Question 5

Intent of Question

The primary intent of this question is to assess a student's ability to make an inference about the difference in two population means using a two-sample *t*-test, including the identification of the null and alternative hypotheses and good communication of test results and conclusions.

Solution

Part 1: States a correct pair of hypotheses

$$\mathrm{H_o}: \mu_S - \mu_N = 0 \text{ versus } \mathrm{H_a}: \mu_S - \mu_N < 0$$

OR

$$\mathrm{H_o}: \mu_N - \mu_S = 0 \text{ versus } \mathrm{H_a}: \mu_N - \mu_S > 0$$

OR

$$\mathrm{H_o}: \mu_S = \mu_N \text{ versus } \mathrm{H_a}: \mu_S < \mu_N$$

where μ_s = mean decrease in cholesterol for standard drug
and μ_N = mean decrease in cholesterol for new drug.

Part 2: Identifies a correct test (by name or by formula) and checks appropriate assumptions.

Two-sample *t*-test (or z-test) $$t = \frac{\overline{X}_S - \overline{X}_N}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_N^2}{n_N}}}$$

Assumptions:

1. random assignment of subjects to treatments;
2. normal population distributions or large samples.

Both sample sizes were large (50), and there was random assignment of subjects to treatments.

NOTE: A two-sample z-test is acceptable as long as the large sample sizes are noted. A pooled *t*-test is also acceptable, but the student must also state and comment on the plausibility of the equal population variances assumption.

Part 3: Correct mechanics, including the value of the test statistic, df (stated or implied by calculator work), and *p*-value (or rejection region).

$$t = \frac{\overline{X}_S - \overline{X}_N}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_N^2}{n_N}}} = \frac{10 - 18}{\sqrt{\frac{8^2}{50} + \frac{12^2}{50}}} = -3.92$$

df = 85, p-value = 0.000088, or from table p-value < 0.001
OR
df = 49, p-value = 0.00014.
For two-sample z-test, $z = -3.92$, , p-value = 0.000044.
For pooled t-test, $t = -3.92$, df = 98, p-value = 0.000081.

Rejection regions:

$\alpha = 0.10$: $t < -1.303$ (df = 40), $t < -1.292$ (df = 80), $t < -1.290$ (df = 100) OR $z < -1.28$
$\alpha = 0.05$: $t < -1.684$ (df = 40), $t < -1.664$ (df = 80), $t < -1.660$ (df = 100) OR $z < -1.645$
$\alpha = 0.01$: $t < -2.423$ (df = 40), $t < -2.374$ (df = 80), $t < -2.364$ (df = 100) OR $z < -2.33$

Part 4: Stating a correct conclusion in the context of the problem, using the result of the statistical test.

Because the p-value < selected α (or because the p-value is so small), reject H_o. There is convincing evidence that the mean cholesterol reduction is greater for the new drug.

Scoring

Each part is scored as either essentially correct (E), partially correct (P), or incorrect (I).

Part 1 is essentially correct (E) if the response:
1. includes the correct pair of hypotheses;
2. defines the parameters in the hypotheses in the context of the problem.

Part 1 is partially correct (P) if the hypotheses are stated correctly, but notation is not defined.

Part 2 is essentially correct (E) if the response:
1. identifies the correct test by name or formula;
2. checks appropriate assumptions (including equal variance if pooled t-test is used).

Part 2 is partially correct (P) if it includes only one of the two elements above.

Part 3 is essentially correct (E) if the response includes:
1. correct mechanics, including the value of the test statistic;
2. df and p-value or rejection region consistent with the hypotheses in part 1.

Part 3 is partially correct (P) if it includes only one of the two elements above.

Part 4 is essentially correct (E) if the response includes:
1. a conclusion in context consistent with the hypotheses in part 1;
2. linkage between the results of the test in part 3 and the conclusion, and this is communicated well.

Part 4 is partially correct (P) if it includes only one of the two elements above.

Question 5 (continued)

NOTES:

- If both an α and a *p*-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the *p*-value or explaining how the conclusion follows from the *p*-value.
- If the *p*-value in part 3 is incorrect but the conclusion is consistent with the computed *p*-value, part 4 can be considered essentially correct (E).

4 Complete Response

All four parts essentially correct

3 Substantial Response

Three parts essentially correct and no parts partially correct

OR

Two parts essentially correct and two parts partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and two parts partially correct

OR

Four parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

If a response is between two scores (for example, 2½ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

5. A serum cholesterol level above 250 milligrams per deciliter (mg/dl) of blood is a risk factor for cardiovascular disease in humans. At a medical center in St. Louis, a study to test the effectiveness of a new cholesterol-lowering drug was conducted. One hundred people with cholesterol levels between 250 mg/dl and 300 mg/dl were available for this study. Fifty people were assigned at random to each of two treatment groups. One group received the standard cholesterol-lowering medication and the other group received the new drug. After taking the drug for three weeks, the 50 subjects who received the standard treatment had a mean decrease in cholesterol level of 10 mg/dl with a standard deviation of 8 mg/dl, and the 50 subjects who received the new drug had a mean decrease of 18 mg/dl with a standard deviation of 12 mg/dl.

Does the new drug appear to be more effective than the standard treatment in lowering mean cholesterol level? Give appropriate statistical evidence to support your conclusion.

Ho: the new drug's effectiveness equals that of the old drug. $\mu_o = \mu_n$

Ha: the new drug is more effective than the old drug. $\mu_o < \mu_n$

μ_o = mean decrease in cholestrol level for old drug

μ_n = mean decrease in cholestrol level for new drug

$\bar{x}_n = 18$ $\quad \bar{x}_o = 10$

$s_n = 12$ $\quad s_o = 8$

$n_n = 50$ $\quad n_o = 50$

$$t^* = \frac{\bar{x}_n - \bar{x}_o}{\sqrt{\frac{s_n^2}{n_n} + \frac{s_o^2}{n_o}}} = \frac{18-10}{\sqrt{\frac{144+64}{50}}} = \frac{8}{2.04} = 3.92$$

$t^* = 3.92$

$df = 50 - 1 = 49$

df	.0005	> p-value
49	3.496	3.92

Assumptions

- the simple random sample of this test is assumed as stated above.
- $n_n = 50 \geq 30$ — For both samples, the
  $n_o = 50 \geq 30$ — Central Limit Theorem is satisfied for normality

population$_n \geq 10n_n = 10(50) = 500$

population$_o \geq 10n_o = 10(50) = 500$

p-value < .0005 which means that the probability is significantly small. Reject Ho, assume that Ha is true.

∴ the new drug seems to be more effective at lowering mean cholesterol level.

GO ON TO THE NEXT PAGE.

5. A serum cholesterol level above 250 milligrams per deciliter (mg/dl) of blood is a risk factor for cardiovascular disease in humans. At a medical center in St. Louis, a study to test the effectiveness of a new cholesterol-lowering drug was conducted. One hundred people with cholesterol levels between 250 mg/dl and 300 mg/dl were available for this study. Fifty people were assigned at random to each of two treatment groups. One group received the standard cholesterol-lowering medication and the other group received the new drug. After taking the drug for three weeks, the 50 subjects who received the standard treatment had a mean decrease in cholesterol level of 10 mg/dl with a standard deviation of 8 mg/dl, and the 50 subjects who received the new drug had a mean decrease of 18 mg/dl with a standard deviation of 12 mg/dl.

Does the new drug appear to be more effective than the standard treatment in lowering mean cholesterol level? Give appropriate statistical evidence to support your conclusion.

$H_0: \mu_1 = \mu_2$ The standard cholesterol-lowering medication and the new drug have the same effectivness.

$H_a: \mu_1 < \mu_2$ The standard cholesterol lowering medication is less effective than the new drug.

Two-Sample T-Test

Conditions
SRS ✓
100 x 10 ≤ pop ✓
normal ✓

$$t = \frac{10 - 18}{\sqrt{\frac{8^2}{50} + \frac{12^2}{50}}} = -3.922$$

$p = .0000883$

With a p-value of .0000883 we can assume that our results were significant and not due to chance. The new drug is more effective than the standard cholesterol lowering medication.

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GO ON TO THE NEXT PAGE.

5. A serum cholesterol level above 250 milligrams per deciliter (mg/dl) of blood is a risk factor for cardiovascular disease in humans. At a medical center in St. Louis, a study to test the effectiveness of a new cholesterol-lowering drug was conducted. One hundred people with cholesterol levels between 250 mg/dl and 300 mg/dl were available for this study. Fifty people were assigned at random to each of two treatment groups. One group received the standard cholesterol-lowering medication and the other group received the new drug. After taking the drug for three weeks, the 50 subjects who received the standard treatment had a mean decrease in cholesterol level of 10 mg/dl with a standard deviation of 8 mg/dl, and the 50 subjects who received the new drug had a mean decrease of 18 mg/dl with a standard deviation of 12 mg/dl.

Does the new drug appear to be more effective than the standard treatment in lowering mean cholesterol level? Give appropriate statistical evidence to support your conclusion.

$n = 100$ (number of subjects available)
$n_1 = n_2 = 50$ (the two different treatment groups)

Let: n_1 be the group that takes the standard treatment
n_2 be the group that takes the new drug.
$\alpha = .10$

Let: μ_1 be the mean decrease of n_1.
μ_2 be the mean decrease of n_2.
σ_1 be the standard deviation of n_1
σ_2 " " " " " n_2.

Since the sample sizes are greater than 30, we can use a normal distribution as an appraxiamation.

Null hypothesis: $\mu_1 - \mu_2 = 0$
Alternate " $\mu_1 - \mu_2 < 0$

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{8^2}{50} + \frac{12^2}{50}} = 2.03961,$$ since we want $\mu_1 - \mu_2 < 0$, we use -2.04

Using the z tables, we find that the p-value is .0207, which is smaller than .10, therefore we reject the null hypothesis and the new drug seems to be more effective.

GO ON TO THE NEXT PAGE.

Question 5

Sample: 5A
Score: 4

This is a complete response. It clearly identifies the appropriate null and alternative hypotheses in the context of comparing mean cholesterol reduction provided by two drugs. A two-sample *t*-test is identified by presenting the formula for the test with the correct degrees of freedom. This response uses the moderately large sample sizes of 50 in each treatment group to justify use of the *t*-test, although use of the *t*-test could be justified by the act of randomly assigning treatments to subjects. There is no information in the statement of the problem about whether or not the subjects were sampled from some larger population. The value of the test statistic is correctly computed, and an upper bound for a *p*-value is obtained up to the accuracy allowed by the *t*-table provided with the exam. The conclusion that the new treatment provides a greater mean decrease in cholesterol level than the old treatment is well supported by indicating that the *p*-value is sufficiently small.

Sample: 5B
Score: 3

This is a substantial response. It identifies the appropriate null and alternative hypotheses using both words and symbols. A two-sample *t*-test is indicated, but the justification for using a two-sample *t*-test is incomplete. The problem provides no information about normal distributions or sampling the subjects from any population as indicated in the condition checks provided in the response. The random assignment of subjects to treatment groups and sufficiently large numbers of subjects should have been used to justify the tests. The value of the *t*-statistic and corresponding *p*-value are correctly computed. The conclusion that the new treatment provides a greater mean decrease in cholesterol level than the old treatment is not supported by comparing the *p*-value to a level of significance.

Sample: 5C
Score: 2

This is a developing response that does not correctly perform the *t*-test. It identifies the appropriate null and alternative hypotheses and identifies notation corresponding to the two treatment groups. There is some indication that sample sizes larger than 30 are needed, but this is not explicitly linked to the use of the two-sample *t*-test. No testing procedure is explicitly identified. A formula for the standard error of the difference between the two sample means is given, and it is incorrectly used as a test statistic. This leads to an incorrect *p*-value. The conclusion is consistent with the computed *p*-value and it is given in context, but the level of communication is not as good as in the previous two responses.