Illustrative Mathematics

A-SSE Profit of a company

Alignment 1: A-SSE.B.3

The profit a company makes selling an item depends on the price $\$p$ of the item. Three equivalent forms for the profit are:

$$\begin{aligned} \text{Standard form: } & -2p^2 + 24p - 54 \\ \text{Factored form: } & -2(p-3)(p-9) \\ \text{Vertex form: } & -2(p-6)^2 + 18. \end{aligned}$$

Which form is most useful for finding

a. The prices that give a profit of zero dollars?

b. The profit when the price is zero?

c. The price that gives the maximum profit?

Commentary:
This task compares the usefulness of different forms of a quadratic expression. Students have to choose which form most easily provides information about the maximum value, the zeros and the vertical intercept of a quadratic expression in the context of a real world situation. Rather than just manipulating one form into the other, students can make sense out of the structure of the expressions.

(From *Algebra: Form and Function, McCallum et al., Wiley 2010*)

Solution: Structure of quadratic expression

a. The factored form gives the values of p that make the profit zero. Since factored form is $-2(p-3)(p-9)$, the profit is zero when $p=3$ or $p=9$. The company breaks even if the price charged for the product is $\$3$ or $\$9$.

b. The standard form is the easiest one to use to find the profit when the price is zero. Substituting $p=0$ into the standard form $-2p^2+24p-54$, we see that the profit is -54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses $\$54{,}000$.

c. The vertex form shows us what price maximizes profit. From the expression $-2(p-6)^2+18$, we see that the maximum profit is 18 thousand dollars, and it occurs when $p=6$. The company should charge a price of $\$6$ for this product.