APS DISTRICT HIGH SCHOOL MATHEMATICS CURRICULUM FRAMEWORK

Course Title: Geometry **Course Number:** SEE BELOW

Department: Mathematics **ADS Number:** SEE BELOW

Prerequisites: Successful completion of Algebra I

Length of Course: One Year **Credit/PRI Area:** .50 per Sem/Mathematics **Grade Level(s):** 9 - 12

COURSE AND ADS NUMBERS

Geometry	35040	20344131
	060C6	20342133
	061C3	20342133
	062C3	20342133
Geometry Bilingual	3504A	20348131

Important Notes:

This course requires student access to a graphing calculator.

COURSE DESCRIPTION: In Geometry the student learns abstract and logical thinking through inductive and deductive reasoning. The student uses lines, planes, polygons, circles, and three-dimensional figures for representing and solving a variety of problems. The student uses calculators, computers and software programs (e.g., Geometer's Sketchpad, Cabri Geometry), construction tools (e.g., compass, protractor, straight edge), and graphing utilities as tools in problem solving. Other areas of study include global processes; algebra, functions, and graphs; and data analysis and probability. Literacy strategies are integrated throughout the curriculum.

References in parentheses following each performance standard align with the National Council of Teachers of Mathematics Standards (NCTM), the State of New Mexico Mathematics Standards (NM), the Albuquerque Public Schools Mathematics Standards (APS), and the Albuquerque Public Schools Language Arts Standards (APS – LA).

A group of Geometry teachers from across the district met to identify the power standards for this course. Those standards the group identified are all in Strand III and are italicized. Focus of instruction should be on #6, #10, #12, #13, #15, #18, and #19 of that strand.

STRATEGIES:
The "Illustrations" column in the *Program of Studies* provides exemplars of the performance standards, strategies, and best practices suggested by mathematics teachers in the Albuquerque Public Schools (APS).

ASSESSMENTS:
Assessments may include: authentic and performance-based assessment, cooperative learning, teacher observations, checklists, tests and exams, formal and informal writing, small group and full class discussions, oral and multimedia presentations, projects, demonstrations, and portfolios. Assessments are based on appropriate rubrics.

SUGGESTED TEXTBOOKS AND INSTRUCTIONAL MATERIALS:
- Current state adopted mathematics textbooks
- Graphing calculators
- Geometer's Sketchpad
- Cabri Geometry

SUGGESTED TITLES/AUTHORS WEB SITES:
- Rubistar4teachers.com
- Nctm.org

Approved by HSCA: December, 2004

STRAND I: GLOBAL MATHEMATICS PROCESSES
CONTENT STANDARD: The student understands and uses mathematical processes.

BENCHMARK: The student uses problem solving, reasoning and proof, communication, connections, and representations as appropriate in all mathematical experiences.

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
		NOTE: Illustrations include suggested activities for attaining each performance standard. A check (√) refers to a key feature to look for while assessing student performance.
	1. Prepares mathematically for future careers (APS – I. 14).	1 – 11. The student creates a project (e.g., topic – architecture) using presentation software to include some or all of the following: a) scanned information to slide (e.g., student sketch of home) b) download pictures from the Internet to another slide (e.g., historical buildings from around the world c) include text with each picture using geometric terms d) cite and support 3 – 5 conjectures showing geometric relevance to everyday situations (e.g., proportionality, congruence, angle measurements √ all required components √ use of technology √ conjectures √ connections √ effective presentation
	2. Uses graphing technology throughout the curriculum (APS – I.9, III.21L; NM – IC.2).	
	3. Applies the "rule of four" (i.e., represents mathematics graphically, symbolically, verbally, numerically) (APS – All of Strand I, III.20L).	3 – 7, 9. The student finds the solutions to the following situation: A cylindrical tank is laying horizontally on the ground. Its diameter is _____ feet, its length _____ feet, and the depth of the water in the tank is _____ feet. (The numbers can be varied where the blanks are.) b) How many gallons of water are in the tank? c) How many more gallons of water does it take to fill the tank? In finding the solution to this problem, the student is to clearly communicate on paper how the problem is solved so that anyone can follow his/her thought process. The work is to be organized, clearly communicated, neat, and accurate. √ adherence to criteria √ accuracy
	4. Uses reasoning and problem-solving strategies to solve new problems [APS – I.3; NM – IIA (5-7)].	
	5. Makes connections among mathematical concepts (APS – I.12; NM – IA.6).	
	6. Works in teams to share ideas, to develop and coordinate group approaches to problems, and to share and learn from each other in communicating findings (APS – I.4, I.8).	

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	7. Develops resourcefulness and perseverance in problem solving by working with everyday problems and applications including integration with other subject areas studied at the same grade level (APS – I.1). 8. Makes and investigates mathematical conjectures and uses them successfully in developing and evaluating mathematical arguments and proof [APS – I.5; NM – IIA (5-7)]. 9. Recognizes when to use previously learned strategies to solve new problems (APS – I.2; NM – IC.1, IID.2). 10. Uses the concept of counterexample to test the legitimacy of an argument (APS – I.6; NM – IIA.5). 11. Develops a logical sequence of arguments leading to a valid conclusion or solution to a problem (APS – I.7; NM – IIA.5).	**Note: The global processes are not taught in isolation but should be integrated in all strands throughout the curriculum. They are universal to all math courses, and all are, consequently, important.**

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: The student understands algebraic concepts and applications.

BENCHMARKS: A. The student represents and analyzes mathematical situations and structures using algebraic symbols.
B. The student understands patterns, relations, functions, and graphs.
C. The student uses mathematical models to represent and understand quantitative relationships.
D. The student analyzes changes in various contexts.

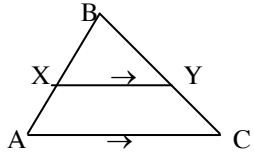
GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark A: The student represents and analyzes mathematical situations and structures using algebraic symbols. 1. Represents and analyzes relationships using written and verbal expressions, tables, equations, and graphs, and describes the connections among those representations (NM – IA.6): • translates from verbal expression to algebraic formulae (e.g., "Set up the equations that represent the data in the following equation: John's father is 23 years older than John. John is 4 years older than his sister Jane. John's mother is 3 years younger than John's father. John's mother is 9 times as old as Jane. How old are John, Jane, John's mother, and John's father?"), • given data in a table, constructs a function that represents these data (linear only), and • given a graph, constructs a function that represents the graph (linear only). 2. Knows, explains, and uses equivalent representations for the same real number including (NM – IA.7): • integers, • decimals, • percents, • ratios, • scientific notation, • numbers with integer exponents, • inverses (reciprocal), and • prime factoring.	1. The student writes a mathematical sentence/equation to represent the following relationship among integers and solves it justifying his/her work. The sum of three consecutive integers is 30 more than the smallest integer. √ translation from word to symbol √ accurate solution √ documentation of work OR Using the information from the table, the student graphs the data and writes the function that represents the data. x | y 0 | -3 2 | 1 4 | 5 -1 | -5 √ accuracy of graph √ correct function 2, 6. The student simplifies a variety of expressions similar to: a) $(610)(2{,}500{,}000{,}000)$ b) $\frac{2.0286 \times 10^{8}}{3.15 \times 10^{3}}$ c) $\frac{4x^3y^3}{2xy^{-1}} \cdot \frac{-5xy^2}{2y}$ √ accuracy √ applications of laws of exponents

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	3. Simplifies square roots and cube roots with monomial radicands that are perfect squares or perfect cubes (e.g., $9a^2x^4$) (NM – IA.11).	3, 5. See Strand III, the illustrations for performance standards #16 and # 19. The student expresses the answer in simplified radical form. As an extension, the student estimates the values both mentally and with the use of the calculator and compares the results. √ accuracy √ multiple representations √ understanding of radicals
	Benchmark B: The student understands patterns, relations, functions, and graphs. 4. Understands symmetry of graphs (NM – IB.9).	4. See Strand III, the illustration for performance standards #11, #12. As an extension to that exercise the student describes and discusses if the drawings have symmetry and identifies the type of symmetry. √ individual participation in discussion √ understanding of symmetry √ clear communication
	Benchmark C: The student uses mathematical models to represent and understand quantitative relationships. 5. Uses a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools) (NM – IC.2).	
	Benchmark D: The student analyzes changes in various contexts. 6. Solves routine two- and three-step problems relating to change using concepts such as (NM – ID.2): • exponents, • factoring, • ratio, • proportion, • average, and • percent.	

STRAND III: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: The student understands geometric concepts and applications.

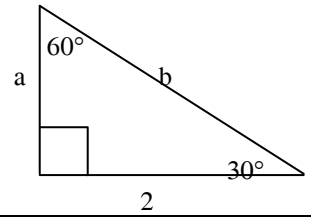
BENCHMARKS: A. The student analyzes characteristics and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships.
B. The student specifies locations and describes spatial relationships using coordinate geometry and other representational systems.
C. The student applies transformations and uses symmetry to analyze mathematical situations.
D. The student uses visualization, spatial reasoning, and geometric modeling to solve problems.

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark A: The student analyzes characterisitcs and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships.	
	1. Interprets and draws two-dimensional objects and finds the area and perimeter of basic figures (e.g., rectangles, circles, triangles, other polygons [e.g., rhombi, parallelograms, trapezoids]) (NM – IIA.1).	1, 2. Using rectangles, the student determines the approximate area of an irregular shape provided by the teacher and justifies his/her work. √ accuracy of solution √ approach to the problem √ documentation of work
	2. Finds the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles, and/or semicircles with just edges in common (NM – IIA.2).	
	3. Finds and uses measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., scalene, isosceles, and equilateral triangles; rectangles [square and non-square]; other convex polygons) (NM – IIA.3).	3. Using the information that the interior angle of a regular polygon is 144°, the student determines what kind of a polygon it is and justifies his/her answer in writing. √ accuracy √ clear explanation √ understanding of key concepts
	4. Interprets and draws three-dimensional objects and finds the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and calculates the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures (NM – IIA.4).	4. The student finds the volume and surface area of a prism with a height of four inches and a three inch square base. He/She then compares the results with the volume and surface area of a cylinder with a height of 5.1 inches and a diameter of three inches and uses the results to explain why canned goods are usually packed in cylindrical containers. √ accuracy in calculations √ justifications √ comparisons
	5. Demonstrates an understanding of simple aspects of a logical argument: • identifies the hypothesis and conclusion in logical deduction, and • uses counterexamples to show that an assertion is false and	5. The student considers the statement: If a figure is a triangle, then it is a polygon. He/She: • identifies the hypothesis and the conclusion, • determines if the statement is true or false,

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	recognizes that a single counterexample is sufficient to refute an assertion (NM – IIA.5).	• writes the converse of the statement, and • shows that the converse is false by drawing a counterexample. √ correct identification of parts of a conditional sentence √ accuracy √ appropriate counterexample
	6. *Demonstrates an understanding of inductive and deductive reasoning, explains the difference between inductive and deductive reasoning, and identifies and provides examples of each (NM – IIA.6):* • *for inductive reasoning, demonstrates understanding that showing a statement is true for a finite number of examples does not show it is true for all cases unless the cases verified are all cases and* • *for deductive reasoning, proves simple theorems.*	6. The student finds the next number in the sequence 1, 1/2, 1/4 … Ask, "Did you use deductive or inductive reasoning? Explain your answer in writing verifying your logic." √ correct type of reasoning √ justifications
	7. Writes geometric proofs (including proofs by contradiction), including (NM – IIA7): • theorems involving the properties of parallel lines cut by a transversal line and the properties of quadrilaterals, • theorems involving complementary, supplementary, and congruent angles, • theorems involving congruence and similarity, and • the Pythagorean theorem (tangram proof).	7. The student does the following proof using direct/indirect methods: Given: XY//AC Prove: Δ XBY ~ Δ ABC  √ logical reasoning
	Benchmark B: The student specifies locations and describes spatial relationships using coordinate geometry and other representational systems. 8. Determines the midpoint and distance between two points within a coordinate system and relates these ideas to geometric figures in the plane (e.g., finds the center of a circle given two endpoints of a diameter of the circle) (NM – IIB.2).	8. The student: a) graphs segment AB with endpoints at A (3, -5) and B (0, - 1) b) finds the distance between the two points, and c) finds the segment's midpoint. √ application of formulas √ accuracy √ graphical representation
	9. Given two linear equations, determines whether the lines are parallel, perpendicular, or coincide (NM – IIB.3). 10. *Uses basic geometric ideas (e.g., the Pythagorean Theorem, area, and perimeter of objects) in the context of the Euclidean Plane, and calculates the perimeter of a rectangle with integer coordinates and sides*	9, 10. The student determines what type of quadrilateral ABCD is if A (7,5), B (8,3), C (0,-1), and D (-1,1) and justifies answers without graphing. √ justifications √ understanding of properties of quadrilaterals √ use of appropriate formulas

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	parallel to the coordinate axes and with sides not parallel (NM – IIB.4). **Benchmark C: The student applies transformations and uses symmetry to analyze mathematical situations.** 11. Describes the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections (NM – IIC.1): • determines whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation and • sketches the planar figure that is the result of a given transformation of this type. 12. *Deduces properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system (NM – IIC.2):* • *identifies congruency and similarity in terms of transformations and* • *determines the effects of the above transformations on linear and area measurements of the original planar figure.* **Benchmark D: The student uses visualization, spatial reasoning, and geometric modeling to solve problems.** 13. *Solves real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow) (NM – IID.1).* 14. Solves problems involving complementary, supplementary, and congruent angles (NM – IID.2).	11, 12. Using computer drawing software, the student draws five different geometric figures, and rotates each 90°. Selecting the calculation application from the software, the student creates the equation of each shape. √ required transformations √ accuracy √ equations Extension: The student reflects and translates each drawing according to specified instructions [e.g., reflect about the x-axis, slide each point by (-2, 5)]. 12, 13. The student responds to the following problem and justifies his/her answer. A solid figurine is 4" tall and weighs five pounds. What is the weight of a similar figure of the same material if it is 12" tall? √ understanding of similarity √ justification for answer √ accuracy 14. The student works in a small group to discuss the following situations. Each student practices the drawings, answers the questions, evaluates the drawings and answers, and offers alternatives. The student sketches possible drawings of $\angle 1$ and $\angle 2$ to show: • $\angle 1$ and $\angle 2$ are supplementary and adjacent, • $\angle 1$ and $\angle 2$ are complementary and not adjacent, and • $\angle 1$ and $\angle 2$ are adjacent complementary and have the same measure. √ teamwork/collaboration √ accurate response to questions √ appropriate drawings √ relevant alternatives √ individual participation

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	15. *Solves problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., "Determine the surface area of a can of height h and radius r. How does the surface area change when the height is changed to 3h? How does the surface area change when the radius is changed to 3r? How does the surface area change when both h and r are doubled?") (NM – IID.3).*	15. The student determines the volume of a rectangular prism using arbitrary values for the length, width, and height and records his/her result. Then he/she recalculates the volume by doubling each dimension and then by halving each. The student compares all three volumes and makes a conjecture. √ accuracy √ generalization Extension: The student also finds the perimeter, area, and surface area for the same figure using the same dimensions.
	16. Solves problems using the Pythagorean Theorem (e.g., "Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder") (NM – IID.4).	16. The student solves a variety of problems where he/she applies the Pythagorean Theorem and justifies his/her work. An example: The base of a 10-foot ladder is placed two feet away from a wall. How high up the wall will the ladder reach? √ correct application of the Pythagorean Theorem √ accuracy √ justification of work
	17. Understands and uses elementary relationships of basic trigonometric functions defined by the angles of a right triangle (NM – IID.5).	17. The student determines the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet and justifies his/her work. √ trigonometric applications √ accuracy √ justification of work
	18. *Uses trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., "A surveyor determines that the angle subtended by a two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?") (NM – IID.6).*	18. The student determines how tall a tree is if he/she views from eye level five feet above the ground and looks up at an angle of 35° to see the top of the tree. The student justifies his/her work including a sketch of the situation. √ trigonometric applications √ reasonable sketch √ justification of work √ accuracy

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	19. *Knows and uses angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles) (NM – IID.7).*	19. The student solves for the missing sides in the given figure. √ all the missing parts √ accuracy 

STRAND IV: DATA ANALYSIS AND PROBABILITY
CONTENT STANDARD: The student understands how to formulate questions, analyze data, and determine probabilities.

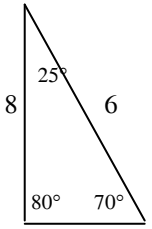
BENCHMARK: A. The student understands and applies basic concepts of probability.

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	Benchmark D: The student understands and applies basic concepts of probability. 1. Understands the concept of probability as relative frequency (NM – IIID.2). 2. Distinguishes between independent and dependent events (NM – IIID.4). 3. Understands how to compute the probability of an event using the basic rules of probability (NM – IIID.5): • complement rule, • addition rule (disjoint and joint events), • multiplication rule (independent events), and • conditional probability.	1 - 3. The student sketches a target which consists of 10 concentric circles (e.g., circles having the same center). The inner circle is labeled 10 with each succeeding circle labeled 9, 8, with the last circle labeled 1. The fifth and sixth-point circles are shaded yellow, all other regions are white. The sketch provides a visualization aspect that helps the student respond to the various parts of the problem. Scenario: Imagine that an arrow hitting the target shown is equally likely to hit any point on the target. The 10-point circle has a 4.8 inch diameter and each of the other rings is 2.4 inches wide. Find the probability that the arrow hits the region described. a) the 10-point region b) the yellow region c) the white region d) the 5-point region e) accuracy OR The student determines the solution to the following situation and explains it to someone else in the class. Buses arrive at a resort hotel every 15 minutes, wait for three minutes while passengers get on and get off, and then depart. What is the probability that there is a bus waiting when a hotel guest walks out the door at a randomly chosen time. (Hint: Drawing a sketch helps determine the solution.) √ accuracy √ clarity of communication

STRAND V: LITERACY
CONTENT STANDARD: The student communicates mathematical principles through reading, writing, and speaking opportunities.

BENCHMARK: The student demonstrates through a variety of writing and speaking requirements proficiency in reading comprehension, specialized vocabulary, and reasoning.

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
	The following performance standards align with the Albuquerque Public Schools 10th grade Language Arts Standards.	**Note:** The very nature of mathematics courses require the student to read the textbook (e.g., word problems); to learn the vocabulary of mathematics; to communicate symbolically, orally, and in written formats; and to think critically through problem solving. Through consistent integration of the mathematical processes, the student works collaboratively with other students, requiring whole or small group discussions; listens to other's viewpoints whether it be via print, technology, or guest speaker; displays data in an organized fashion; and makes connections. Consequently, literacy strategies are integrated and reflected in every strand. The following citations illustrate specific examples of these strategies although numerous opportunities are presented throughout the year and throughout the curriculum.
	1. Prioritizes and organizes information to construct a complete and reasonable interpretation of a given situation (APS – LA I.3). 2. Expands vocabulary using knowledge of the origins and meanings of common, learned, and foreign words used frequently in written and spoken English (APS – LA I.4). 3. Reads critically and independently to draw conclusions from research (APS – LA II.9). 4. Analyzes how the historical context of a literary work affects its meaning (APS – LA II.11). 5. Develops increased competence and fluency in using the writing process to create a final product (APS – LA III.1).	1 – 4. When a student studies mathematics, he/she learns a new "language". Reading the textbook requires a different level of comprehension from reading a literature book. Because there is so much terminology to learn, the student takes notes as he/she reads the text, keeps a list of new vocabulary words and possible origins of the words, and draws and labels pictures to help him/her increase his/her comprehension as well as help him/her visualize new concepts. He/She can also read the text aloud cooperatively and discuss key ideas and derivation of formulas. √ reading analysis √ individual participation in discussions √ comprehension of key ideas √ vocabulary compilation 5, 7. In a standards-based mathematics classroom the student expresses himself/herself more through writing and communicates comprehension in a variety of ways. He/She writes: • explanations of key concepts • metaphors related to a geometric concept (e.g., My life is like a hexagon because…) • poems, stories, autobiographies

GRADE 9 - 12	PERFORMANCE STANDARDS	ILLUSTRATIONS
		• compiles a portfolio to include reflection pieces on each entry √ effective writing elements √ expression of ideas Examples: The student explains in writing why a right triangle can be isosceles, but not equilateral and why if a triangle is equilateral it is also isosceles. √ expression of ideas √ accuracy OR  2 The student writes a paragraph stating what is wrong with the figure. √ understanding of theorems regarding angles and sides of a triangle √ clear communication
	6. Develops increased competence in using a variety of technology to present information appropriate for the intended purpose and audience (APS – LA III.3). 7. Develops increased competence and fluency in using writing conventions (APS – LA III.4). 8. Develops increased competence with speaking strategies (APS – LA IV.1). 9. Analyzes an instance of public speaking or media presentation (APS – LA V.1). 10. Uses a variety of information resources to critically interpret and evaluate experiences, language, and ideas (APS – VI.2). 11. Uses multiple resources to gather information to evaluate problems, examine cause and effect relationships, and answer research questions to inform an audience (APS – LA VI.3). 12. Defends positions on research issues (APS – LA VI.7).	6. Using geometry software, the student draws an obtuse, a right, and an acute triangle. He/She then sketches and labels appropriately in each triangle an altitude, a median, an angle bisector, and a perpendicular bisector. √ use of technology √ accurate identification of parts 8 – 12. The student selects and researches a topic (teacher-approved) that has geometrical significance (e.g., George Seurat's work, Golden ratio, Maurits Escher, Euclid) and presents findings to the class incorporating visuals. √ thorough research √ relevance √ effective visuals √ compelling presentation √ audience response