Inventories, Unobservable Heterogeneity and Long Run Price Elasticities*

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Abstract

Studies of competition when information on actual firm costs is unavailable require consistent estimates of long run demand price-elasticities. When consumers stockpile, traditional static discrete-choice models overestimate long-term price responses. In this paper, we develop a dynamic model of demand with inventories and estimate the structural parameters fully accounting for consumers' unobservable heterogeneity, but without having to solve the dynamic programming. We find a significant quantitative difference between the price-elasticities yielded by the static and inventory model, pointing to the risks of making wrong policy recommendations based on short run measures.

1 Introduction

Consistently estimating demand is an empirical task of great importance in the area of Industrial Economics. Since information on production costs and wholesale prices is rarely available, the study of market structures requires the use of estimated preference parameters from which sample market shares and price cost margins can be recovered. The traditional empirical literature in the

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area has relied on standard static discrete choice models such as described in McFadden (1980, 1984). Examples are Berry, Levinsohn and Pakes (1995), who study the automobile industry, and Nevo (2001), Berto Villas-Boas (2007), and Bonnet and Dubois (2006) who estimate demand for ready-to-eat cereal, yogurt, and bottled water respectively, among many others.

While the static discrete-choice models are an interesting and relatively simple framework for studying individual demand, the recent literature casts doubts on the appropriateness of the resulting estimated price-elasticities for most demand applications. The study of mergers and market structure, for example, requires knowledge of long run responsiveness to price. As noticed by Hendel and Nevo (2006a, 2006b), if consumer behavior includes stockpiling, for instance, then short and long run elasticities differ, and static models will yield short-run elasticities which overestimate the long-run measures.

A number of papers bring evidence that dynamics in the form of inventories are an important component of individual demand. If consumers stockpile then the decision to purchase today is affected by past prices and future expected prices. Reduced-form studies that test whether consumers stockpiling behavior is relevant include Boizot, Robin and Visser (2001) and Hendel and Nevo (2006a).

Hendel and Nevo (2006a) document purchasing patterns that are consistent with stockpiling behavior. More specifically, they find that duration since last sale positively affects aggregate quantity purchased, both during sales and non-sales periods. This is consistent with a model of stockpiling where consumers follow a s-S inventory rule. The longer the duration since previous sale, the closer consumers will on average be to the lower-bound inventory threshold, making purchase more likely. Furthermore, they find that indirect measures of storage costs are negatively correlated with the probability that households buy large quantities on sale. The authors also report a significant difference between sales and non-sales purchase in what concerns duration from previous purchase and duration to next purchase. Duration to previous purchase is shorter during sales periods than during non-sales. The idea is that during sales consumers will take advantage of the lower prices and buy at higher levels of current inventory, i.e., duration to previous purchase is shorter. Since inventories will be at higher levels, on average it will take longer for consumers to reach purchase threshold and duration to next purchase will therefore be longer.

A very similar model is used by Boizot et al. (2001). The only differences are that in the latter time is continuous and consumption is assumed to be exogenous and constant (actually, Hendel and

Nevo also assume exogenous constant consumption when they estimate demand. The theoretical model itself however determines consumption endogenously). They perform reduced form empirical analysis, finding evidence that corroborates the consumer inventory theory.

More recently, some researchers have structurally estimated demand under stockpiling behavior of consumers. This is the case of Hendel and Nevo (2006b), Erdem, Imai, and Keane (2003), and Sun (2005).

Hendel and Nevo (2006b) structurally estimate a model of household demand for a storable product that incorporates the dynamics dictated by stockpiling behavior. Their goal is to assess and quantify the implications of stockpiling on demand estimation and, in particular, compare the resulting estimates to the ones obtained from standard static models. In the dynamic model, households purchase both for current consumption and for inventory building. Consumers increase inventory when the difference between current and expected future price is lower than the cost of holding inventory. To estimate the model, the authors use weekly scanner data on laundry detergents collected in nine supermarkets of a large U.S. mid-west city. The state space includes prices and advertising expenditures for all brands in all sizes of the products. Hendel and Nevo suggest an interesting method to reduce the state space: in their model, the probability of choosing a brand, conditional on quantity, does not depend on dynamic considerations. Therefore, a large number of parameters can be estimated from a static brand-choice model, without solving the dynamic program. Estimation follows an adjusted version of the "nested algorithm", as proposed by Rust (1987), where the value function is approximated by policy function iterations as suggested by Benitaz-Silva et al. (2000). Results suggest that ignoring dynamics has strong implications on demand estimation. The static model overestimates own price elasticities, underestimates cross-price elasticities to other products, and overestimates substitution to no purchase outside option. Resulting estimated price-cost margins using the figures yielded by standard static models will be biased downwards.

Close to the work of Hendel and Nevo is that of Erdem, Imai and Keane (2003). The main difference between the two papers is related to how they reduce the complexity of the state space. Erdem et al. assume that once consumption is determined, each brand in stock is consumed at a rate proportional to the share of that brand in storage. Their method is more flexible in modelling unobserved consumer heterogeneity but at a higher computational cost. Which method is more convenient will depend on the market or industry under study. In particular, for applications where the choice sets are large, the Erdem et al.'s method is difficult to apply.

Finally, Sun (2005) studies promotion effects on consumption, which is endogenous but not uncertain as in Hendel and Nevo (2006b). The shock to utility is assumed to be logistically distributed so that product choice probabilities are multinomial logit. To solve the dynamic program, Sun adopts simulated maximum likelihood techniques employing Monte Carlo methods (Keane, 1993) in addition to the interpolation method (Keane and Wolpin, 1994) to estimate parameters. The model is applied to individual purchases of packaged tuna and yogurt.

Notice that a common aspect of the models above is that estimation of structural parameters require solving the dynamic program, which is a complicated numerical problem, as well as very costly in terms of computer time (see Rust, 1996). In this paper we develop a way of structurally estimating long run price responses of consumers under inventory behavior without having to solve the dynamic program. That is one of the main contributions of our work. We use a model of dynamic consumer demand similar to Hendel and Nevo (2006a)'s model, where an additional assumption on consumption permits not only to empirically test the validity of the model but also to estimate the structural parameter needed to calculate the long run price elasticities.

Our methodology is extremely flexible with respect to consumer unobserved heterogeneity: the main estimated parameter is household-specific, thus yielding household-specific price elasticities. Developing a method that fully accounts for heterogeneity is the second important contribution of this paper. We compare the long run and short run elasticities, finding that the short run measure overestimates long run elasticities by more than 80% on average, making it clear that considering consumer stockpiling behavior makes a significant quantitative difference in price elasticities. Results are robust to different price expectation hypothesis as well as different estimation methods.

We use a home scan survey. Household level information on store visits, purchases, and prices paid were collected during three years (1999, 2000, 2001) from a nationally representative survey. Data on household characteristics, including characteristics of the home and of the individuals composing the household, and store characteristics were also collected.

This work is organized as follows. In the next section, we present the inventory model and derive the testable implications, the purchase decision equations, and the long and short run demand price elasticities implied by the model. In section 3, we compare our methodology to that in Hendel and Nevo (2006b) and in Erdem et al. (2003). Data description as well as descriptive statistics for some of the variables used in the empirical analysis can be found in Section 4. Section 5 brings the econometric implementation and empirical results, while the sixth section checks the robustness of the results. Section 7 concludes.

2 The Model

Empirical studies of market competition need an unbiased estimate of the long-run demand price elasticity in order to calculate price cost margins when information on actual costs is not available. The usual discrete-choice static models of demand yield estimates of the short-term price elasticities, which will be different from the long-run elasticities if some dynamic component influences consumers' purchase choice. In particular, if consumers hold inventories, the static model will not yield the desired elasticities. To correct for this problem, we propose a dynamic demand model with random prices which takes into account the possibility that consumers stockpile. Making an assumption on consumption at periods without purchases, we are able not only to derive testable implications of the model but also to structurally estimate model parameters without having to solve the dynamic program. Finally, we derive the short and long-run demand price elasticities implied by the dynamic model and show they are indeed different.

2.1 Consumer Behavior with Inventories

The net per period utility of consumer i is equal to $u\left(c_{it}\right)$, where u is an increasing and concave function of consumption at period t, c_{it}. At each period, consumer i must decide how much to purchase of a certain good, how much to consume, and how much to stock as inventory. The law of motion of inventories is:

$$y_{it} = q_{it} - c_{it} + x_{it-1} \tag{1}$$

where y_{it} is the end of the period level of inventories, and x_{it} is the beginning of the period level, i.e., before consumption c_{it} and purchases q_{it}. Notice that

$$y_{it-1} = x_{it}$$

The problem of the consumer i at any period t is:

$$\max_{\{c_{it},q_{it},y_{it}\}} E_t \left\{ \sum_{t=\tau}^{\infty} \delta^t \left[u\left(c_{it}\right) - \alpha_i p_t q_{it} - \Phi\left(y_{it}\right) \right] \right\} \tag{2}$$

$$s.t. \quad y_{it} = q_{it} - c_{it} + x_{it} \quad (\lambda_{it})$$

$$y_{it} = x_{it-1}$$

$$q_{it} \geqslant 0 \quad (\Psi_{it})$$

$$y_{it} \geqslant 0 \quad (\mu_{it})$$

where α_i is the marginal utility of revenue, and the parameters between brackets are the Lagrange multipliers of each constraint. The function $\Phi\left(y_{it}\right)$ represents the cost of storing inventory, which is an increasing and convex function of inventories. Assume $\Phi\left(y_{it}\right) = \phi y_{it}^2$.

At the beginning of each period (before purchases) consumers learn the practised prices at the period. The Lagrangian of the problem is:

$$\pounds = \max_{\{c_{it},q_{it},y_{it}\}} E_t \left\{ \sum_{t=0}^{\infty} \delta^t \left[u\left(c_{it}\right) - \alpha_i p_t q_{it} - \phi y_{it}^2 + \lambda_{it}\left(q_{it} - c_{it} + x_{it} - y_{it}\right) + \Psi_{it} q_{it} + \mu_{it} y_{it} \right] \right\}$$

The first order conditions with respect to consumption at period t (c_{it}), purchase at period t (q_{it}), and end of the period inventory at period t (y_{it}), are, respectively:

$$\frac{\partial \pounds}{\partial c_{it}} = 0 \Rightarrow u'\left(c_{it}\right) = \lambda_{it} \tag{3}$$

$$\frac{\partial \pounds}{\partial q_{it}} = 0 \Rightarrow \alpha_i p_t = \lambda_{it} + \Psi_{it} \tag{4}$$

$$\frac{\partial \pounds}{\partial y_{it}} = 0 \Rightarrow y_{it} = \frac{\delta E_t\left(\lambda_{it+1}\right)}{2\phi_i} - \frac{\lambda_{it}}{2\phi_i} + \frac{\mu_{it}}{2\phi_i} \tag{5}$$

We assume positive consumption every period and that consumers always expect future consumption to be positive. This assumption can be strong for certain product categories. But here, as we will see later, we only consider products that are consumed at regular basis and for which we expected households to consume a positive amount at every period.

Manipulating the first order conditions and given the assumptions above, we get our main result:

Proposition 1 *In periods with purchases ($q_{it} > 0$), if the price is higher than the discounted expected price for the following period ($p_t > \delta E_t\left(p_{t+1}\right)$), the utility maximizing end of the period inventory level is equal to zero ($y_{it} = 0$). Furthermore, in this case, next period purchased quantity (q_{it+1}) will be positive. On the other hand, if $p_t \leq \delta E_t\left(p_{t+1}\right)$, then $y_{it} > 0$ in periods with purchases* *and periods without purchases, and next period purchased quantity can be either positive or equal to zero.*

Proof. When $q_{it} > 0$ ($\Psi_{it} = 0$), we have: $y_{it} = \frac{\alpha_i(\delta E_t(p_{t+1})-p_t)}{2\phi_i} - \frac{E_t\Psi_{it+1}}{2\phi_i} + \frac{\mu_{it}}{2\phi_i}$. Assume $y_{it} > 0$ when $p_t > \delta E_t(p_{t+1})$. Then, $\mu_{it} = 0$ and $y_{it} = \frac{\alpha_i(\delta E_t(p_{t+1})-p_t)}{2\phi_i} - \frac{E_t\Psi_{it+1}}{2\phi_i}$, which implies $y_{it} < 0$ (since $p_t > \delta E_t(p_{t+1})$ and $\frac{E_t\Psi_{it+1}}{2\phi_i} \geqslant 0$), which contradicts $y_{it} > 0$. Thus $y_{it} = 0$ when $p_t > \delta E_t(p_{t+1})$ and $q_{it} > 0$.

When $y_{it} = 0$, next period purchase is going to be equal to: $q_{it+1} = y_{it+1} + c_{it+1}$. The end of the period inventory level is non-negative, while consumption is assumed to be positive at every period. Therefore, if $y_{it} = 0$, $q_{it+1} > 0$.

Now, take periods when $q_{it} > 0$ ($\Psi_{it} = 0$) and $p_t \leq \delta E_t(p_{t+1})$. Assume that $\delta E_t(p_{t+1})$, in this case, $y_{it} = 0$ ($\mu_{it} > 0$). Then $y_{it} = \frac{\alpha_i(\delta E_t(p_{t+1})-p_t)}{2\phi_i} - \frac{E_t\Psi_{it+1}}{2\phi_i} + \frac{\mu_{it}}{2\phi_i}$. Now, $E_t\Psi_{it+1}$ will be greater than zero if and only if the expected purchase at period $t+1$ (q_{it+1}) is equal to zero. When $y_{it} = 0$, the expected purchase next period is equal to: $E_t(q_{it+1}) = E_t(y_{it+1} + c_{it+1}) = E_t(y_{it+1}) + E_t(c_{it+1})$. Notice that this expectation is going to be greater than zero since we assume consumers always expect next period consumption to be positive and since end of the period inventories are non-negative. But if $E_t(q_{it+1}) > 0$, then $E_t\Psi_{it+1} = 0$ and $y_{it} = \frac{\alpha_i(\delta E_t(p_{t+1})-p_t)}{2\phi_i} + \frac{\mu_{it}}{2\phi_i}$, which implies $y_{it} > 0$ (since $p_t \leq \delta E_t(p_{t+1})$ and $\mu_{it} > 0$), contradicting $y_{it} = 0$. Therefore, it must be that $y_{it} > 0$ in periods when purchases are positive and the price is lower than the regular price.

Now, let $q_{it} = 0$ ($\Psi_{it} > 0$) and $p_t \leq E_t(p_{t+1})$. Assume $y_{it} = 0$. Then $\mu_{it} > 0$ and $E_t\Psi_{it+1} = 0$, and $y_{it} = \frac{\alpha_i(\delta E_t(p_{t+1})-p_t)}{2\phi_i} + \frac{\Psi_{it}}{2\phi_i} + \frac{\mu_{it}}{2\phi_i}$, which implies $y_{it} > 0$, contradicting the initial hypothesis $y_{it} = 0$. Thus $y_{it} > 0$ when $q_{it} = 0$ and $p_t \leq \delta E_t(p_{t+1})$.

The next period purchase is going to be equal to: $q_{it+1} = y_{it+1} - x_{it+1} + c_{it+1}$, which is equal to zero if $y_{it+1} \geqslant y_{it} - c_{it+1}$, and greater than zero otherwise. ■

Although prices are random, we assume consumers always expect prices to return to their regular level p_r (including the discount[1]), which is equal to the mean price they pay for the good. At first view, this may seem a strong assumption, given that stores are known to frequently offer discounted prices. However, notice that the regular price, defined as the mean price paid by the consumer across all her purchase occasions, already incorporates discount prices and their probabilities. Therefore, what may be controversial in our hypothesis of expected next-period price being always equal to

[1] Since the time period we consider is short (one week), we let the discount rate be very close to 1. So $E_t(p_{t+1}) = \delta^{-1}p_r \simeq p_r$, where p_r is the regular price.

regular price is the assumption that future prices expectations are independent of price realizations today[2]. Our assumption is valid if consumers are unable to predict the timing of sales. Indeed, a number of theoretical studies show that in an important number of situations sales are necessarily random. See, for example, Braido (2009) and references therein.

Anyway, we do not need the assumption that price expectations are equal to the observed average price. For instance, we could assume price expectations are rational and estimate a Markov price process. The rational expectation hypothesis would then imply that expected prices would be equal to the predicted price process. Although in general this may be a preferred solution, we only do that in the robustness check section of the paper. We chose an alternative route of action because we are not sure how much we can trust our price process estimates since we do not observe prices, only prices paid by the households. But if our model is to be applied to a database which brings enough information to assure consistent estimates of the price process, there is no reason why the rational expectation hypothesis should not be used.

Finally, notice that the absence of "precautionary" stocking is not implied by the assumptions on expectations. Rather, it is due to the assumption that preferences are quasi-linear, which is quite a standard assumption in the literature without which it would be much harder to estimate model parameters. The quasi-linearity of preferences imply that the marginal utility of consumption is separable from the marginal utility of income and linear in prices. Hence, there is no concavity which would trigger precautionary behavior.

From the proposition above, we know that, given purchases today at non-discounted prices ($p_t > p_r$), consumers will certainly purchase next period. On the other hand, if they purchase today at a lower than regular price, they will hold positive inventories at the end of the period and they may not have to purchase next period. Thus, the probability of purchasing next period is higher when the consumer purchases today at higher prices. This is the idea behind the first implication of the model:

Implication 1 Conditional on inventories, duration until next purchase is higher at discounted price periods.

Purchases are going to be positive when $q_{it} = y_{it} - x_{it} + c_{it} > 0$. Hence, the level of inventory at the beginning of the period, x_{it}, that triggers purchase is $\tilde{x}_{it} = y_{it} + c_{it}$, which is lower at discounted prices than at regular or higher prices since both y_{it} and c_{it} decrease with current prices. Thus:

[2] The standard assumption (see Hendel and Nevo, 2006) is that price expectations are first order Markov.

Implication 2 Conditional on inventories, duration from last purchase is lower at periods of discounted price.

Moreover, the higher the marginal cost of holding inventories ϕ_i, the lower the chosen inventory level y_{it}, and the lower $\tilde{x}_{it}$ that triggers purchases. Therefore, the lower ϕ_i, the higher the frequency of purchases. If we compare two households, l and j, such that $\phi_l > \phi_j$, we should expect household l to purchase more frequently than household j:

Implication 3 Conditional on inventories, households with a high marginal cost of holding inventories purchase more frequently than households with low marginal costs.

Of course, we do not observe the marginal cost of holding inventories. However, we can make some reasonable assumptions on the ordering of the marginal costs. Take the purchase of butter, for example, which must be stocked in the refrigerator. Then stocking butter is certainly more costly for a household who has a refrigerator than for a household who does not. In general, an important part of the cost of stockpiling is the space cost. The less space a household has available for stocking, the higher the marginal costs of stocking. Thus, households that live in bigger houses probably have lower marginal costs of holding inventories. We can therefore test Implication 3 using some observable characteristics of the households' homes as indirect measures of space availability.

Notice that testing the above implications is equivalent to testing the dynamic model of consumer decision. The alternative hypothesis is the static model, where duration is independent of prices because a price variation will be completely translated into consumption variation, and will not affect decisions in other periods.

2.2 The Purchase Decision

From Proposition 1, we know that $\Pr(y_{it} = 0 \mid q_{it} > 0, p_t > p_r) = 1$. Thus

$$E_t(q_{it} \mid q_{it} > 0, p_t > p_r, y_{it} = 0) = E_t(q_{it} \mid q_{it} > 0, p_t > p_r)$$

Furthermore, if $y_{it} = 0$ then $q_{it} = c_{it} - x_{it}$, where $c_{it} = h(\alpha_i p_t)$ and $h = u'^{-1}$. Therefore, conditional on purchases and prices being higher than regular:

$$\begin{aligned} E_t(q_{it} \mid q_{it} > 0, p_t > p_r) &= E_t(c_{it} - x_{it} \mid q_{it} > 0, p_t > p_r) \qquad (6) \\ &= E_t(h(\alpha_i p_t) - x_{it} \mid q_{it} > 0, p_t > p_r) \end{aligned}$$

When $p_t \leq p_r$ and $q_{it} > 0$ ($\Psi_{it} = 0$) on the other hand, y_{it} is always positive ($\mu_{it} = 0$). In this case, we have $q_{it} = y_{it} - x_{it} + c_{it}$, where $y_{it} = \frac{\alpha_i(p_r - p_t)}{2\phi_i} - \frac{E_t \Psi_{it+1}}{2\phi_i}$ and $c_{it} = h(\alpha_i p_t)$. Hence

$$E_t (q_{it} \mid q_{it} > 0, p_t \leq p_r) = E_t \left(\frac{\alpha_i (p_r - p_t)}{2\phi_i} - \frac{E_t \Psi_{it+1}}{2\phi_i} + h(\alpha_i p_t) - x_{it} \mid q_{it} > 0, p_t \leq p_r \right) \quad (7)$$

Now, assume that at each t we observe the purchased quantity q_{it} with an error v_{it}. Substituting the observed quantity $q^*_{it} = q_{it} - v_{it}$ into (6) and (7), we get:

$$\begin{aligned} E_t (q^*_{it} \mid q_{it} > 0, p_t > p_r) &= E_t (q_{it} - v_{it} \mid q_{it} > 0, p_t > p_r) \\ &= E_t (c_{it} - x_{it} - v_{it} \mid q_{it} > 0, p_t > p_r) \end{aligned} \quad (8)$$

and

$$\begin{aligned} E_t (q^*_{it} \mid q_{it} > 0, p_t \leq p_r) &= E (q_{it} - v_{it} \mid q_{it} > 0, p_t \leq p_r) \\ &= E_t \left(\frac{\alpha_i (p_r - p_t)}{2\phi_i} - \frac{E_t \Psi_{it+1}}{2\phi_i} + h(\alpha_i p_t) - x_{it} - v_{it} \mid q_{it} > 0, p_t \leq p_r \right) \end{aligned} \quad (9)$$

The observable variables in (8) are q^*_{it}, the regular price p_r, and the price at period t, p_t. Although the beginning of the period level of inventories x_{it} is not directly observable, we know, from the law of motion of inventories that:

$$\begin{aligned} x_{it} &= x_{i1} + \sum_{n=1}^{t-1} q_{in} - \sum_{n=1}^{t-1} c_{in} \\ &= x_{i1} + \sum_{n=1}^{t-1} q^*_{in} - \sum_{n=1}^{t-1} c_{in} - \sum_{n=1}^{t-1} v_{in} \end{aligned}$$

Moreover, we know from Proposition 1 that at periods t' with purchases ($q_{it'} > 0$) and price higher than regular price ($p_{t'} > p_r$), the chosen end of the period inventory of consumer i is going to be equal to zero ($y_{it'} = 0$), and thus at the beginning of the next period, inventories ($x_{it+1'}$) will also be equal to zero. Or, more formally, if at $t_0(i) < t, q_{i0(i)} > 0$ and $p_{0(i)} > p_r$, then $y_{i0(i)} = 0$ and $x_{i1(i)} = 0$. Therefore, we will consider household i's period zero as the period $t_1(i)$ immediately following period $t_0(i)$,which is the first period when prices are higher than regular and household i purchases. In that way, we are sure that x_{i1} is equal to zero and we can write:

$$x_{it} = \sum_{n=t_1(i)}^{t-1} (q^*_{in} - c_{in} - v_{in}) \quad (10)$$

where we observe q^*_{it} for all t and can thus calculate $\sum\limits_{n=t_1(i)}^{t-1} q_{in}$.

In what concerns consumption, its utility maximizing level at periods $\tilde{t}$ with purchases ($q_{i\tilde{t}} > 0$), $\tilde{t} \in \{1, 2...\}$, is $c_{i\tilde{t}} = h(\alpha_i p_{\tilde{t}})$. However, for periods $\bar{t}$ without purchases ($q_{i\bar{t}} = 0$), we only know that $c_{i\bar{t}} = x_{i\bar{t}} - y_{i\bar{t}}$, which we cannot use in (10) to calculate beginning of the period inventories. We assume, therefore, that at periods without purchase, consumption will be equal to consumption at regular prices , that is $c_{i\bar{t}} = h(\alpha_i p_r)$.

2.2.1 Utility Specification

Assume $u(c_{it}) = -(1/\rho)\exp[-\rho c_{it}]$, where ρ is a positive parameter, so that $h(\alpha_i p_t) = -\frac{1}{\rho}\ln \alpha_i - \frac{1}{\rho}\ln p_t$. Let $T_{i1}^{t-1} \in \{t_1(i), ..., t-1\}$ be the set of periods where consumer i purchased, and $T_{i0}^{t-1} \in \{t_1(i), ..., t-1\}$, the set of periods where i does not purchase. Then:

$$\sum_{n=t_1(i)}^{t-1} c_{in} = -\frac{1}{\rho}\left(T_i^{t-1}\ln(\alpha_i) + \sum_{n\in T_{i1}^{t-1}} \ln p_n + T_{i0}^{t-1}\ln p_r\right) \tag{11}$$

where T_i^t is the total number of periods for household i ($T_i^t = T_{i1}^{t-1} + T_{i0}^{t-1} + 1 = t - t_1(i)$).

Substituting (10) and (11) into the purchased quantity equations (8 and 9) yields, respectively:

$$E_t(q_{it}^* \mid q_{it} > 0, p_t > p_r) = -\frac{1}{\rho}T_i^{t-1}\ln(\alpha_i) - \frac{1}{\rho}\left[\ln p_t + \sum_{n\in T_{i1}^{t-1}}\ln p_n + T_{i0}^{t-1}\ln p_r\right] - \sum_{n=t_1(i)}^{t-1} q_{in}^* +$$
$$E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t > p_r\right)$$

and

$$E_t(q_{it}^* \mid q_{it} > 0, p_t \leq p_r) = \frac{\alpha_i}{2\phi_i}(p_r - p_t) + \frac{1}{\rho}T_i^t\ln(\alpha_i) + \frac{1}{\rho}\left[\ln p_t + \sum_{n\in T_{i1}^{t-1}}\ln p_n + T_{i0}^{t-1}\ln p_r\right] -$$
$$\sum_{n=t_1(i)}^{t-1} q_{in}^* + E_t\Psi_{it+1} + E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t \leq p_r\right)$$

Let $Q_{it}^* = \sum_{n=t_1(i)}^{t} q_{in}^*$. Moving $\sum_{n=t_1(i)}^{t-1} q_{in}^*$ to the left hand side, we get:

$$E_t(Q_{it}^* \mid q_{it} > 0, p_t > p_r) = -\frac{1}{\rho}T_i^{t-1}\ln(\alpha_i) - \frac{1}{\rho}\left[\ln p_t + \sum_{n\in T_{i1}^{t-1}}\ln p_n + T_{i0}^{t-1}\ln p_r\right] + \tag{12}$$
$$E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t > p_r\right)$$

and

$$E_t\left(Q_{it}^* \mid q_{it} > 0, p_t \leq p_r\right) = \frac{\alpha_i}{2\phi_i}\left(p_r - p_t\right) + \frac{1}{\rho}T_i^t \ln\left(\alpha_i\right) + \frac{1}{\rho}\left[\ln p_t + \sum_{n \in T_{i_1}^{t-1}} \ln p_n + T_{i0}^{t-1} \ln p_r\right] \tag{13}$$

$$+E_t\Psi_{it+1} + E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t \leq p_r\right)$$

We assume that the errors v_{it} are mean independent of q_{it}, p_t, T_{i0}^{t-1}, T_{i1}^{t-1}, and p_r for all t, which implies that $E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t \leq p_r\right) = E_t\left(\sum_{n=t_1(i)}^{t} v_{in} \mid q_{it} > 0, p_t \leq p_r\right) = 0$.

This means that we are able to estimate the marginal utility of income α_i and the parameter ρ using (12). Unfortunately, we cannot estimate ϕ_i in (13) because of $E_t\Psi_{it+1}$, which is an unknown function varying on period t and on household i. However, α_i is the most important parameter to be estimated when we are ultimately interested in price elasticities because, as will be seen in the next subsection, the marginal utility of income is the model parameter needed to calculate the long run price elasticity.

Instead of a CARA utility, Hendel and Nevo (2006) consider a logarithm utility function ($u(c_{it}) = \ln(cit)$). But in our case the logarithm utility function is too restrictive in terms of price elasticities because in our model they are always equal to 1. Examples of alternative utility functions can be found in Sun (2005), where utility is a second-degree polynomial of consumption, and Erdem et al. (2003), where utility is linear in consumption.

2.3 Price Elasticities: Long Run versus Short Run

If consumers stockpile, short run and long run price elasticities will differ. The long run price-elasticity should only take into account the effect of a price variation on consumption, not in purchases in a certain period, since part of the variation in purchases in a certain period will be due to variation in stocks[3]. The long run price elasticity is therefore the price elasticity of consumption in the inventory model, where purchase and consumption are not the same. What we call the short run price elasticity, on the other hand, measures the responsiveness of purchases to variation in prices. It can be calculated as the price elasticity of purchases in the inventory model, or as the price elasticity of demand in a static model where purchases and consumption are equal at every period.

[3] See Hendel and Nevo (2006a, 2006b). A similar argument is developed in Feenstra and Shapiro (2001).

In this subsection, we compare short (purchases) and long run (consumption) price responses in the inventory model, showing that these two measures are not the same. We also show the expressions for the short run price elasticities yielded by the static model of demand.

Short Run The short run price-elasticity of demand will capture the effect of a variation in prices on the purchased quantity, which in the presence of stockpiling behavior is not necessarily equal to consumption. Hence, the short run price elasticity is actually the price elasticity of purchase in the inventory model and, as shown in the Appendix, it is equal to

$$\begin{aligned} \epsilon_{it}^{SR_d} &= -\frac{h(\alpha_i p_t)}{(V_{it} - v_{it})} \Pr(p_t > p_r) \left[\Phi(V_{it} \mid p_t > p_r) + (V_{it} - v_{it}) \Phi'(V_{it} \mid p_t > p_r) \right] \\ &\quad - \frac{p_t}{V_{it} - v_{it}} \left(\frac{\alpha_i}{\theta_i} + \frac{1}{\alpha_i p_t^2} \right) \Pr(p_t \leq p_r) \left[\Phi(U_{it} \mid p_t \leq p_r) + (U_{it} - v_{it}) \Phi'(U_{it} \mid p_t \leq p_r) \right] \\ &\quad + p_t \left[\Phi(V_{it} \mid p_t > p_r) - \Phi(U_{it} \mid p_t \leq p_r) \right] \frac{d \Pr(p_t > p_r)}{d p_t} \end{aligned} \tag{14}$$

where Φ is the cumulative distribution function of $\sum_{n=t_1(i)}^{t} v_{in}$, and V_{it} and U_{it} are, respectively:

$$V_{it} = h(\alpha_i p_t) - x_{it}$$

and

$$U_{it} = \frac{\alpha_i}{2\phi_i}(p_r - p_t) - E_t \Psi_{it+1} + h(\alpha_i p_t) - x_{it}$$

As can be seen below, the expression for the short run (purchases) price elasticity is different from the expression for the long run elasticity of demand (consumption) under stockpiling behavior.

Unfortunately, we are unable to compute or estimate the elasticity in (14). However, we are able to estimate the short run elasticity implied by a static model of demand behavior. We would like to compare the measures thus obtained with the long run elasticities resulting from the inventory model. The static (short run) elasticities are obtained from a model identical to the inventory model described above, with the exception that dynamics are now ignored. Thus, consumers choose today whether to purchase and how much to purchase taking into account only the current price and the utility realization shock. Furthermore, in the static model, quantity purchased is equal to quantity consumed since there is no stockpiling. Considering $u(c_{it}) = -(1/\rho)\exp[-\rho c_{it}]$ leads to the following first order condition:

$$c_{it} = -\frac{1}{\rho} \ln \beta_i - \frac{1}{\rho} \ln p_t \tag{15}$$

where β_i is the marginal utility of income in the static model and c_{it} equals quantity purchased. Equation (15) implies that the price elasticity in the static model is thus:

$$\epsilon_{it}^{SR_s} = \frac{1}{\ln \beta_i + \ln p_t} \tag{16}$$

The parameter β_i can be identified in (15). The estimated parameters can then be plugged into (16) to obtain measures of the short price elasticities.

Long Run Inventories are a form of intertemporal substitution but in the long run, everything which is purchased will be consumed, since it is from consumption that the individual extracts utility. Therefore, the long run purchased quantity, or the long run demand, depends only on consumption, not on inventories. Hence, a measure of the long run price elasticity should take into account only the effect of prices on consumption. We shall consider the effect of prices on consumption even when there are no purchases because we assume consumption is positive in every period.

When $u(c_{it}) = -(1/\rho)\exp[-\rho c_{it}]$, the long run price elasticity (the consumption price elasticity in the inventory model) of individual i at period t is:

$$\begin{aligned} \epsilon_{it}^{LR} &= \frac{dc_{it}}{dp_t}\frac{p_t}{c_{it}} \\ &= \frac{1}{\rho p_t}\frac{p_t}{\frac{1}{\rho}(\ln \alpha_i + \ln p_t)} \\ &= \frac{1}{\ln \alpha_i + \ln p_t} \end{aligned} \tag{17}$$

Thus, to calculate the long run price elasticity implied by the inventory model, we only need to identify one of the parameters of the model, the α_i. The estimated values can then be directly plugged into (17) to obtain household specific measures of the long run price elasticities.

Notice that the price elasticities of the dynamic model have exactly the same functional form as in the static model (equation 16). What will differ between the short and long run measures is the estimated coefficients for the marginal utility of income ($\hat{\beta}_i$ in the static model and $\hat{\alpha}_i$ in the dynamic model).

3 Comparison with other Methods

In this section, we compare our work to other studies that structurally estimate parameters of the inventory model, namely Erdem et al. (2003) and Hendel and Nevo (2006b)[4]. We are mainly interested in comparing the characteristics of the different models in what concerns simplicity of the estimation method, flexibility of consumer heterogeneity, and product differentiation.

Hendel and Nevo's inventory model is very similar to ours: per period utility is a concave function, there is no stock out or purchase costs, holding inventory is costly, and prices are random. An important difference is that they consider brand choice. Product differentiation takes place only at the time of purchase. Literally, product differences affect the behavior of the consumer at the store but do not give different utilities at the time of consumption. This assumption reduces the state space because instead of the whole vector of brand inventories, only the total quantity in stock matters. Hence, they are able to separate the product (brand) and quantity decisions. Their approach leads to an important computational simplification, which is the main contribution of the paper. However, their model is very restrictive in terms of observable consumer heterogeneity, and it does not allow for unobservable heterogeneity, which would break down the complete separation of brand and quantity.

A less important difference between Hendel and Nevo's model and ours is related to the random term. In their model, the randomness is included as a preference shock . That is, per period utility is a function not only of consumption but also of an additive random shock, $u\left(c_{it}+v_{it}\right)$. Equations to be estimated are exactly the same whether we consider a preference shock on consumption as they do or a measurement error on purchase as we do, only interpretation changes. However, in our model, if we consider v_{it} to be a preference shock on consumption, at each period the decision to purchase a positive quantity will depend on that period's preference shock. Therefore, in the purchase decision equations (12) and (13), the random component (which includes past shocks) would be correlated to the number of periods the household decided to purchase (T_{i1}) and with the number of periods the household decided not to purchase (T_{i0}), creating an endogeneity problem.

In Erdem et al.'s model, the consumption function is linear and consumers have an exogenous stochastic per period usage requirement for the good, which is only revealed after the purchase

[4] We chose to compare our work to those two paper because we believe they represent the state of the art in what concerns the study of consumer inventory behaviour. Another paper that structurally estimate demand parameters is Sun (2005). Reduced form estimates can be found in Ailawadi and Neslin (1998), and Boizot et al. (2001).

decision is made. Thus consumers run a risk of stocking out, which is costly, if they maintain an inadequate inventory to meet the usage requirement. Notice that the usage rate assumption means that consumption is independent of prices in the short run. However, if prices remain high for a long period, consumption will adjust accordingly through more frequent stock outs.

To reduce the complexity of the state space, they assume that once quantity to be consumed is determined, each brand in storage is consumed at a rate which is proportional to the share of that brand in storage. Together with the assumption that brand differences enter linearly in the utility function, it implies that only the total inventory and a quality weighted inventory matter as state variables.

Finally, they incorporate consumer heterogeneity by allowing for 16 types of consumers which differ in terms of taste for the brands and in terms of usage rates.

The approach in Erdem et al. is computationally more complicated than that on Hendel and Nevo and consumption is exogenous. However, it allows for some degree of unobservable heterogeneity.

The main drawback of our model is not considering product differentiation. Another weakness is the ad hoc hypothesis on consumption when there is no purchase. However, these restrictions are counterbalanced by an extremely flexible consumer heterogeneity structure and a very simple and fast way of computing structural estimates. Furthermore, in what concerns consumption, our assumption is not stronger than the usage rate hypothesis of Erdem et al. Indeed, in our case, consumption always responds to prices at periods with purchases. The way consumption in periods without purchases react to prices is similar to the Erdem et al.'s, i.e., adjustment happens following long term price changes (for instance, if prices increase and remain high for a long time, consumption without purchases will go down through the increase in regular prices).

4 Data

The database is a representative survey of households distributed across all regions of France. We use information on three years: 1999, 2000, and 2001. Each household was given a scanner with which to register every food product purchased. For each product purchased, we have information on its brand and characteristics, including price and pack size, the date of the purchase and the brand of the retailer where it was purchased. We also have comprehensive information on household demographics, and on home characteristics, such as if the household has a storage room, a bathroom, a fridge, pets etc.

In the database, one observation is one purchase made by the household. For each product category under study, we consider a sub-sample of households that purchased that product category at least once in the three years. The product categories that we study are milk, coffee, canned tuna, pasta, yogurt, and butter. Table 1 through 6 bring descriptive statistics on these product categories, including number of households that purchased the product at least once during the three-year time span, total quantity purchased, average quantity per purchase, average duration between purchases, and average price paid per pack-size.

Table 1: Descriptive Statistics - Butter

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6695	-	-	-	372869
total qty (kg)	151743.488	-	-	-	372869
qty/purchase (kg)	0.407	264.394	0.025	10.000	372869
durationlast (days)	15.99	26.91	0	1057	366174
durationnext (days)	15.99	26.91	0	1057	366174
price (€/kg)	4.848	0.72	0.899	119.520	372869
price/pack (€)	4.848	0.03	3.872	5.564	372869

Table 2: Descriptive Statistics - Yogurt

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6814	-	-	-	646560
total qty (kg)	759114.624	-	-	-	646560
mean day (kg)	1.174	686.825	0	16.000	
durationlast (days)	9.60	17.62	0	903	639746
durationnext (days)	9.60	17.62	0	903	639746
price (€/kg)	2.028	0.61	0.107	17.623	646532
price/pack (€)	2.028	0.35	1.204	4.025	646560

The choice of product categories was made according to three criteria. First of all, we chose products that are consumed in a regular basis. Our model does not apply for products that are infrequently consumed. Second, we chose products that differ in terms of storage costs. While

Table 3: Descriptive Statistics - Coffee

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6548	-	-	-	231609
total qty (kg)	119226.560	-	-	-	231609
qty/purchase (kg)	0.514	374.540	0	12.000	231609
duration last (days)	24.17	37.42	0	1071	225061
duration next (days)	24.17	37.42	0	1071	225061
price (€/kg)	7.211	6.17	0.610	16.891	231597
price/pack (€)	7.211	0.91	1.387	3.765	231609

Table 4: Descriptive Statistics - Milk

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6741	-	-	-	432267
total qty (kg)	2372314.368	-	-	-	432267
qty/purchase (kg)	5.488	4890.45	0	216.000	432267
duration last (days)	14.43	20.07	0	1078	425526
duration next (days)	14.43	20.07	0	1078	425526
price (€/kg)	0.640	0.21	0.061	16.007	432264
price/pack (€)	0.640	0.05	0.503	0.69	432267

Table 5: Descriptive Statistics - Pasta

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6834	-	-	-	330253
total qty (kg)	231704.048	-	-	-	330253
qty/purchase (kg)	0.702	496.101	0	16.000	330253
duration last (days)	18.62	32.16	0	973	323419
duration next (days)	18.62	32.16	0	973	323419
price (€/kg)	1.753	0.71	0.229	24.392	330047
price/pack (€)	1.753	0.02	1.418	2.256	330253

Table 6: Descriptive Statistics - Tuna

Variable	Mean	Std. Dev.	Min.	Max.	N
number of households	6598	-	-	-	124859
totql qty (kg)	38255.404	-	-	-	124859
qty/purchase (kg)	0.306	206.130	0.054	8.760	124859
duration last (days)	40.06	66.5	0	1026	118261
duration next (days)	40.06	66.5	0	1026	118261
price (€/kg)	6.921	2.26	0.335	89.244	124859
price/pack (€)	6.921	0.26	4.695	8.187	124859

butter and yogurt need to be stocked in a refrigerated area, this is not the case for tuna, coffee, and pasta. Butter and yogurt are thus more costly to stock than tuna, coffee or pasta. Milk is more costly to stock than tuna since it requires more space per pack etc. Third, we chose products that differ in terms of storability, or how long or well a product can be stored. Pasta and tuna can be stored for a longer period than coffee which can be stored for a lot longer than yogurt, for instance[5].

Moreover, we took into account potential measurement errors arising from the fact that we use a broad definition of product. We consider each category as a single product, capturing the fact that different brands are substitutes (although not necessarily perfect substitutes). If, however, the consumption of one of the category brands is, for a certain household, independent from the consumption of another brand, then by treating both brands as substitutes, we introduce measurement error in the definition of inter-purchase duration and underestimate the true effects. To try to avoid these, we chose categories which are relatively homogeneous, increasing the probability that different brands will be substitutes. An exception is, perhaps, yogurt. Yogurt is sold in different brands and pack sizes, but the main differentiation is between plain yogurt and non plain. It is not clear that all households will regard plain and, for instance, fruit yogurt as substitutes. In general, we expect more homogenous product categories to present stronger evidence consistent with the dynamic model than less homogenous product categories.

Table 7 through 12 present, for each product category sample, descriptive statistics of the household characteristics included in estimations as controls for observable heterogeneity.

[5] It would have been interesting to apply the model to a product that is not at all storable or that has an infinite storage cost. However, we could not find product categories presenting those characteristics.

Table 7: Descriptive Statisticts of Characteristics of Households who buy Butter

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.21	0.43	0	1	372869
house (1) vs apartment (0)	0.71	0.45	0	1	372507
car ownership	0.94	0.24	0	1	372869
household size	3.15	1.40	1	9	372869
responsible for purchases is a man	0.03	0.17	0	1	372869

Table 8: Descriptive Statisticts of Characteristics of Households who buy Yogurt

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.21	0.41	0	1	646560
house (1) vs apartment (0)	0.67	0.47	0	1	645931
car ownership	0.94	0.24	0	1	646560
household size	3.21	1.39	1	9	646560
responsible for purchases is a man	0.03	0.16	0	1	646560

Table 9: Descriptive Statisticts of Characteristics of Households who buy Coffee

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.23	0.42	0	1	231609
house (1) vs apartment (0)	0.71	0.45	0	1	231395
car ownership	0.94	0.24	0	1	231609
household size	3.07	1.38	1	9	231609
responsible for purchases is a man	0.03	0.17	0	1	231609

Table 10: Descriptive Statisticts of Characteristics of Households who buy Milk

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.19	0.39	0	1	432267
house (1) vs apartment (0)	0.66	0.47	0	1	431405
car ownership	0.93	0.26	0	1	432267
household size	3.23	1.43	1	9	432267
responsible for purchases is a man	0.03	0.17	0	1	432267

Table 11: Descriptive Statisticts of Characteristics of Households who buy Pasta

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.19	0.4	0	1	330253
house (1) vs apartment (0)	0.70	0.46	0	1	329878
car ownership	0.95	0.22	0	1	330253
household size	3.47	1.39	1	9	330253
responsible for purchases is a man	0.02	0.15	0	1	330253

Table 12: Descriptive Statisticts of Characteristics of Households who buy Tuna

Variable	Mean	Std. Dev.	Min.	Max.	N
home has a cellar	0.19	0.39	0	1	124859
house (1) vs apartment (0)	0.68	0.47	0	1	124732
car ownership	0.94	0.24	0	1	124859
household size	3.28	1.4	1	9	124859
responsible for purchases is a man	0.03	0.18	0	1	124859

5 Econometric Implementation and Empirical Results

5.1 Model Implications

The first two implications derived from the model relate duration (from last purchase and to next purchase) and prices. *Implication 1* states that duration from last purchase is lower if the price today is lower than regular. *Implication 2*, on the other hand, states that duration until next purchase is higher if the current price is lower than the regular price. Therefore, in a regression of duration from last purchase (until next purchase) on a dummy indicating if current price is lower then the regular price, the coefficient of the dummy should be negative (positive).

Regular price is defined as the mean price per pack size paid by the household during the three years considered. Therefore, each household i has a regular price p_{ris}, where s is the pack size. By defining regular price per household, we hope to be partly controlling for the relevant consumption set of the household, since what matters for the consumer when deciding to purchase and to stock is a lower or higher price than the one she is used to pay, not the mean price paid by all households. Suppose for instance that consumer i never buys store brands, which are usually cheaper. Then the price of store brand products should not affect her purchase and quantity decisions because store brands do not enter her consumption set. Analogously, if consumer i never shops at store A (too far away from home, for example) then the average price of products in store A should not affect her decisions, for the same reason as prices in northern France, for instance, should not enter the regular price index of consumers living in the South of France.

The definition of regular price at the household level, however, has its problems. Households that have a high cost of stocking have a harder time trying to coincide purchases and sales. Their level of inventories is on average lower and they have to purchase more frequently, making it harder for them to wait for the next low price. Therefore, on average, the regular price of high stock cost households will be higher, meaning there is a correlation, by definition, of regular price and the household's costs of stockpiling. We believe this problem is not very important since empirical results remain basically the same when we use a regular price which is not household specific and thus free of the correlation with household costs.

We consider pack size in the definition of regular price because we want to differentiate situations when the household purchased at a lower price because the product was really on a sale, from situations when households purchased at a lower price only because they bought larger packs. Since quantity discounts are extremely frequent, the price per quantity paid when a larger pack is purchased is lower than the price per quantity paid when small packs are purchased, even if the product category was not on sale. The use of the regular price defined by household and size permits to separate sales from quantity discounts.

In the Appendix (Table 19), we show results on a regression of regular prices per household and pack size on household characteristics. The regular price decreases with family size, and increases with the age of the head of the household. Households where a woman is the shopper have higher regular prices than household where a man shops. Having a child of 6 or less years of age positively affects regular prices paid. Interestingly, regular prices paid for butter, yogurt, and pasta decrease with household income whereas regular prices paid for milk, coffee, and tuna increase with income. Finally, having a cellar and having a car, which can be considered as indication of low cost of stockpiling, have a positive effect on the regular price, which we interpret as evidence that the correlation between household costs and regular prices is not very important.

We define a price discount as a price 5% or more lower than the regular price. We could have defined discounted price simply as any price lower than regular. With the five percent margin, however, we try to avoid confounding regular price fluctuations to actual discounts. The 5%

margin is of course quite arbitrary, but we have performed the tests with different discounted price definitions (lower than regular, at least 2% lower than regular, at least 10% lower than regular, and at least 15% lower than regular), and the results are qualitatively the same (coefficient signs do not change). Table 13 below shows the proportion of total quantity purchased during sales[6].

Table 13: Proportion of Quantity Purchased during Sales

Product	Proportion
Milk	0.332
Coffee	0.340
Tuna	0.470
Pasta	0.608
Butter	0.340
Yogurt	0.404

One main concern when studying the correlation between interpurchase duration and price is household heterogeneity with respect to inventory costs. If household heterogeneity is important, and uncrontrolled for, the estimated coefficient will be biased. To correct for this, we include household fixed effects, as well as household characteristics that are potentially correlated with inventory costs, such as if the household has a car or not, and variables that are proxies for stock space availability. In general, the bigger the home, the more space available for stocking inventories, and the lower the cost of inventories. We use two measures of space availability: if the household has an extra room for storage (a cellar), and if the household lives in a house or in an apartment.

Results for the estimation of the effect of discounted price on interpurchase duration are presented in Table 14. The first and third columns show results for simple OLS regressions where the dependent variable is, respectively, duration since last purchase and duration until next purchase. Coefficients displayed in the second and fourth columns are from regressions with fixed effect where the dependent variables are again duration from last purchase and until next purchase. The three last columns present, respectively, the number of households and the number of observation included in the regressions with and without household fixed effects. All regressions include controls for region of residence, family size, presence of a child (of 16 or less and 6 or less years of age), household income, car ownership, age and education of the head of the household, and gender of

[6] Proportion of total quantity purchased per product during whole time span which was made in periods when prices (per pack) were at least 5% lower than regular price.

the person responsible for purchases. They also include space availability controls, namely, whether the family lives in a house or an apartment and whether there is a special room for storage.

Results highly corroborate the inventory model. Almost every coefficient presents the correct sign. The only exceptions are pasta and yogurt for which there is no empirical evidence that duration from last purchase is negatively affected by lower prices today (although duration to next purchase is higher if the product is on sale today). Notice that, in general, evidence in favor of the model is stronger once household fixed effects are included (for instance, in the regressions of duration from last purchase for milk and butter, and duration until next purchase for tuna, the coefficient has the right sign and is significant only when we include fixed effects), which underline the importance of household unobservable heterogeneity.

Table 14: Effect of discounted price on duration from last purchase and duration until next purchase

Product	Coefficient Estimates: duration on discount prices						
	last(1)	last (2)	next (1)	next (2)	Nb hh	N (1)	N (2)
Milk	0.058	-0.254	0.688	0.359	6729	424674	425526
	(0.89)	(4.08)**	(10.60)**	(5.81)**			
Coffee	-0.098	-0.211	0.356	0.423	6478	224857	225061
	(0.59)	(1.55)	(2.14)*	(3.10)**			
Tuna	-1.062	0.686	-0.589	1.504	6474	118144	118261
	(2.74)**	(1.94)	(1.52)	(4.26)**			
Pasta	0.300	0.026	1.127	0.955	6819	323054	323419
	(2.73)**	(0.25)	(10.24)**	(9.17)**			
Butter	0.128	-0.422	1.412	0.672	6645	365822	366174
	(1.37)	(5.19)**	(15.14)**	(8.28)**			
Yogurt	0.032	0.232	0.197	0.372	6798	639128	639746
	(0.71)	(5.73)**	(4.40)**	(9.19)**			

Notes: (i) Absolute value of t statistics in parentheses, (ii) * significant at 5%, ** significant at 10%; (iii) (1) regression without fixed effects, and (2) with fixed effects; (iv) controls are: whether the household has a car, family size, region of residence, income, whether the person responsible for purchases is a male, presence of a child of 16 or less years of age, presence of a child of 6 or less years of age, and educational level of the head; (v) the column "Nb hh" (5th column) presents the number of households in the sample, and the two last columns, the number of observations in the regression without and with fixed effects, respectively.

To investigate the relationship between inventory costs, as proxied by space availability at home and frequency of purchases (*Implication 3*), we run regressions for each product subsample in which the dependent variable is the number of times the household purchased the product during the three year time span. In this exercise, the focus is on frequent consumers. Therefore, for each product category, we consider a subsample of households that purchased the product at least 12 times during the three years (approximately once every three months). The regressions include controls for income level, household size, age and education of the person of reference, gender of the person responsible for the purchases, presence of a child of 6 or less years of age, presence of a child of 16 or less years of age, as well as the region where the household lives. The variables that proxy for available space are the same as used above, i.e., dummy variables indicating if the household has a cellar, and if the home of the household is a house or an apartment[7]. We also include a dummy indicating whether the household owns a car. Although a car is not necessarily correlated with space availability, we conjecture that having a car decreases storage costs simply because it decreases the cost of bringing home large quantities of the product. Estimated coefficients are in Table 15.

Table 15: Effect of Space Availability on Frequency of Purchases

	Milk	Coffee	Tuna	Pasta	Butter	Yogurt
Cellar	-7.882	-2.189	-2.163	-5.294	0.916	-6.013
	(5.61)**	(1.95)	(2.43)*	(4.32)**	(0.64)	(2.67)**
House	-4.455	0.734	-0.220	0.821	2.444	-2.234
	(3.34)**	(0.66)	(0.26)	(0.70)	(1.75)	(1.04)
Car	-9.511	-0.210	-1.256	-0.775	-0.582	-2.860
	(3.90)**	(0.10)	(0.75)	(0.34)	(0.22)	(0.73)
Other Controls				Yes		
Obs	6292	4920	3632	6019	5721	6414

[7] Ideally, we would include a measure of total space availability (i.e., the total size of the home) but the data do not include this information. Alternatively, we could use census data and compute square footage of living area by zip code, as in Bell and Hilber (2005). However, we believe that may be substantial heterogeneity in home sizes within a zip code in France and preferred to stick to the extra room and house or apartment measures. We could also have included other space availability measures, such as wether the home has a garden, a dog (as in Hendel and Nevo, 2003), a bathroom etc., but decided not not to in order to avoid multicollinearity.

Notes: (i) Dependent variable is frequency of purchase per household (ii) Absolute value of t statistics in parentheses, (iii) * significant at 5%, ** significant at 10%; (iv) Other controls are: income level, household size, region, age and education of person of reference, gender of person responsible for purchases, presence of child of 6 or less years of age, presence of child of 6 or less years of age.

Clearly, having a cellar negatively affects the frequency of purchases of all product categories, except for butter. Living in a house instead of an apartment and owning a car also seems to negatively affect the frequency of purchase, although the negative coefficient is significant only for milk. Interestingly, among the product categories considered, milk is the one that occupies the most space (i.e., one standard-size pack of milk is larger than one standard-size pack of coffee, or butter, or tuna) and that weighs the most (thus the importance of owning a car when deciding to purchase for storage).

5.2 Structural Analysis

5.2.1 Identification and Estimation

In this section, we structurally estimate demand for the case where price is higher than regular price (equation 12). Unfortunately, we cannot estimate the structural parameter ϕ_i because its identification relies on the estimation of equation (13), which includes an unknown and unobservable function, namely $E_t\Psi_{it+1}$.

Rewriting (12), we get the equation to be estimated:

$$E_t\left(Q_{it}^* \mid q_{it} > 0, p_t > p_r\right) = -\frac{1}{\rho}T_i^{t-1}\ln\left(\alpha_i\right) - \frac{1}{\rho}\left[\ln p_t + \sum_{n\in T_{i1}^t}\ln p_n + T_{i0}^{t-1}\ln p_r\right] \quad (18)$$

Identification of the parameters of the model is standard. Variation over time of prices and the periods with and without purchases ensure the semi-parametric identification of the model parameters. This means that we can identify and estimate α_i and ρ without specifying the distribution function of ε_{it}. We do so running an OLS.

To allow for consumer-specific marginal utility of income (α) we interact T_i^{t-1} in (18) with a set of dummy variables indicating the household. In this way, instead of getting one estimated α per product category, we get as many α estimates as there are households in each product category sample. Individual specific parameters are an important contribution of our work and can be obtained thanks to the large number of observations in the dataset.

Remember that we do not observe inventories. Instead of assuming an arbitrary level for the initial inventory level, we perform the estimation on a subsample which begins, for each consumer i at the first purchase occasion following a purchase at higher than regular prices. When prices are higher than regular and the consumer purchases a positive amount of the product, Proposition 1 says that the end of the period level of inventories is equal to zero. Therefore, eliminating the observations before the first purchase at high prices, we can comfortably assume that initial stocks are equal to zero, avoiding the initial condition problem.

Another problem we have to deal with in the estimation is the fact that we only observe prices paid by households. This means that when a household does not purchase, we do not have information on the price she would have paid. Then, in periods when the household does not purchase, we consider as the price she would have paid the mean price paid that week by households living in the same region.

Finally, to control for potential seasonal effects, we include dummies indicating each one of the four seasons of the year in all regressions.

5.2.2 Results

Estimated ρ are all positive as predicted and significant at 5%. We do not report the estimated values because the absolute value of ρ has no special meaning. In this context, it is not actually the risk aversion but the concavity of the utility function for a certain product, and it depends not only on the product category but also on the unit of measurement considered for q_{it}.

Table 16 brings the mean estimated α_i per percentile. The estimated coefficients show the signs predicted by the model. Furthermore, except for a few exceptions (for less than 10% of the households), they are significant at least at 5%.

Below, estimated $\alpha_i s$ are used to simulate the long run price elasticities derived from the model and to study whether household observable characteristics can explain differences in estimated parameters.

5.2.3 Price Responsiveness in the Dynamic and Static Models

To obtain a measure of the long-run and short run price elasticities, we plug the estimated $\alpha_i s$ and $\beta_i s$ into (17) and (16). The $\beta_i s$ are estimated by OLS using (15).

Table 17 below shows elasticities for each product when considering estimated parameters sig-

Table 16: Percentiles of Estimated α_i

Product	Obs	Percent5	Percent25	Median	Percent75	Percent95
Milk	534172	2.66e-23	1.74e-4	2.20	110.4	717.2
Coffee	560578	3.27e-12	0.002	0.804	11.9	83.4
Tuna	417879	1.23e-06	0.084	3.00	23.0	101.1
Pasta	711960	3.08e-21	0.001	3.09	60.6	252.2
Butter	604873	2.25e-11e-21	0.005	2.14	30.4	120.1
Yogurt	834354	3.00e-14	4.39e-4	0.605	24.7	179.7

nificant at 5%. In both tables, the first column shows the average long run price elasticity, while the second column shows the short run price elasticity calculated using the estimated $\beta_i s$. The third column presents an alternative measure of the price-elasticities: the estimated coefficient of the regression of the log of the individual quantity purchased on the log of the price, denoted ϵ^R. We have decided to include this last column for illustrative purposes. However, we believe that to have an idea of the difference between the short and long run measures of price elasticities, it is best to compare the first two columns of Table 17. These two columns show price elasticities which yield from nested models (one is the static version of the other).

Table 17: Estimated Long and Short Run Price Elasticities per Product Category

Products	Average Price Elasticities		
	ϵ^{LR}	ϵ^{SR}	ϵ^{R}
Milk	-0.058	-0.122	-0.737
Coffee	-0.090	-0.190	-0.506
Tuna	-0.090	-0.181	-0.409
Pasta	-0.065	-0.148	-0.504
Butter	-0.084	-0.161	-0.413
Yogurt	-0.093	-0.188	-0.609

Note: Estimated parameter used to calculate elasticities yields from inventory model $\left(\epsilon^{LR}\right)$, static version of demand model with inventories $\left(\epsilon^{SR}\right)$, and the coefficient of the regression of $\log$ of quantity purchased on $\log$ of price $\left(\epsilon^{R}\right)$. Utility Specification: $u\left(c_{it}\right) = -\left(1/\rho\right)\exp\left[-\rho c_{it}\right].$

The short run measures are consistently higher than their long run counterparts. The upper bias varies from 80% up to more than 100% depending on the product. The difference in measures is a result of the short run elasticities capturing not only consumption responses to price variation, but also inventory responses.

Interestingly, the difference between the long and short run price elasticities is lowest in the case of yogurt and butter. Yogurt is less storable than other products (yogurt cannot be stored for a long time), and is expensive to store (needs a refrigerator). Butter needs to be frozen to be stored thus requiring the household to have a freezer and increasing storage costs. We expect therefore inventories to be less relevant in the case of those two product categories and this seems indeed to be the case.

5.2.4 Consumers Heterogeneity

We regress the estimated $\alpha_i s$ on a number of household characteristics in order to assess if observables explain at least partially individual differences in the theoretical model parameter. Notice that a higher α_i for a certain product indicates that the household is willing to spend a lower proportion of its income on that product. Therefore, differences in the coefficients across products shed light on relative preferences or taste over products.

The variables included in the regression are: dummy variables indicating whether the household has a cellar, lives in a house or an apartment, has a car, includes a child of 6 years of age or less, includes a child of 16 years of age or less. Furthermore, we control for region of residence, household size, age, gender and education of the person of reference, and household income. The tables with all the estimated coefficients is in the Appendix (Table 22 - 24). Very few coefficients are significant indicating that unobservables play an important role in explaining the marginal utility of income.

6 Robustness Check

To check the robustness of the model results, we consider an alternative procedure for estimating model parameters. Furthermore, we estimate the marginal utility of revenue and calculate price elasticities under different price expectation assumptions.

6.1 Alternative Estimation Procedure

Suppose a certain consumer purchases today at a price higher than regular. Proposition 1 says that her end of the period inventory level will be equal to zero and she will purchase again next period since her beginning of next period level of inventory is going to be zero. If next period, the price is still higher than the regular price, she will again purchase only enough to cover consumption, choosing to hold no stocks. This means that next period, her consumption is going to be equal to the quantity purchased, since she had nothing stored at home and she will not store anything either. More formally, let T_i' be the set of consecutive periods such that $p_{(t-1)'} > p_r$ and $p_{t'} > p_r$ and $q_{i(t-1)'} > 0$ and $q_{it'} > 0$. Then, for all $(t-1)'$ and t' belonging to T_i', $y_{i(t-1)'} = y_{it'} = 0$, and therefore, $q_{it'} = c_{it'}$. This observation suggests a rather simple alternative for estimating the parameters of the model which considers only the subset of observation in T'. The equation to be estimated is:

$$q_{it'}^* = h\left(\alpha_i p_{t'}\right) - v_{it'} \tag{19}$$

When the utility function is a CARA, as considered before, (19) becomes:

$$q_{it'}^* = -\frac{1}{\rho}\ln\alpha_i - \frac{1}{\rho}\ln p_{t'} - v_{it'} \tag{20}$$

We estimate α_i and ρ in (20) and calculate price elasticities as before. We then compare the long and short run measures of price elasticity, where the short run measure is the price elasticity of the static model. Results do not change much. However, they are less precise because standard errors are bigger since here we use less observations to estimate the model parameters.

6.2 Alternative Price Expectation Hypothesis

The assumption that consumers always expect prices to return to its regular level may be too strong. Here, we re-estimate the parameters of the model and calculate price-elasticities under an alternative price expectation hypothesis. We start by estimating two different Markov price processes. We then assume consumers have rational expectations and that they expect prices to follow the estimated price processes.

The first price process we consider is[8]:

$$\text{Process 1: } p_{it} = (a_h + b_h p_{it-1})h + (a_l + b_l p_{it-1})l + (a_r + b_r p_{it-1})r \tag{21}$$

[8] Higher order processes do not alter results in a relevant manner.

where h, l, and r indicate that p_{t-1} is either higher, lower or equal to the regular price, respectively. The second process is a VAR(1):

$$\text{Process 2: } p_{it} = a + bp_{it-1} \tag{22}$$

The problem is we do not observe prices, only prices paid by the households. We could have defined p_t and p_{t-1} to be the average price paid at periods t and $t-1$. However, we believe this would fail to capture important regional and per brand price variations. We thus preferred to consider the prices paid by each household, thus the i index on the price variables in (21) and (22). For periods where household i did not purchase (hence we do not observe the price paid by i), we consider the price paid by i to be equal to the average price paid at the same period by households that live in the same region as i.

Table 18 brings the long run price elasticities implied by the inventory model when we consider Process 1 and Process 2 (ϵ^{LR1} and ϵ^{LR2}). Only estimated parameters which are significant at 5% are used (for all product categories, this respresents more than 90% of the estimated parameters). The third column brings the price elasticities yielded by the static model of demand. Those are the same measures seen in the second column of Table 17 since the static estimates are not affected by price expectations. We re-include them here to facilitate comparison with the long run measures.

Table 18: Estimated Long and Short Run Price Elasticities under Alternative Price Expectation Assumptions

Products	Average Price Elasticities		
	ϵ^{LR1}	ϵ^{LR2}	ϵ^{S}
Milk	-0.073	-0.089	-0.122
Coffee	-0.159	-0.142	-0.190
Tuna	-0.127	-0.148	-0.188
Pasta	-0.126	-0.109	-0.148
Butter	-0.148	-0.129	-0.161
Yogurt	-0.116	-0.114	-0.142

Note: Estimated parameters used to calculate price elasticities yielded from inventory model under price process 1 $\left(\epsilon^{LR1}\right)$ and price process 2 $\left(\epsilon^{LR2}\right)$, and the static version of the model $\left(\epsilon^{SR}\right)$.

Under both price expectation hypothesis, the results of the model are maintained. Even though the upperbias of the static measures is smaller than before, it is still very significative, with the difference between the short and long run price elasticities varying from 9% to 42% under the first price process, and from 16% to 30% under the second price process.

7 Conclusion

Ignoring dynamics in the demand behavior of consumers may lead to biased estimates of the long run demand price elasticities. We propose a model of demand where consumers stockpile and prices are random. An assumption on the level of consumption at periods without purchases enables identification of the long run price elasticity without having to solve the dynamic program We also derive and test implications of the model.

The empirical analysis is performed using a comprehensive dataset on household food products purchases. We estimate individual specific marginal utilities of income from the purchase probability equations yielded by the model. The estimates are then used to simulate the long run demand price elasticities. We find that price elasticities resulting from a static demand model significantly overestimate the long run price elasticities. Finally, we show that results are robust to different price expectation assumptions.

8 References

Aguirregabiria, Victor (1999) “The Dynamics of Markups and Inventories in Retailing Firms’, *Review of Economic Studies*, 66, 275-308.

Aguirregabiria, Victor (2005) “Retail Stockouts and Manufacture Brand Competition”, mimeo, Boston University.

Ailawadi, Kusum and Scott A. Neslin (1998) "The Effect of Promotion on Consumption: Buying More and Consuming it Faster", *Journal of Marketing Research*, 35, 390-398.

Assunção, João, Meyer, Robert (1993) "The Rational Effect of Price Promotions on Sales and Consumption", Management Science, 39 (5), pp. 517-535.

Bell, David R., and Christian A.L. Hilber (2005) "An Empirical Test of the Theory of Sales: Do Household Storage Constraints Affect Consumer and Store Behavior?", mimeo.

Benitaz-Silva, H. G. Hall, G. Hilsch, G. Pauletto and J. Rust (2000) "A Comparison of Discrete and Parametric Aproxximation Methods for Continuous-State Dynamic Programming Problems." Yale University, mimeo.

Berry, S., J. Levinsohn, A. Pakes (1995). “Automobile Prices in Market Equilibrium”, *Econometrica*, 63(4), pp. 841-90.

Berto Villas-Boas, Sofia (2007). “Vertical Contracts between Manufacturers and Retailers: Inference with Limited Data”, mimeo, University of California, Berkeley.

Bierlaire, Michel, Denis Bolduc, Daniel McFadden (2003). “Characteristics of Generalized Extreme Value Distributions”, Working Paper.

Boizot, C., J.M. Robin, and M. Visser (2001). "The Demand for Food Products: An Analysis of Interpurchase Times and Purchased Quantities," *Economic Journal*, 111(470), pp. 391-419.

Bonnet, C.and P. Dubois (2006). "Non-linear Contracting and Endogenous Market Power between Manufacturers and Retailers: Identification and Estimation on Differentiated Products," mimeo, Université de Toulouse.

Braido, L.H.B. (2009) "Multiproduct Price Competition with Heterogeneous Consumers and Nonconvex Costs", *Journal of Mathematical Economics* (2007), forthcoming.

Erdem, T., S. Imai, and M. Keane (2003). “Brand and Quantity Choice Dynamics under Price Uncertainty”, *Quantitative Marketing and Economics*, 1, pp. 5-64.

Feenstra, Robert C. and Matthew D. Shapiro (2001) "High-Frequency Substitution and the Measurement of Price Indexes." NBER Working Paper 8176.

Hendel, N. and A. Nevo (2006a) "Sales and consumer Inventory." NBER Working Paper 9048.

Hendel, N. and A. Nevo (2006b) “Measuring the Implications of Sales and consumer Inventory Behavior”, Econometrica, 74 (6), 1637-73.

Keane, Michael P. (1993) "Simualtion Estimation for Panel Data Models with Limited Independent Variables." G.S. Maddala, C.R. Rao, H.D. Vinod, eds. Handbook of Statistics, North Holland, 545-572.

Keane, Michael P. and Kenneth I. Wolpin (1994) "Solution and Estimation of Dynamic Programming Models by Simulation", *Review of Economics and Statistics*, 76, 684-672.

McFadden, Daniel (1980) “Econometric Models for Probabilistic Choice Among Products”, *Journal of Business*, 53(3), Part 2: Interfaces between Marketing and Economics, 13-29.

McFadden, D. (1984) “Econometric Analysis of Qualitative Response Models”, in Z. Griliches and M. Intilligator, eds., Handbook of Econometrics, Vol. II, Amsterdam: North-Holland, 1396-1456.

Nevo, Aviv (2001) “Measuring Market Power in the Ready-to-Eat Cereal Industry.” *Econometrica*, Vol. 69 (2), pp. 307-42.

Pesendorfer, Martin (2002). “Retail Sales. A Study of Pricing Behavior in Supermarkets,” *Journal of Business* , 75, 33-66.

Rust, John (1987) “Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zucher.” *Econometrica*, 55(5), pp. 999-1033.

Rust, John (1996) "Numerical Dynamic Programming in Economics", in H.M. Amman, D.A. Kendrick, and J. Rust, eds., Handbook of Computational Economics, Vol. I, Elsevier, 731-800.

Sun, Baohong (2005) "Promotion Effect on Endogenous Consumption." *Marketing Science*, 24 (3), pp. 420-43.

9 Appendix

Short Run Price Elasticities in the Inventory Model

Individual demand at period t (the short run demand) is equal to:

$$
\begin{aligned}
D_{it}^{ST}(p_t) &= q_{it}\Pr(q_{it}>0 \mid p_t) \\
&= q_{it}\Pr(q_{it}>0 \mid p_t>p_r)\Pr(p_t>p_r)+q_{it}\Pr(q_{it}>0 \mid p_t\le p_r)\Pr(p_t\le p_r) \\
&= (c_{it}-x_{it})\Pr(q_{it}>0 \mid p_t>p_r)\Pr(p_t>p_r)+ \\
&\quad (y_{it}+c_{it}-x_{it})\Pr(q_{it}>0 \mid p_t\le p_r)\Pr(p_t\le p_r)
\end{aligned}
$$

where we use the fact that $p_t > p_r$ and $p_t \le p_r$ are complementary events and apply Bayes' Theorem. We also use the law of motion of inventories (equation 1) to write $q_{it} = y_{it} + c_{it} - x_{it}$, as well as Proposition 1 which says that $y_{it} = 0$ when prices are higher than regular prices.

We assume v_{it} is normally distributed. Moreover, the first order conditions for the dynamic model imply that in periods with purchases, $c_{it} = h(\alpha_i p_t) - v_{it}$ and, $y_{it} = \frac{\alpha_i(p_r-p_t)}{2\phi_i} - \frac{E_t\Psi_{it+1}}{2\phi_i}$. Therefore, we can write:

$$
\begin{aligned}
\Pr(q_{it}>0|p_t>p_r) &= \Pr(c_{it}-x_{it}>0 \mid p_t>p_r) \\
&= \Pr(h(\alpha_i p_t)-v_{it}-x_{it}>0 \mid p_t>p_r) \\
&= \Pr(v_{it}<h(\alpha_i p_t)-x_{it} \mid p_t>p_r) \\
&= \Phi(V_{it}|p_t>p_r)
\end{aligned}
$$

and

$$
\begin{aligned}
\Pr(q_{it}>0|p\le p_r) &= \Pr(y_{it}+c_{it}-x_{it}>0 \mid p\le p_r) \\
&= \Pr\left(\frac{\alpha_i(p_r-p_t)}{2\phi_i}-\frac{E_t\Psi_{it+1}}{2\phi_i}+h(\alpha_i p_t)-v_{it}-x_{it}>0 \mid p\le p_r\right) \\
&= \Pr\left(v_{it}<\frac{\alpha_i(p_r-p_t)}{2\phi_i}-\frac{E_t\Psi_{it+1}}{2\phi_i}+h(\alpha_i p_t)-x_{it}>0 \mid p\le p_r\right) \\
&= \Phi(U_{it} \mid p_t\le p_r)
\end{aligned}
$$

where Φ is the Normal cumulative distribution function, and V_{it} and U_{it} are, respectively:

$$V_{it} = h(\alpha_i p_t) - x_{it}$$

and

$$U_{it} = \frac{\alpha_i(p_r-p_t)}{2\phi_i} - \frac{E_t\Psi_{it+1}}{2\phi_i} + h(\alpha_i p_t) - x_{it}$$

Substituting V_{it}, U_{it}, $\Phi(V_{it} \mid p_t > p_r)$ and $\Phi(U_{it} \mid p_t \leq p_r)$ in (**??**) we get:

$$D_{it}^{ST}(p_t) = (V_{it} - v_{it})\,\Phi(V_{it} \mid p_t > p_r)\Pr(p_t > p_r) + (U_{it} - v_{it})\,\Phi(U_{it} \mid p_t \leq p_r)\Pr(p_t \leq p_r)$$

Hence, the short run elasticity of demand (or the price elasticity of purchases) in the inventory model is equal to:

$$\begin{aligned}
\epsilon_{SR} &= \left[\begin{array}{c} \frac{dV_{it}}{dp_t}\Phi(V_{it}|p_t > p_r)\Pr(p_t > p_r) + (V_{it} - v_{it})\,\Phi'(V_{it}|p_t > p_r)\frac{dV_{it}}{dp_t}\Pr(p_t > p_r) \\ + (V_{it} - v_{it})\,\Phi(V_{it}|p_t > p_r)\frac{d\Pr(p_t > p_r)}{dp_t} \end{array}\right]\frac{p_t}{V_{it} - v_{it}} \\
&+ \left[\begin{array}{c} \frac{dU_{it}}{dp_t}\Phi(U_{it}|p_t \leq p_r)\Pr(p_t \leq p_r) + (U_{it} - v_{it})\,\Phi'(U_{it}|p_t \leq p_r)\frac{dU_{it}}{dp_t}\Pr(p_t \leq p_r) \\ + (U_{it} - v_{it})\,\Phi(U_{it}|p_t \leq p_r)\frac{d\Pr(p_t \leq p_r)}{dp_t} \end{array}\right]\frac{p_t}{V_{it} - v_{it}} \\
&= \frac{p_t}{V_{it} - v_{it}}\frac{dV_{it}}{dp_t}\Pr(p_t > p_r)\left[\Phi(V_{it}|p_t > p_r) + (V_{it} - v_{it})\,\Phi'(V_{it}|p_t > p_r)\right] \\
&\quad + p_t\Phi(V_{it}|p_t > p_r)\frac{d\Pr(p_t > p_r)}{dp_t} \\
&\quad + \frac{p_t}{V_{it} - v_{it}}\frac{dU_{it}}{dp_t}\Pr(p_t \leq p_r)\left[\Phi(U_{it}|p_t \leq p_r) + (U_{it} - v_{it})\,\Phi'(U_{it}|p_t \leq p_r)\right] \\
&\quad + p_t\Phi(U_{it}|p_t \leq p_r)\frac{d\Pr(p_t \leq p_r)}{dp_t} \\
&= \frac{p_t}{V_{it} - v_{it}}\frac{dV_{it}}{dp_t}\Pr(p_t > p_r)\left[\Phi(V_{it}|p_t > p_r) + (V_{it} - v_{it})\,\Phi'(V_{it}|p_t > p_r)\right] \\
&\quad + \frac{p_t}{V_{it} - v_{it}}\frac{dU_{it}}{dp_t}\Pr(p_t \leq p_r)\left[\Phi(U_{it}|p_t \leq p_r) + (U_{it} - v_{it})\,\Phi'(U_{it}|p_t \leq p_r)\right] \\
&\quad + p_t\left[\Phi(V_{it}|p_t > p_r) - \Phi(U_{it}|p_t \leq p_r)\right]\frac{d\Pr(p_t > p_r)}{dp_t} \\
&= -\frac{1}{\alpha_i p_t (V_{it} - v_{it})}\Pr(p_t > p_r)\left[\Phi(V_{it}|p_t > p_r) + (V_{it} - v_{it})\,\Phi'(V_{it}|p_t > p_r)\right] \\
&\quad - \frac{p_t}{V_{it} - v_{it}}\left(\frac{\alpha_i}{\theta_i} + \frac{1}{\alpha_i p_t^2}\right)\Pr(p_t \leq p_r)\left[\Phi(U_{it}|p_t \leq p_r) + (U_{it} - v_{it})\,\Phi'(U_{it}|p_t \leq p_r)\right] \\
&\quad + p_t\left[\Phi(V_{it}|p_t > p_r) - \Phi(U_{it}|p_t \leq p_r)\right]\frac{d\Pr(p_t > p_r)}{dp_t}
\end{aligned}$$

Note: Absolute values of t statistics in parentheses, (i) * significant at 5%, ** significant at 10%.

Note: Absolute values of t statistics in parentheses, (i) * significant at 5%, ** significant at 10%.

Table 19: Regular Price on Household Characteristics (I)

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Extra Room	0.04	0.103	0.035	0.037	0.037	0.048
	(7.66)**	(8.28)**	(17.62)**	(21.89)**	(21.89)**	(3.94)**
House	-0.011	0.012	0.001	-0.016	-0.016	0.012
	(20.95)**	(0.93)	(0.52)	(10.50)**	(10.50)**	(1.08)
Car	0.002	0.137	0.008	0.031	0.031	0.233
	(1.73)	(5.91)**	(2.28)*	(9.75)**	(9.75)**	(11.06)**
Fam Size=2	-0.036	-0.582	-0.137	-0.100	-0.100	-0.521
	(39.00)**	(26.88)**	(39.07)**	(30.26)**	(30.26)**	(25.38)**
Fam Size=3	-0.064	-0.696	-0.253	-0.152	-0.152	-0.647
	(60.05)**	(28.27)**	(62.87)**	(42.06)**	(42.06)**	(28.27)**
Fam Size=4	-0.094	-0.840	-0.327	-0.232	-0.232	-0.822
	(83.13)**	(32.30)**	(78.18)**	(62.35)**	(62.35)**	(34.41)**
Fam Size=5	-0.109	-1.063	-0.435	-0.317	-0.317	-1.125
	(89.40)**	(36.96)**	(94.91)**	(80.19)**	(80.19)**	(43.39)**
Fam Size=6	-0.127	-1.453	-0.491	-0.380	-0.380	-1.332
	(79.25)**	(37.10)**	(80.44)**	(78.35)**	(78.35)**	(39.10)**
Fam Size=7	-0.126	-1.933	-0.550	-0.507	-0.507	-1.660
	(49.85)**	(28.55)**	(49.08)**	(68.84)**	(68.84)**	(30.46)**
Fam Size=8	-0.146	-1.108	-0.666	-0.390	-0.390	-1.590
	(31.45)**	(9.68)**	(33.08)**	(33.42)**	(33.42)**	(13.02)**
Fam Size=9	-0.191	-2.878	-0.742	-0.758	-0.758	-2.308
	(29.75)**	(16.95)**	(37.26)**	(26.11)**	(26.11)**	(13.40)**
Inc 2	-0.048	-0.118	-0.285	-0.531	-0.531	-0.137
	(3.31)**	(0.42)	(4.01)**	(11.51)**	(11.51)**	(0.43)
Inc 3	0.007	0.530	-0.236	-0.314	-0.314	0.280
	(0.52)	(1.94)	(3.38)**	(7.02)**	(7.02)**	(0.89)
Inc 4	-0.000	0.542	-0.298	-0.400	-0.400	0.641
	(0.01)	((2.00)**	(4.27)**	(9.00)**	(9.00)**	(2.07)*
Inc 5	0.011	0.448	-0.268	-0.393	-0.393	0.502
	(0.76)	(1.66)	(3.86)**	(8.88)**	(8.88)**	(1.62)

Table 20: (Cont) Regular Price on Household Characteristics (II)

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Inc 6	0.013	0.640	-0.187	-0.306	-0.306	0.655
	(0.89)	(2.37)*	(2.68)**	(6.91)**	(6.91)**	(2.12)*
Inc 7	0.031	0.952	-0.259	-0.324	-0.324	0.655
	(2.21)*	(3.53)**	(3.73)**	(7.34)**	(7.34)**	(2.12)*
Inc 8	0.032	0.909	-0.189	-0.325	-0.325	0.846
	(2.25)**	(3.37)**	(2.71)**	(7.36)**	(7.36)**	(2.73)**
Inc 9	0.030	0.951	-0.194	-0.297	-0.297	0.657
	(2.12)*	(3.52)**	(2.80)**	(6.73)**	(6.73)**	(2.12)*
Inc 10	0.037	1.024	-0.146	-0.258	-0.258	0.904
	(2.59)**	(3.80)**	(2.10)*	(5.85)**	(5.85)**	(2.92)**
Inc 11	0.052	1.264	-0.105	-0.227	-0.227	0.991
	(3.66)**	(4.69)**	(1.51)	(5.13)*	(5.13)**	(3.21)**
Inc 12	0.052	1.321	-0.087	-0.227	-0.277	1.012
	(3.70)**	(4.89)**	(1.25)	(5.13)**	(5.13)**	(3.27)**
Inc 13	0.059	1.471	0.000	-0.177	-0.177	1.372
	(4.19)**	(5.45)**	(0.00)	(4.00)**	(4.00)**	(4.44)**
Inc 14	0.066	1.587	-0.031	-0.152	-0.152	1.494
	(4.70)**	(5.87)**	(0.44)	(3.43)**	(3.43)**	(4.83)**
Inc 15	0.078	1.845	0.021	-0.085	-0.085	1.579
	(5.52)**	(6.81)**	(0.31)	(1.91)	(1.91)	(5.10)**
Inc 16	0.087	1.392	0.085	-0.130	-0.130	1.491
	(6.09)**	(5.09)**	(1.21)	(2.92)**	(2.92)**	(4.78)**
Inc 17	0.101	1.837	0.036	-0.049	-0.049	2.024
	(7.06)**	(6.70)**	(0.51)	(1.10)	(1.10)	(6.47)**
Inc 18	0.327	2.316	0.349	0.173	0.173	2.767
	(21.90)**	(8.01)**	(4.92)**	(6.25)**	(3.73)**	(8.54)**
Man	-0.002	-0.374	-0.025	-0.068	-0.086	-0.270
	(1.30)	(11.16)**	(4.63)**	(16.56)**	(17.85)**	(9.16)**
Child<16	-0.010	-0.064	-0.041	0.001	-0.025	-0.063
	(13.56)**	(3.84)**	(15.26)**	(0.52)	(12.33)**	(4.33)**
Child<6	0.040	0.066	0.018	0.000	0.014	0.009
	(53.93)**	(3.42)**	(6.10)**	(0.04)	(6.46)**	(0.55)

Table 21: (Cont) Regular Price on Household Characteristics (III)

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Age 30	0.007	0.120	0.034	0.030	0.063	0.108
	(6.94)**	(3.97)**	(7.28)**	(9.56)**	(19.83)**	(4.77)**
Age 40	0.024	0.313	0.076	0.063	0.105	0.374
	(21.00)**	(9.89)**	(15.53)**	(18.83)**	(30.78)**	(15.54)**
Age 60	0.045	0.727	0.147	0.047	0.159	0.996
	(35.66)**	(21.98)**	(28.69)**	(13.15)**	(42.32)**	(38.16)**
Educ 1	0.029	0.444	0.095	0.049	0.137	0.307
	(31.12)**	(20.69)**	(27.43)**	(20.07)**	(49.87)**	(15.66)**
Educ 2	0.014	0.474	0.076	0.039	0.053	0.173
	(19.58)**	(29.78)**	(29.23)**	(19.84)**	(25.48)**	(11.80)**
Educ 3	0.010	0.125	0.046	0.023	0.022	0.009
	(17.19)**	(9.52)**	(21.33)**	(13.69)**	(12.73)**	(0.76)
Region Dummies	Yes					
Observations	431405	231383	327507	645906	329878	124732
R-squared	0.11	0.05	0.13	0.07	0.18	0.13

Table 22: Alpha on Household Characteristics I

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Extra	5.04e+3	7.52e+16	-2.87e+17	-3.43e+21	-6.04e+21	-2.69e+14
	(1.91)	(1.50)	(0.31)	(0.03)	(0.84)	(0.66)
House	1.29e+3	-1.28e+17	-1.52e+18	-1.48e+23	4.85e+21	-5.07e+14
	(0.51)	(2.62)**	(1.58)	(1.46)	(0.70)	(1.27)
Car	-1.36e+3	3.08e+16	4.76e+17	1.96e+23	-1.87e+20	5.29e+14
	(0.27)	(0.32)	(0.25)	(1.05)	(0.01)	(0.69)
Fam Size=2	-8.27e+2	7.34e+16	4.69e+17	-3.99e+23	8.25e+21	-6.65e+14
	(0.17)	(0.76)	(0.28)	(2.46)*	(0.73)	(0.99)
Fam Size=3	-8.18E+3	3.16e+16	2.78e+18	-4.91e+23	-8.38e+21	-9.04e+14
	(0.15)	(0.29)	(1.47)	(2.56)*	(0.63)	(1.15)
Fam Size=4	1.86e+3	3.44e+16	1.09e+18	-5.04e+23	-1.01e+22	-8.98e+14
	(0.33)	(0.31)	(0.54)	(2.47)*	(0.71)	(1.08)
Fam Size=5	-1.81e+3	4.48e+16	1.32e+18	-5.12e+23	-1.14e+22	-8.62e+14
	(0.29)	(0.37)	(0.60)	(2.24)*	(0.72)	(0.94)
Fam Size=6	-332	5.13e+16	1.27e+18	-5.05e+23	-1.2e+22	-9.20e+14
	(0.04)	(0.34)	(0.43)	(1.62)	(0.55)	(0.75)
Fam Size=7	-871	6.83e+16	1.726e+18	-5.14e+23	-1.11e+22	-1.03e+15
	(0.07)	(0.28)	(0.33)	(0.98)	(0.32)	(0.48)
Fam Size=8	4.5e+3	2.62e+16	8.17e+17	-6.40e+23	-1.236e+22	-1.27e+15
	(0.23)	(0.06)	(0.11)	(0.69)	(0.21)	(0.34)
Fam Size=9	1.9e+3	7.33e+15	1.56e+18	-6.92e+23	-1.43e+22	-5.21e+14
	(0.06)	(0.01)	(0.11)	(0.43)	(0.12)	(0.08)
Region=2	329	5.06e+16	4.27e+18	7.06e+22	-1.75e+21	6.20e+14
	(0.07)	(0.60)	(2.56)*	(0.40)	(0.14)	(0.88)
Region=3	26.6	7.81e+16	7.63e+17	5.163+23	-1.38e+21	6.88e+14
	(0.01)	(0.88)	(0.44)	(2.80)**	(0.11)	(0.94)
Region=4	232	1.62e+17	6.12e+17	9.48e+22	1.13e+22	7.08e+14
	(0.06)	(2.13)*	(0.43)	(0.62)	(1.08)	(1.18)
Region=5	109	5.89e+16	5.50e+17	7.50e+22	-1.47e+21	2.39e+15
	(0.02)	(0.64)	(0.31)	(0.40)	(0.12)	(3.27)**
Region=6	6.6e+3	3.86e+16	3.32e+17	3.52e+22	-1.43e+21	4.20e+14
	(1.68)	(0.49)	(0.22)	(0.23)	(0.13)	(0.68)
Region=7	992	2.97e+16	2.80e+17	1.83e+22	-1.88e+21	3.58e+14
	(0.23)	(0.35)	(0.17)	(0.11)	(0.16)	(0.53)
Region=8	669	5.74e+16	3.45e+17	5.60e+22	-2.93e+21	6.00e+14
	(0.14)	(0.64)	(0.20)	(0.31)	(0.23)	(0.80)

Table 23: (cont) Alpha on Household Characteristics II

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Man	-77.3	-1.12e+16	-3.06e+17	-3.83e+23	-6.69e+21	4.09e+15
	(0.01)	(0.08)	(0.13)	(1.68)	(0.40)	(4.13)**
Child⩽16	2.6e+3	3.77e+12	-1.45e+18	-7.47e+21	-1.76e+21	4.33e+13
	(0.81)	(0.00)	(1.13)	(0.05)	(0.18)	(0.08)
Child⩽6	-7.1e+33	-4.78e+15	9.65e+16	8.55e+22	4.24e+21	1.60e+14
	(2.01)*	(0.07)	(0.07)	(0.59)	(0.42)	(0.27)
age30	1.2e+3	2.12e+16	9.38e+17	8.45e+22	2.99e+21	5.75e+13
	(0.23)	(0.20)	(0.39)	(0.42)	(0.20)	(0.06)
age40	-5.0e+3	3.30e+16	1.27e+18	2.34e+23	1.00e+22	5.51e+14
	(0.88)	(0.29)	(0.51)	(1.11)	(0.64)	(0.58)
age60	-4.5e+3	1.20e+17	8.39e+17	5.58e+22	-5.02e+21	1.21e+14
	(0.73)	(0.99)	(0.32)	(0.25)	(0.30)	(0.12)
educ1	-2.7e+3	3.03e+16	8.61e+17	1.49e+22	4.22e+21	8.28e+14
	(0.62)	(0.35)	(0.53)	(0.09)	(0.35)	(1.20)
educ2	1.4e+3	1.23e+17	6.78e+17	2.06e+23	3.00e+21	-2.97e+14
	(0.44)	(1.90)	(0.54)	(1.52)	(0.32)	(0.56)
educ3	-1.5e+3	2.86e+16	1.16e+18	3.27e+22	8.65e+21	-1.56e+14
	(0.54)	(0.53)	(1.12)	(0.28)	(1.10)	(0.35)
Inc2	-3.4e+3	7.95e+16	-7.70e+17	4.65e+22	-4.13e+21	-9.18e+14
	(0.06)	(0.06)	(0.03)	(0.02)	(0.03)	(0.08)
Inc3	-1.8e+3	7.36e+16	-1.44e+18	6.40e+22	-3.63e+21	-7.05e+14
	(0.03)	(0.05)	(0.06)	(0.03)	(0.03)	(0.06)
Inc4	-2.0e+3	7.74e+16	-1.50e+18	1.53e+23	-4.10e+21	-4.81e+14
	(0.04)	(0.06)	(0.06)	(0.08)	(0.04)	(0.04)
Inc5	-338	6.48e+16	-1.57e+18	1.25e+23	-5.70e+21	-2.47e+14
	(0.01)	(0.05)	(0.07)	(0.07)	(0.05)	(0.02)
Inc6	-1,5e+3	5.93e+16	-1.65e+18	1.10e+23	-3.86e+21	-1.28e+14
	(0.03)	(0.04)	(0.07)	(0.06)	(0.03)	(0.01)
Inc7	-1.4e+3	6.6e+16	-1.85e+18	1.63e+23	-4.47e+21	-3.79e+14
	(0.03)	(0.05)	(0.08)	(0.09)	(0.04)	(0.03)

Table 24: (cont) Alpha on Household Characteristics III

Charac	Products					
	Milk	Coffee	Butter	Yogurt	Pasta	Tuna
Inc8	-1.8e+3	7.99e+16	-1.92e+18	9.07e+23	-4.65e+21	-7.23e+13
	(0.03)	(0.06)	(0.08)	(0.48)	(0.04)	(0.01)
Inc9	-2.0e+3	7.62e+16	-1.94e+18	1.70e+23	-4.17e+21	-2.94e+12
	(0.04)	(0.06)	(0.08)	(0.09)	(0.04)	(0.00)
Inc10	-2.52e+3	6.93e+16	-1.8e+15	2.06e+23	1.12e+22	1.10e+14
	(0.04)	(0.05)	(0.00)	(0.11)	(0.10)	(0.01)
Inc11	-2.8e+3	6.97e+16	-2.18e+18	2.07e+23	-4.37e+21	1.67e+14
	(0.05)	(0.05)	(0.09)	(0.11)	(0.04)	(0.02)
Inc12	-2.78e+3	7.69e+16	-2.17e+18	2.16e+23	-4.17e+21	1.92e+14
	(0.05)	(0.06)	(0.09)	(0.12)	(0.04)	(0.02)
Inc13	-2.54e+3	2.98e+17	-2.09e+18	2.13e+23	-4.21e+21	2.42e+14
	(0.04)	(0.22)	(0.09)	(0.11)	(0.04)	(0.02)
Inc14	9.38e+3	5.73e+16	-2.07e+18	2.00e+23	-2.62e+21	2.14e+15
	(0.17)	(0.04)	(0.09)	(0.11)	(0.02)	(0.20)
Inc15	-3.09e+3	7.28e+16	-2.15e+18	2.32e+23	-4.22e+21	2.91e+14
	(0.05)	(0.05)	(0.09)	(0.12)	(0.04)	(0.03)
Inc16	-3.13e+3	5.37e+16	-2.15e+18	2.17e+23	-3.53e+21	1.04e+14
	(0.05)	(0.04)	(0.09)	(0.12)	(0.03)	(0.01)
Inc17	-2.80e+3	6.89e+16	-2.64e+18	2.11e+23	-1.76e+21	1.52e+14
	(0.05)	(0.05)	(0.11)	(0.11)	(0.01)	(0.01)
Inc18	-4.55e+3	9.97e+16	-1.95e+18	2.34e+23	-7.14e+20	3.05e+14
	(0.07)	(0.07)	(0.08)	(0.12)	(0.01)	(0.03)
Const	4.46e+3	-2.32e+17	-4.10e+17	-1.70e+23	-3.48e+21	-5.28e+14
	(0.08)	(0.17)	(0.02)	(0.09)	(0.03)	(0.05)
Obs	3083	4569	3716	6110	4396	3086
R2	0.01	0.01	0.01	0.01	0.01	0.00