Answer Key for Selected-Response Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the selected-response questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the selected-response questions at the bottom of the *Answer/Scoring Sheet*.

SELECTED RESPONSE QUESTIONS / QUESTIONS À RÉPONSE CHOISIE

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

12	**A** B C D	**17**	A **B** C D	**22**	**A** B C D
13	A B C **D**	**18**	A B C **D**	**23**	A B **C** D
14	A **B** C D	**19**	**A** B C D	**24**	A B C **D**
15	**A** B C D	**20**	A B **C** D	**25**	A **B** C D
16	A **B** C D	**21**	**A** B C D	**26**	A B **C** D

Selected-Response Questions			
Question	***Outcome***	***Question***	***Outcome***
12	C1	20	D8
13	C2	21	D3
14	A1	22	E4
15	A3	23	E3
16	A5	24	E1
17	B3	25	G5
18	B1	26	F3
19	H1		

Part 1—Open-Response Questions

Question No. 1

Outcome: E2

A baseball coach needs to assign one player to each of the following positions: pitcher, catcher and first base. In how many ways can Dean, Dolores, Carlos, Carmine, Gus and Olga be assigned to those positions?
Give your answer as a whole number.

Solution

Method 1

$= {}_6P_3$	1 mark for permutation
$= 120$	1 mark for consistent answer
	2 marks

Method 2

$\frac{6}{\text{P}}\ \frac{5}{\text{C}}\ \frac{4}{\text{F}} = 120$	1 mark for factors 1 mark for consistent answer
	2 marks

Question No. 2

Outcome: F1

The equation of a circle is given by: $x^2 + y^2 - 4x + y - 1 = 0$.

a) State the coordinates of the centre of the circle.

b) Calculate the radius of the circle.

Solution

a)

$$\left(x^2 - 4x \quad\right) + \left(y^2 + y \quad\right) = 1$$

$$x^2 - 4x + 4 + y^2 + y + \frac{1}{4} = 1 + 4 + \frac{1}{4}$$

1 mark for completing the square
(½ mark for left side, ½ mark for right side)

$$\left(x-2\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{21}{4}$$

$$\mathrm{C}\left(2, -\frac{1}{2}\right)$$

1 mark for consistent centre

$\boxed{\text{2 marks}}$

b)

$$r = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2} = 2.291$$

1 mark for consistent radius

$\boxed{\text{1 mark}}$

Note(s):

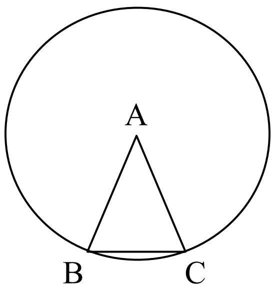
- if correct centre is found using an alternate method give full marks

Question No. 3

Outcome: A1

Points B and C lie on a circle with centre A.

The measure of $\angle A = \dfrac{\pi}{7}$. Find the measure of $\angle B$ to the **nearest degree**.



Solution

$$\pi - \angle A = (\angle B + \angle C)$$

$$\pi - \frac{\pi}{7} = \frac{7\pi}{7} - \frac{\pi}{7} = \frac{6\pi}{7}$$

½ mark for sum $\angle B + \angle C$

$$\angle B = \angle C$$

$$\therefore \angle B + \angle B = \frac{6\pi}{7}$$

$$2\angle B = \frac{6\pi}{7}$$

$$\angle B = \frac{6\pi}{14} = \frac{3\pi}{7}$$

½ mark for solving for $\angle B$

$$\angle B = \frac{3\pi}{7}\left(\frac{180}{\pi}\right) \approx 77^\circ$$

1 mark for conversion to degrees

$\boxed{\text{2 marks}}$

Question No. 4

Outcome: A5

Solve the following equation, where $\theta \in \mathbb{R}$.
State all solutions in radians correct to 3 decimal places.

$$4\cos(2\theta) + 3 = 0$$

Solution

Method 1

$$4\cos 2\theta = -3$$

$$\cos 2\theta = -\frac{3}{4}$$

$$\cos^{-1}\left(\frac{3}{4}\right) = 0.722\ 734$$

$$\left.\begin{aligned} 2\theta &= 2.418\ 86 + 2k\pi \\ &\ \ 3.864\ 33 + 2k\pi \end{aligned}\right\} k \in \mathrm{I}$$

1 mark for 2nd quadrant value
1 mark for 3rd quadrant value
1 mark for correct general solution

$$\left.\begin{aligned} \theta &= 1.209 + k\pi \\ &\ \ 1.932 + k\pi \end{aligned}\right\} k \in \mathrm{I}$$

1 mark for dividing by 2

$\boxed{\text{4 marks}}$

Method 2

$$\cos 2\theta = -\frac{3}{4}$$

$$\left(2\cos^2\theta - 1\right) = -\frac{3}{4}$$

1 mark for identity

$$2\cos^2\theta = \frac{1}{4}$$

$$\cos^2\theta = \frac{1}{8}$$

$$\cos\theta = \pm\frac{1}{\sqrt{8}}$$

$$\left.\begin{aligned} \theta &= 1.209 \\ \theta &= 1.932 \\ \theta &= 4.351 \\ \theta &= 5.074 \end{aligned}\right\}$$

½ mark for each solution in the first positive revolution

$$\left.\begin{aligned} \theta &= 1.209 + 2k\pi \\ \theta &= 1.932 + 2k\pi \\ \theta &= 4.351 + 2k\pi \\ \theta &= 5.074 + 2k\pi \\ &\text{or} \\ &5.073 + 2k\pi \\ &(\text{truncated}) \end{aligned}\right\} k \in \mathrm{I}$$

1 mark for stating general solution correctly

$\boxed{\text{4 marks}}$

More methods on the next pages.

Question No. 4

Outcome: A5

Method 3 (graphing calculator)

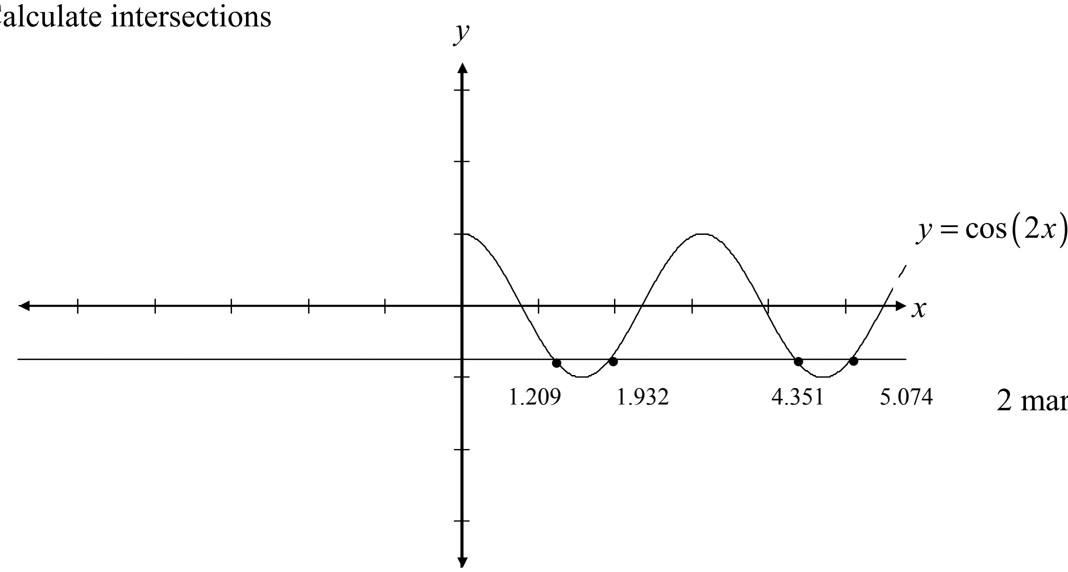
Let $\theta = x$

$y = \cos(2x)$

$y = \frac{-3}{4}$

1 mark for explanation

Calculate intersections



2 marks for solutions

Then generalize:

$$\left.\begin{aligned}\theta &= 1.209 + 2k\pi \\ \theta &= 1.932 + 2k\pi \\ \theta &= 4.351 + 2k\pi \\ \theta &= 5.074 + 2k\pi\end{aligned}\right\} k \in \mathrm{I}$$

1 mark for general solution

$\boxed{\text{4 marks}}$

Question No. 4

Outcome: A5

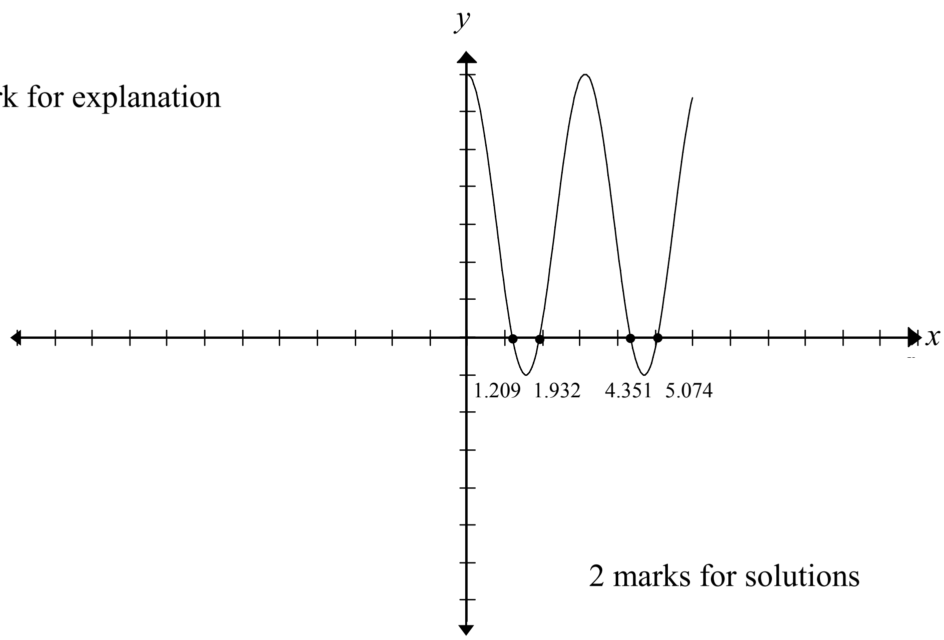
Solve the following equation, where $\theta \in \mathbb{R}$.
State all solutions in radians correct to 3 decimal places.

$$4\cos(2\theta) + 3 = 0$$

Solution

Method 4 (graphing calculator)

$$\left.\begin{array}{l}\text{Let } \theta = x \\ y = 4\cos(2x) + 3 \\ \text{Calculate zeroes}\end{array}\right\}$$ 1 mark for explanation



2 marks for solutions

Then generalize:

$$\left.\begin{array}{l}\theta = 1.209 + 2k\pi \\ \theta = 1.932 + 2k\pi \\ \theta = 4.351 + 2k\pi \\ \theta = 5.074 + 2k\pi\end{array}\right\} k \in \mathrm{I}$$ 1 mark for general solution

$\boxed{\text{4 marks}}$

Note(s):

- give maximum ½ mark if only reference angle $\cos^{-1}\left(\frac{3}{4}\right) = 0.723$ stated
- **deduct** ½ mark if $k \in \mathrm{I}$ is omitted
- give a maximum score of 3 out of 4 if the student has solved correctly for θ and the student hasn't placed the answer as a general solution
- **deduct** 1 mark if final answers are correct but stated in degrees; $\left.\begin{array}{l}\theta = 69.295 + 180°k \\ = 110.705 + 180°k\end{array}\right\} k \in \mathrm{I}$
- give maximum of 2 marks for answers in degrees with no general solution

Question No. 5

Outcome: D6

Solve for x **algebraically**.
Give your answer correct to 3 decimal places.

$$3^{x+4} = 7^{2x+1}$$

Solution

$\log 3^{x+4} = \log 7^{2x+1}$	½ mark for taking the log of both sides
$(x+4)\log 3 = (2x+1)\log 7$	½ mark for log theorem
$x\log 3 + 4\log 3 = 2x\log 7 + \log 7$	½ mark for distributing
$x\log 3 - 2x\log 7 = \log 7 - 4\log 3$ $x(\log 3 - 2\log 7) = \log 7 - 4\log 3$	½ mark for isolating and factoring x
$x = \dfrac{\log 7 - 4\log 3}{\log 3 - 2\log 7}$	½ mark for solving for x
$x = 0.877$	½ mark for consistent answer

$\boxed{\text{3 marks}}$

Note(s):

- $\log 3 = 0.477\ 121$ $\qquad \ln 3 = 1.098\ 612$
  $\log 7 = 0.845\ 098$ $\qquad \ln 7 = 1.945\ 910$
- if brackets are omitted in the second step, but assumed to be there, **deduct** ½ mark
- the final answer must be correct to at least 3 decimal places

Question No. 6

Outcome: a) G2, b) G3

Tickets numbered 3, 6, 9, 12, 15 and 18 are placed in Box A. Tickets numbered 6, 12, 18, 24 and 30 are placed in Box B. A ticket is chosen at random from each box.
Find the probability:

a) that both tickets have the same number.

b) that there are different numbers on the two tickets.

Solution

Method 1

a) $P(\text{both } 6) = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$ ½ mark

$P(\text{both } 12) = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$ ½ mark

$P(\text{both } 18) = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$ ½ mark

$P(\text{same number}) = 3\left(\frac{1}{30}\right) = \frac{1}{10}$ ½ mark

$\boxed{\text{2 marks}}$

b) $\left.\begin{aligned} P(\text{different number}) &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}\right\}$ 1 mark

$\boxed{\text{1 mark}}$

Method 2

a) $\frac{3}{6} \cdot \frac{1}{5} = \frac{3}{30} = \frac{1}{10}$ ½ mark for choosing 6, 12, or 18

choose 6, 12, 18 — choose a match

½ mark for choosing a match

½ mark for multiplying

½ mark for consistent answer

$\boxed{\text{2 marks}}$

b) $\left.\begin{aligned} P(\text{different number}) &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}\right\}$ 1 mark

$\boxed{\text{1 mark}}$

Question No. 6

Outcome: a) G2, b) G3

Method 3

a) $_6C_1 \bullet {_5C_1} = 30$ in sample space — ½ mark for each combination

$3 \text{ favorable} = (6,\ 6)\ (12,\ 12)\ (18,\ 18)$ — ½ mark for multiplying

$P(\text{match}) = \dfrac{3}{30} = \dfrac{1}{10}$ — ½ mark for consistent answer

$\boxed{\text{2 marks}}$

b) $\left.\begin{aligned} P(\text{different number}) &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}\right\}$ 1 mark

$\boxed{\text{1 mark}}$

Method 4

a)

	3	**6**	**9**	**12**	**15**	**18**
6	6, 3	6, 6	6, 9	6, 12	6, 15	6, 18
12	12, 3	12, 6	12, 9	12, 12	12, 15	12, 18
18	18, 3	18, 6	18, 9	18, 12	18, 15	18, 18
24	24, 3	24, 6	24, 9	24, 12	24, 15	24, 18
30	30, 3	30, 6	30, 9	30, 12	30, 15	30, 18

$P(\text{same number}) = \dfrac{3}{30} = \dfrac{1}{10}$

1 mark for drawing the sample space
½ mark for the numerator
½ mark for the denominator

$\boxed{\text{2 marks}}$

b) $\left.\begin{aligned} P(\text{different number}) &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}\right\}$ 1 mark

$\boxed{\text{1 mark}}$

Question No. 7

Outcome: D5

If $\log_a 2 = 0.356\ 2$ and $\log_a 5 = 0.827\ 1$ show that $\log_a 40 = 1.895\ 7$.

Solution

Method 1

$\log_a 40 = \log_a \left(2^3 \bullet 5\right)$	1 mark for factoring 40
$= \log_a 2^3 + \log_a 5$	1 mark for log theorem
$= 3\log_a 2 + \log_a 5$	1 mark for log theorem
$= 3(0.356\ 2) + (0.827\ 1)$	½ mark for substitution
$= 1.895\ 7$	½ mark for final step
	$\boxed{\text{4 marks}}$

Method 2

$\log_a 2 = 0.356\ 2$	
$a^{0.356\ 2} = 2$	1 mark for changing to exponential
$0.356\ 2 \log a = \log 2$	½ mark for log theorem
$\log a = \dfrac{\log 2}{0.356\ 2}$	
$\log a = 0.845\ 1$	½ mark for isolating $\log a$
$a = 10^{0.845\ 1}$	½ mark for exponential
$a = 7.000\ 0$	½ mark for solving for a
$\log_7 40 = \dfrac{\log 40}{\log 7}$	½ mark for change of base
$= 1.895\ 7$	½ mark for consistent answer
	$\boxed{\text{4 marks}}$

Question No. 7

Outcome: D5

Method 3 (graphing calculator)

$\log_a 2 = 0.356\ 2$

$a^{0.356\ 2} = 2$ — 1 mark for exponential form

$$\left.\begin{array}{l} \quad y = x^{0.356\ 2} \\ \quad y = 2 \\ \text{Intersection} \\ \quad x = 7.000\ 274\ 8 \end{array}\right\}$$ 1 mark for calculator explanation

$\therefore a = 7.000\ 274\ 8$ — 1 mark for value of a

$\log_a 40 = \dfrac{\log 40}{\log a} =$ — ½ mark for change of base

$\dfrac{\log 40}{\log 7.000\ 274\ 8} = 1.895\ 7$ — ½ mark for consistent answer

$\boxed{\text{4 marks}}$

Note(s):

- to evaluate "a", students may use a graphing calculator to find zeros of $y = x^{0.356\ 2}-2$ or use SOLVER to solve $0 = x^{0.356\ 2}-2$

Question No. 8

Outcome: D8

A new automobile costs \$24,000. Its value after t years is given by: $V = 24\ 000(0.8)^t$.

a) Determine the value after 8 years.

b) How many years will it take for its value to decrease to one-eighth of its initial value? State your answer correct to 3 decimal places.

Solution

a)

$$V(8) = 24\ 000(0.8)^8$$ ½ mark for substitution

$$\approx \$4,026.53$$ ½ mark for consistent answer

1 mark

b)

$$24\ 000 \bullet \frac{1}{8} = \$3,000$$ ½ mark for calculation

$$3\ 000 = 24\ 000(0.8)^t$$

$$0.125 = 0.8^t$$ ½ mark for calculation

$$\ln 0.125 = t \ln 0.8$$ 1 mark for log theorem

$$t = \frac{\ln 0.125}{\ln 0.8}$$ ½ mark for isolating "t"

$$t = 9.319 \text{ years}$$ ½ mark for consistent answer

or

9.318 (truncated)

3 marks

Note(s):

- final answer must be correct to at least 3 decimal places

Question No. 9

Outcome: E3

Karl has written 20 songs and must choose 12 of them to record in his studio.

a) In how many ways can he choose 12 songs for his CD?
Express your answer as a whole number.

b) The songs *Miracle* and *Bright Beginning* are very similar. If Karl uses **no more than one** of these two songs, in how many ways can he choose 12 songs for his CD?
Briefly describe your calculations.

Solution

a) $_{20}C_{12} = 125\ 970$

½ mark for combination
½ mark for consistent answer

1 mark

b)

$\text{Case 1 - use one of M or BB: } {}_2C_1 \bullet {}_{18}C_{11} = 63\ 648$
$\text{Case 2 - use neither: } {}_{18}C_{12} = 18\ 564$
$\text{Total number of ways } = 82\ 212$

1 mark for Case 1
½ mark for Case 2
½ mark for addition of cases

2 marks

OR

$_{20}C_{12} - {}_{18}C_{10} = 82\ 212$

1 mark ½ mark ½ mark for consistent answer

Note(s):

- **deduct** ½ mark if explanation of cases is not given

Question No. 10

Outcome: E1

Using the letters from the word PORTAGE:

a) How many 5 letter arrangements are possible?
Express your answer as a whole number.

b) How many 7 letter arrangements are possible if “P” must be the first letter and the letters “T” and “E” must be together?
Briefly explain your calculations.

Solution

a) ${}_7P_5 = 2\ 520$ — ½ mark for permutation

or

$\underline{7}\ \ \underline{6}\ \ \underline{5}\ \ \underline{4}\ \ \underline{3} = 2\ 520$ — ½ mark for consistent answer

1 mark

b) $\underset{\text{P}}{\underline{1}}\ \ \underset{\text{TE}}{\underline{5}}\ \ \underline{4}\ \ \underline{3}\ \ \underline{2}\ \ \underline{1} \cdot \underset{\text{TE or ET}}{\underline{\quad 2 \quad}}$

$= 240 \text{ ways}$

½ mark for 5!
½ mark for the factor of 2
½ mark for multiplication
½ mark for consistent answer

2 marks

Question No. 11

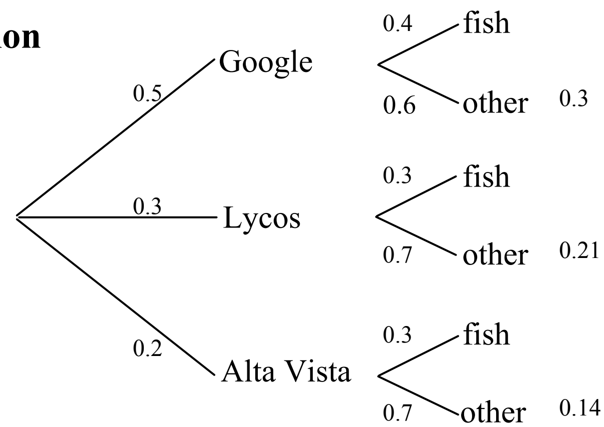
Outcome: a) G2, b) G4

John uses Google 50% of the time for Internet research. He uses Lycos 30% of the time and Alta Vista 20% of the time. If he is using Google, there is 40% probability that he is searching for information about fish. If he is using either Lycos or Alta Vista, the probability he is searching for information about fish is 30%.

a) What is the probability that John decides to use Lycos and searches for information about fish?

b) Given that you see John looking at a Website on the Internet that is **not** about fish, what is the probability that he used Google to find it?

Solution



a) (0.3) (0.3) = 0.09

½ mark for correct factors
½ mark for consistent answer

$\boxed{\text{1 mark}}$

b) P(Google/Other)

$$= \frac{(0.5)(0.6)}{(0.5)(0.6)+(0.3)(0.7)+(0.2)(0.7)}$$

$$= \frac{0.30}{0.30+0.21+0.14}$$

$$= \frac{30}{65}$$

$$= \frac{6}{13}$$

$$= 0.462$$

1 mark for numerator

1½ marks $\begin{cases} \text{½ mark for each} \\ \text{term in denominator} \end{cases}$

½ mark for consistent answer

$\boxed{\text{3 marks}}$

Note(s):

- give maximum 1 mark if only tree diagram given with no calculations

Part 2—Restricted-Response Questions

Question No. 27

Outcome: A1

How many radians are there between the minute and hour hands of a clock at 5:00?

Answer

$\frac{5\pi}{6}$ or $\frac{7\pi}{6}$ or $\frac{-5\pi}{6}$ or $\frac{-7\pi}{6}$ $\boxed{\text{1 mark}}$

Note(s):

- give ½ mark for equivalent answer in degrees: $\pm 150°$ or $\pm 210°$

Question No. 28

Outcome: A4

Find the period of the graph whose equation is:

$$y = \tan\theta$$

Answer

Period = π or $180°$ $\boxed{\text{1 mark}}$

Question No. 29

Outcome: A3

Find the value of $\sin\left(\frac{-11\pi}{6}\right)$.

Answer

$\frac{1}{2}$ $\boxed{\text{1 mark}}$

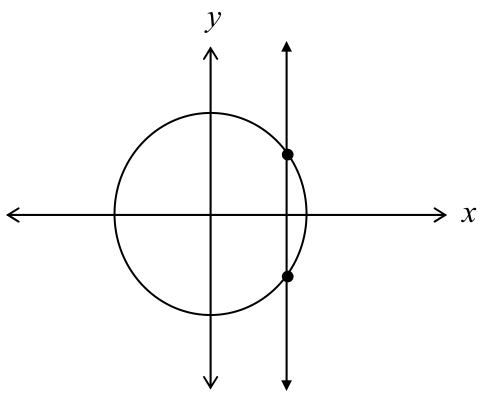
Note(s):

- do not give marks for an answer of $-\frac{1}{2}$

Question No. 30

Outcome: A2

State the coordinates of a point where the unit circle given by the equation $x^2 + y^2 = 1$ and the line given by $x = \frac{\sqrt{3}}{2}$ intersect.



Answer

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ OR $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$ 1 mark

Note(s):

- give ½ mark for $\frac{1}{2}$ or $\frac{-1}{2}$ as final answer
- **deduct** ½ mark for missing brackets

Question No. 31

Outcome: B1

The graph of $y = x^2 - 4x - 5$ crosses the x-axis at -1 and 5.

Where does the graph of $y = (x+10)^2 - 4(x+10) - 5$ cross the x-axis?

Answer

$-11, -5$ ½ mark for each correct answer

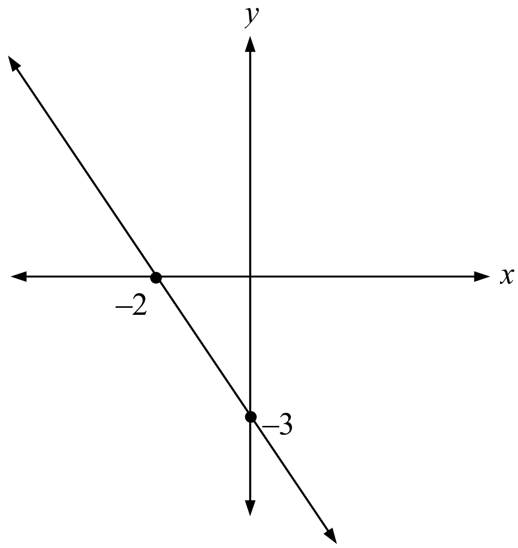
$\boxed{\text{1 mark}}$

Question No. 32

Outcome: B4

The graph $y = g(x)$ is sketched below.

What is the domain of the function $y = \dfrac{1}{g(x)}$?



Answer

$\{x \mid x \neq -2\}$

or

$(-\infty, -2) \cup (-2, \infty)$ $\boxed{\text{1 mark}}$

or

$x \in \mathbb{R},\ x \neq -2$

Note(s):

- **deduct** ½ mark for $(-\infty, -2) \cap (-2, \infty)$

Question No. 33

Outcome: B7

The graph of the function $f(x) = 5\sin x + p$ touches the x-axis once in the interval $[0, 2\pi]$. State a possible value of p.

Answer

5 or –5 $\boxed{\text{1 mark}}$

Question No. 34

Outcome: D3, A3

Evaluate $\log\left(100\sin\dfrac{\pi}{2}\right)$.

Answer

2 $\boxed{\text{1 mark}}$

Note(s):

- give ½ mark for $\log(100)$

Question No. 35

Outcome: D1, B3

State the range of the function $f(x) = 2^{-x}$.

Answer

$(0, \infty)$ OR $y > 0$ 1 mark

Note(s):

- give ½ mark for $[0, \infty)$ or $y \geq 0$

Question No. 36

Outcome: H1

Given the geometric sequence $-18, 6, -2, \frac{2}{3},...$

What is the value of r?

Answer

$r = -\frac{1}{3}$ 1 mark

Question No. 37

Outcome: E1

There are 3 different roads connecting St. Malo with Rosa and 4 different roads connecting Rosa with Tolstoi. In how many different ways can a person drive from St. Malo to Tolstoi, passing through Rosa on the way?

Answer

$4(3) = 12$ 1 mark

Question No. 38

Outcome: G1

A 6-sided die is rolled twice. List all the ordered pairs of the sample space that represent a sum greater than 10.

Answer

(6, 5) (6, 6) (5, 6) $\boxed{\text{1 mark}}$

Note(s):

- give ½ mark for any two of three outcomes

Question No. 39

Outcome: E2

You have 2 different pictures and 5 different frames. In how many different ways can you frame the 2 pictures?

Answer

5×4 or ${}_5P_2$ or $\dfrac{5!}{3!}$ or 20 $\boxed{\text{1 mark}}$

Question No. 40

Outcome: G2

Two traffic lights on Broadway operate independently.
The probability of the first one being red is 0.4.
The probability of the second one being red is 0.7.
What is the probability of neither light being red?

Answer

$= (0.6)\,(0.3)$
$= 0.18$

1 mark

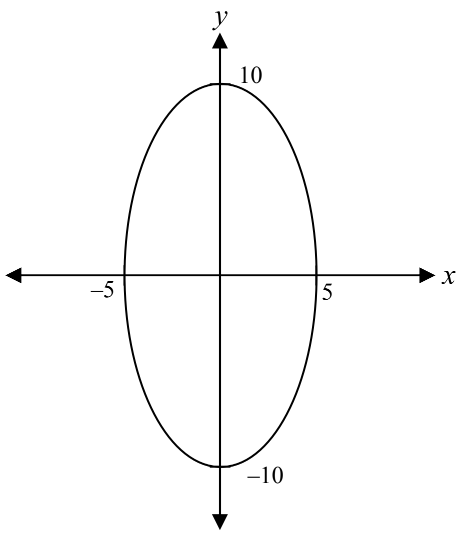
Note(s):

- give ½ mark for $(0.6)(0.3)$

Question No. 41

Outcome: F3

Write the equation of the ellipse shown in the diagram.



Answer

$$\frac{x^2}{25} + \frac{y^2}{100} = 1$$

1 mark

Note(s):

- do not give marks for any equation that is not an ellipse
- give ½ mark for $\frac{x^2}{100} + \frac{y^2}{25} = 1$

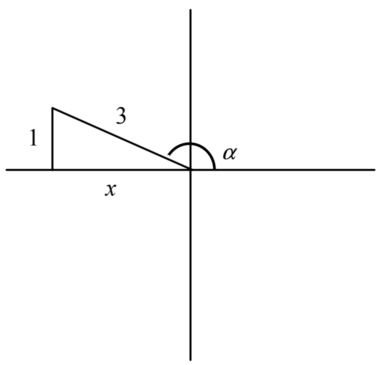
Part 2—Open-Response Questions

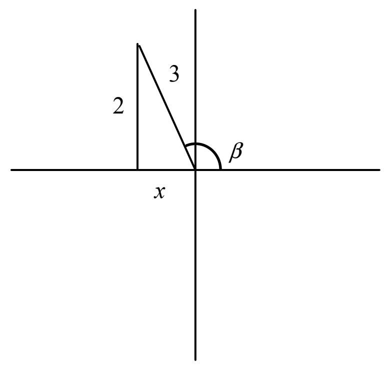
Question No. 42

Outcome: C2

If α and β are both angles in the second quadrant and $\sin\alpha = \frac{1}{3}$, $\sin\beta = \frac{2}{3}$, find the exact value of $\cos(\alpha + \beta)$.

Solution





$$x^2 + 1^2 = 3^2$$
$$x^2 = 8$$
$$x = -2\sqrt{2}$$
$$\therefore \cos\alpha = \frac{-2\sqrt{2}}{3}$$

1 mark for $\cos\alpha$

$$x^2 + 2^2 = 3^2$$
$$x^2 = 5$$
$$x = -\sqrt{5}$$
$$\cos\beta = \frac{-\sqrt{5}}{3}$$

1 mark for $\cos\beta$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) - \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$

1 mark for substitution into the correct formula

$$= \frac{2\sqrt{10}}{9} - \frac{2}{9}$$

$$= \frac{2\sqrt{10} - 2}{9}$$

½ mark for simplification
½ mark for consistent answer

$\boxed{\text{4 marks}}$

Note(s):

- if either $\cos\alpha$ and $\cos\beta$ are given as a positive value or if both are given as a positive value **deduct** 1 mark
- if the student only calculates x values, with the correct signs, give 1 mark

Question No. 43

Outcome: C1

Prove:

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\tan^2\theta+2$$

Solution

Method 1

LHS	$=\dfrac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$	1 mark for common denominator
	$=\dfrac{2}{\cos^2\theta}$	1 mark for identity
	$=2\sec^2\theta$	1 mark for identity
	$=2\left(\tan^2\theta+1\right)$	1 mark for identity
	$=2\tan^2\theta+2=\text{RHS}$	4 marks

Method 2

RHS	$=\dfrac{2\sin^2\theta}{\cos^2\theta}+2$	½ mark
	$=\dfrac{2\sin^2\theta}{\cos^2\theta}+\dfrac{2\cos^2\theta}{\cos^2\theta}$	½ mark
	$=\dfrac{2\sin^2\theta+2\cos^2\theta}{\cos^2\theta}$	
	$=\dfrac{2\left(\sin^2\theta+\cos^2\theta\right)}{\cos^2\theta}$	½ mark for factoring
	$=\dfrac{2}{\cos^2\theta}$	½ mark for simplifying

LHS	$=\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}$	
	$=\dfrac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}$	1 mark for common denominator
	$=\dfrac{2}{\cos^2\theta}$	1 mark for identity
LHS	$=\text{RHS}$	

4 marks

Question No. 44

Outcome: A4

Solve for x over the interval $\left[0, \frac{3\pi}{2}\right]$ for:

$$\cos x = \cos^2 x$$

Solution

$0 = \cos^2 x - \cos x$

$0 = \cos x(\cos x - 1)$ ½ mark for factoring

$\cos x = 0 \qquad \cos x - 1 = 0$

$\cos x = 1$ 1 mark for solving for $\cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = 0$ 1½ marks (½ mark for each answer)

$\boxed{\text{3 marks}}$

Note(s):

- give a maximum of 1 mark out of 3 if the student divides both sides by $\cos x$ in the first step

Question No. 45

Outcome: A6

Consider the function $y = \sin(x)$ defined **only** on the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

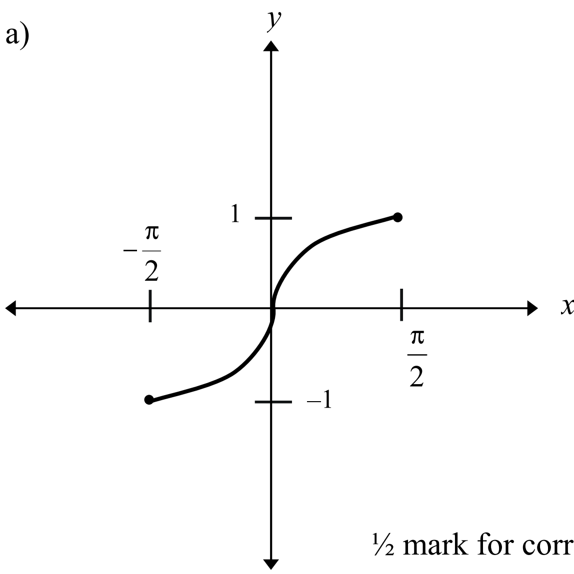
a) Sketch a clearly labelled graph of this function on its restricted domain.

b) Sketch a clearly labelled graph of the inverse function $y = \sin^{-1}(x)$.

c) State the domain of $y = \sin^{-1}(x)$.

Solution

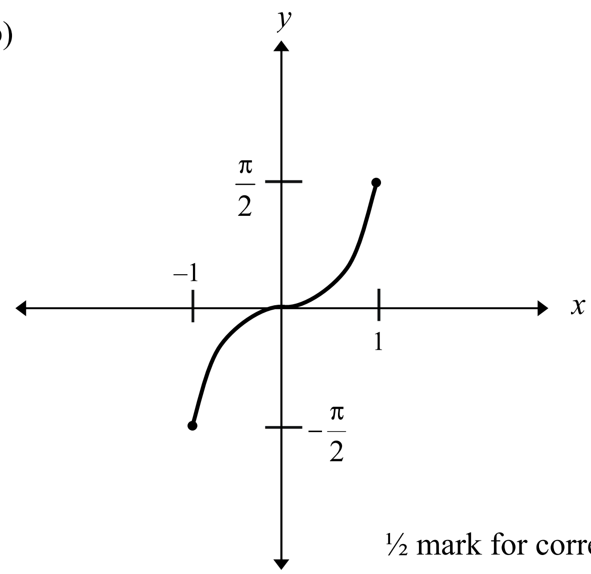
a)



½ mark for correct shape
½ mark for labelling

$\boxed{\text{1 mark}}$

b)



½ mark for correct shape
½ mark for labelling

$\boxed{\text{1 mark}}$

c) $[-1, 1]$

1 mark

$\boxed{\text{1 mark}}$

Note(s):

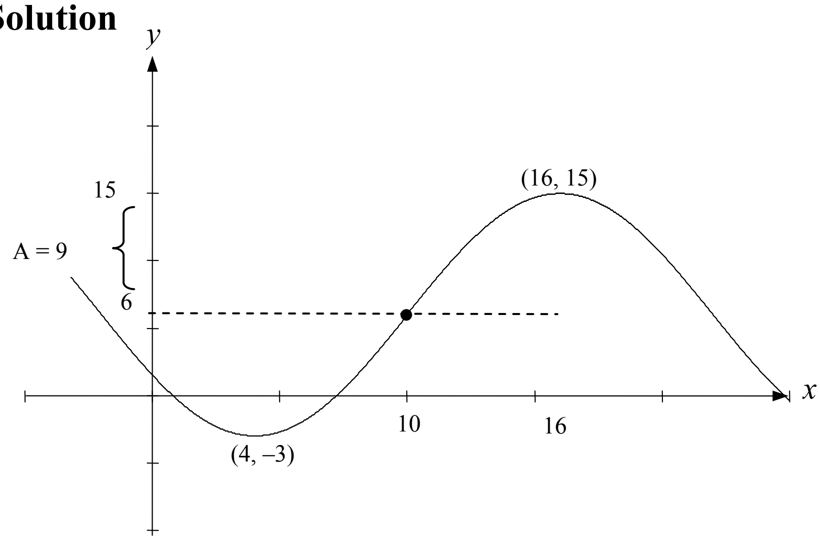
- **deduct** ½ mark for $y = \sin(x)$ extending beyond the interval
- for part b), **deduct** ½ mark if arrows are placed on the graph
- for part c) give ½ mark for $(-1, 1)$

Question No. 46

Outcome: B7

A minimum point on a sinusoidal graph occurs at $(4, -3)$ and the next maximum occurs at $(16, 15)$. If the equation of this function is written as $y = A\sin\left[B(x - C)\right] + D$, determine a set of possible values for A, B, C and D.

Solution



$A = 9$ 1 mark

$\text{period} = 24 = \dfrac{2\pi}{B}$ ½ mark

$B = \dfrac{\pi}{12}$ ½ mark

$C = 10$ 1 mark

$D = 6$ 1 mark

$\boxed{\text{4 marks}}$

Note(s):

- $C = 10 + 24k$ if $A = 9$ $(k \in I)$
- $C = -2 + 24k$ if $A = -9$ $(k \in I)$

Question No. 47

Outcome: D2

Solve for x:

$$\left(\frac{1}{3}\right)^{2x} = 27^{x-5}$$

Solution

Method 1

$\left(3^{-1}\right)^{2x} = \left(3^{3}\right)^{x-5}$ 1 mark (½ mark for each side)

$3^{-2x} = 3^{3x-15}$ ½ mark for simplifying powers

$-2x = 3x - 15$ 1 mark for equating exponents

$15 = 5x$

$x = 3$ ½ mark for consistent answer

3 marks

Method 2

$2x\log\left(\frac{1}{3}\right) = (x-5)\log 27$ 1 mark for log theorem

$2x\log\left(\frac{1}{3}\right) = x\log 27 - 5\log 27$ ½ mark for distributing

$2x\log\left(\frac{1}{3}\right) - x\log 27 = -5\log 27$ ½ mark for isolating and factoring x

$x\left(2\log\left(\frac{1}{3}\right) - \log 27\right) = -5\log 27$ ½ mark for solving for x

$x = \dfrac{-5\log 27}{2\log\left(\frac{1}{3}\right) - \log 27}$ ½ mark for consistent answer

3 marks

Question No. 48

Outcome: B2, B3

Given the graph of $y = f(x)$:

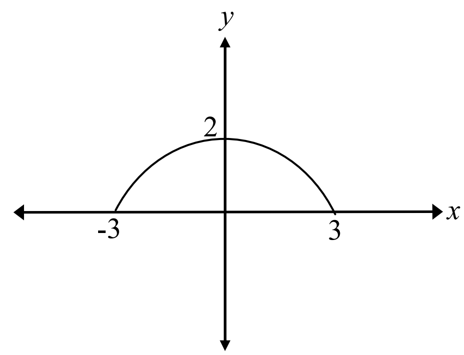
Sketch a clearly labelled graph of each of the following:

a) $y = 2f(x)$

b) $y = f(2x)$

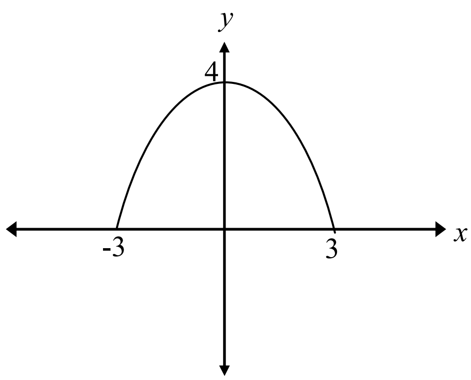
c) $y = -f(x)$

d) $y = f(-x)$

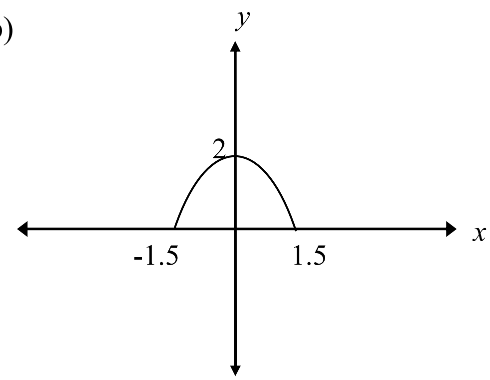


Solution

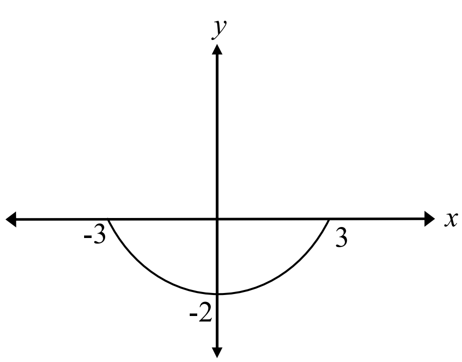
a)



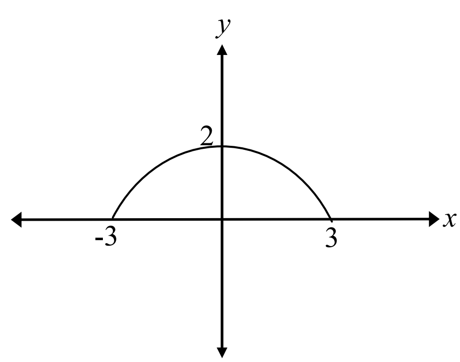
b)



c)



d)

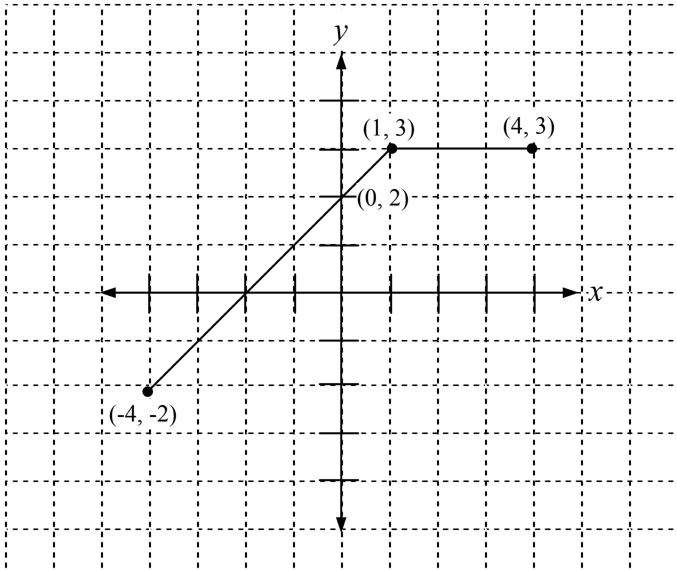


4 marks

Question No. 49

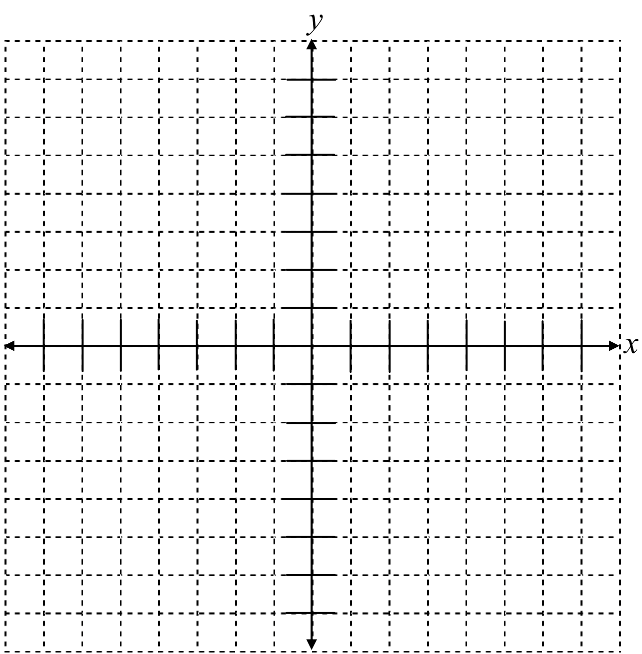
Outcome: B1, B5

Given the graph of $y = f(x)$:



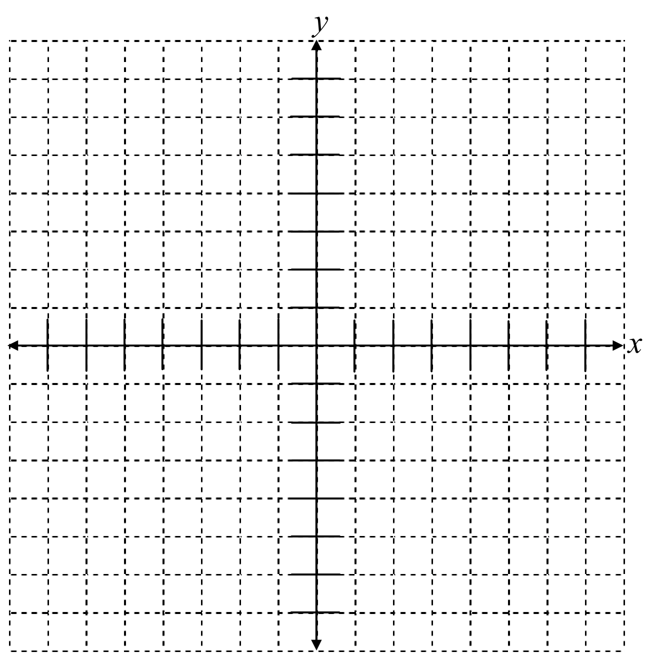
a) Sketch the clearly labelled graph of:

$$y = f(x+3)$$



b) Sketch the clearly labelled graph of:

$$y = |f(x)| - 2$$

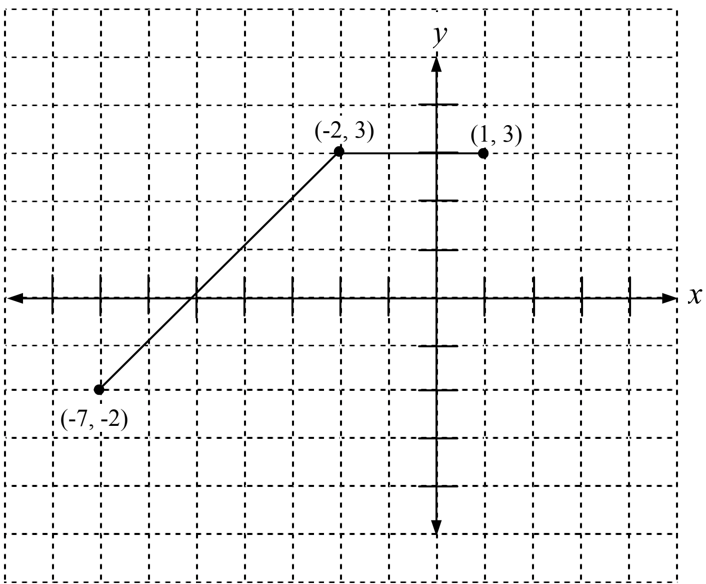


Question No. 49

Outcome: B1, B5

Solution

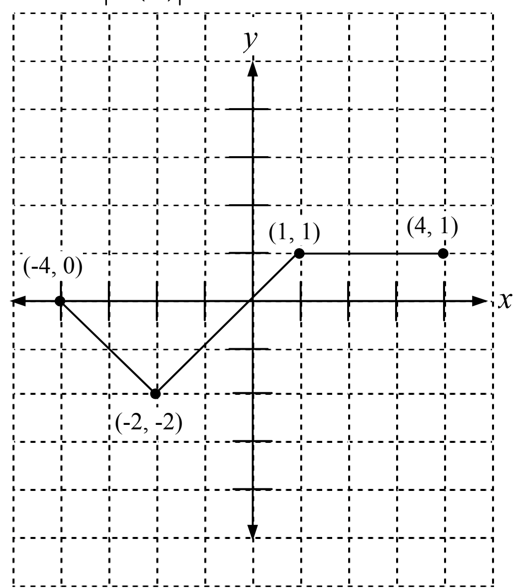
a) $y = f(x+3)$



1 mark

$\boxed{\text{1 mark}}$

b) $y = |f(x)| - 2$



$\left.\begin{array}{l}\text{1 mark for absolute value}\\ \text{1 mark for vertical shift}\end{array}\right\}$ 2 marks

$\boxed{\text{2 marks}}$

Note(s):

- in a) **deduct** ½ mark for one incorrect point
- in b) **deduct** 1 mark if they are done in the wrong order

Question No. 50

Outcome: E1

Solve for n **algebraically**:

$$(n-1)! = 6(n-3)!$$

Solution

Method 1

$$\frac{(n-1)!}{(n-3)!} = 6$$

$(n-1)(n-2) = 6$ 1 mark for simplification

$\left.\begin{aligned} n^2 - 3n + 2 &= 6 \\ n^2 - 3n - 4 &= 0 \\ (n-4)(n+1) &= 0 \end{aligned}\right\}$ 1 mark for factoring

$n = 4$ $\cancel{n = -1}$ 1 mark (½ mark for each answer)

$\boxed{\text{3 marks}}$

Method 2

$$\frac{(n-1)!}{(n-3!)} = 6$$

$(n-1)(n-2) = 6$ 1 mark for simplification

$\left.\begin{aligned} n-1 &= 3 \\ n-2 &= 2 \end{aligned}\right\}$ $\left.\begin{array}{l}\text{2 consecutive positive} \\ \text{numbers whose product is 6}\end{array}\right\}$ 1 mark for justification

$n = 4$ 1 mark for consistent answer

$\boxed{\text{3 marks}}$

Note(s):

- **deduct** ½ mark if extraneous root is not discarded
- give 1 mark for correct answer of 4 with no supporting work

Question No. 51

Outcome: F1, F3

The equation of a conic section is $\dfrac{(x-3)^2}{1} - \dfrac{(y+1)^2}{4} = 1$.

a) Identify this conic section.

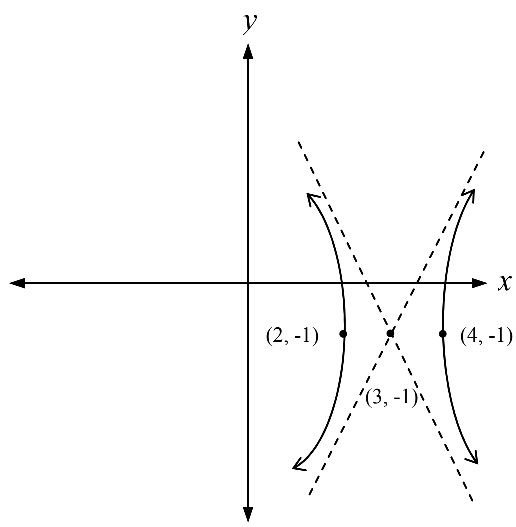
b) Sketch a clearly labelled graph of this conic section.

c) Give its domain.

Solution

a) hyperbola — 1 mark $\boxed{\text{1 mark}}$

b)



½ mark for horizontal hyperbola

½ mark for vertices consistent with the centre

½ mark for centre

½ mark for asymptotes

$\boxed{\text{2 marks}}$

c) $(-\infty, 2\,] \cup [4, \infty)$ — 1 mark for domain consistent with the graph

$\boxed{\text{1 mark}}$

Note(s):

- **deduct** ½ mark for $(-\infty, 2] \cap [4, \infty)$, $(-\infty, 2) \cup (4, \infty)$