $x^2 + 13x - 30$ — $(x + 15)(x - 2)$
27. $c^2 - 13c + 36$ — $(c - 9)(c - 4)$
28. $y^2 - 12y + 32$ — $(y - 8)(y - 4)$

29. Simplify and factor the polynomial $n(n + 7) + 12$. Show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3$, and 4. — $n^2 + 7n + 12$; $(n + 3)(n + 4)$

8-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

30. $2a^2 + 5a + 2$ — $(2a + 1)(a + 2)$
31. $6x^2 + 11x + 4$ — $(2x + 1)(3x + 4)$
32. $3x^2 - x - 10$ — $(3x + 5)(x - 2)$
33. $4x^2 - 14x + 12$ — $2(2x - 3)(x - 2)$
34. $6c^2 - 12c - 18$ — $6(c - 3)(c + 1)$
35. $12y^2 - y - 20$ — $(3y - 4)(4y + 5)$

36. The area of a rectangle is $(10x^2 + 21x + 9)$ cm^2. The length is $(5x + 3)$ cm. What is the width of the rectangle? — $(2x + 3)$ cm

SECTION 8B Ready To Go On? Quiz

8-5 Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $a^2 + 6a + 9$ — Yes; $(a + 3)^2$
2. $16x^2 - 40x + 25$ — Yes; $(4x - 5)^2$
3. $w^2 - 12w + 4$ — No; $12w \neq 2(w)(2)$
4. $5y^2 - 14y + 16$ — 5 is not a perfect square.
5. $x^2 + 4x + 4$ — Yes; $(x + 2)^2$
6. $25h^2 - 70h + 49$ — Yes; $(5h - 7)^2$

7. An architect is designing rectangular windows with an area of $(x^2 + 22x + 121)$ ft^2. The dimensions of the windows are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the windows. Find the perimeter of a window when $x = 3$ ft. — $4(x + 11)$; 56 feet

Determine whether each trinomial is the difference of two squares. If so, factor it. If not, explain why.

8. $r^2 - 144$ — Yes; $(r - 12)(r + 12)$
9. $4a^2 - 30$ — 30 is not a perfect square.
10. $1 - 25a^4$ — Yes; $(1 - 5a^2)(1 + 5a^2)$
11. $36k^2 - 9k^6$ — Yes; $(6k - 3k^3)(6k + 3k^3)$
12. $49a^2 + 64$ — The terms are added.
13. $w^4 - a^2$ — Yes; $(w^2 - a)(w^2 + a)$

14. The area of a square is $(49x^2 - 28x + 4)$ in^2.
    a. What is the length of a side of the square? — $(7x - 2)$
    b. What is the perimeter of the square? — $4(7x - 2)$
    c. What are the length of a side, the perimeter, and the area of the square when $x = 4$ in.? — 26 in.; 104 in.; 676 in.^{2}

SECTION 8B Ready To Go On? Quiz continued

8-6 Choosing a Factoring Method

Tell whether each polynomial is completely factored. If not, factor it.

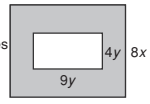
15. $7x^2 + 35x + 7 = 7(x^2 + 5x + 1)$ — Yes
16. $16x^3 - 24x^2 = 8x(2x^2 - 3x)$ — $8x^2(2x - 3)$
17. $4x^5 - 16x = 4x(x^4 - 4)$ — $4x(x^2 - 2)(x^2 + 2)$
18. $3a^2 - 42a + 147 = 3(a^2 - 14a + 49)$ — $3(a - 7)(a - 7)$
19. $8y^3 - 8y^2 - 12y + 12 = 4(2y^2 - 3)(y - 1)$ — Yes
20. $3y^2 + 17y + 10 = (3y + 2)(y + 5)$ — Yes

Factor each polynomial completely. Check your answer.

21. $4x^3 - 24x^2 + 36x$ — $4x(x - 3)^2$
22. $3x^2y - 12xy^2$ — $3xy(x - 4y)$
23. $3a^2 + a + 1$ — Unfactorable
24. $12x^3 - 3x$ — $3x(2x + 1)(2x - 1)$
25. $4x^2 + 12x - 112$ — $4(x - 4)(x + 7)$
26. $x^5 - 16x$ — $x(x - 2)(x + 2)(x^2 + 4)$

Write an expression for each situation. Factor your expression.

27. The difference of the square of a pipe's length and 49. — $\ell^2 - 49 = (\ell + 7)(\ell - 7)$
28. The square of Catherine's age plus 18 times Catherine's age plus 81. — $x^2 + 18x + 81 = (x + 9)^2$
29. Three times the square of a truck's speed minus three times the trucks speed plus 18. — $3x^2 - 3x + 18 = 3(x - 3)(x + 2)$
30. Four times the square of cherries on a tree minus 11 times cherries minus 21. — $4x^2 - 11x - 21$; not factorable



31. Write an expression for the area of the shaded region. Then factor the expression. — $64x^2 - 36y^2 = 4(4x - 3y)(4x + 3y)$

SECTION 7A Ready To Go On? Enrichment

Digits

Using the digits 1, 2, 3 and 4 and addition, subtraction, multiplication, division, parentheses, and exponents, write an expression equivalent to the numbers 1 to 20.

- You must use all four digits in each expression.
- You may use any of the operations but each symbol may be used only once in each expression.

An example has been done for you. There may be more than one correct expression for a given number. Sample answers given

1 = $3 \cdot 2 - (4 + 1)$

2 = $\frac{3 \cdot 2}{1} - 4$

3 = $4 - 3 + 2 \cdot 1$

4 = $(3 - 2) \cdot \frac{4}{1}$

5 = $3 \cdot 1 + 4 - 2$

6 = $3^1 \cdot (4 - 2)$

7 = $4 + 3 \cdot 1^2$

8 = $(4 - 2) \cdot (3 + 1)$

9 = $(4 + 1 - 2) \cdot 3$

10 = $4 \cdot 3 - \frac{2}{1}$

11 = $2^3 + (4 - 1)$

12 = $2^3 + (4 \cdot 1)$

13 = $2^4 - (3 \cdot 1)$

14 = $(4 + 3) \cdot \frac{2}{1}$

15 = $(4 + 2 - 1) \cdot 3$

16 = $(3 + 2 - 1) \cdot 4$

17 = $(4 + 2) \cdot 3 - 1$

18 = $2^4 + (3 - 1)$

19 = $2^4 + (3 \cdot 1)$

20 = $(3 + 2) \cdot \frac{4}{1}$

SECTION 7B Ready to Go On? Enrichment

Pascal's Triangle

Pascal's Triangle is a geometric arrangement of numbers. These numbers represent the binomial coefficients. That is, they represent the coefficients of the terms of the expansion of $(x + y)^n$. The first seven rows of Pascal's Triangle look like this.

Row 0 1

Row 1 1 1

Row 2 1 2 1

Row 3 1 3 3 1

Row 4 1 4 6 4 1

Row 5 1 5 10 10 5 1

Row 6 1 6 15 20 15 6 1

Notice that each number is the sum of the two numbers above it.

For example what two numbers were added to get 10 in the 5th row? 4 and 6

What are the numbers for the 7th row? 1, 7, 21, 35, 35, 21, 7, 1

As an example, find $(x + y)^3$.

Look at row 3, what are the coefficients of the expansion? 1, 3, 3, 1

The first term of the expansion starts with the highest power of x, namely x^3, and the lowest power of y, namely $y^0 = 1$. The power of x increases by 1 for each successive term and the power of y increases by 1 for each successive term.

$(x + y)^3 = \underline{1 \cdot x^3 \cdot y^0} + \underline{3 \cdot x^2 \cdot y^1} + \underline{3 \cdot x^1 \cdot y^2} + \underline{1 \cdot x^0 \cdot y^3}$

$= x^3 + 3x^2y + 3xy^2 + y^3$

Expand each of the following polynomials.

1. $(x + y)^4$ — $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
2. $(x + 1)^5$ — $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$
3. $(x + 3)^4$ — $x^4 + 12x^3 + 54x^2 + 108x + 81$
4. $(x + 2)^3$ — $x^3 + 6x^2 + 12x + 8$
5. $(x + 1)^9$ — $x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1$
6. $(x + 2y)^5$ — $x^5 + 10x^4y + 40x^3y^3 + 80x^2y^3 + 80xy^4 + 32y^5$

SECTION 8A Ready To Go On? Enrichment

Sum and Differences of Cubes

While it is possible to factor the sum and differences of two squares, it is also possible to factor the sum and differences of two cubes.

The sum of two cubes can be factored in the following way:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

The differences of two cubes can be factored in the following way:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor each of the following.

1. $r^3 - s^3$ — $(r - s)(r^2 + rs + s^2)$
2. $x^3 + y^3$ — $(x + y)(x^2 - xy + y^2)$
3. $x^3 + 8$ — $(x + 2)(x^2 - 2x + 4)$
4. $n^3 - 64$ — $(n - 4)(n^2 + 4n + 16)$
5. $8y^3 + 27$ — $(2y + 3)(4y^2 - 6y + 9)$
6. $pq^3 - 64p$ — $p(q - 4)(q^2 + 4q + 16)$

Express each of the following as the sum or difference of two cubes.

7. $(m - 1)(m^2 + m + 1)$ — $m^3 - 1$
8. $(2 + 3t)(4 - 6t + 9t^2)$ — $8 + 27t^3$
9. $(b - 64)(b^2 + 4b + 16)$ — $b^3 - 4^3$
10. $(x + 7)(x^2 - 7x + 49)$ — $x^3 + 7^3$
11. $(2y - 1)(4y^2 + 2y + 1)$ — $8y^3 - 1$
12. $(3 + 2t)(9 - 6t + 4t^2)$ — $27 + 8t^3$
13. $(s + 10)(s^2 - 10s + 100)$ — $s^3 + 1000$
14. $2(x - 4)(x^2 + 4x + 16)$ — $2x^3 - 128$

SECTION 8B Ready To Go On? Enrichment

Fourth Degree Trinomials

Sometimes it is possible to write a trinomial of the fourth degree, $a^4 + a^2b^2 + b^4$, as a difference of two squares and then factor.

Example: Factor $4a^4 - 21a^2b^2 + 9b^4$.

Step I Find the square roots of the first and last terms.

$\sqrt{4a^4} = 2a^2$ $\sqrt{9b^4} = 3b^2$

Step II Find twice the product of the square roots from the terms in Step 1.

$2(2a^2)(3b^2) = 12a^2b^2$

Step III Split the middle term of the trinomial into two parts. One part is either the answer from the Step II or its opposite. The other part should be the opposite of a perfect square.

$-21a^2b^2 = -12a^2b^2 - 9a^2b^2$

Step IV Rewrite the trinomial as the difference of two squares and then factor.

$4a^4 - 21a^2b^2 + 9b^4 = (4a^4 - 12a^2b^2 + 9b^4) - 9a^2b^2$

$= (2a^2 - 3b^2)^2 - 9a^2b^2$

$= [(2a^2 - 3b^2) - 3ab][(2a^2 - 3b^2) - 3ab]$

$= (2a^2 + 3ab - 3b^2)(2a^2 - 3ab - 3b^2)$

Factor each trinomial.

1. $16d^4 + 7d^2 + 1$ — $(4d^2 - d + 1)(4d^2 + d + 1)$
2. $p^4 + p^2 + 1$ — $(p^2 + p + 1)(p^2 - p + 1)$
3. $4x^4 - 13x^2 + 1$ — $(2x^2 + 3x - 1)(2x^2 - 3x - 1)$
4. $4x^4 - 9x^2y^2 + 16y^4$ — $(2x^2 - 5xy + 4y^2)(2x^2 + 5xy + 4y^2)$
5. $9r^4 + 26r^2s^2 + 25s^4$ — $(3r^2 + 2rs + 5s^2)(3r^2 - 2rs + 5s^2)$
6. $4a^4 - 5a^2c^2 + 25c^4$ — $(2a^2 - 5ac + 5c^2)(2a^2 + 5ac + 5c^2)$