

Course: Algebra II

Unit # 7: Trigonometric Functions

Overarching Question:

How can the unit circle be used to develop a circular definition of trigonometric functions?

Previous Unit:

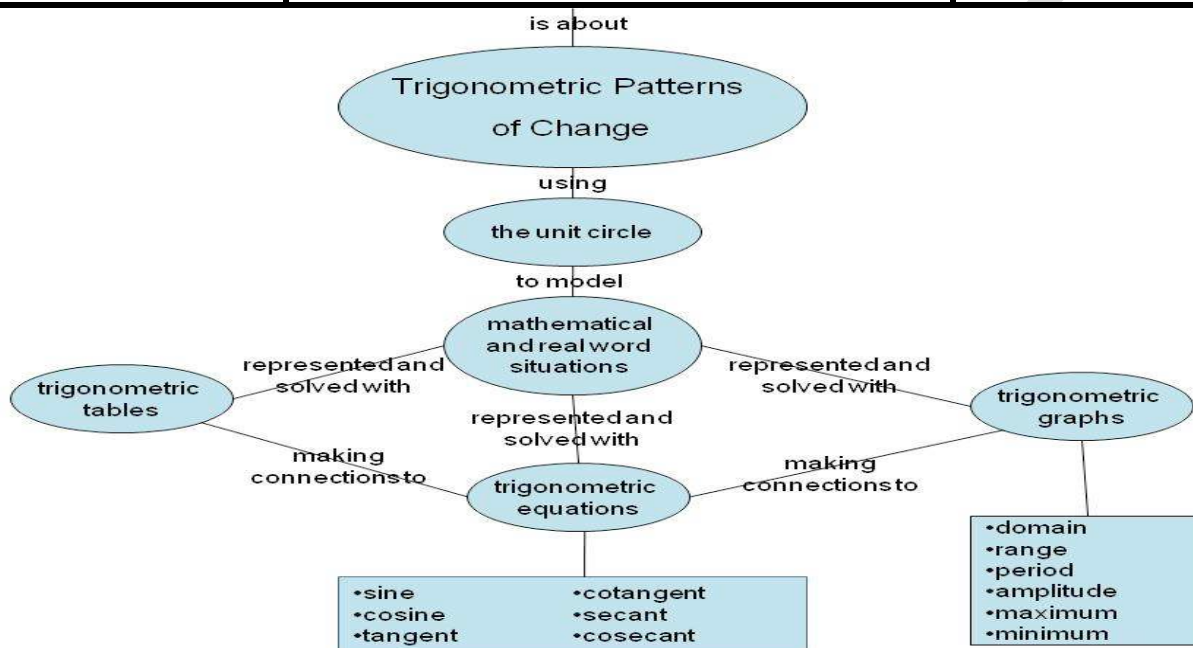
Conic Sections

This Unit:

Trigonometric Functions

Next Unit:

Probability



Questions to Focus Assessment and Instruction:

1. What is the relationship between degree and radian measures of angles? Why or when would you use degree or radian?
2. How can the effect of transformations on the sine and cosine curves be seen in the graphs and tables of these functions?
3. How can the unit circle be used to generate the sine and cosine graphs?
4. Why are the trigonometric functions periodic?
5. How do you know if a function is periodic?

Intellectual Processes (Standards for Mathematical Practice):

Model with mathematics: Use trigonometric functions and transformations of these functions to model periodic behavior.

Look for and express regularity in repeated reasoning: Observe the relationships between the sine and cosine using the unit circle to generate the periodic graphs of these functions.

Use appropriate tools strategically: Use available tools to determine trigonometric values including exact values of special angles, as well as tables, calculators and computers.

Key Concepts

Sine

Cotangent

Unit Circle

Period

Maxima

Cosine

Secant

Domain

Phase Shift

Minima

Tangent

Cosecant

Range

Amplitude

Asymptote

Degree and radian relationship and conversion

Transformations of trig function from the parent functions

Relate graphs of trigonometric functions to their inverses

Unit Abstract

Previous studies of trigonometry have included right triangle trigonometric relationships and the Laws of Sines and Cosines. In this unit students develop a circular definition of trigonometric functions using both degree and radian measure. The unit circle is the basis for these definitions and leads to graphs of the sine and cosine curves. Students can then define the tangent and inverse trigonometric functions based on the sine and cosine. Making connections between the other trigonometric functions and the graphs and tables of the sine and cosine functions provides a context for students to develop the graphs and tables of the tangent, cotangent, secant, and cosecant functions. Building on their previous knowledge of transformations (both algebraic and geometric), students explore and graph the effects of transformations (amplitude, period, and phase shift) on the sine and cosine curves.

Common Core State Standards

Interpreting Functions (F-IF)

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions (F-BF)

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
 - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*

Trigonometric Functions (F-TF)

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x , $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.
9. (+) Prove the addition and subtraction formula

Instructional Resources

NCTM Illuminations (<http://illuminations.nctm.org>)

Graphs from the Unit Circle: In exploring the unit circle, students will discover the sine and cosine curves, and their properties.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L785>

Seeing Music: After calculating frequencies of the notes in a chromatic scale, students will compare the sine waves to see and hear the trigonometry behind harmonious and dissonant note combinations.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L686>

Trigonometry for Solving Problems: This lesson offers a pair of puzzles to enforce the skills of identifying equivalent trigonometric expressions.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L383>

Trigonometric Graphing: This applet allows students to explore the effect of the parameter changes in the graph of the function $y = a \cdot \sin(bx+c)+d$.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=174>

Squares on a Triangle: Exploration of the Law of Cosines and its' relationship to the Pythagorean Theorem are the focus of this lesson.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L716>

NCTM Reasoning and Sense Making Task Bank (www.nctm.org/hsfocus)

Tidal Waves: Students analyze a problem faced by the captain of a shipping vessel. They use a range of functions to model the situation and reflect on their usefulness. Because trigonometric functions can be useful, this task would be particularly appropriate for students who have had an introduction to graphing sine and cosine functions. Online applet (optional; <http://MathRSM.net/applets/tidal>)

http://www.nctm.org/uploadedFiles/Journals_and_Books/Books/FHSM/RSM-Task/Tidal_Waves.pdf

Texas Instruments (<http://education.ti.com>)

The Transform App on the Ti-84 family can be used to explore changes in trigonometric functions. Select the application on your calculator, and then enter the appropriate trig function in the $y=$, (e.g. $y = a \sin(b(x-c))+d$)

The Sound of Music: This activity uses tuning forks and the CBL to gather sound wave data for students. They will then write Sine (or Cosine) functions to best match the collected data (without using the regression capability of the calculator).

<http://education.ti.com/calculators/downloads/US/Activities/Search/Subject?s=5022&sa=5025&t=5075&d=9>

Biorhythms and Sinusoidal Functions: Students use information about their own biorhythms to create three sine curves that are graphed on the same axes. They write equations for each curve and make predictions about future days based on the graphs.

<http://education.ti.com/calculators/downloads/US/Activities/Detail?id=7576&ref=%2fcalculators%2fdownloads%2fUS%2fActivities%2fSearch%2fSubject%3fs%3d5022%26sa%3d5025%26t%3d5075%26d%3d9>

Find that Sine: Students will use sinusoidal regression to determine equations to model various data sets and use the equations to make inferences.

http://education.ti.com/xchange/US/Math/PrecalculusTrig/9733/FindSine_TI84.pdf

NUMB3RS Activity: Changing Sines Episode: “Counterfeit Reality”:

Students are presented with a real-life application of sine curves and asked to explore changes in the sine function as various parameters are modified. On the worksheet provided, students are asked for explanations about why the graph moves for each transformation.

http://education.ti.com/xchange/US/Math/PrecalculusTrig/7716/Ep107_ChangingSines_CounterfeitReality.pdf

Geometer’s Sketchpad

Exploring Algebra 2 with The Geometer's Sketchpad This book, published by Key Curriculum Press, contains activities exploring trigonometry. Demonstrations and activities help users explore Algebra 2 topics, including functions and relations, transformations of functions, systems of equations, matrices, and fitting functions to data. **Chapters include:** Functions and Relations, Systems, Quadratic Functions, Algebraic Transformations, Other Functions, Trigonometric Functions, Probability and Data, Vectors and Matrices

Worksheets exploring a variety of trigonometric topics such as “A Sine Wave Tracer” and “Six Trigonometric Functions” are found here. Each worksheet describes the steps necessary for the activity to be used with Geometer’s Sketchpad.

<http://www.dynamicgeometry.com/documents/recentTalks/trigNCTM08/Wave%20Activity%20Worksheets.pdf>

Geogebra

This is an index page for a wide variety of applets useful in trigonometry and written in Geogebra.

<http://www.geogebra.org/en/wiki/index.php/Trigonometry>

Explore the Trigonometric Functions and Their Graphs: This applet allows exploration of the effect of changing parameters in the six trigonometric functions using either degrees or radians. Questions pertaining to this exploration are also provided.

http://www.geogebra.org/en/upload/files/couture_daniel/TrigFunctions.html

Sine and Cosine Activity: A worksheet is provided to guide students through their exploration of the functions $y = \sin(b(x+c))+d$ and

$y = \cos(b(x+c))+d$. Sliders are used to demonstrate the impact of changing the parameters a , b , c , and d to transform the functions.

<http://www.geogebra.org/en/wiki/index.php/Trigonometry>

Other Applets

A game where students estimate the angle that yields a specific y -value helps students connect the unit circle to the sine curve.

<http://www.ies.co.jp/math/java/trig/sineshot/sineshot.html>

Trigonometric and Inverse Trigonometric Function Gallery provides demos illustrating selected families of trigonometric functions.

http://mathdemos.gcsu.edu/mathdemos/family_of_functions/trig_gallery.html

A variety of math applets related to trigonometry are found on this index page.

<http://www.ies.co.jp/math/java/trig/index.html>

The applet Function Plotter, allows entry of a variety of functions, including trigonometric functions, and allows exploration of the functions entered. Multiple functions from any function family can be entered on the same graph screen.

<http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan>

Many applications of trigonometric functions are discussed on this site. In particular there is an interactive applet, which gives a sample sine or cosine curve for students to manipulate the data in a set of exercises.

<http://mathdemos.org/mathdemos/sinusoidapp/sinusoidapp.html>

Assessment

The applet at the bottom of this webpage gives students a parent function on the graph and then asks them to transform the graph to a new equation. The applet is self-checking and has multiple difficulty levels. Another self-checking applet asks students to calculate amplitude, period or phase shift of a given function. A third applet allows students to explore trig identities by transforming the graphs. All are useful for formative assessments.

<http://colalg.math.csusb.edu/~devel/precalcdemo/circtrig/src/sineshift.html>

Twenty different sinusoid curves are provided with this applet which allows students to manipulate the values of a , b , and c in the equations

$y = a \sin(bx) + c$ or $y = a \cos(bx) + c$ to match the graph displayed.

<http://mathdemos.gcsu.edu/mathdemos/sinusoidapp/sinusoidpractice.html>

The applet Recognize Function 3, found on this website, provides equations of functions. The graphs of the functions are then dragged over to the appropriate equation. A check is provided.

<http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan>

The applet Recognize Graphs 3, found on this website, provides graphs of functions. The goal is to drag the equations provided over to the correct graph. Answers can be checked as you go.

<http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan>

Professional Resources

NCTM (www.nctm.org)

Focus in High School Mathematics: Reasoning and Sense Making: This publication elevates reasoning and sense making to a primary focus of secondary mathematics teaching. It shifts the teachers' role from acting as the main source of information to fostering students' reasoning to make sense of the mathematics.

<http://www.nctm.org/catalog/product.aspx?ID=13494>

Focus in High School Mathematics: Reasoning and Sense Making in Algebra:

Reasoning about and making sense of algebra are essential to students' future success. This book examines the five key elements (meaningful use of symbols, mindful manipulation, reasoned solving, connection algebra with geometry, and linking expressions and functions) identified in Focus in High School Mathematics: Reasoning and Sense Making in more detail and elaborates on the associated reasoning habits.

<http://www.nctm.org/catalog/product.aspx?ID=13524>

Articles from National Council of Teachers of Mathematics (www.nctm.org)

Articles available as free downloads to NCTM members, or for a fee to non-members

Kaplan, G. (2008). Trigonometry through a Ferris Wheel. *Mathematics Teacher*, 102 (2), 138-143. Retrieved February 18, 2011 from:

http://www.nctm.org/eresources/article_summary.asp?from=B&uri=MT2008-09-138a

Calzada, M. and Scariano, S. (2006). A Natural Bridge from Algebra and Geometry to Trigonometry. *Mathematics Teacher*, 99 (6), 450-458. Retrieved February 18, 2011 from:

http://www.nctm.org/eresources/article_summary.asp?from=B&uri=MT2006-02-450a

Bressoud, D. (2010). Historical Reflections on Teaching Trigonometry. *Mathematics Teacher*, 104 (2), 106-112. Retrieved February 18, 2011 from:

http://www.nctm.org/eresources/view_media.asp?article_id=9389

Gutmann, T. (2003). A Direct Approach to Computing the Sine or Cosine of the Sum of Two Angles. *Mathematics Teacher*, 96(5), 314-318. Retrieved February 18, 2011 from:

http://www.nctm.org/eresources/view_media.asp?article_id=6191