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Control Charts for Zero-Inflated Poisson Models

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Abstract

This paper develops three new control charts for processes with nonconformities based on a Zero-Inflated Poisson (ZIP) distribution. The ZIP distribution is approximated by a non-central chi-square distribution with parameterλ (λ_{Chi}). The best fit value of λ_{Chi} is used to replace the estimated mean and variance that are used in the control limits of the traditional Shewhart *c-Chart* by three different methods. The three new charts are called $c_{Chi}-Chart$, $c_{CChi}-Chart$ and $c_{MChi}-Chart$. In the $c_{Chi}-Chart$, the estimated values of the mean and variance in the *c-Chart* are replaced by λ_{Chi}. In the $c_{CChi}-Chart$, they are replaced with the estimators of the mean and the variance, respectively, of the non-central chi-square distribution. In the $c_{MChi}-Chart$, the mean is replaced by the estimated mean of the non-central chi-square and the variance is replaced by the inter-quartile range. Extensive simulations have been carried out to evaluate the performance of the new charts. The Average Run Length (ARL) and the Average Coverage Probability (ACP) have been used to compare the performance of the proposed charts. We have found that for an in-control process (ZIP mean μ_0), the $c_{CChi}-Chart$ is superior to all other charts considered for all μ_0 for a low proportion (ω) of zero non-conformities. For an out-of-control process (ZIP mean μ_1), the $c_{MChi}-Chart$ performs better than the other charts for low values of ω for all μ_1 and mean shifts (ρ). However, for high values of ω, the $c_{Chi}-Chart$ performs better than the other charts considered.

Keywords: non-central chi-square distribution, nonconforming control chart, zero-inflated poisson distribution

1 Introduction

If sample sizes are constant, the traditional Shewhart control chart of nonconformities (*c-Chart*) can be used to monitor the number of nonconformities per unit of product. In this case, the sampling of products takes place as a set of repeated samplings, with each sampling finding either zero nonconformities or a nonzero number of nonconformities. If the nonconformity distribution is a Poisson distribution, then in some sampling processes an excess number of zeros might be observed. In this case, the distribution is called a "Zero-Inflated Poisson (*ZIP*)"and the estimation of the sample mean tends to underestimate the mean of the Poisson distribution. If the estimated variance is greater than the mean (this is called "Over Dispersion"), then the estimated limits in the *c-Chart* are tighter than the correct Poisson distribution limits (Sim and Lim [2]). These tighter limits lead to an increased false alarm rate when the *c-Chart* is used to detect an out-of-control state.

Cohen [1] developed a *ZIP* model in which he estimated the value for the mean λ by using the maximum likelihood estimator (MLE) $\hat{\lambda}$ because he found that the *MLE* value is closer to the actual value. This study has been used in many applications (see, e.g., Gupta et al. [8], Bohning et al. [3]).

Xie et al. [7] constructed a *c-Chart* for the *ZIP* model that they called the $c_{ZIP} - Chart$. They examined the efficiency of this chart for detection of upward shifts of the mean value of number of nonconformities in a process.

Sim and Lim [2] proposed a charting method called a $c_J - Chart$ in which they used a one-sided Jeffreys prior interval (Cai [9]) to detect upward shifts. They compared this $c_J - Chart$ with a usual *c-Chart* and a $c_{ZIP} - Chart$. They showed that the $c_J - Chart$ was appropriate for processes when the mean was in-control. On the other hand, if the process mean was in an out-of-control situation, then the *c-Chart* performed better than the other charts. However, they found that the *c-Chart* yields poor coverage probability.

Peerajit and Mayureesawan [10] extended the research ideas of Sim and Lim by supplying both the proportion of zero nonconformities and the mean shift in a production process. The results obtained for the performance of the *c-Chart*, $c_{ZIP} - Chart$ and $c_J - Chart$ were in agreement with those of Sim and Lim.

In the studies mentioned above, the authors have either studied the performance of the charts for either the average run length (ARL) or the average coverage probability (ACP) but not both, or if they studied both they found that charts might perform well for one measure but poorly for the other measure.

The aims of the present study are to develop modified versions of a *c-Chart* for the *ZIP* model that perform satisfactorily for a range of parameters of the *ZIP* model and to compare the performance of these new charts with the charts mentioned above. The outline of the paper is as follows. We first develop an approximation for the distribution of the *ZIP* model as a non-central chi-square distribution with parameter λ, and we examine how the value of λ varies as the parameter of the *ZIP* model is changed. We then use the λ from the non-central chi-squared distribution to adjust the control limit of *c-Chart* by three different methods to develop three new control charts, which we call $c_{Chi} - Chart$, $c_{CChi} - Chart$ and $c_{MChi} - Chart$. The performance of these newly developed control charts is then compared with the performance of *c-Chart*, $c_{ZIP} - Chart$ and $c_J - Chart$.

2 Materials and Methods

Zero-Inflated Poisson (*ZIP*)

The probability mass function is given by (Gupta et al. [8]):

$$P(Y = y) = \begin{cases} \omega + (1-\omega)exp(-\lambda) & , \quad y = 0 \\ \frac{(1-\omega)exp(-\lambda)\lambda^y}{y\,!} & , \quad y > 0, \end{cases} \tag{1}$$

where Y = the random variables of nonconformities in a sample unit,
λ = the mean of nonconformities in a sample unit,
ω = is a measure of the extra proportion of zero nonconformity in a sample unit, and

$$E(Y) = \mu = (1-\omega)\lambda \textit{ and } V(Y) = \mu + (\frac{\omega}{1-\omega})\mu^2. \tag{2}$$

Note: $\omega = 0$ is the Poisson distribution.

The Non-central Chi-square distribution

The probability function is given by (Krishnamoorthy [6]):

$$f(Y = y) = \sum_{k=0}^{\infty} \frac{exp(-\frac{\lambda_{Chi}}{2})(\frac{\lambda_{Chi}}{2})^k}{k!} \frac{exp(-\frac{y}{2})^{\frac{n+2k}{2}-1}}{2^{\frac{n+2k}{2}}\tau(\frac{n+2k}{2})} \quad , \quad 0 \leq y < \infty, \tag{3}$$

where Y = the random variables of non-centralchi-square distribution,
n = number of degrees of freedom,
λ_{Chi}= the sum of squares of the ratios of means and standard deviations of Y, and

$$E(Y) = n + \lambda_{Chi} \textit{ and } V(Y) = 2(n + 2\lambda_{Chi}). \tag{4}$$

The Shewhart control chart of nonconformities (*c-Chart*)

The control limits are given by (Montgomery [4]):

$$\begin{aligned} UCL &= c + 3\sqrt{c} \\ CL &= c \\ LCL &= c - 3\sqrt{c}. \end{aligned} \tag{5}$$

c is assumed to be the mean number of nonconformities if the mean of the probability distribution is known, otherwise c is estimated as the mean of the number of nonconformities in a sample of observed product units $(\bar{c})$.

The control chart of nonconformities with *ZIP* model $(c_{ZIP} - Chart)$

In 1991, Cohen [1] developed a *ZIP* model for a Poisson probability function g(y,λ), y=0,1,2,.. given by:

$$P(Y = y) = \omega I_{(y,o)} + (1 - \omega)g(y, \lambda) \quad , \quad y = 0, 1, 2, ... \tag{6}$$

where $Y =$ the random variables of nonconformities in a product process, and $I_{(y,0)} = 1$ if $y = 0$ and $I_{(y,0)} = 0$ if $y \neq 0$. The maximum likelihood estimates (*MLEs*) of parameter λ in the *ZIP* model of Cohen is given by:

$$\hat{\lambda} = \bar{y}^{+}[1 - e^{\hat{\lambda}}], \tag{7}$$

where $\bar{y}^{+} =$ the mean of the number of nonconformities in product units that have a nonzero number of nonconformities.

The $\hat{\lambda}$ are then used in the control limits for the $c_{ZIP} - Chart$ (Xie et al. [7]) as follows:

$$\begin{aligned} UCL &= \hat{\lambda} + 3\sqrt{\hat{\lambda}} \\ CL &= \hat{\lambda} \\ LCL &= \hat{\lambda} - 3\sqrt{\hat{\lambda}}. \end{aligned} \tag{8}$$

The control chart of nonconformities with Jeffreys Prior Interval method ($c_J - Chart$)

The one-sided Jeffreys prior interval is given by (Cai [9]):

$$CI_J^{\lambda}(y) = [G(\alpha; y + 0.5, 1), \infty], \tag{9}$$

where y = the number of nonconformities for a Poisson distribution,
λ = the parameter estimate $\hat{\lambda}$ for the *ZIP* model from equation (7).
If $y = 0$ the confidence interval is $[0,\infty)$, if $y \neq 0$ the confidence interval is $[G(\alpha; y + 0.5, 1), \infty]$, where $G(\alpha; a, b)$ is the 100αth percentile of a Gamma distribution with shape parameter $a = y + 0.5$ and scale parameter $b = 1$,

For the control chart of nonconformities with Jeffreys Prior Interval method, i.e., the $c_J - Chart$, the control limit is given by (Sim and Lim [2]):

$$UCL = max[y \mid \lambda > G(\alpha; y + 0.5, 1)]. \tag{10}$$

Development of the new *c-Charts* for nonconforming units in a process

For a given *ZIP* distribution, we first obtain an approximate non-central Chi-square distribution with parameter λ_{Chi} by using the Kolmogorov-Smirnov test [5]. The three new charts for nonconformities are then defined as follows.

1. $c_{Chi} - Chart$. The control limit of this chart is obtained from a *c-Chart* by replacing c with λ_{Chi} The control limit of the $c_{Chi} - Chart$ is therefore given by:

$$\begin{aligned} UCL &= \hat{\lambda}_{Chi} + 3\sqrt{\hat{\lambda}_{Chi}} \\ LCL &= 0, \end{aligned} \tag{11}$$

where $\hat{\lambda}_{Chi} = \sum_{i=1}^{k} (\frac{\mu_i}{\sigma_i})^2$.

2. $c_{CChi} - Chart$ is a modified version of the *c-Chart* obtained by replacing the estimated values of the mean and variance in the upper control limits of a one-sided *c-Chart* with the estimators of the mean and variance of a non-central chi-square distribution with number of degrees of freedom defined to be zero. Therefore the control limit of $c_{CChi} - Chart$ is given by:

$$\begin{aligned} UCL &= E(Y) + 3\sqrt{V(Y)} \\ LCL &= 0, \end{aligned} \tag{12}$$

where $E(Y) = \lambda_{Chi}$ and $V(Y) = 4\lambda_{Chi}$.

3. $c_{MChi} - Chart$ is a modified version of the *c-Chart* obtained by replacing the estimated value of the mean in the *c-Chart* with the estimator of the mean of a non-central chi-square distribution with number of degrees of freedom defined to be zero and with the estimated value of the variance in the *c-Chart* replaced with the inter-quartile range ($IQR(c)$). Therefore the control limit of $c_{MChi} - Chart$ is given by:

$$\begin{aligned} UCL &= E(Y) + 3\sqrt{IQR(c)} \\ LCL &= 0, \end{aligned} \tag{13}$$

where $IQR(c) = Q_3 - Q_1$.

3 Simulation Results

In this section we report the results of tests of the new charts by a simulation study. For the simulations, we assume the following ranges of parameter values. The means for the in-control process are: $(\mu_0) = 4.0(0.5)5.5$. The means for the out-of-control process are: $(\mu_1 = \mu_0 + \rho)$ where the mean shifts are: $(\rho) =$ 0.00, 0.40, 0.80 and 1.20. The proportions of zero nonconformity are: $(\omega) =$ 0.30(0.10)0.90. Finally, the value for the over-dispersion $(\varphi) = 1$.

The evaluation of the performance of the control charts was conducted as follows:

1. The R program was used to simulate the number of nonconforming items for a *ZIP* model with values for the parameters $(n, \mu_0, \varphi, \omega)$ chosen from the set of values given above.

2. The value of the parameter λ_{Chi} which gives a best fit between the distribution of nonconforming items from step 1 and a non-central chi-square distribution with number of degrees of freedom equal to zero was calculated.

3. The Kolmogorov-Smirnov test was used to test the hypothesis that a non-central chi-square distribution with zero degrees of freedom and with the λ_{Chi} value from step 2 could give a reasonable fit to the data obtained in step 1. Based on simulations with 20,000 replications, the results of the test showed that the hypothesis was satisfied for at least 95% of the replications.

4. Based on 100,000 replications, the averaged control limits were calculated for the *c-Chart*, $c_{ZIP} - Chart$, $c_J - Chart$ and $c_{MChi} - Chart$. For the $c_{Chi} - Chart$ and $c_{CChi} - Chart$ the values for λ_{Chi} calculated in step 2 were used for

calculating the control limits.

5. Based on a new set of 100,000 replications, the control limits calculated in step 4 were then used to compute the ARL and the ACP for each chart.

6. Steps 1 to 5 were then repeated for a new set of values for parameters $(n, \mu_0, \varphi, \omega)$.

4 Results

In this section a summary is given of some of the results that were obtained from the simulations.

Table 1 shows the values of $\hat{\lambda}_{Chi}$ for the non-central chi-square distribution that gives the best fit between the chi-square and the *ZIP* distribution for a range of ω and μ values. It can be seen that as the values of ω are increased, the values of $\hat{\lambda}_{Chi}$ vary from approximately 0.7 of the μ value at $\omega = 0.3$ to a constant value of 1.44 independent of μ at the higher values of ω (0.7-0.9).

The process is in the in-control state

The results for the in-control case ($\rho = 0.00$) are shown in table 2. Table 2 shows a comparison of ARL_0 and ACP values for the $c - Chart(c)$, $c_{ZIP} - Chart(c_{ZIP})$, $c_J - Chart(c_J)$, $c_{CChi} - Chart(c_{CChi})$, $c_{Chi} - Chart(c_{Chi})$ and $c_{MChi} - Chart(c_{MChi})$. A comparison of ARL_0 values for the charts is given in Figure 1. It can be seen that for all levels of μ_0 and for ω =0.3-0.5, the $c_{CChi} - Chart$ returns the highest ARL_0 values. Therefore the $c_{CChi} - Chart$ is accepted as the preferred control chart because it detects shifts slowly. However, when ω =0.6-0.9, the $c_{ZIP} - Chart$ and $c_J - Chart$ are more appropriate as control charts because they show the highest ARL_0 values.

Figure 2 shows the absolute values of the differences between the ACP values and the confidence level of 0.9973, which we call the *ACP-DIFF* value, for the preferred charts for the ARL values i.e., for the $c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$. It can be seen that when ω =0.3-0.5, these three charts have similar low *ACP-DIFF* values for all values of μ_0. That is, these control charts all give ACP values close to the target level of 0.9973. However, for higher ω(0.6-0.9), only the $c_{ZIP} - Chart$ and $c_J - Chart$ give ACP values close to the required confidence level.

When both ARL_0 and ACP values are considered, the $c_{CChi} - Chart$ will be the preferred control chart when ω =0.3-0.5 for all levels of μ_0. When μ_0 =4.0, the $c_J - Chart$ will be the preferred control chart for $\omega = 0.6$-0.9. When $\mu_0 = 4.5$-5.5, the $c_{ZIP} - Chart$ and $c_J - Chart$ will be the preferred control charts.

The process is in an out-of-control state ($\rho > 0.00$)

Results for this case are shown in Figures 3 and 4. Figure 3 gives a comparison of ARL_1 values for a range of values of μ_1, ω and ρ. It can be seen that the *c-Chart*, $c_{Chi} - Chart$ and $c_{MChi} - Chart$ return similar low values of ARL_1. That is, they are able to detect shifts faster than the other charts. However, it can be seen from Figure 3, that all control charts detect a shift slowly for values of ω of (0.8, 0.9). Figure 4 gives a comparison of the *ACP-DIFF* values for the preferred charts for the ARL_1 values, that is, for the *c-Chart*, $c_{Chi} - Chart$ and $c_{MChi} - Chart$. It can be seen that when ω =0.3-0.7 the $c_{MChi} - Chart$ returns the lowest *ACP-DIFF* values for all values of μ_1, ρ tested, that is, it gives the ACP value closest to the target value. However, for higher ω(0.8, 0.9), it can be seen that the ACP value for the $c_{Chi} - Chart$ is closer to the target value than the ACP values for the other charts.

When both ARL_1 and ACP values are considered, the $c_{MChi} - Chart$ will be the preferred control chart when ω =0.3-0.7 for all levels of μ_1 and ρ. However, when ω =0.8, 0.9, the $c_{Chi} - Chart$ will be the preferred control chart.

Table 1. The $\hat{\lambda}_{Chi}$ values for the non-central chi-square that give the best fit to the distribution of the *ZIP* model for a range of μ and ω values.

ω	μ			
	4.0	4.5	5.0	5.5
0.30	3.08	3.08	3.36	3.99
0.40	3.08	3.08	3.35	3.34
0.50	2.30	2.50	2.50	2.97
0.60	1.80	1.90	1.90	1.90
0.70	1.44	1.44	1.44	1.44
0.80	1.44	1.44	1.44	1.44
0.90	1.44	1.44	1.44	1.44

Table 2. Comparison of ARL_0 and ACP values of the *c-Chart*, $c_{ZIP}-Chart$, $c_J-Chart$, $c_{CChi}-Chart$, $c_{Chi}-Chart$ and $c_{MChi}-Chart$ for a range of μ_0 and ω values.

μ_0	ω	ARL_0						ACP					
		c	c_{ZIP}	c_J	c_{CChi}	c_{Chi}	c_{MChi}	c	c_{ZIP}	c_J	c_{CChi}	c_{Chi}	c_{MChi}
4.5	0.3	34.4	213.3	210.4	761.8	15.7	82.7	0.9640	0.9953	0.9983	0.9994	0.9391	0.9886
	0.4	18.1	245.1	243.4	790.1	18.2	96.5	0.9451	0.9959	0.9984	0.9994	0.9473	0.9903
	0.5	10.8	289.4	287.5	577.8	10.8	48.4	0.9153	0.9963	0.9985	0.9985	0.9160	0.9798
	0.6	7.4	349.2	347.1	145.7	7.5	28.1	0.8847	0.9976	0.9992	0.9930	0.8807	0.9588
	0.7	6.1	493.6	491.1	81.3	6.1	10.2	0.8614	0.9980	0.9993	0.9884	0.8563	0.9252
	0.8	6.5	607.3	605.5	123.0	9.7	4.4	0.8675	0.9984	0.9993	0.9920	0.9070	0.8137
	0.9	11.0	768.5	765.0	243.5	20.5	9.7	0.9160	0.9990	0.9997	0.9964	0.9519	0.9063
5.0	0.3	20.0	255.0	256.3	789.9	20.1	103.0	0.9576	0.9959	0.9984	0.9994	0.9534	0.9890
	0.4	11.4	293.0	293.0	820.8	23.6	51.2	0.9229	0.9960	0.9985	0.9995	0.9601	0.9820
	0.5	7.4	341.9	342.6	343.2	7.4	28.2	0.8962	0.9976	0.9987	0.9976	0.8828	0.9656
	0.6	5.5	406.2	406.0	77.8	5.5	17.9	0.8679	0.9975	0.9999	0.9861	0.8447	0.9479
	0.7	4.9	493.6	493.1	48.3	4.9	12.9	0.8438	0.9980	0.9991	0.9797	0.8324	0.9216
	0.8	5.8	607.3	606.5	72.1	7.9	4.2	0.8570	0.9990	0.9995	0.9859	0.8839	0.8059
	0.9	10.3	768.5	773.0	145.8	16.7	9.4	0.9143	0.9992	0.9996	0.9940	0.9432	0.9018
5.5	0.3	25.5	306.3	304.9	932.5	25.3	54.8	0.9601	0.9945	0.9987	0.9998	0.9615	0.9818
	0.4	14.8	348.7	346.3	630.7	14.9	65.6	0.9364	0.9952	0.9989	0.9992	0.9367	0.9843
	0.5	9.4	400.1	398.9	402.0	9.4	36.0	0.9017	0.9968	0.9990	0.9976	0.9060	0.9723
	0.6	6.9	466.6	464.4	45.2	4.4	12.1	0.8705	0.9974	0.9994	0.9800	0.8097	0.9465
	0.7	6.0	550.9	548.9	30.6	4.2	9.6	0.8559	0.9981	0.9993	0.9679	0.8099	0.9113
	0.8	6.7	663.0	660.6	46.4	6.8	4.2	0.8560	0.9987	0.9995	0.9785	0.8703	0.8042
	0.9	10.0	807.6	808.2	93.1	14.5	9.3	0.9071	0.9992	0.9997	0.9897	0.9367	0.9032

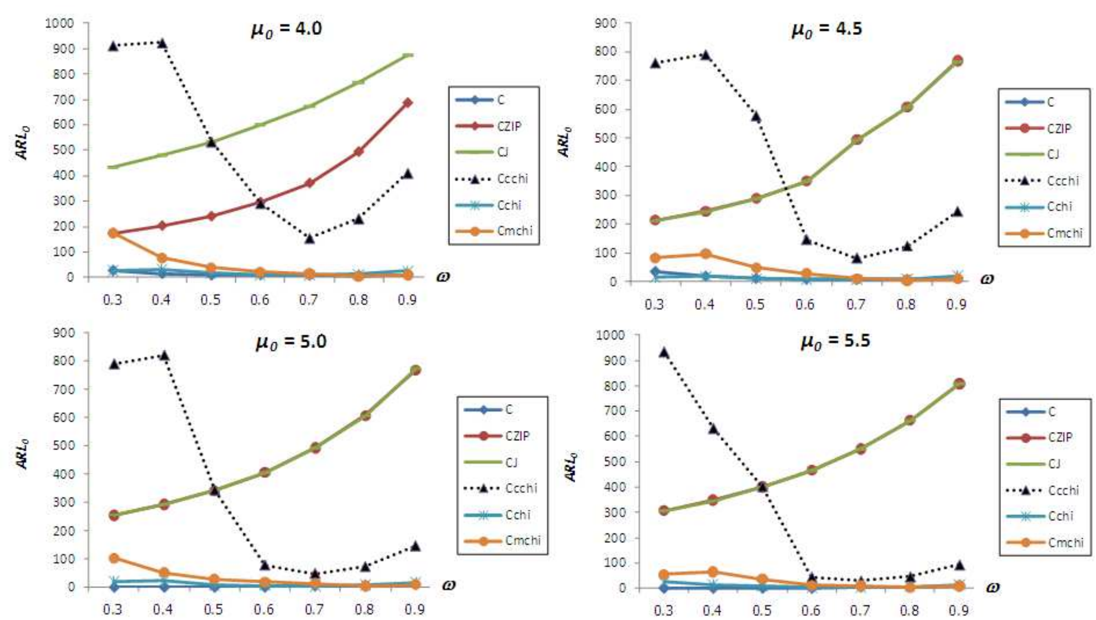

Figure 1: Comparison of ARL_0 of the *c-Chart*, $c_{ZIP}-Chart$, $c_J-Chart$, $c_{CChi}-Chart$, $c_{Chi}-Chart$ and $c_{MChi}-Chart$ for a range of μ_0 and ω values.

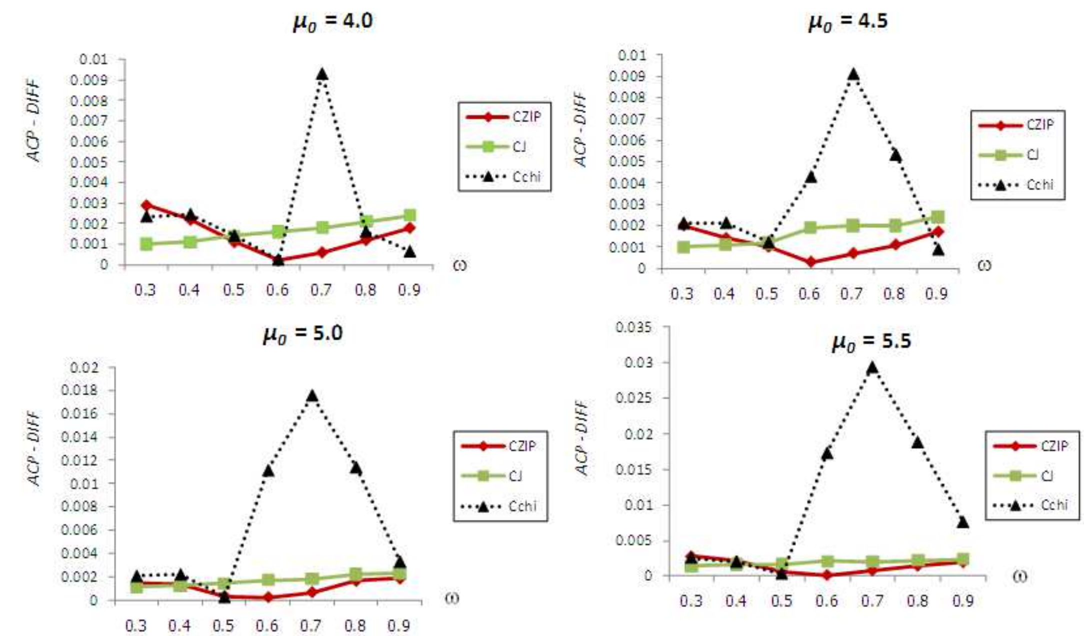

Figure 2: Comparison of the ACP - $DIFF$ of the $c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$ for a range of μ_0 and ω values.

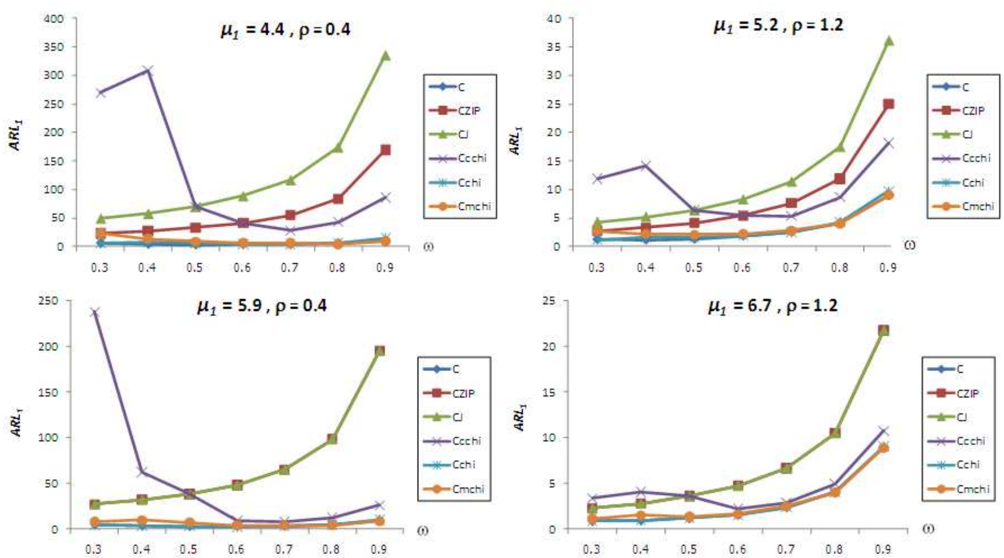

Figure 3: Comparison of ARL_1 of the c-$Chart$, $c_{ZIP} - Chart$, $c_J - Chart$, $c_{CChi} - Chart$, $c_{Chi} - Chart$ and $c_{MChi} - Chart$ for a range of μ_1 and ω and ρ values.

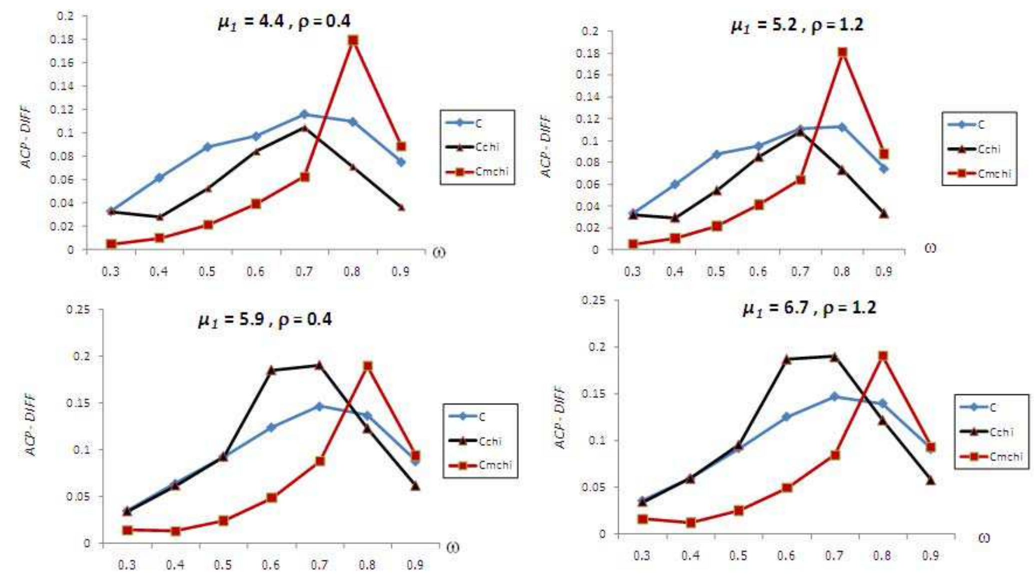

Figure 4: Comparison of the *ACP - DIFF* of the *c-Chart*, $c_{Chi} - Chart$ and $c_{MChi} - Chart$ for a range of μ_1 and ω and ρ values.

5 Conclusion

In this paper, three new control charts have been proposed for a process with number of non-conformities from a *ZIP* distribution. In developing the new charts, the number of non-conformities is modeled as a non-central chi-square distribution with zero degrees of freedom with parameter λ_{Chi}, where λ_{Chi} gives the best fit between the non-central chi-square and *ZIP* distributions. The three new charts are called the $c_{Chi}-Chart$, $c_{CChi}-Chart$ and $c_{MChi}-Chart$. In the $c_{Chi} - Chart$, the estimated value of the mean and the variance in the control limits of the *c-Chart* are replaced by λ_{Chi}. In the $c_{CChi} - Chart$, the estimated values of the mean and variance in the control limit of *c-Chart* are replaced with the estimators of the mean and variance, respectively, of the non-central chi-square distribution. In the $c_{MChi} - Chart$, the estimated value of the mean in the *c-Chart* is replaced with the estimator of the mean of the non-central chi-square distribution, and the variance in the *c-Chart* is replaced by the inter-quartile range.

Extensive simulations have been carried out to compare the performances of the three new control charts with the performances of three other charts: *c-Chart*, $c_{ZIP} - Chart$ and $c_J - Chart$. The average run length (ARL) and average coverage probability (ACP) have been compared. The results of the comparisons are summarized in table 3 which gives a list of preferred control charts for both in-control and out-of control states for a range of values of *ZIP* parameters.

Table 3. Summary of preferred control charts

The mean shift of process	Mean of process (μ_0/μ_1)	Proportion of zero (ω)	Preferred control charts: For *ARL* value	For *ACP* value	For both *ARL* and *ACP* values
In-control	4.0	0.3 - 0.5	$c_{CChi} - Chart$	$c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$	$c_{CChi} - Chart$
		0.6 - 0.9	$c_J - Chart$	$c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$	$c_J - Chart$
	4.5 - 5.5	0.3 - 0.5	$c_{CChi} - Chart$	$c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$	$c_{CChi} - Chart$
		0.6 - 0.9	$c_J - Chart$ and $c_{ZIP} - Chart$	$c_{ZIP} - Chart$, $c_J - Chart$ and $c_{CChi} - Chart$	$c_J - Chart$ and $c_{ZIP} - Chart$
Out-of-control (all level of ρ)	4.0 - 5.5	0.3 - 0.7	$c - Chart$, $c_{Chi} - Chart$ and $c_{MChi} - Chart$	$c_{MChi} - Chart$	$c_{MChi} - Chart$
		0.8 - 0.9	$c - Chart$, $c_{Chi} - Chart$ and $c_{MChi} - Chart$	$c_{Chi} - Chart$	$c_{Chi} - Chart$

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