Tracking detectors for the LHC

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Overview lectures part I

Principles of gaseous and solid state tracking detectors

- Tracking detectors at the LHC
- Drift chambers
- Silicon detectors
- Modeling of a silicon detector

Overview lectures part II

Tracker design and performance

- Momentum and impact parameter measurement
- Optimization of tracker design and G4 simulation

Track reconstruction and fitting

- Principles and basic ideas
- Pattern recognition
- Track fitting

Physics applications

- Primary and secondary vertex reconstruction
- Invariant mass reconstruction
- b-tagging

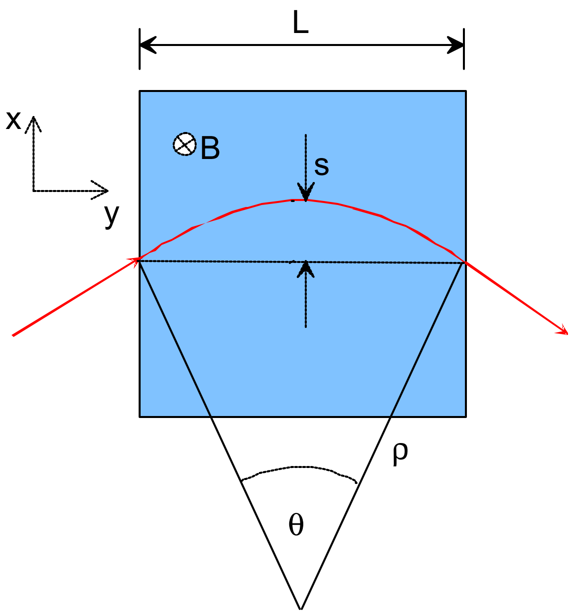
Tracker design and performance

- Study two figures of merit
  - momentum resolution
  - impact parameter resolution
- Impact on physics:
  - invariant mass resolution
    e.g. Higgs mass resolution
  - flight distance of B hadrons precision
    e.g. B_s oscillations, b-tagging for Higgs

Momentum measurement

$$p_T\ (\text{GeV/c}) = 0.3B\rho \quad (\text{T}\cdot\text{m})$$

ρ radius of curvature



3 point sagitta s is: $s = x_2 - \frac{1}{2}(x_1 + x_3)$

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma_s}{s} = \frac{\sigma_x}{s}\sqrt{3/2} = \frac{\sigma_x \cdot p_T}{0.3 \cdot BL^2}\sqrt{96}$$

for N equidistant measurements:

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma_x \cdot p_T}{0.3 \cdot BL^2}\sqrt{720/(N+4)}$$

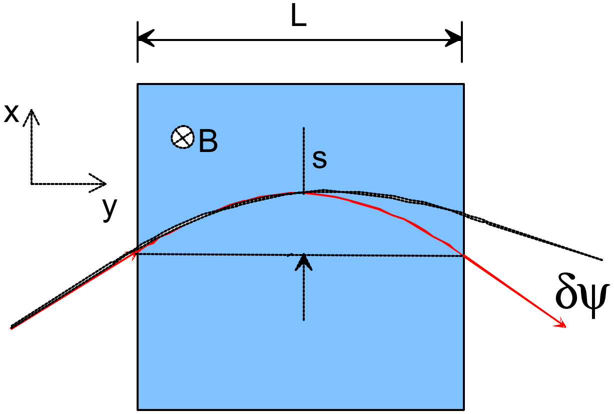
Detector: L=1m, B=1T, σ_x=200μm, N=10

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} \approx 0.5\%\, p_T$$

Influence of material on momentum

- Multiple scattering:
  - middle of volume

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{ms} = \frac{28\,MeV}{0.3\cdot BL}\sqrt{x/X_0}\,\frac{p_T}{\beta c p}$$



$$\delta\psi = \frac{14\,MeV}{\beta c p}\sqrt{x/X_0}$$

- Fluctuations on energy loss

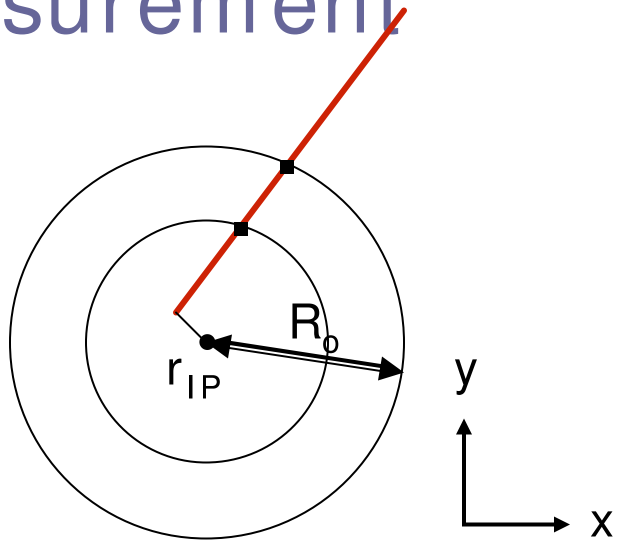
$$\left.\frac{\sigma(p_T)}{p_T}\right|^{Eloss} \sim \frac{x/X_0}{p}$$

Important e.g. ATLAS muon tracker

Impact parameter measurement

- Impact parameter r:
  point of closest approach to the interaction point

$$\sigma_{IP}^{meas.} = \sigma_{hit}\sqrt{1 + 2R_O^{\,2}/(R_O - R_I)^2}$$



- With multiple scattering
  at inner radius

$$\sigma_{IP}^{ms.} = \frac{14\ MeV}{\beta c p}\sqrt{x_I / X_0}\ R_I$$

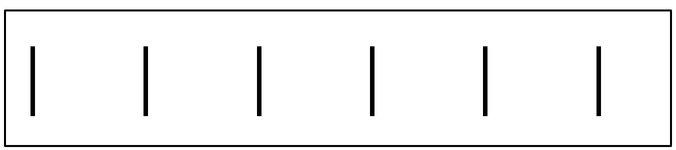
Important contribution

- DELPHI vertex detector: $28 \oplus 71/(p/GeV\ \sin^{\frac{3}{2}}\theta)\mu m$

Optimization of tracker design

- Vertex detector: important material X in first layer
- How many layers?
  - redundancy/missing hits, fake track rate

- Tracker: a 6-point sagitta measurement



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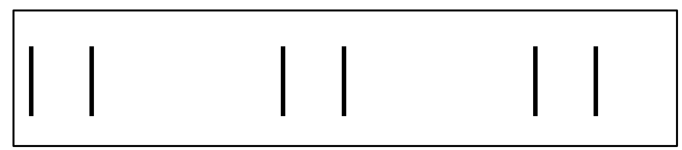


?

  - Need detailed simulations including material and geometry including support, cooling, endcap-barrel transition
  - Full Geant 4 simulation

Geant 4 detector simulation

- Interfaced with generator for physics process input: final state particle four-vectors

- Detector modeling by Geant 4
  - track particles through subdetectors
  - G4 includes geometry, material (multiple scattering)
  - secondary interactions (photon conversion, final state radiation, delta electrons, hadronic interactions)
- User provides compact subdetector response to the particles
  - Full detector response e.g. by Garfield program including ionization, drift and electronics response

What is not discussed

Crucial ingredients for the performance of a tracking detector

- Calibration: e.g. drift detectors, r-t relations
- Alignment: principle, methods and strategies

Track representation

- Five parameters 4π detector ATLAS/CMS/ALICE
  - impact parameters, polar and azimuthal angles and charge/momentum:

$$r_{IP} \quad z_{IP} \quad \theta \quad \phi \quad q/p$$

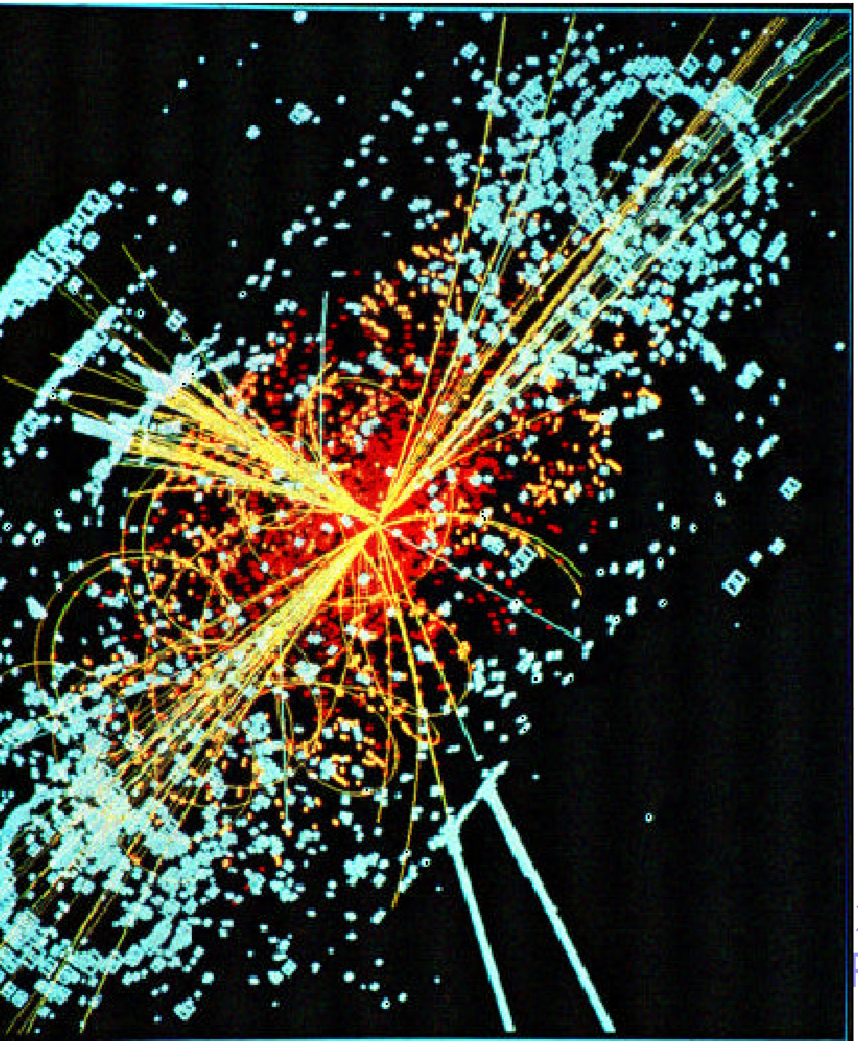
- Forward spectrometer LHCB
  - positions and derivatives $t_x = dx/dz$ $t_y = dy/dz$ and q/p:

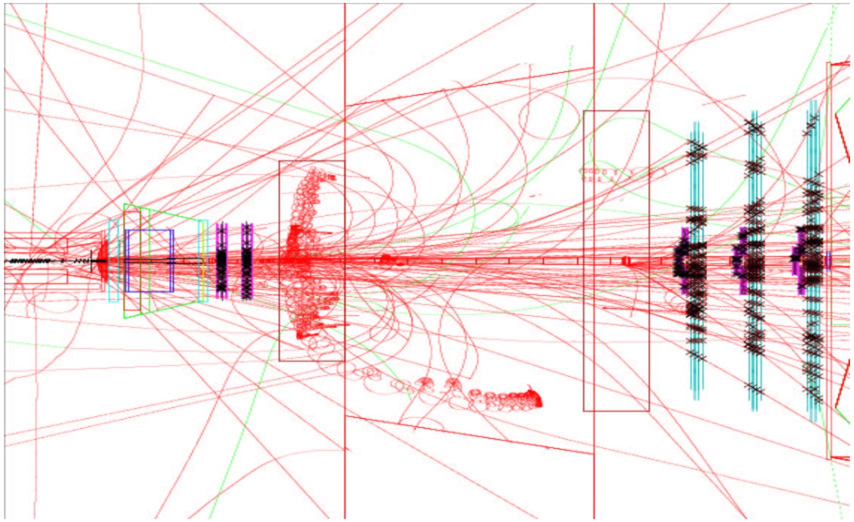
$$x \quad y \quad t_x \quad t_y \quad q/p$$

NB: use q/p ~ gaussian measurement errors

Don't use p or x_{ip} y_{ip} for r_{ip}

Pattern recognition: the problem





ents in CMS and LHCB

Decompose the tracking problem

I Pattern recognition or track finding

II Track parameter estimation or track fitting

III Test of track and pattern

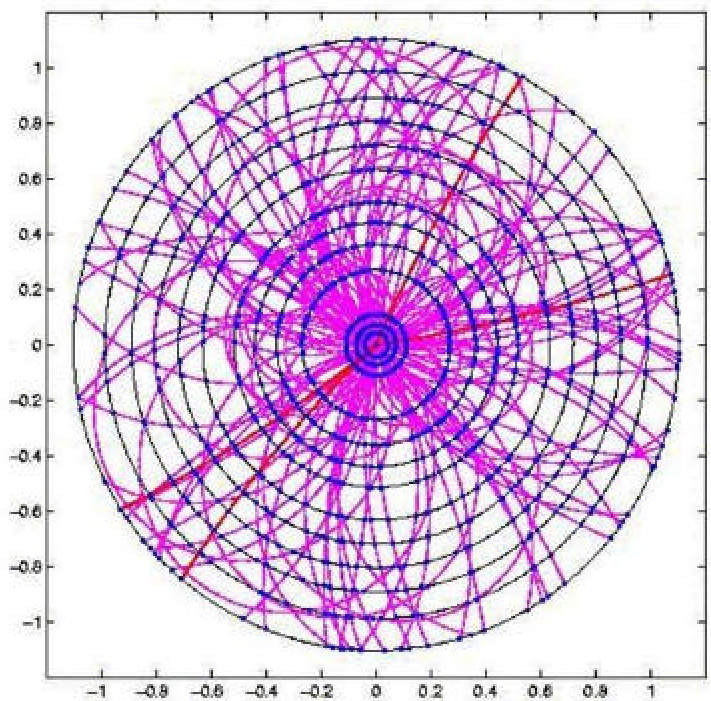
check residuals and remove outliers

Iteratively for particle hypothesis:

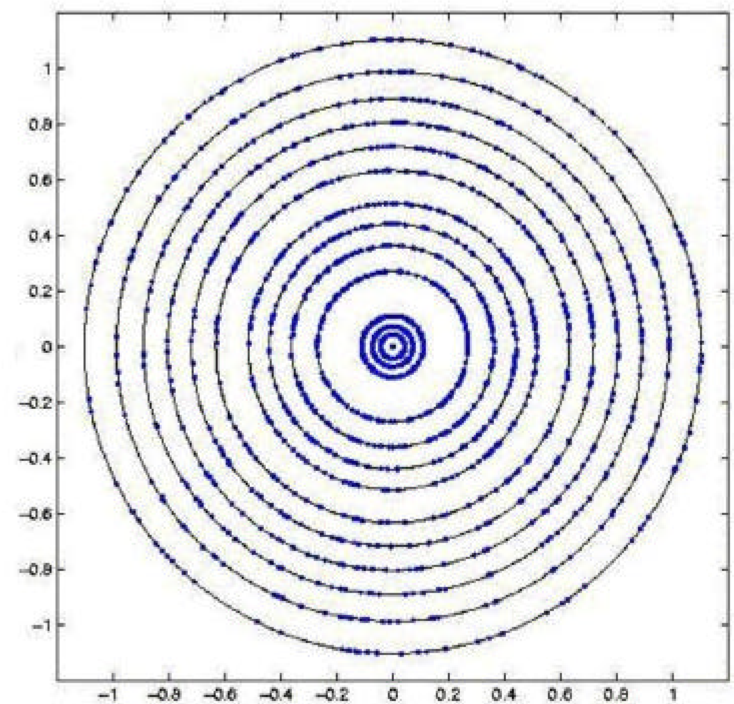
- Muons (pions, kaons, protons)
- Electrons
- Photon conversions ($\gamma \rightarrow e^+ e^-$)
- V_0 decays ($K_0 \rightarrow \pi\pi$, $\Lambda_0 \rightarrow p\,\pi$)
- Hadronic interactions

Task: track finding

Tracks with hits



hits only



A simulated Higgs event in CMS X-Y view

Track finding methods

Global methods: start from all points

- Hough transform/ histograming method
- Hopfield neural network etc.

Local Methods: start from track candidates

- Track following: candidate extrapolation
  collect hits and reiterate
- Kalman filter, track road interpolation etc.

Critical points:

- Track model approximations
- Track seed finding
- Avoid testing all combinations of hits: CPU ~ n!
- Need fast approximate tracking for first stage / trigger

Track finding

- No general solution: combine different methods
  - geometry/configuration
  - B field
  - precision tracker/detector
  - occupancy
- Strategy: modular stages (first/second stage)

Local: rough patterns

modules of a subdetector (pixel layers)
full subdetector (pixel detector)
full tracking detector (inner tracker)
add different detectors (muon + tracker)

Global:refined

Simple track models

- Straight line approximation
  - no magnetic field (2 par's)
    - $x = r_0 \sin\phi + L \cos\phi$
    - $y = -r_0 \cos\phi + L \sin\phi$
- Helix (constant solenoidal B field // z)
  - Circle in xy plane (3 par's)
    - $(x-x_c)^2+(y-y_c)^2 = R^2$ R radius of curvature
    - $r_0^{2} = R^2 - x_c^2 - y_c^2$, $\phi = \mathrm{atan}(x_c, y_c)$

  (some times a parabola is used)

The Hough transform for straight lines

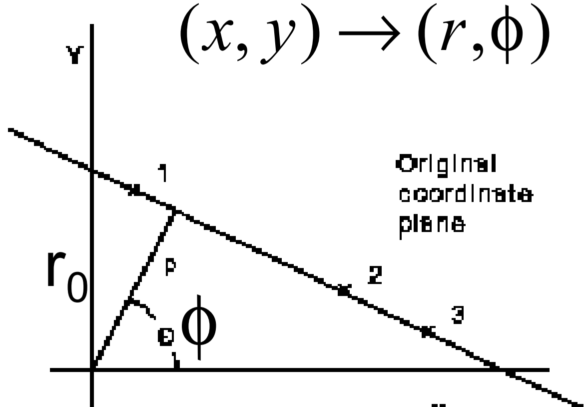
P V Hough 1959 CERN Bubble chamber

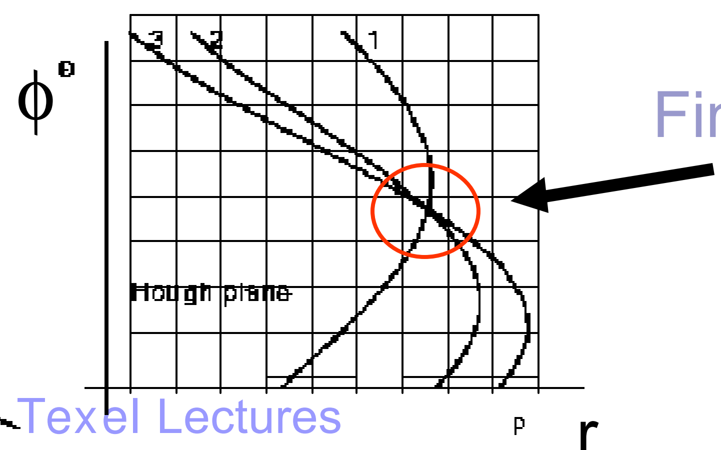
- Line in xy has two parameters r_0 , ϕ
- Find transformation for hits in xy that determines r if one scans over ϕ:

$$r = x \sin\phi - y \cos\phi$$

- Make binned histogram in $r\phi$ space

$$(x, y) \rightarrow (r, \phi)$$





Find maximum

Some remarks: the Hough transform

- Mathematical form important:
  - x_0 wouldn't work: 8
  - flat noise transforms to flat $r\phi$ plane
- Use measurement plane e.g. xy or Rz:

  e.g. straight line is deformed in Rz plane
- Carefully optimized binning in r and ϕ
- A helix has a different Hough transform
- Very fast and efficient: each hit used "once"
- High background rejection
- Used in LHC trigger and reconstruction

More on Hough transforms

- Use of filters to exploit the structure of the maximum: "butterfly" weighting bins around
- Simplest 1D Hough is histogram of the angle e.g. ϕ assuming interaction point
- By filling twice hits one can solve left-right ambiguities using a 2D SL transform
- Generalized Hough transforms (any model)
- With the 2D form one can determine helix with interaction point
- With 3D reconstruct V_0 and γ conversions

Track extrapolation

I For straight line and helix (solenoidal field): analytical expressions (fast tracking)

II High precision extrapolation in varying B field need numerical method

Runge-Kutta integration is used

- Extrapolate to the measurement surface
- Transform track into measurement plane: Transformation Matrix
- Determine residual: predicted – hit distance in this plane

Error propagation and material

- Error matrix of the track:

Symmetric

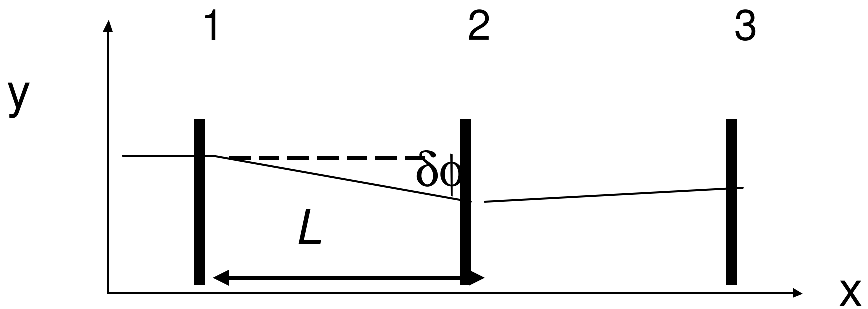
$$M = \begin{pmatrix} \delta r\delta r & \delta r\delta z & \delta r\delta\phi & \delta r\delta\theta & \delta r\delta q/p \\ & \delta z\delta z & \delta z\delta\phi & \delta z\delta\theta & \delta z\delta q/p \\ & & \delta\phi\delta\phi & \delta\phi\delta\theta & \delta\phi\delta q/p \\ & & & \delta\theta\delta\theta & \delta\theta\delta q/p \\ & & & & \delta q/p\delta q/p \end{pmatrix}$$

- Covariance Matrix C inverse error Matrix M
- Extrapolation in by simple matrix multiplication
- Extrapolation through material:
  - Additional contribution to error matrix is added
  - Energy (momentum) loss in the material

Off-diagonal elements and correlations

- Very important to treat correctly
- Due to measurements that are performed in different plane than track representation

  As e.g. below y mst plane ↔ r_0 track
- Correlation between measurements due to multiple scattering



$$y_1 = y_1$$
$$y_2 = y_1 + L \sin \delta\phi_1$$
$$y_3 = y_2 + L \sin \delta\phi_2$$

- Use matrix formalism

Controlling measurement and extrapolation

- Checking the pull distributions in mst plane
- pull = (extrapolation – hit)/error$_{total}$

  $error_{total}^2 = error_{extrap}^2 + error_{mst}^2$

  Gaussian distribution centered at 0 with width 1

  Can be done on data and simulation
- Checking the covariance or error matrix compare reconstructed and simulated values
  - Diagonal elements make pull distribution
  - Off-diagonals calculate e.g. $dzd\theta = (z_{rec} - z_{sim})(\theta_{rec} - \theta_{sim})$ and divide by error matrix element $\delta z \delta \theta$

Checking the pattern recognition

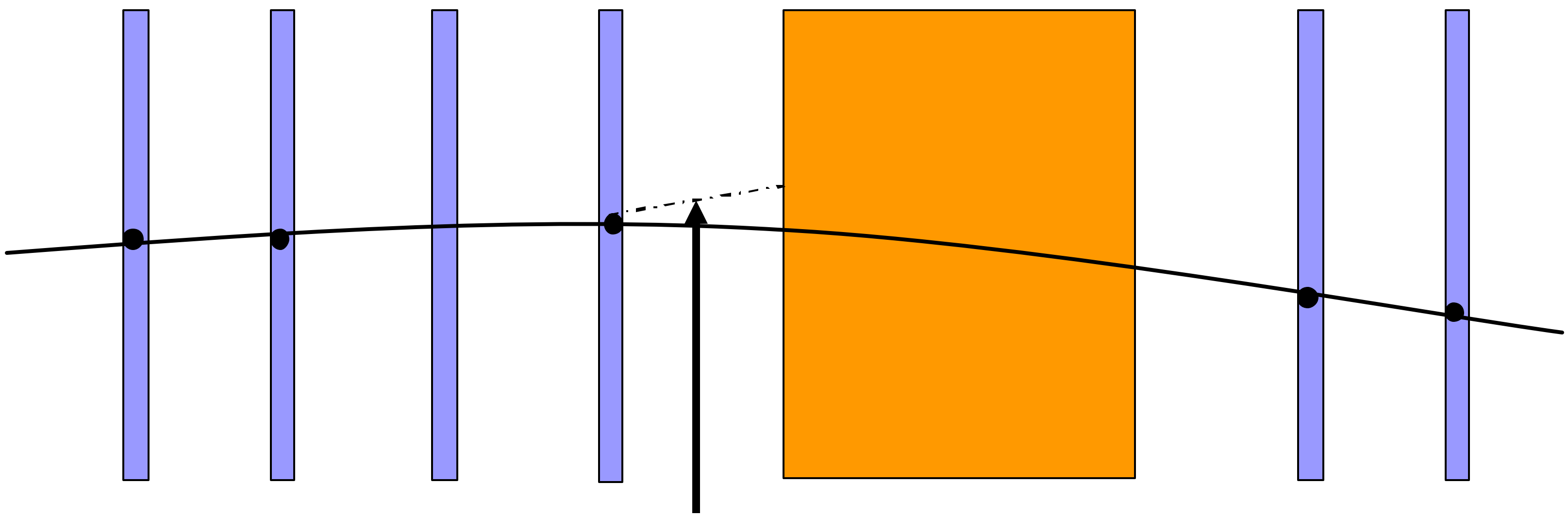
- Event display very powerful
  - show hits and pattern in mst plane
  - offline selection 'wrong' patterns

    in simulation or reconstruction
- Note 1: that if a wrong pattern or wrong seed is given to the fit large probability that it is rejected
- Note 2: if a pattern is not correctly found (e.g. a γ conversion or $V_{0)}$) gives confusion, wrong hits are associated to the track

Track fitting

- Track parameter estimation
  - parameters and errors along fitted trajectory
  - determine residuals/outliers
  - estimate goodness of fit
- Take into account measurements, scattering surfaces and brehmstrahlung

A typical track trajectory



Hard radiation

- Measurements

thin scatterers

thick scatterer

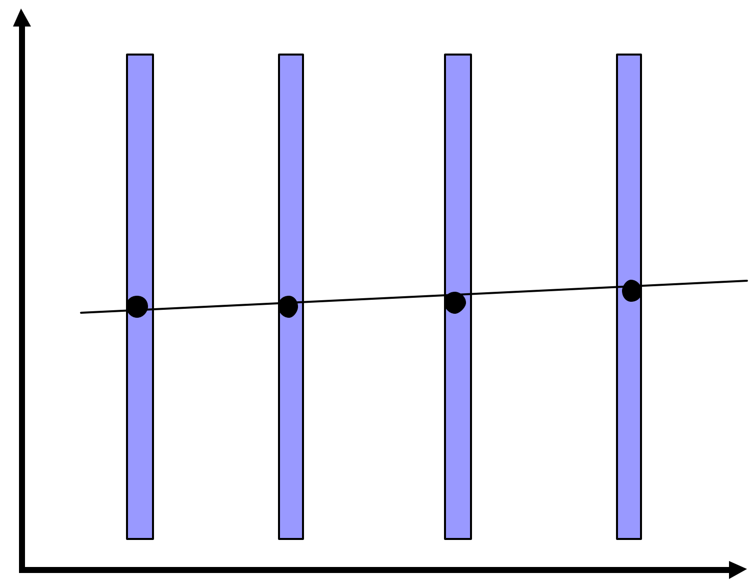
Continuous multiple scattering

Multiple scattering

Track fits

- Classical track fit
- Kalman fit
- Common features
  - based on least square estimator
  - linearize the problem
  - uses track model and extrapolator
  - input seed/initial parameters
  - for final result re-extrapolate and refit

Classical track fit



- Track model

$$\begin{pmatrix} x(z) \\ y(z) \end{pmatrix} = \begin{pmatrix} x_0 + t_x z \\ y_0 + t_y z \end{pmatrix}$$

- Measurement m and σ

Strip detector angle α

$$m_i = x(z_i)\cos\alpha - y(z_i)\sin\alpha$$

- Express in matrix form

$$m_i = A\,\vec{x}_t = \begin{pmatrix}\cos\alpha & -\sin\alpha & z_i\cos\alpha & -z_i\sin\alpha\end{pmatrix}\vec{x}_t \qquad V_i = \sigma^2(z_i)$$

track parameters $\vec{x}_t^{\,T} = \begin{pmatrix} x_0 & y_0 & t_x & t_y \end{pmatrix}$

Classical track fit

- Construct χ^2, measurements m in one vector:

$$\chi^2 = \left(\vec{m} - A\,\vec{x}_t\right)^T V^{-1} \left(\vec{m} - A\,\vec{x}_t\right)$$

- Minimize χ^2 solution for track parameters:

$$\vec{x}_t = \left(A^T V^{-1} A\right)^{-1} A^T V^{-1} \vec{m}$$

- Error matrix for track:

$$V_t = \left(A^T V^{-1} A\right)^{-1}$$

- Here inversion on 4 x 4 matrix
  if momentum q/p included 5 x 5 matrix

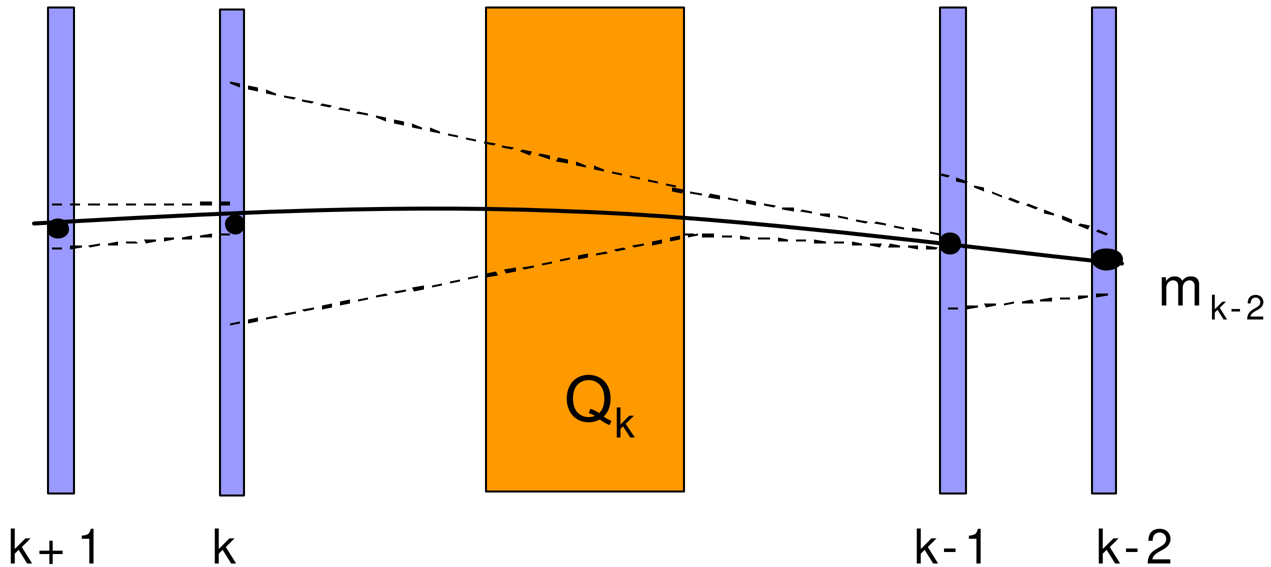
Multiple scattering in a classical fit

- Add two scattering angles per plane
  - Same formalism
  - Matrix 5x5 → (5+2*n) x (5+2*n)
  - Matrix inversion CPU consuming with many planes
- Thick material approximated by two thin planes
  - positioned at + and – L/v 12
- Can take into account hard radiation
  - well suited for electrons
- Need re-extrapolation and refit (non linear part)

Kalman fit

- Pattern recognition Kalman filtering
- Kalman fit: a progressive way of making a least square fit
- Kalman (Hungary, 1930)
  - paper published 1960 for tracking rockets
  - many applications aerospace, marine
  - 1983 first used in HEP DELPHI

Procedure of Extended Kalman fit



- The prediction step
- The filter step: add measurements m one-by-one
- The smoother step

I Prediction step

1. Propagate track state at k-1 to predicted state at k: $\vec{x}_k^{\,k-1} = f_k(\vec{x}_{k-1})$
2. Propagate Covariance and add multiple scattering: $C_k^{\,k-1} = F_k C_{k-1} F_k^{\,T} + Q_k$
3. Project prediction in measurement plane, estimate residual and cov:

$$r_k^{\,k-1} = m_k - h_k(\vec{x}_k^{\,k-1}) \qquad R_k^{\,k-1} = V_k - H_k C_k^{\,k-1} H_k^{\,T}$$

NB: Classical fit: matrix A -> f_k and h_k

II Filter step

Update track state at k with measurement

$$\vec{x}_k = \vec{x}_k^{\,k-1} + K_k r_k^{\,k-1}$$

$$C_k = (1 - K_k H_k)\, C_k^{\,k-1}$$

K_k is a 5x1 Gain matrix:

$$K_k = C_k^{\,k-1} H_k^{\,T} (R_k^{\,k-1})^{-1}$$

III Smoother step

Gives best estimate of track state on previous surfaces (k->k+1)

$$\vec{x}_k{}^n = \vec{x}_k + A_k(\vec{x}^n{}_{k+1} - \vec{x}^k{}_{k+1})$$

$$C_k{}^n = C_k + A_k(C^n{}_{k+1} - C^k{}_{k+1})A_k{}^T$$

with: $$A_k = C_k F_k{}^T (C^k{}_{k+1})^{-1}$$

Expressions for residuals and χ^2

Material and energy loss

- Multiple scattering taken care of in Q_k
- Energy loss can is corrected for

$$\Delta E = c\rho\, l \frac{Z}{A}$$

- Electrons – by hypothesis – correct for bremsstrahlung:

$$\Delta E = E\left(1 - e^{\frac{-l}{X_0}}\right)$$

correct covariance matrix δq/p

Functionality of Kalman fit

- Kalman prediction and filter can be used for pattern recognition
- Only smoother involves 5x5 matrix inversion
- No limit on number of scattering surfaces
- Expression for track state at all surfaces
- NB classical fit adds degrees of freedom

  needs a refit at different surfaces

Outlier removal, ambiguities and refit

- In fit residuals are calculated
- Used for removing bad hits
- Refit and reiterate
- Kalman and classical: critical start with seed and good pattern
- Try to solve left-right ambiguities locally
- Can allow to share hits with different tracks and solve this at later stage: modular pattern recognition

Physics applications

- Primary and secondary vertex search
- Invariant mass reconstruction
- b quark tagging

Explain where at LHC precise and well understood tracking matters

Vertex reconstruction

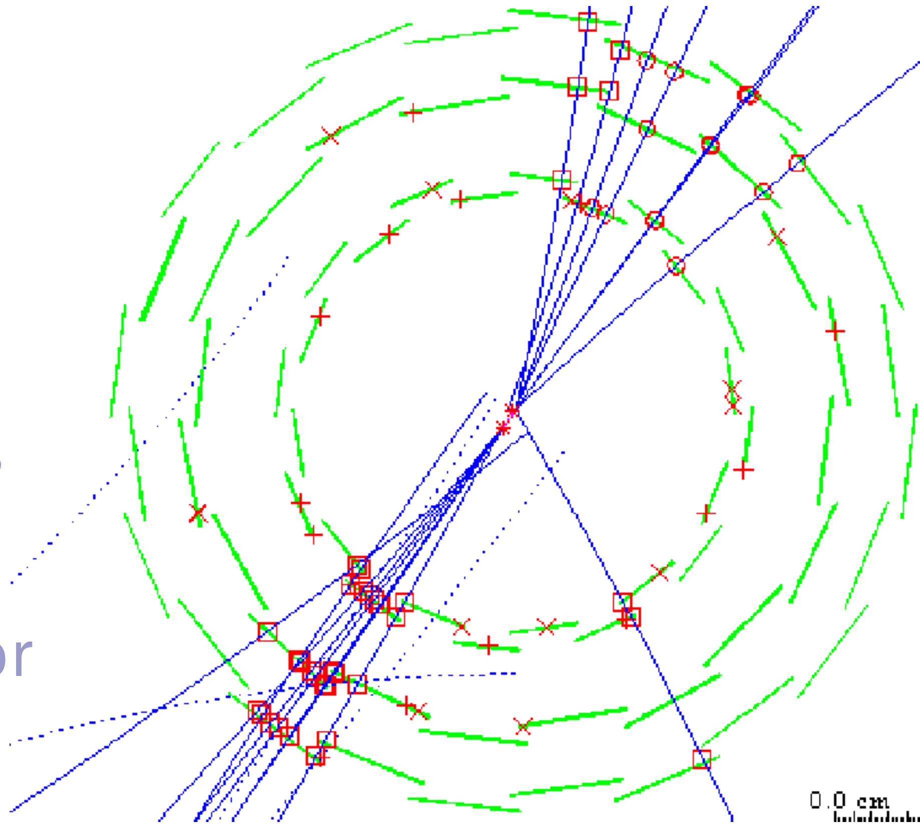
- Primary vertex:

interaction point beam of particles

- Secondary vertex:

decay vertices of unstable particles: B and C hadrons (no V_0s)

typical decay length B or C hadron few mm



Procedure for vertex reconstruction

- Vertex finding (clustering problem)
- Extrapolation of tracks to vertex
  - point of closest approach and error
- Association to vertex e.g. χ^2 cut (pull)
- Fit of tracks to common vertex
- Estimate vertex position and covariance
- Iterative procedure with outlier removal

Primary and secondary search vertex

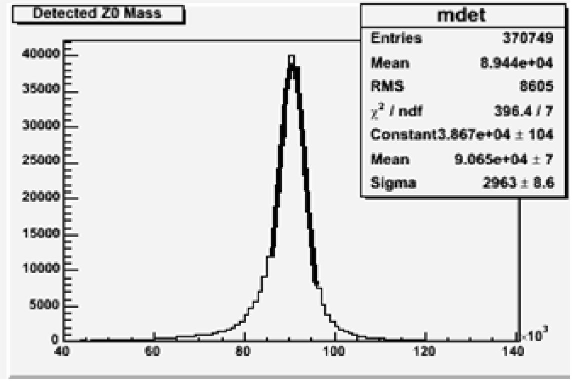
- Primary vertex search "easy"
  - Many tracks from interaction point
  - Seed: fill average beamspot
- Secondary vertex search difficult
  - A few ~ 5 tracks from a B/C decays
  - Seed helps: e.g. lepton from B/C decay
  - Two or n track vertex using tracks with large impact parameters
  - Problems: rejection of V_0 particles, secondary interactions, badly reconstructed tracks

Vertex fit

- Classical approach:
  - Input track pars and covariance matrix
  - Solve vertex by matrix inversion
- Kalman fit:
  - Add one by one tracks to vertex and update vertex parameters
  - Smoothing gives the updated track pars
- Adaptive Kalman or classical fit:
  - Uses probabilities for a track to belong to vertex

Invariant mass estimation

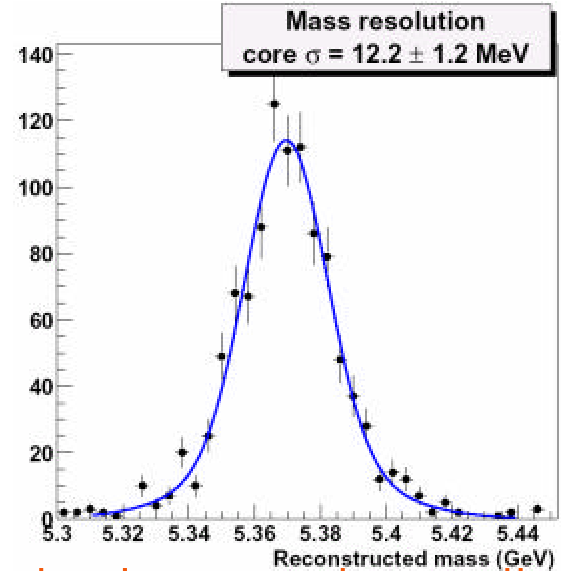
- No additional constraints
  - Reconstruction of Z or $J/\psi \longrightarrow \mu^+\mu^-$
    - mass from measured momenta and E-p conservation
    - propagate errors momenta to mass
  - Allows to control tracking parameters and errors by:
    - Z and J/ψ mass shift, width and pull
      - Diagnostics difficult: material, B field, alignment etc



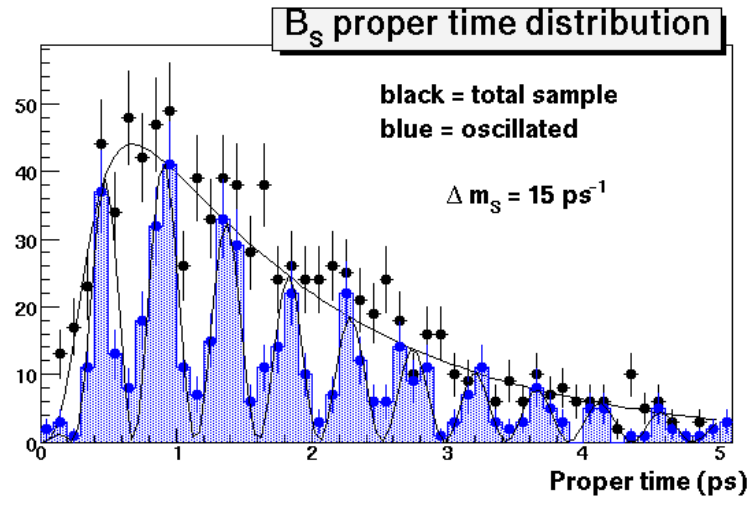
$\mu^+\mu^-$ invariant mass

Invariant mass with constraints

- Full B decay chain
- $B_s \longrightarrow D_s\ \pi$
  $\quad\quad\quad\quad \lfloor\!\!\longrightarrow \Phi\ \pi \longrightarrow KK\ \pi$
- Vertices: primary, B decay and C decay
- Masses Φ, D_s, B_s
- Complex set of equations Minimisation programme
  - Improvement in the purity of B_s particles
  - Best proper time B_s : L p

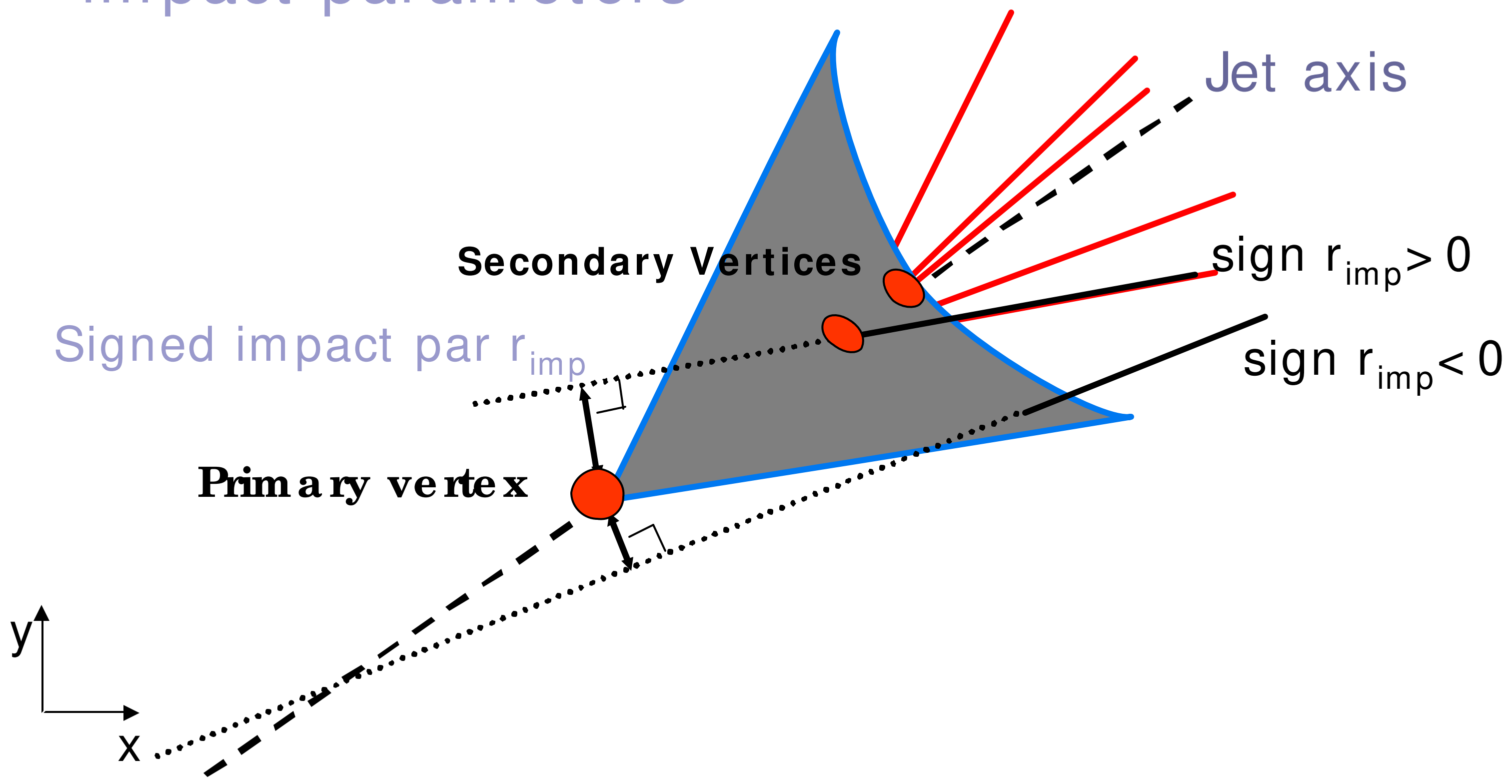


B physics very demanding



b quark tagging

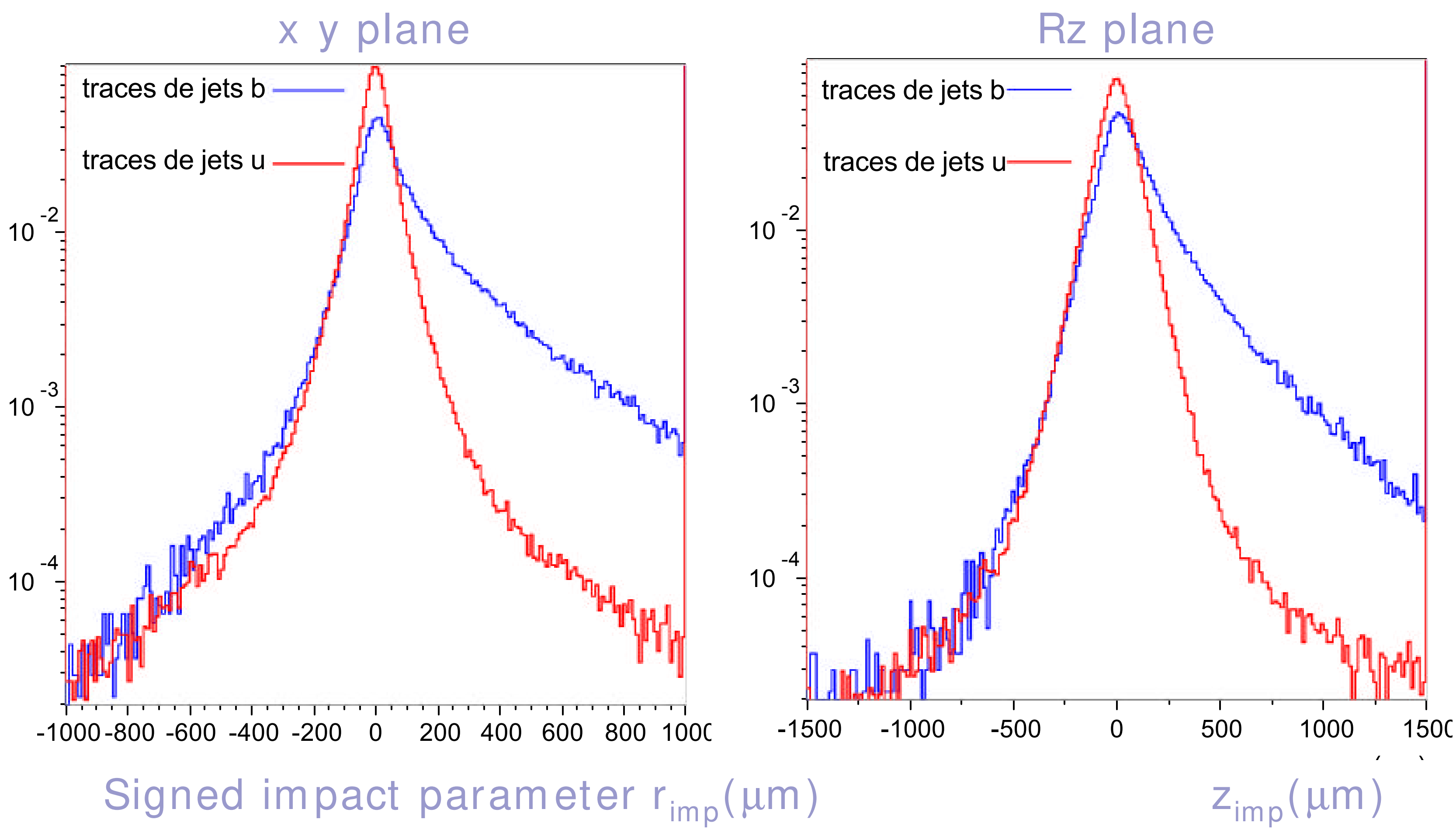
- Exploit the vertex structure of B decays
- Tagging based on the lifetime signed track impact parameters



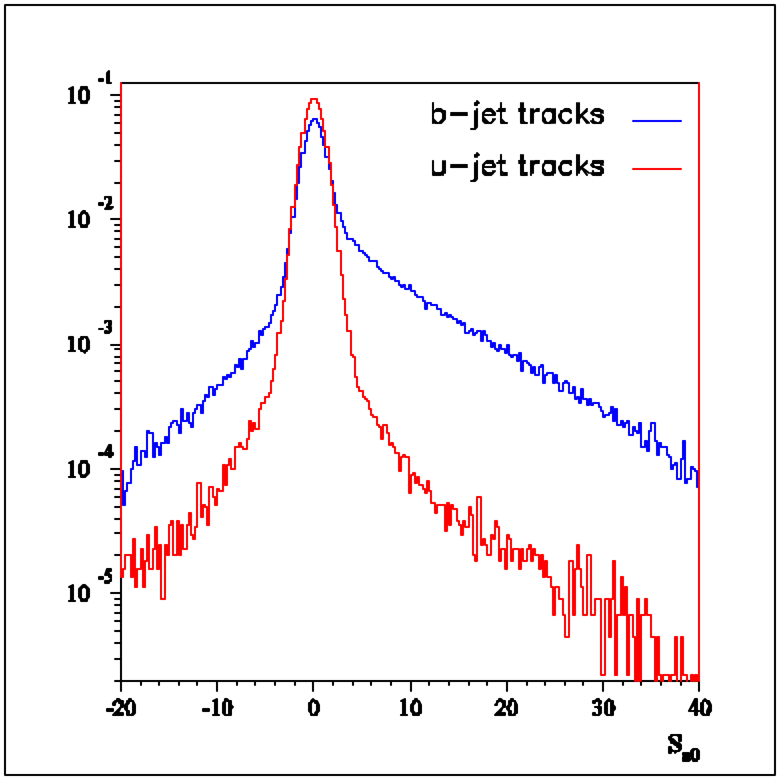
b quark tagging

- Typical procedure impact parameter tag:
  - reconstruct primary vertex
  - Select tracks for a jet
  - Require a high quality tracks: pixel hits, remove: V_0s, conversions, short badly measured tracks
  - Determine lifetime signed impact parameters and errors wrt primary vertex
    - Lifetime sign: angle track wrt jet
- Refinements:
  - Leptons from B or C decays
  - Secondary vertex search

2D impact parameter



Impact parameter significance tag



- Significance S:

  $S = r_{imp}/\sigma_{r_{imp}}$ or $z_{imp}/\sigma_{z_{ip}}$

  expect (i.e. can check):
  - Gaussian distribution for negative part
  - Gaussian distribution for light (uds) quarks
- Calculate rejection:

  $R(S) = P_b(S)/P_{uds}(S)$
  - Take into account tails
- Combine rejections variable

$R_{jet} = \prod R_i(S)$; $w_{jet} = \ln(R_{jet})$

Physics with a b tag

- Historically developed at LEP
  - precise electroweak measurements
    - Measurement $R_b = 0.21629 \pm 0.00066$
- Role at LHC:
  - Important LHCb trigger
  - ATLAS and CMS
    - B physics
    - Higgs couplings tt H with H->bb
    - Top physics t->bW
    - SUSY particles, New Physics? X->bx

Some literature

- CERN lectures on detectors by C. Joram (2003) and O. Ullaland (2004)
- Lectures on detectors by Spieler http:/www-physics.lbl.gov/~spieler
- W. Blum and L. Rolandi, “Particle Detection with Drift Chambers”, Springer, 1994
- CERN lectures data analysis from R. Fruhwirth (2004), theory lectures by D. Hall (2004)
- W. Regler et al., “Data analysis techniques for high energy physics” - 2nd ed Cambridge (2000)
- LHCb theses R van der Eijck en J van Tilburg

Excursie naar de slufter 14.00