MULTILEVEL ANALYSIS OF DELAMINATION INITIATED NEAR THE EDGES OF COMPOSITE STRUCTURES

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SUMMARY

The present study is aimed at developing a method to describe delamination initiated near the edges on composite structures. In this study, two aspects are under investigation : (i) the modeling of the singular stress fields near the edges and (ii) two complementary methods to predict the onset of delamination.

Keywords: edge effects, cohesive zone models, fracture mechanics, delamination, multilevel strategy

INTRODUCTION

Due to their high mechanical properties, composite materials are more and more employed in many aerospace applications. However, the present simulation tools used during design and conception of new structures do not take into account the complexity of damage/rupture mechanisms and the multiscale nature of composites. Moreover, some key problems remain very sensitive. The first one concerns the prediction of the strength of high stress gradient parts of the structures. The second one concerns the modeling of delamination which could not be, contrary to in-ply damage, described by a continuous damage model. Obviously, these two problems arise in laminate composites due to a mismatch in elastic properties between plies. In such a case, singular interlaminar stresses are indeed created leading to interply debonding. The aim of this paper is to propose a robust methodology to analyze the delamination (onset threshold and if necessary propagation) initiated near the edges and the possible interaction with damage inside the plies. This paper is devided in x sections. The first one is devoted to the strategy developed to predict the delamination initiation near the edges. In this section, the FE method to calculate the singular stress field near the edges is presented. Two complementary approaches are developed to predict the onset of delamination. The second section is devoted to the identification and the comparison with experimental results. Finally, the last section is devoted to discussion and conclusion.

STRATEGY FOR THE PREDICTION OF DELAMINATION INITIATED NEAR THE EDGES

Calculation of singular stress field near edges

Due to elastic mismatch between the plies, stress field near the edges are singular. Various approaches have been proposed in the literature in order to investigate the stress field near the edges of a composite laminate based on linear assumptions [1]. However, the aim of this paper is to propose a methodology that could be applied to predict the onset of inter-ply damage (i.e. delamination) and intra-ply damage (matrix cracking). This is the reason why, the elastic assumption is no more valid. A Finite Element modeling is thus necessary to calculate the stress fields. In order to capture the singularities, it is necessary to use very fine meshes leading to high computational cost. Two approaches are developed:

- The first one is a 2D approach, based on a variationnal approach initially proposed in [2]. It consists in a 2.5D model that supposes a uniform strain on the length of the laminate. Forces, which depend on the elastic properties of the plies, are applied to the boundary of the mesh (see Figure 1).

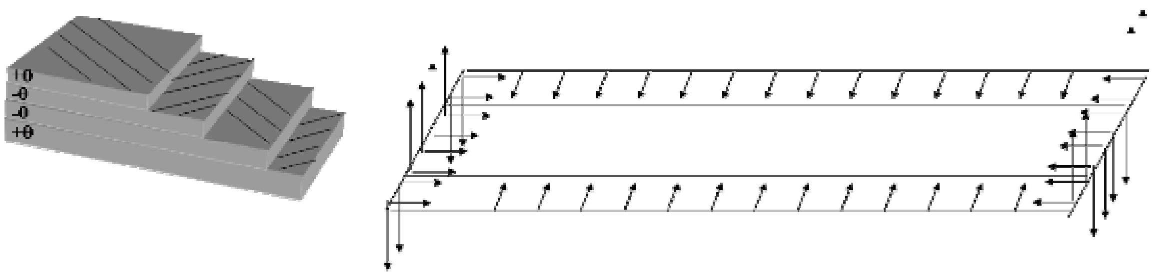


Figure 1 : Principle of the 2D approach proposed in [2].

- The second approach consists in using 3D volume elements. The boundary conditions and a mesh are presented in **Figure 2**.

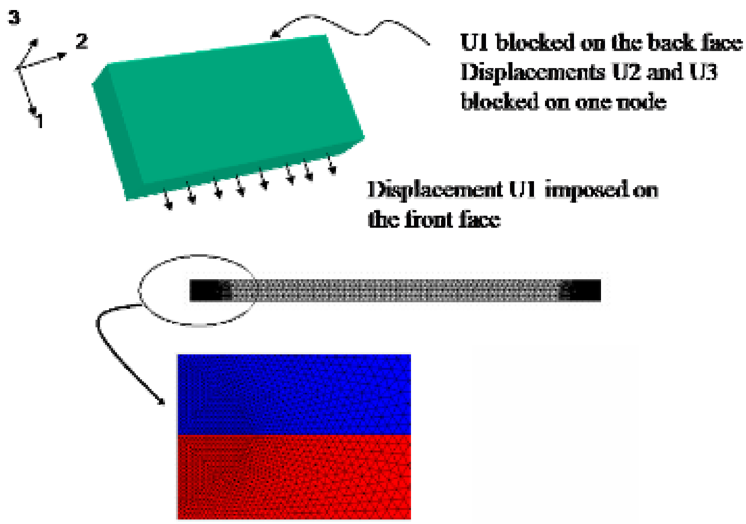


Figure 2 : Boundary conditions and mesh for the 3D FE modelling

The results obtained by these two methods are compared with those obtained in the literature for instance with CLEOPS [3]. See Figure 3 for a comparison between the 3D FE approach and the exact results provided by CLEOPS. The two approaches presented in this paper lead to very similar results. However, the computational cost of the 2D approach is very low as compared with the 3D one. On the other hand, the 3D approach is more general and could be applied to every geometry (for instance edges of a hole) and loadings while the 2D model is restricted to plane plates subjected to in-plane loadings.

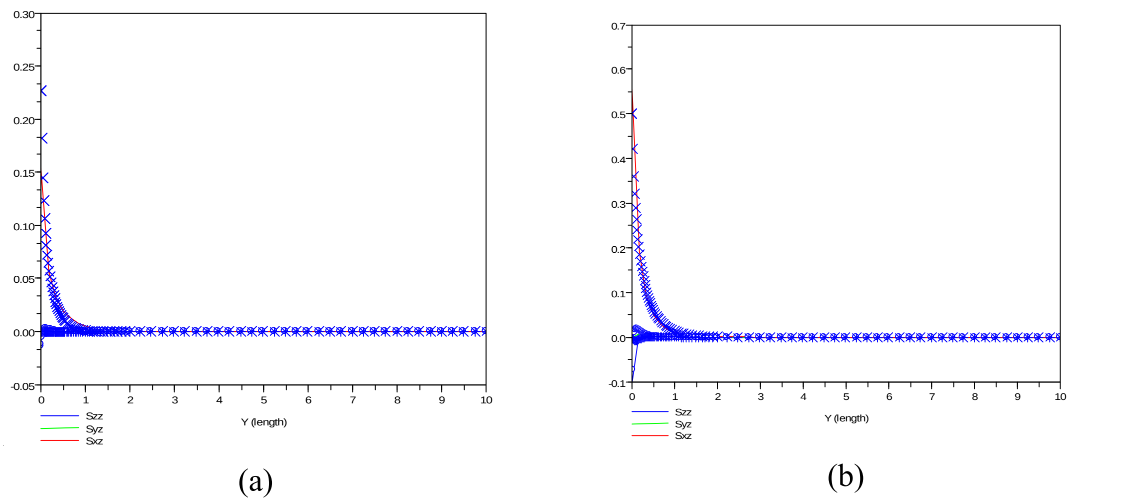


Figure 3 : Comparison between a 3D FE model (dot in the figure) and the exact soltuin provided by CLEOPS [3] for (a) [+/-10°]s (b) [+/-30°]s laminates.

Prediction of the onset of delamination

The stress fields being singular near the edges it is not possible to apply a simple stress or strain based criterion. Different methods could be found in the literature to overcome this problem for instance by averaging, over a length *L*, the shear stress at the interface [3]. However, this approach requires to identify the parameter *L* which has no physical sense. This is the reason why, in the present study, two complementary approaches are proposed: the first one is based on a mixed stress and energy criterion and the second one involves cohesive zone models.

Mixed criterion to describe the onset of damage

The mixed criterion is based on an energetic condition and a stress criterion.

The energetic criterion compared the change of the potential energy ΔW and the material toughness G^c. ΔW is expressed as a variation between the final state (with a crack of length d) and the initial state (without a crack). This energetic condition is given by the following equation:

$$G^{inc}(d) = \frac{W(0) - W(d)}{d} = \overline{A}(\underline{l}, d) R E \varepsilon^2 \geq G^c \quad (1)$$

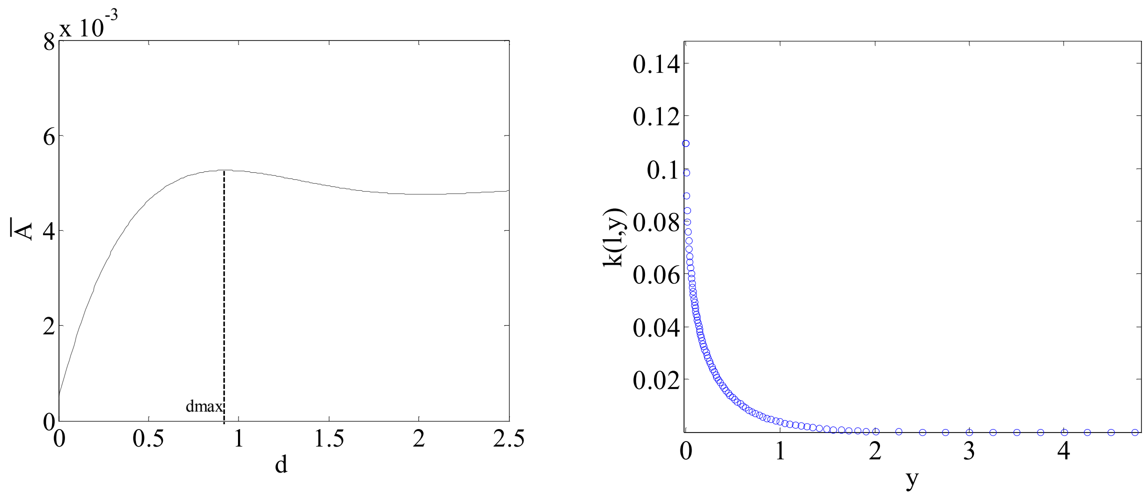
where W is the potential energy for a constant external loading ε, G^c is the interfacial toughness. $\overline{A}(\underline{l}, d)$ is a dimensionless parameter which depends only on the geometry ($\underline{l}$) and of the length crack (d). R is the thickness of the plies. This relation involves an incremental energy release rate $G^{inc}(d)$ since the classical infinitesimal increment of crack is replaced here by a finite increment of crack.

In the present study, the stress criterion is a simple maximum criterion. It implies that, prior to the rupture, a state of stress σ greater than the strength of the interface σ^c takes place on a distance at least equal to the length of the initiated crack:

$$\sigma(l, y) \geq k(\underline{l}, y)E\varepsilon \geq \sigma^c \tag{2}$$

where $k(\underline{l}, y)$ is a dimensionless parameter which depends only on the geometry ($\underline{l}$) and of the coordinates along the interface (y). The procedure to calculate the dimensionless parameters could be found in [4].

Characteristic evolutions of the energy release rate (represented by $\overline{A}(\underline{l}, d)$, see eq 1) and the stress intensity factor ($k(\underline{l}, y)$) in the case of a delamination initiated from the edge are given in Figure 4a and b.



(a)

(b)

Figure 4 : $\overline{A}(\underline{l}, d)$ as a function of the crack length (a) and $k(\underline{l}, y)$ as a function of the y abscissa (b).

Figure 4a shows an increase of the energy release rate as a function of the crack length d. $\overline{A}(\underline{l}, d)$ is maximal for a crack length noted d_{max}. For a monotonic loading, the energy criterion is satisfied for

$$\overline{A}(\underline{l}, d_{max})RE\varepsilon^2 = G^c \tag{3}$$

The initiation of the crack with a length d_{max} is possible if :

$$k(\underline{l}, d_{max})E\varepsilon = \sigma^c \tag{4}$$

(3) and (4) lead to

$$L^c = \frac{EG^c}{\sigma^c} \geq R\frac{\overline{A}(\underline{l}, d_{max})}{k^2(\underline{l}, d_{max})} = L_{max} \tag{5}$$

Two cases must be considered:

1. $L^c \geq L_{max}$. In this case, the stress criterion (4) is satisfied, on a length greater than d_{max}, before the energy criterion (3). In this case, the crack is initiated on a length $d^* = d_{max}$, and the critical loading is no more a function of σ^c
2. $L^c < L_{max}$. In this case, the critical loading is a function of σ^c, G^c and the crack is initiated on a length d^* which corresponds to the solution of

$$\frac{\overline{A}(\underline{l}, d_{max})}{k^2(\underline{l}, d_{max})} = \frac{Lr^c}{R} \tag{6}$$

Finally, the critical loading is calculated :

$$\frac{\varepsilon^c}{\varepsilon^0} = \frac{1}{\sqrt{\overline{A}(\underline{l}, d^*)}} \text{ with } \varepsilon^0 = \sqrt{\frac{G^c}{RE}} \tag{7}$$

Cohesive zone models to describe de delamination

Cohesive zone elements are used to simulate the interfaces between the plies. The behaviour of these interfaces is elastic with softening damage rules in normal tension and shear; moreover, friction is taken into account in shear. The mechanical behaviour of the interface is described by the relations between the normal and tangential relative displacements (U_n, U_t) of these nodes, and their respective normal and tangential tractions (T_n, T_t). The damage evolution is taken into account by the damage variable λ which combines the tension and the shear damages as follows [5]:

$$\lambda = \sqrt{\left(\frac{\langle U_n \rangle_+}{\delta_n}\right)^2 + \left(\frac{U_t}{\delta_t}\right)^2}$$

The non-linear relations between (U_n, U_t) and (T_n, T_t) have the form:

$$\begin{cases} T_n = \dfrac{U_n}{\delta_n} F(\lambda) \\ T_t = \alpha \dfrac{U_t}{\delta_t} F(\lambda) \end{cases}$$

and $F(\lambda)$ is chosen as $F(\lambda) = \dfrac{27}{4}\sigma_{max}(1-\lambda)^2$ for $0 \leq \lambda \leq 1$, where σ_{max} and $\alpha\sigma_{max}$ are respectively the maximum values of T_n and T_t in pure modes. The damage parameter λ varies continuously from 0 (locally bonded case) to 1 (locally debonded case). The complete separation between two corresponding nodes occurs for λ=1 ($F(\lambda)$=0) so that δ_n and δ_t are the maximum values of the relative displacements U_n and U_t in pure normal and pure shear modes respectively. In the present study, no distinction is made between normal and tangential loadings, it means that $\alpha = 1$ and $\delta_t = \delta_n$. Two parameters have to be identified: σ_{max} and δ_n.

A very important point concerns the definition of the initiation of the crack and of its length. In the present paper, it has been supposed that a crack is created as soon as the damage variable λ is equal to 1 in (at least) one Gauss Point, the crack tip is located at the last damage Gauss Point.

IDENTIFICATION AND COMPARISON WITH EXPERIMENTAL RESULTS

The aim of this section is to identify and validate these two approaches against experimental results provided in [6]. The material under investigation is a carbon/epoxy G947/M18 laminate. The mechanical properties of the ply are given in Table 1. The ply thickness is equal to 0.190 mm.

Table 1 : Mechanical properties of the G947/M18 ply material

E_{11}(GPa)	E_{22} (GPa)	E_{33} (GPa)	G_{12} (GPa)	G_{23} (GPa)	G_{13} (GPa)	n_{12}	n_{23}	n_{13}
97.6	8	8	3.1	2.7	3.1	0.37	0.5	0.37

$[+/-\theta_n]_s$ laminates are investigated in the present paper. The results obtained in [6] are reported in Table 2. Two important points are evidenced in this table (i) an increase in thickness of the specimen (it means n increases) leads to a decrease of the critical stress, and (ii) an increase in θ leads to a decrease of the critical load.

Table 2 : Experimental results from [6] for [+/-θn]s laminates

Stacking sequence	*Mode of delamination*	*Critical stress (MPa)*	*Std deviation*
$[+/-10]_s$	III	812	10
$[+/-10_2]_s$	III	723	13
$[+/-10_3]_s$	III	690	10
$[+/-10_4]_s$	III	672	16
$[+/-20_2]_s$	Quasi pure III	481	10
$[+/-20_4]_s$	Quasi pure III	453	8
$[+/-30]_s$	Quasi pure III	391	8
$[+/-30_2]_s$	Quasi pure III	351	5
$[+/-30_3]_s$	Quasi pure III	339	5
$[+/-30_4]_s$	Quasi pure III	328	8

Identification of the models

The identification of the mixed criterion model necessitates, first of all, to perform FE calculations to calculate $\overline{A}(\underline{l}, d)$ and $k(\underline{l}, y)$. In order to identify the couples (G^c,σ^c) that correspond to the experimental critical load ε^c leading to the initiation of the delamination, G^c (and then ε^0) is fixed in order to calculate d^* (see eq. 5 and 6). Finally, L^c (σ^c) is calculated using eq. 7. It is worth mentioning that only a few elastic FE calculations are necessary to calculate $\overline{A}(\underline{l}, d)$ and $k(\underline{l}, y)$. Using these two parameters, the identification of all the couples (G^c,σ^c) leading to the correct experimental critical load, involved only some analytical treatments. Table 3 shows the identification for the [+/-10n]s and the [+/-20n]s laminates.

Table 3: Identification of the best couples (G^c,σ^c) for the $[+/-10_n]_s$ and the $[+/-20_n]_s$ laminates

Stacking sequence	± 10	± 20
σ^c (MPa)	344	406
G^c (Jm^{-2})	86.5	60.7
Error between experimental and numerical critical load (%)	1.18	3.44
a*(μm)	1-2	0.5-1.5

The identification of the cohesive zone models, involving non-linear FE calculations, is more complex. Indeed, (G^c,σ^c) couples are iteratively chosen thanks to an optimization algorithm. A finite element calculation is performed until the onset of damage. The critical numerical load is compared to the experimental one. The iterative process is performed until the convergence between the numerical and the experimental load. This approach could lead to a high computational time and provides only one couple (G^c,σ^c) leading to the experimental critical load. In the present paper, a strategy, based on the use of a surrogate model (also called response surface), is used. A surrogate model consists in building an analytical relation (most generally polynomial) between some inputs (in the present case G^c,σ^c) and the outputs (in the present case the critical load) using a limited number of runs of the reference modelling (see Figure 5). Finally, this analytical approximation is used to determine the values of the outputs as a function of the variation of the inputs. In this study, the approximation is performed thanks to a sparse polynomial expansion. It is worth mentioning that the analytical model is just an approximation and some tools have been developed in order to increase the confidence in this approximation [7].

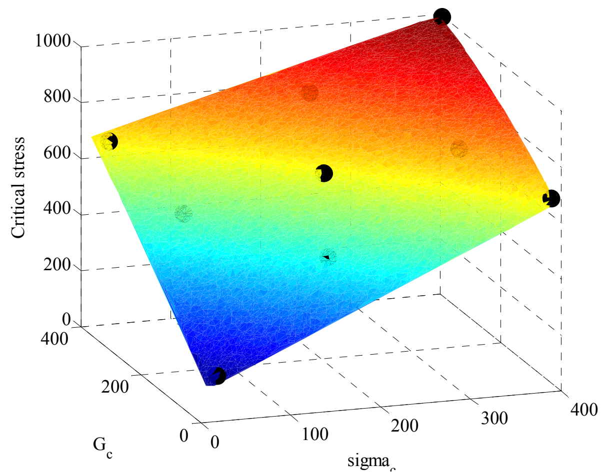


Figure 5 : Principle of the response surface method (RSM) to assess the evolution of the critical stress as a function of the interfacial parameters G^c,σ^c. The black dots correspond to the identification points. The surface corresponds to the results obtained by the RSM.

Figure 6a and Figure 6b show the couples (G^c,σ^c) leading to a critical load with an error of 10% as compared with the experimental ones for the [+/-10]s and [+/-10$_2$]s laminates. It is worth mentioning that a large number of couples (G^c,σ^c) permits to predict the experimental results for the two laminates. The value identified from the mixed criterion is close to (but lower) some values given by this analysis on cohesive zone models.

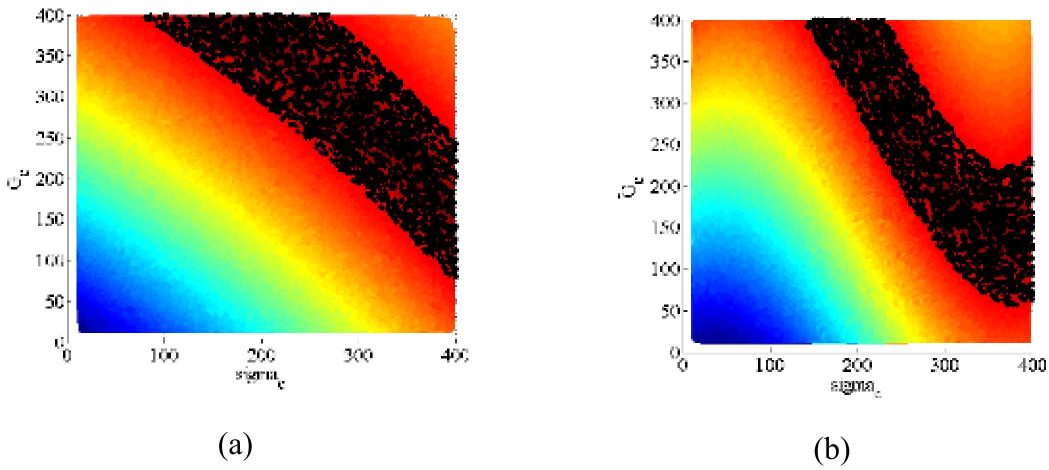


Figure 6 : Identification of the couples (G^c,σ^c) corresponding to the experimental critical load for a $[+/-10]_s$ (a) and a $[+/-10_2]_s$ (b) laminate. The black dots correspond to (G^c,σ^c) parameters leading to an error lower than 10%.

The evolution of the critical load as a function of the thickness is shown in Figure 7. The model permits to reproduce in a correct manner the main experimental evidences i.e. a decrease of the critical load if the thickness and/or the angle of the plies increase.

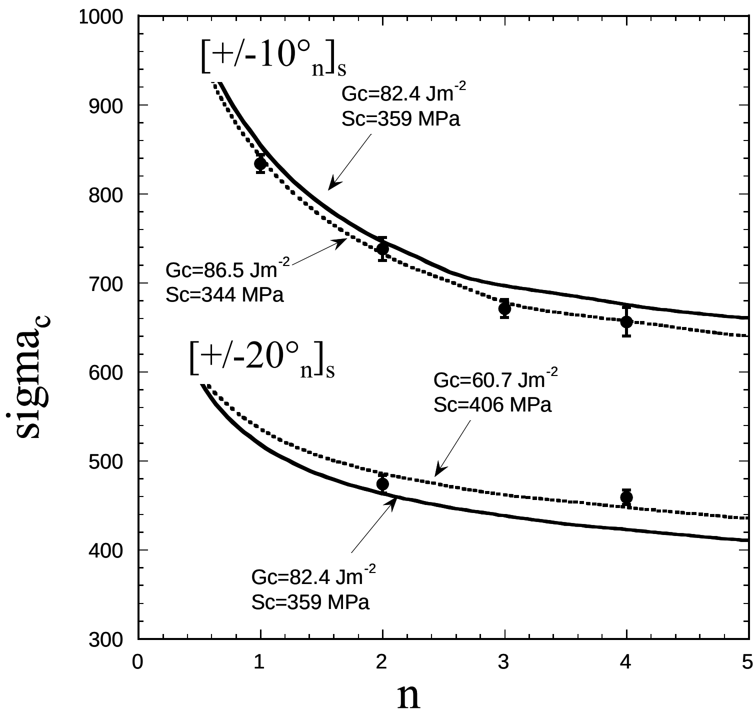


Figure 7 : Evolution of the critical load as a function of the number of plies for the $[+/-10_n]_s$ and $[+/-20_n]_s$ laminates. The dashed curves are predicted with the (G^c,σ^c) provided in Table 3. The continuous line corresponds to the best fit for the two laminates.

DISCUSSION AND CONCLUSION

A strategy has been presented to predict the onset of delamination initiated from the edges. Two approaches have been proposed to attain this goal. The first is based on a mixed criterion that necessitates only elastic FE calculations to be applied. The second approach used 3D FE calculations with cohesive zone elements. As compared with the classical methods used in the literature, these two approaches do not necessitate the identification of a critical length. Only the critical energy release rate G^c and the strength σ^c of the interface necessitate to be identified.

These two approaches have permitted to reproduce experimental evidences such as an increase in thickness of the specimen leading to a decrease of the critical stress, and (ii) an increase in θ leading to a decrease of the critical load. However, it has been shown that :

- In order to use these methods, very fine meshes are needed. Indeed, the crack length is very small (from 0.5 to 2 µm). This is the reason why, in order to apply this approach to a complex structure, a computational strategy is needed (for instance a global->local approach). A 2D FE approach has also been presented in order to reduce drastically the computational cost.

- The mixed criterion approach and the cohesive zone models are complementary. Indeed, the mixed criterion dimensionless parameters are only a function of the elastic properties and of the geometry. These parameters are calculated using simple elastic modelings. Thanks to these parameters, analytical equations permit to assess the effect of (G^c,σ^c) on the critical loading leading to the onset of delamination. On the other hand, cohesive zone models necessitate expensive non-linear FE calculations which are a function of the elastic properties, the geometry and the interfacial parameters (G^c,σ^c). In order to assess the effect of the interfacial parameters, a strategy based on surrogate model is necessary. However, contrary to the mixed criterion, cohesive zone models permit to take into account all the non-linearities (viscosity, in-plane damage in the plies) and the coupling between intra-ply damage and delamination.

- In order to reproduce the experimental results, is necessary to identify the interfacial parameters of each stacking sequence. This problem is probably due to the evolution of the mixed-mode delamination (see Table 2). Indeed, it is well known that the critical energy release rate is a function of this mixed-mode delamination, and this point must be introduced in the present analysis for both the energy release rate and the critical strength of the interface.

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